EXCHANGE RATE, INCOME DISTRIBUTION AND TECHNICAL CHANGE IN A BALANCE-OF-PAYMENTS CONSTRAINED GROWTH MODEL

Rafael Saulo Marques Ribeiro
(corresponding author)
Ph.D. student at the Department of Land Economy, University of Cambridge, UK
rsmr2@cam.ac.uk
+44 075 6461 4097

Gilberto Tadeu Lima
Department of Economics, University of São Paulo, Brazil
giltadeu@usp.br

John S. L. McCombie
Department of Land Economy, University of Cambridge, UK
jslm2@cam.ac.uk

Abstract

This paper sets out a formal model to account for what would be the net effect of an exchange rate devaluation on both income elasticities of demand for export and imports and, consequently, on the long-term balance-of-payments constrained growth rate. Such a model shows how the exchange rate impacts on the home country non-price competitiveness via changes in the variables of the economic structure such as the income distribution and technological change. It is built upon two basic hypotheses. Firstly, it assumes technological improvements impact positively on the income elasticity of demand for exports and negatively on the income elasticity of demand for imports. Secondly, it assumes here that improvements in income distribution have an ambiguous impact on the income elasticities ratio. The model shows that the net impact of a currency devaluation on growth is ambiguous and depends on several conditions.

Keywords: income distribution, technological progress, real exchange rate and balance-of-payments constrained growth.

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I. INTRODUCTION

A topic of main concern among economists nowadays is the impacts of exchange rate on growth. We might say the exchange rate is, in an open economy, one of the most important macroeconomic policy tools due to its capacity of affecting a country's relative competitiveness in international trade. After the Word War II, Keynes strongly advocated the adoption of fixed exchange rates within the rules set forth in the Bretton Woods agreement. In the early 1950s, the fixed exchange rate system worked pretty well, for the US had the major share of the world's official gold reserves. However, during the 1960s, this scenario began to drastically change. The low interest rates employed by the US government drained out their gold reserves, thereby reducing their financial account surplus. Besides, the Vietnam War-related expenditures and US foreign aid to other countries were pointed out as the main cause of the country's ever-growing fiscal and current account deficits. Lastly, the US also had to struggle with a sharp increase in the price of some inputs and the oil crisis (Glyn *et al*, 1990).

As a result of these events, the Bretton Woods system fell apart. At this moment, the majority of the economists began to claim countries would be better off with a floating exchange rate regime instead. They believed that such a regime would correct real exchange rate (RER) misalignments, thereby preventing balance-of-payments crisis and bringing autonomy back to monetary policy. According to the traditional mainstream theory, in an open economy with a floating exchange rate system, the balance-of-payments is, by definition, always in equilibrium. It happens because, given any foreign exchange fluctuation, the money exchange rate and the domestic prices adjust themselves automatically, thus keeping the RER constant and the balance-of-payments in equilibrium. In other words, the conventional wisdom argues that relative prices remain practically unchanged due to the so-called neoclassical "law of one price". Underlying such an argument there is a strong belief that free, self-regulating markets, instead of governments for instance, can determine more efficiently the equilibrium exchange rate yielding the best possible result for the economy as a whole. Therefore, the floating exchange rate regime should be the best way to avoid balance-of-payments crisis and promote economic growth. Thus, it is clear that, according to this school of thought, the exchange rate adjusts itself endogenously and, consequently, can play no role as a macroeconomic policy tool. It means only market is in charge of correcting any possible misalignments and hence boosting growth.

On a theoretical level, the law of one price requires infinite price elasticities of demand for exports and imports, and negligible income elasticities of demand for exports and imports. As a result, growth cannot be balance-of-payments constrained. However, "[s]ince the onset of floating exchange rates in 1972, it is clear from the historical evidence that the massive nominal exchange rate movements that have taken place have not rectified balance-of-payments disequilibria" (McCombie and Thirlwall, 1999, p. 51). Empirical studies strongly suggest that price elasticities are rather low or statistically non-significant, whereas the opposite is observed for the income elasticities (Moreno-Brid, 1999; Moreno-Brid and Perez, 1999; Léon-Ledesma, 2002; Perraton, 2003; Razmi, 2005; Carvalho and Lima, 2009; Gouvea and Lima, 2013). It is income that adjusts to correct balance-of-payments misalignments, not relative prices.

Since no country can run an ever-increasing level of overseas debt to GDP ratio, the balance-of-payments constrained growth rate is the maximum a country is permitted to grow[1]. Thirlwall (1979) demonstrates that a country's equilibrium growth rate is determined by the growth of the world income and the trade income elasticities ratio reflecting the non-price competition factors affecting its performance in the international trade, such as technological capabilities, tastes, stock of knowledge, and so on . In this context, relative prices have been pushed aside from the center of the discussion on growth[2]. Thirlwall states "that a once-for-all depreciation (or devaluation) will not put a country on a permanently higher growth rate. For this to happen, the depreciation would either have to be continuous, or affect the parameters of the model favourably" (Thirlwall, 2011, p. 323). However, since in the standard Thirlwall's (1979) growth model income elasticities are assumed to be exogenous and the growth of relative prices is mitigated over time, the RER plays no role in determining the equilibrium growth rate.

Nonetheless, even though a very weak, or no relationship, is found between relative prices and the volume of exports and imports in the trade demand functions, a statistically significant connexion between a devalued currency and higher growth rates has been observed in the literature. There is a vast range of empirical evidences demonstrating a meaningful connection between exchange rate volatility and a poor growth performance for

[1] Setterfield (2006) and McCombie (2011) convincely argue that the supply side, given by the growth of the productive potential, adjusts to the balance-of-payments constrained growth rate.

[2] An exception to this is Porcile and Lima (2010). They proposed a model where the RER actually plays an important role in a balance-of-payments constrained growth model. In their model the level of employment and the elasticity of labour supply can prevent the currency appreciation, thereby impacting directly on income distribution and growth in developing countries.

several countries since the adoption of the floating exchange rate regime in the 1970s has been brought to light (Dollar, 1992; Razin and Collins, 1997; Aguirre and Calderón, 2005). The empirical literature also shows that countries that sustained their currency at competitive levels over relatively long periods experienced, by and large, higher growth rates (Sachs, 1985; Dollar, 1992; Loayza *et al*, 2004; Aguirre and Calderón, 2005; Rodrik, 2007, Gala 2007). In the light of Thirlwall's growth model, these results indicate that the level of the RER may have some impact on the country's income elasticities. In other words, even if the growth rate of the RER is zero in the long run, temporary shocks in a country's price-competition factors may have permanent effects on the non-price competition characteristics of its goods.

In this spirit, a more recent literature of balance-of-payments constrained growth models has been exploring the links between a competitive currency and non-price competition factors. Barbosa-Filho (2006) says that a devalued RER over a prolonged period of time favours the relative prices of tradable goods, hence promoting a structural change in the economy, since technological progress and productivity are usually higher in these sectors. Considering the multi-sectoral Thirlwall's Law, Araújo (2012) discusses how a currency appreciation affects a country's equilibrium growth rate. He shows that the country might lose its comparative advantages in some sector due to a currency appreciation, and hence damage its capacity to export. As a result, the country undergoes an unfavourable structural change slowing down its equilibrium growth rate. Ferrari *et al* (2013) assume a two-sector economy producing only high- and low-technology tradables, and suggest that a currency devaluation increases the high-technology tradables share of total exports and decreases the high-technology tradables share of total imports, thus raising the country's elasticities ratio. Missio and Jayme Jr (2012) state that a currency devaluation, by increasing the profit share in detriment of the real wages, accelerates technological progress, improves exports competitiveness, and modifies the country's specialization pattern through changes in the income elasticities of demand for exports and imports ratio, therefore boosting long-term growth.

However, the literature has ignored how a changes in the income distribution may have some impact on the consumption patterns, domestic firms' production decisions and so the country's non-price competitiveness. Assuming that consumers tend to demand relatively more luxury than necessity goods as their income grows, an increase in the wage share of income at the expense of the profit share induces to changes in the average consumer preferences, hence affecting qualitatively the country's imports. Moreover, high levels of income inequality might also encourage domestic firms to avail of cheap labour and specialize in the production of labour-intensive goods, which traditionally tend to have relatively low income elasticity of demand, thereby damaging the country's capacity to compete in international trade. Here, we work on the hypothesis that the RER affects not only the country's technological progress, but also its income distribution, thus changing its non-price competitiveness and, consequently, its long-term growth rate. Therefore, we set out a more general theoretical framework that enables us to assess the conditions under which a currency devaluation might either boost or harm long-term growth. Without the key-hypothesis that income elasticities are also endogenous to the income distribution, we can only find a positive relationship between a devalued currency and the long-term growth rate, as in the previous works aforementioned. Our model also provides some insights on the dynamics between income inequality and technical change. In our framework, a once-for-all devaluation provokes perturbations in this dynamics that might lead to a virtuous (vicious) circle of increased (decreased) technological progress and redistribution of income in favour of workers (capitalists) or simply to a trade-off within the stable area between the level of technological capabilities and the wage share of income, depending on the initial conditions. Our model also contributes to the literature by allowing for an exchange rate pass-through mechanism into the standard Thirlwall's (1979) formal model. The previous models state that by keeping the currency at competitive levels for a long period of time a country can achieve higher growth rates. However, the question is: how long is "long"? We attempt to provide an answer for that by considering the role played by the labour market and the collective wage bargaining for the relative price adjustment process. We demonstrate that the higher the workers' bargaining power, the faster the relative prices converge toward their long-term equilibrium level.

The next section presents the revisited version of the balance-of-payments constrained growth model and discusses the dynamics between technical change and income distribution. Section III shows the impact of a currency devaluation on the dynamics between technological progress and income distribution. Section IV lays out the conditions under which a devaluation can boost or harm long-term growth. Finally, section V summarizes and draws some conclusions.

II. THE MODEL

The balance-of-payments constrained growth model revisited

Let us assume the global economy consists of basically two different countries: a rich foreign country and a poor home country. The foreign country is a large economy that issues the international currency and the home country is a small economy facing a balance-of-payments constraint. There are no international capital flows between the two countries. The foreign country is a two-sector economy which produces and exports consumption goods and raw materials. The home country is a one-sector economy that produces and exports only one sort of consumption good and there is imperfect substitutability between the foreign and domestic consumption goods. It is also assumed that the home country imports consumption goods and intermediate inputs from the foreign country. That is to say, the home country imports are disaggregated in two different categories, namely, imported consumption good (M^c) and imported raw materials (M^r), that is, $M = M^c + M^r$. By doing so, we have now an extended balance-of-payments identity

$$P_d X = E\left(P_f M^c + P_f^r M^r\right) \tag{1}$$

where X is the quantity of exports; P_d is the domestic price; P_f is the imported consumption goods price in foreign currency; and P_f^r is the imported raw material prices in foreign currency; and E is the nominal exchange rate. Hereafter, let us assume $P_f = P_f^r$ over time. In rates of change

$$p_d + x = e + p_f + \phi_c m^c + (1 - \phi_c) m^r \tag{2}$$

where $\phi_c = M^c/M$; $(1 - \phi_c) = M^r/M$; and $p_f = p_f^r$.

The export demand function is

$$X = \left(\frac{P_d}{P_f E}\right)^{\eta} Z^{\varepsilon} \qquad \eta < 0, \varepsilon > 0 \tag{3}$$

where Z is the level of the foreign income; η is the price elasticity of demand for exports (here the own and cross price elasticities are assumed to be equal in absolute values); ε is the income elasticity of demand for exports. In rates of change

$$x = \eta(p_d - p_f - e) + \varepsilon z \tag{4}$$

The import demand functions are

$$M^c = M_0^c \left(\frac{EP_f}{P_d}\right)^{\psi} Y^{\pi_c} \qquad \psi < 0, \pi_c > 1 \tag{5}$$

$$M^r = \mu Y \tag{6}$$

where M_0^c and μ are constants; ψ is the price elasticity of demand for imports; and π_c is the income elasticity of demand for imported consumption goods. Equation (5) is a stable multiplicative demand function for imported consumption goods, whereas equation (6) expresses a linear relationship – one can view it as a linear approximation to a long run multiplicative function at a specific point in time. It will be assumed here that technological innovations are neutral with respect to the amount of raw materials utilized in the production process, and hence the ration of imported raw materials to domestic output (M^r/Y) does not change over time. In this scenario, it is reasonable to assume the coefficient μ is constant.

In rates of change

$$m^c = \psi(e + p_f - p_d) + \pi_c y \tag{7}$$

$$m^r = y \tag{8}$$

Now let us extend the balance-of-payments constrained growth model by making domestic product prices a function of the exchange rate and foreign prices. To do so, at first the unit variable costs must be disaggregated in two parts, namely the unit labour costs and unit imported raw material costs:

$$P_d = T\left(\frac{W}{a} + P_f E\mu\right) \tag{9}$$

Where T is the mark-up factor; $P_f E\mu$ is the unit imported intermediate input (raw materials) cost in domestic currency; and W/a is the unit labour cost. In rates of change:

$$p_d = \tau + \varphi(w - \hat{a}) + (1 - \varphi)(p_f + e) \tag{10}$$

where $\varphi = W/a/(W/a + P_f E\mu)$. The share φ is positively related to the degree of the workers' bargaining power and inversely related to the proportion of imported raw materials in the total unitary cost of production of domestic firms. Alternatively, it can be best demonstrated by rewriting the unit labour cost share in total unit cost as $\varphi = \sigma_L / [\Theta\mu + \sigma_L]$, where $\sigma_L = W/P_d a$ is the wage share of income, and $\Theta = P_f E / P_d$ is the RER in level, which means that $\Theta\mu$ is the imported intermediate input share of income. Workers bargain for a higher wage share of total income. Thus, the higher their bargaining power, the higher the share φ.

Now we must show the process of relative price adjustment. The exchange rate pass-through mechanism states that an increase in the rate of change of imported raw material prices in domestic currency caused by a nominal devaluation feeds through into the rate of change of domestic prices with a lag. From equation (10), we can point out basically two other transmission channels through which a permanent positive shock in the nominal exchange rate can impact on the domestic prices: (i) the rate of change of money wage increases with a lag to match the increased inflation of imported raw material prices and, consequently, the inflation of domestic prices, until the rate of change of the real wage equals the rate of change of labour productivity; (ii) a currency devaluation increases the market share for domestic goods, thus allowing exporters to raise the mark-up factor and, consequently, domestic prices.

With respect to the first transmission channel, it is assumed that the rate of change of the nominal wage is positively related to the rates of change of domestic price and labour productivity, as follows

$$w = p_d - \upsilon\hat{a} \tag{11}$$

where $\upsilon = 2/\{1 + exp[\kappa(\varphi^e - \varphi)]\}$, κ is an adjustment parameter and φ^e is the unit labour cost share of unit variable cost expected by workers in the long run. Given $\nu \in (0,1)$, we assume that the path followed by ν over time can be proxied by a logit-like function. This specification guarantees that increases in the growth of labour productivity progressively pass through into the growth of nominal wages in the equation (11). Since a currency devaluation increases domestic prices, via pass-through mechanism, and labour productivity, via Verdoorn's Law, the parameter ν must reduce accordingly to keep the equality (11). Therefore, we must assume that a currency devaluation increases the gap $(\varphi^e - \varphi)$ by reducing relatively more φ than φ^e. For now, it is assumed here for convenience that the wage share of income expected by workers is exogenously given in the long run (such a hypothesis will be flexibilized leter), which means that a devaluation reduces φ^e only by increasing the unit imported intermediate cost share of unit variable cost $\left(P_f E\mu/P_d\right)$.

However, the reduction in the share φ caused by the increased imported raw material share $(P_f E\mu/P_d)$ after a devaluation is augmented by the initial decrease in the current wage share $(W/P_d a)$, due to the existence of imperfect flexibility of nominal wages. Accordingly, in the long run, as the wage share converges to its equilibrium level, $\upsilon \to 1$ and the growth of real wage keeps up with the growth of labour productivity.

The second transmission channel concerns the mark-up factor. First of all, let us redefine the mark-up as a function of the RER. A devalued currency increases the monopoly power of domestic firms and enables them to raise their mark-up. Therefore, if we rewrite the mark-up as a positive function of the RER and take into account the extended mark-up price equation (9), we have:

$$T = \delta\Theta = \delta\left[\frac{EP_f}{T\left(\frac{W}{a} + P_f E\mu\right)}\right] \tag{12}$$

where $\delta > 0$ is a parameter. Given $EP_f/\left[(W/a) + P_f E\mu\right] = (1-\varphi)/\mu$, we can rewrite equation (12) as follows:

$$T = [(\delta/\mu)(1-\varphi)]^{1/2} \tag{13}$$

From equation (13) we find $dT/d\varphi < 0$. It means the higher the workers' bargaining power (or the higher the share φ), the lesser the mark-up of domestic firms. Since $\varphi = \sigma_L/[\Theta\mu + \sigma_L]$, equation (13) basically highlights that the home country is price-taker, which means the mark-up factor is determined, not by the capitalists, but by the wage bargaining process and the prices of imported raw materials in domestic currency.

In rates of change we have:

$$\tau = -\frac{\varphi}{2(1-\varphi)}\frac{d\varphi}{\varphi} \tag{14}$$

The rate of change of the unit labour cost share in the unit variable cost is given by:

$$\frac{d\varphi}{\varphi} = (1-\varphi)\left[(w-\hat{a}) - \left(p_f + e\right)\right] \tag{15}$$

see appendix I. Substituting equation (15) into (14), we obtain:

$$\tau = -(\varphi/2)\left[(w-\hat{a}) - \left(p_f + e\right)\right] \tag{16}$$

Ergo, when the nominal wages reach the level desired by workers in the long run and the unit imported raw material costs cease to change, the mark-up factor also remains unchanged ($\tau = 0$). Substituting equations (16) and (11) into (10), we have the extended mark-up pricing equation with endogenous mark-up in the short run:

$$p_d = (\varphi/2)[p_d + (\upsilon - 1)\hat{a}] + (1 - \varphi/2)(p_f + e) \tag{17}$$

Rearranging equation (17) we have:

$$p_d = [(\upsilon - 1)\hat{a}/(1 - \varphi/2)] + p_f + e \tag{17*}$$

which means that in the long run, given $\upsilon = 1$, we have $p_d = e + p_f$. In other words, when the wage share matches the wage share desired by workers in the long run, relative prices do not change. Alternatively, we can also say from the equation (10) that when the share φ reaches its stable equilibrium level in the long run, relative prices also remain unchanged, that is, we obtain $p_d = e + p_f$ (see appendix II). It is worth noting the major role played by the labour market in this framework. The higher the workers' bargaining power, the faster the share φ, and, consequently, the relative prices adjust to their equilibrium level.

Now, we have to define the growth of the labour productivity. Kaldor (1966) persuasively argues that the growth of the labour productivity is positively related to the growth of output (the so-called Verdoorn's Law). Drawing upon the formalization of Kaldor's theory proposed by Dixon and Thirlwall (1975), we have the following equation in the long term:

$$\hat{a} = a_0 + \lambda y \tag{18}$$

where a_0 is rate of autonomous productivity growth; and $\lambda > 0$ is the Verdoorn coefficient.

That said, if we allow equations (17*), (18), (8), (7) and (4) into (2), we find the growth rate of the revisited model:

$$y = \frac{(1 - \varphi/2)\varepsilon z + (1 + \eta + \theta\psi)(\upsilon - 1)a_0}{(1 - \varphi/2)\pi - (1 + \eta + \theta\psi)(\upsilon - 1)\lambda} \tag{19}$$

where $\pi = \phi_c(\pi_c) + (1 - \phi_c)$. Since $\upsilon = 1$ in the long term, we can derive the equilibrium balance-of-payments constrained growth rate from equation (19):

$$y_{BP} = \frac{(1-\varphi/2)\varepsilon z}{(1-\varphi/2)\pi} = \frac{\varepsilon z}{\pi} \tag{20}$$

Equation (20) describes the widely known Thirlwall's Law. This law states that the domestic growth is directly related to the foreign demand growth rate. It also states that a country's output growth rate depends positively on its existing non-price competition factors, here expressed by the ratio ε/π. This ratio reflects disparities between countries with respect to factors determining the demand for a country's exports and imports, such as technological capabilities, product quality, stock of knowledge, and consumer preferences, for instance. Before we continue, it is worth noting that the income elasticity of demand for total imports (π) is given by the weighted average of income elasticities of demand for imported consumption goods (π_c) and imported raw materials (which is equal to unity). In this model it will be assumed that the impact of changes in the RER and on the weight ϕ_c is negligible.

Endogenizing the income elasticities

In this analysis, we intend to consider the net effect of a currency devaluation on the home country non-price competitiveness via changes in the technology gap and income distribution.

To commence, let us assume there are technological disparities between the foreign and the home countries. The foreign country pushes forward the technological frontier while the home country is lagging behind. Therefore, the inverse of the technology gap between foreign and home countries (S) can be proxied by

$$S = \frac{S_d}{S_f}, \quad S \in (0,1)$$

where S_d and S_f are the technological capabilities of the home and the foreign countries, respectively.

It is also assumed that there are two classes in the economy, capitalists and workers. Since we are assuming workers' consumption equals to wages, the economy saving is the capitalists' saving. In an economy that allows for imported raw materials into the unit production cost, the wage share of income can be defined as follows

$$\sigma_L = \frac{W}{P_d}\frac{1}{a} = 1 - \sigma_K - \left(\frac{EP_f}{P_d}\mu\right) = 1 - \sigma_K - \Theta\mu \tag{21}$$

where σ_K is the share of profits in income. Once we know the wage share (σ_L) and the raw materials share $(\Theta\mu)$, the profit share (σ_K) is determined as a residue. Now we must address two key hypotheses of the model concerning the impact of technological change and income distribution on a country's non-price competitiveness.

The first assumption is that an increase in the home country's relative technological capabilities (S) improves its non-price competitiveness (ε/π). This hypothesis is strongly supported by the literature in a theoretical and empirical level. Fagerberg (1988) questions the traditional wisdom by suggesting that technology and the ability to compete on delivery are the main factors affecting differences in international competitiveness, rather than relative unit labour costs reflecting differences in price-competitiveness. He also finds evidence for 15 industrial countries during the period 1960-83 supporting his arguments. Amable and Verspagen (1995) find strong empirical evidence of the positive impact of technological progress on exports-market shares for 5 industrialized countries and 18 industries over the period 1970-91. Hughes (1986) proposes the hypothesis that there is a two-way relationship between exports and innovation due to differences in the specificities of demand between export and domestic markets in a study for 46 UK manufacturing industries. Léon-Ledesma (2002) extends the Dixon and Thirlwall's (1975) model and also finds a positive and statistically significant impact of technological innovations on exports and labour productivity growth for 17 OECD countries from 1965-94. Araujo and Lima (2007) developed a disaggregated multi-sectoral version of the Thirlwall's Law, where a country can reach higher growth rates only by specializing in sectors with relatively high (low) income elasticities of demand for exports (imports). Gouvea and Lima (2010, 2013) test the multi-sectoral model and their results, in general, support the hypothesis that goods from relatively high technology-intensive sectors have higher (lower) income elasticities of demand for exports (imports).

Another key assumption is that changes in income distribution affect the country's non-price competitiveness. From a historical perspective, in 1961 Linder observed that international trade in manufactured goods amongst developed countries was heavily determined by within-country income levels and income distribution. He postulated that, given the existence of non-homothetic preferences[3], the more unequal the income distribution of a country is, the greater its expenditures on luxury goods (Francois and Kaplan, 1996). Latin American Structuralists argue that in undeveloped countries an increase in the wage share at the expense of the profit share of income would reduce capitalists' saving and consequently their capability of importing superfluous and highly technological products from developed countries (Furtado 1968, 1969; Tavares and Serra, 1976; Pinto, 1976). A more recent literature emphasizes the impact of an increasing income inequality on foreign trade (Mitra and Trindade, 2005; Bohman and Nilsson, 2007; Dalgin et al, 2008). They conclude that, given the non-homothetic preferences, more unequal countries tend to export relatively more necessity goods and import more luxury goods. Without doubt changes in income distribution reshape patterns of consumption and foreign trade. Since we are assuming that the home country exports and imports only one type of consumption goods with imperfect substitutability between them, we do not divide consumption goods in necessity and luxury. In this work the relationship between income distribution and the country's non-price competitiveness is twofold. First, on the impact of income distribution on the income elasticity of demand of imports, it will be assumed that an increased wage share reduces the capitalist demand for luxury goods (and increase their demand for necessity goods) and increase the worker demand for luxury goods (at the expense of their demand for necessity goods). Also, assuming that the foreign developed economy has the capacity of constantly transforming the quality of its exports and its productive structure in order to match the changing consumer demand in its markets (Setterfield, 1997), if the reduction in capitalist demand for luxury goods overcompensates the increase in worker demand for luxury goods, then a higher wage share reduces the home country's total consumer demand for luxury goods, thus lowering the income elasticity of imports; conversely, if an increase in worker demand for luxury goods outweighs the decrease in capitalist demand for luxury goods, then an increased wage share increases the country's total consumer demand for luxury goods and, consequently, the income elasticity for imports. As for the relationship between income distribution and income elasticity of exports, it will be assumed that relatively low real wages simultaneously lower the wage share and encourage domestic firms to specialize in the production of labour-intensive goods, which traditionally tend to have relatively low income

[3] In 1857 the statistician Ernst Engel presented his seminal theory on consumer behaviour. He states that, as income grows, consumers tend to substitute luxury for necessity goods, where luxury goods have income elasticity of demand greater than unity and necessity goods have income elasticity of demand less than unity. Here, non-homothetic preferences basically mean that the proportion of income that consumers spend on luxury and necessity goods varies as income increases.

elasticity of demand (Gouvea and Lima, 2010, 2013). Since it is assumed the home country produces and exports only one sort of consumption good, a worsened income distribution impacts negatively on the income elasticity of exports, independently of the demand structure of the foreign country.

As a result of these hypotheses, we can define the country's elasticity ratio as a linear function, for expositional ease, of the wage share of its workers in income and its relative technological capabilities, as follows:

$$\frac{\varepsilon}{\pi} = \beta_0 + \beta_1 \sigma_L + \beta_2 S \tag{22}$$

Here it will be assumed without loss of generality that $\beta_0, \beta_1, \beta_2 > 0$. $\beta_0 > 0$ is sufficiently high in absolute value in order to rule out the possibility of a negative elasticity ratio; $\beta_1 > 0$ means that any negative effect of a possible increase in the country's total consumer demand for luxury goods on the income elasticity of imports, caused by an increased wage share, is not enough to mitigate the positive impact of a more even distribution of income on the income elasticity of exports; and $\beta_2 > 0$ falls in line with the hypothesis that a country's relatively high technological capabilities impact positively on its non-price competitiveness.

However, in order to analyse the net effect of RER on the elasticities ratio and, consequently, on long-term growth rate, we must describe how technological innovation and income distribution interact over time and also what is the very role of the RER in such a dynamics.

Technological progress

Let us assume that the rate of change of technological progress can be expressed as

$$\dot{S} = \alpha_0 + \alpha_1 \sigma_K - \alpha_2 \sigma_K^2 - \alpha_3 S \tag{23}$$

where $\dot{S}$ is the derivative of S with respect o time; and $\alpha_0 \lesseqgtr 0$ and $\alpha_1, \alpha_0, \alpha_3 > 0$ are parameters. Following Lima (2004), we assume a non-linear relationship between profit rate and technological progress. Since there is no financial market in the model, investments in technology are funded by the profit rate. At low levels of profit rate capitalists have incentives to invest in technology but run into saving shortage to do so. As the profit share in income increases capitalists raise their capacity to innovate and the technology gap reduces until the point where higher profit rates become ineffective to boost technological progress. At high levels of profit share of income, capitalists lack the incentives to introduce new technologies. We also assume that the rate of change of technical progress is a negative function of the inverted technology gap. "This happens because the higher the technology gap, the higher the opportunities for learning related to imitation, international technological spillovers and catching up" (Cimoli and Porcile, 2014, p. 217).

Substituting σ_K with its very definition $(1 - \sigma_L - \Theta\mu)$ in (23) and assuming $\alpha_1 = \alpha_2$ for convenience, we have after some rearrangements

$$\dot{S} = \rho_0 + \rho_1\sigma_L - \alpha_2\sigma_L^2 - \alpha_3 S \tag{24}$$

where $\rho_0 = \alpha_0 + \alpha_2(1 - \Theta\mu)\Theta\mu$ and $\rho_1 = \alpha_2(1 - \Theta\mu)$ (see appendix III). We are also assuming henceforth $\rho_0 < 0$ without loss of generality, which means α_0 is negative and sufficiently large in absolute value. By considering $\alpha_0 < 0$, we are assuming the autonomous technological progress in the foreign country grows faster than in the home country.

The *locus* $\dot{S} = 0$ describes the relation between technological progress and wage share, provided a constant technology gap over time. This equation is given by

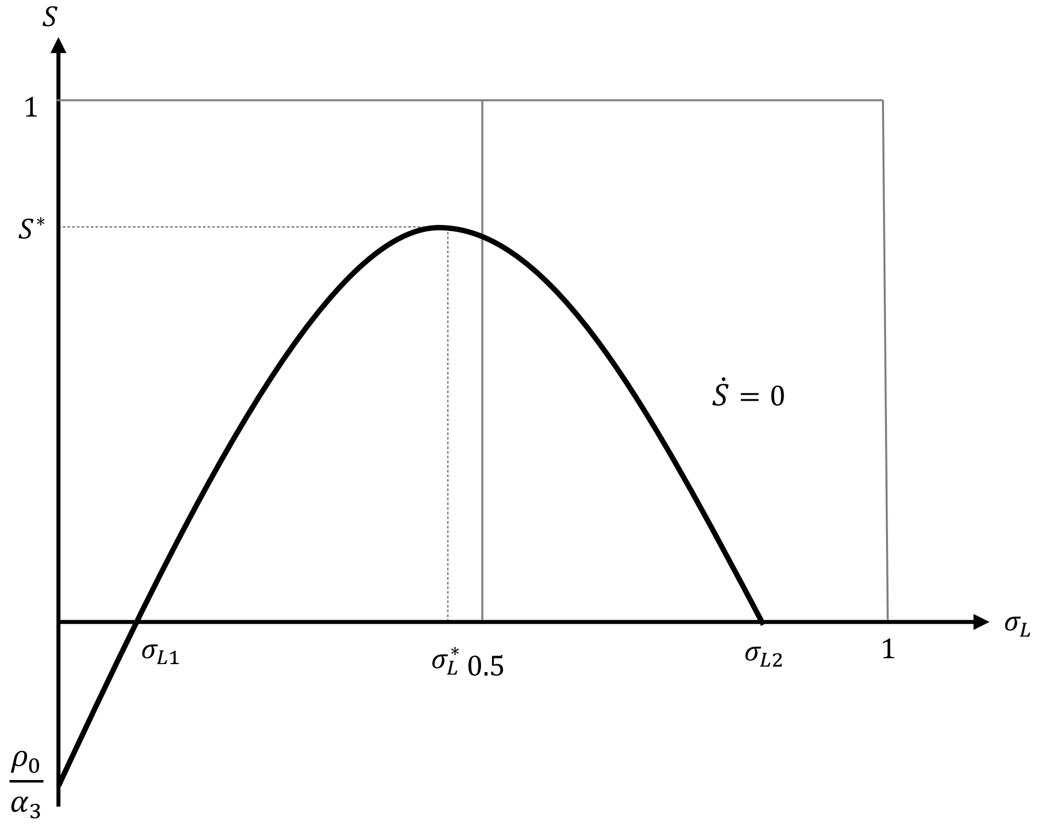
$$S = \frac{\rho_0}{\alpha_3} + \frac{\rho_1}{\alpha_3}\sigma_L - \frac{\alpha_2}{\alpha_3}\sigma_L^2 \tag{25}$$

Accordingly, we can picture an inverted U-shaped curve relating wage share and technological progress. Taking the first derivative we obtain the point of maximum

$$\sigma_L^* = \frac{\rho_1}{2\alpha_2} = \frac{1 - \Theta\mu}{2} \tag{26}$$

since $\Theta\mu \in (0,1)$, then $\sigma_L^* \in (0,1/2)$. Once the second derivative is negative, $-2\,\alpha_2/\alpha_3 < 0$, then σ_L^* is a point of maximum. Now we have to impose some constraints over the parameters in order to obtain meaningful results within the domain of the variables under discussion, that is, $S, \sigma_L \in (0,1)$. Assuming equation (25) has two distinct real roots, we have it pictured in Figure 1 below. See appendix IV and check the conditions for existence of equation 25 within the bounded domain of the endogenous variables.

FIGURE 1 – The *locus* $\dot{S} = 0$



Functional Income Distribution

Let us assume that the time derivative of the wage share of income is a positive function of the gap between the wage share expected or desired by workers (σ_L^e) and the current wage share, as follows

$$\dot{\sigma}_L = \theta(\sigma_L^e - \sigma_L) \tag{27}$$

where $\theta > 0$ is an adjustment parameter. Now, we must endogenize σ_L^e. To do so, let us redefine first σ_L^e, as follows

$$\sigma_L^e = 1 - \sigma_K^e - \Theta\mu \tag{28}$$

where σ_K^e is the residual expected profit share with respect to the wage share expected by workers (σ_L^e) which, by its turn, depends on the workers' bargaining power. Once $\Theta\mu$ is constant at first and exogenously given, that is, neither workers nor capitalists have any influence on the imported raw materials share of variable unit costs of production, then we can say σ_L^e and σ_K^e are inversely related.

If σ_L^e depends positively on workers' bargaining power, then σ_K^e, on the other hand, is negatively related to workers' bargaining power

$$\sigma_K^e = \omega_0 - \omega_1(l - n) \tag{29}$$

where l is the rate of change of employment, n is the exogenously given potential workers growth rate and $\omega_0, \omega_1 \geq 0$ are parameters. The underlying assumption in equation (29) is that the higher the employment rate, the higher the workers' bargaining power. Plugging equations (29) and (28) into (27) we have

$$\dot{\sigma}_L = \theta[1 - \omega_0 + \omega_1(l - n) - \Theta\mu - \sigma_L] \tag{30}$$

From the definition of the long term rate of change of labour productivity, we have the following identity

$$l = y_{BP} - \hat{a} \tag{31}$$

Substituting the Verdoorn's Law from equation (18) in (31) yields

$$l = (1 - \lambda)y_{BP} - a_0 \tag{32}$$

Substituting equation (22) in the Thirlwall's Law $y = \varepsilon z/\pi$, we have

$$y_{BP} = (\beta_0 + \beta_1\sigma_L + \beta_2 S)z \tag{33}$$

If we substitute (33) in (32), and then in (30), we obtain

$$\begin{aligned}\dot{\sigma}_L = \theta\{1 - \omega_0 + \omega_1[(1 - \lambda)(\beta_0 + \beta_1\sigma_L + \beta_2 S)z - a_0 - n] \\ -\Theta\mu - \sigma_L\}\end{aligned} \tag{34}$$

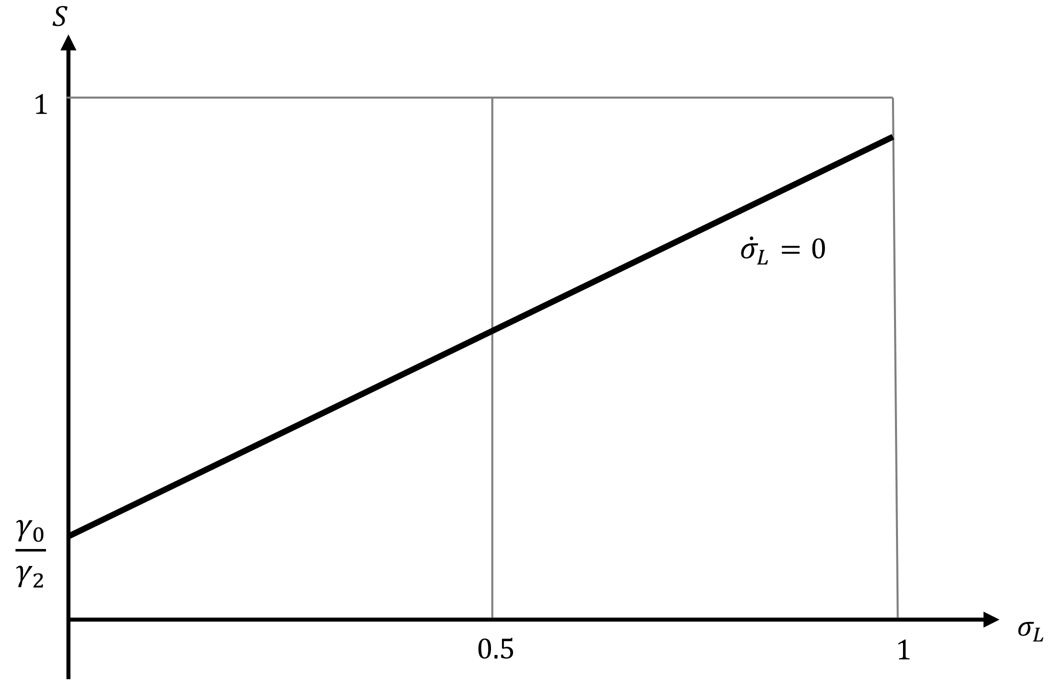
In the long term we expect the real wage and the labour productivity will grow at the same rate. Therefore, in the *locus* $\dot{\sigma}_L = 0$, the equation that describes the relationship between technology gap and wage share is given by

$$S = \frac{\gamma_0}{\gamma_2} + \frac{\gamma_1}{\gamma_2}\sigma_L \tag{35}$$

where $\gamma_0 = \Theta\mu - 1 + \omega_0 - \omega_1[(1 - \lambda)\beta_0 z - a_0 - n] \gtreqless 0$, $\gamma_1 = 1 - \omega_1(1 - \lambda)\beta_1 z \gtreqless 0$, $\gamma_2 = \omega_1(1 - \lambda)\beta_2 z > 0$. Here we will assume $\gamma_0 > 0$ without loss of generality. It is also assumed $\gamma_1 > 0$ which means $z < 1/\omega_1(1 - \lambda)\beta_2$ is the condition to be fulfilled.

Therefore, given these constraints on the parameters, we make sure that the equation (35) falls within a meaningful economic domain, as represented in Figure 2 below.

FIGURE 2 – The *locus* $\dot{\sigma}_L = 0$



The Technology Gap and Income Distribution Dynamics

First of all, we must find the non-trivial solutions for the system determined by equations (25) and (35). Taking the difference between these equations and rearranging the terms, we have

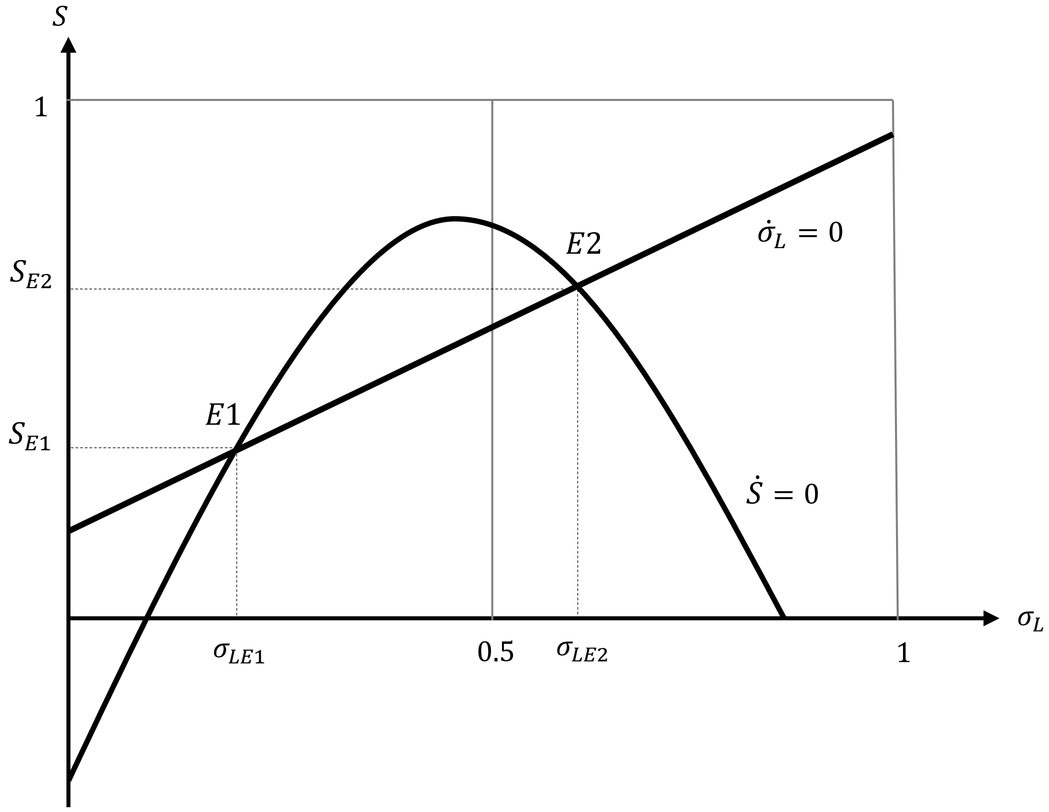
$$H(\sigma_L) = \left(\frac{\rho_0}{\alpha_3} - \frac{\gamma_0}{\gamma_2}\right) + \left(\frac{\rho_1}{\alpha_3} - \frac{\gamma_1}{\gamma_2}\right)\sigma_L - \left(\frac{\alpha_2}{\alpha_3}\right)\sigma_L^2 \tag{36}$$

The real roots of the quadratic equation (36) represent the equilibrium points where both the isoclines ($\dot{S} = 0$ and $\dot{\sigma}_L = 0$) cancel out. Assuming $H(\sigma_L)$ has two different real roots in the meaningful domain, we obtain

$$\sigma_{LEi} = \frac{\gamma_2\rho_1 - \alpha_3\gamma_1}{2\alpha_2\gamma_2} \pm \frac{\left[(\gamma_2\rho_1 - \alpha_3\gamma_1)^2 + 4\alpha_2\gamma_2(\gamma_2\rho_0 - \alpha_3\gamma_0)\right]^{1/2}}{2\alpha_2\gamma_2} \tag{37}$$

where σ_{LEi}, for $i = \{1,2\}$, stands for generic equilibrium values of σ_L. In equilibrium $E1$, $\sigma_L = \sigma_{LE1}$ and $S(\sigma_{LE1}) = S_{E1}$ whereas in $E2$, $\sigma_L = \sigma_{LE2}$ and $S(\sigma_{LE2}) = S_{E2}$. It is assumed $\sigma_{LE1}, \sigma_{LE2}, S_{E1}, S_{E2} \in (0,1)$, so we can have meaningful results. See Figure 3 below:

FIGURE 3 – The *loci* $\dot{S} = 0$ and $\dot{\sigma}_L = 0$



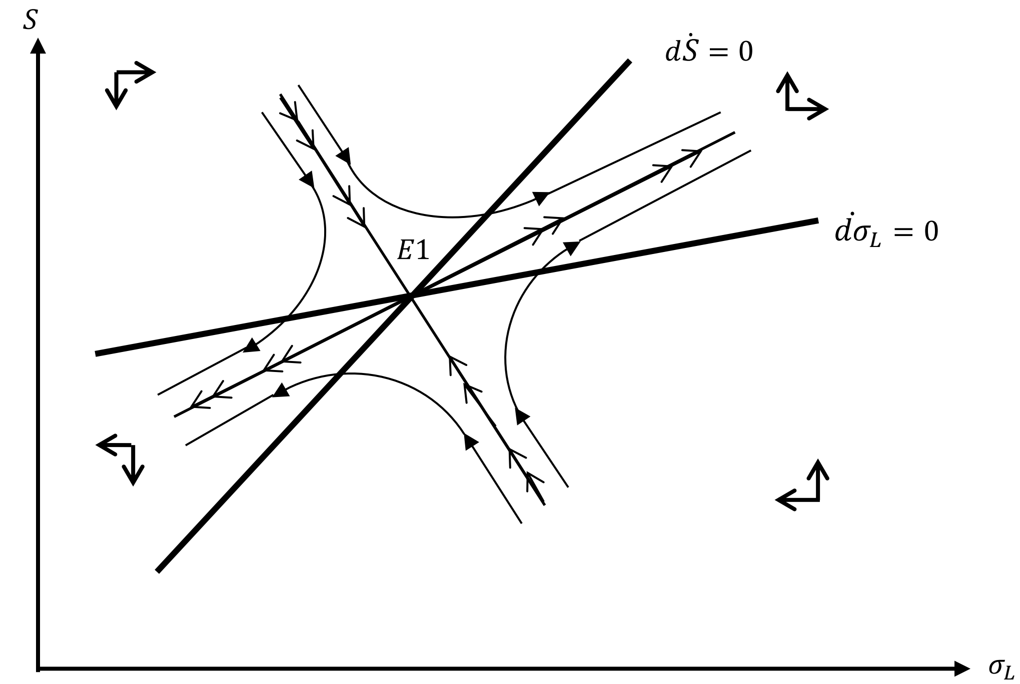
Now we must analyse the local stability conditions around the equilibrium points. From equations (24) and (34) we form a 2x2 non-linear dynamic system for the technology gap and income distribution. The linear version of the system is formed by the terms of the Jacobian matrix (see appendix V):

$$\begin{bmatrix} d\dot{S}/dt \\ d\dot{\sigma}_L/dt \end{bmatrix} = \begin{bmatrix} -\alpha_3 & 2\alpha_2(\sigma_L^* - \sigma_{LEi}) \\ \theta\gamma_2 & -\theta\gamma_1 \end{bmatrix} \begin{bmatrix} S - S_{Ei} \\ \sigma_L - \sigma_{LEi} \end{bmatrix} \tag{38}$$

where S_{Ei}, for $i = \{1,2\}$, stands for generic equilibrium values of S. Before we start the stability analysis, let us follow Lima (2004) and divide the domain into low wage share (LWS) and high wage share (HWS) regions. In the LWS region $\sigma_{LE} < \sigma_L^*$, which means the innovation process is wage-led, since $\partial\dot{S}/\partial\sigma_L = 2\alpha_2(\sigma_L^* - \sigma_{LE}) > 0$, whereas in the HWS region $\sigma_L^* < \sigma_{LE}$, and the technological progress becomes profit-led, for $\partial\dot{S}/\partial\sigma_L < 0$. As we can see in Figure 3, $E1$ is placed within the LWS region, whereas $E2$ is in the HWS region.

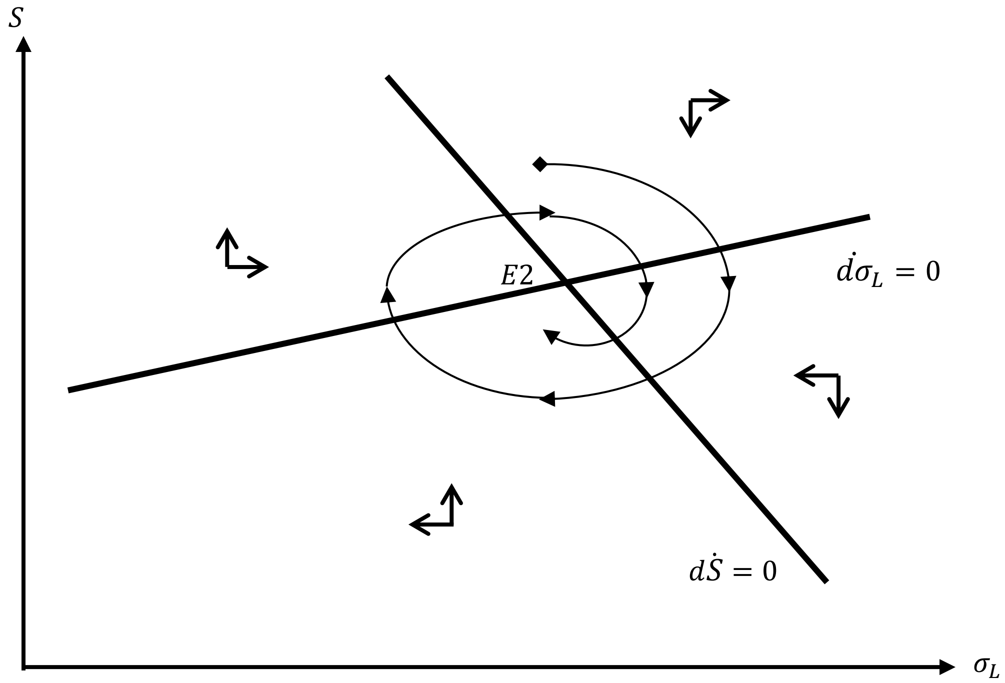
First, we will analyse the stability conditions around the point $E1$ in the LWS region. For the point $E1$ in the LWS region, the trace of the Jacobian matrix is negative $(-\alpha_3 - \theta\gamma_1 < 0)$ and the determinant is ambiguous $[\alpha_3\theta\gamma_1 - \theta\gamma_2 2\alpha_2(\sigma_L^* - \sigma_{LE1}) \lesseqgtr 0]$, given $\sigma_L^* > \sigma_{LE1}$. If the determinant is negative, then $E1$ is a saddle-point; otherwise, it is a stable point. Here, it will be assumed, without loss of generality, that the determinant is negative and $E1$ is a saddle-point. Alternatively, the stability condition can be analysed by taking into account the slope of the linear approximation of the curves $\dot{S} = 0$ and $\dot{\sigma}_L = 0$ around the equilibrium solution of the dynamical system (38). The slope of the isocline $d\dot{S} = 0$, given by $(2\alpha_2(\sigma_L^* - \sigma_{LE1})/\alpha_3)$, is positive. Therefore, once $\partial\dot{S}/\partial\sigma_L > 0$, an increase in σ_L increases S. It means the sign of $d\dot{S} = 0$ is positive to the right and negative to the left. As for the isocline $d\dot{\sigma}_L = 0$, since its slope (γ_1/γ_2) is also positive, this curve is upward slopping. Once $\partial\dot{\sigma}_L/\partial S > 0$, $d\dot{\sigma}_L = 0$ will be negative to the right and positive to the left of the curve. Thus, $E1$ in the LWS region is a saddle-point. See Figure 4 below.

FIGURE 4 – The low wage share region



The same analysis can be extended to the point $E2$ in the HWS region. Since the trace of the Jacobian matrix in $E2$ is also negative and, given $\sigma_L^* < \sigma_{LE2}$, the determinant is unambiguously positive, the point E2 is locally stable. The slope of the isocline $d\dot{S} = 0$ is now negative, for $\sigma_L^* < \sigma_{Lt}$. Since $\partial\dot{S}/\partial\sigma_L < 0$, $d\dot{S} = 0$ is negative to the right and positive to the left. The slope of $d\dot{\sigma}_L = 0$ remains the same. Therefore, $E2$ in the HWS region is a stable point. See Figure 5 below.

FIGURE 5 – The high wage share region



Lastly, it is worth noting that if the parameter γ_1 was negative, then isocline $d\dot{\sigma}_L = 0$ would be downward slopping. As a result, the signs of the trace and the determinant of the Jacobian matrix from the dynamical system (38) would be ambiguous, and hence the local stability of the solutions in the LWS and HWS regions would depend on the magnitude of the parameters constituting the matrix.

III. REAL EXCHANGE RATE, TECHNOLOGICAL PROGRESS AND INCOME DISTRIBUTION

If we substitute equation (16) in (10), and then calculate the rate of change of the real exchange rate, we obtain:

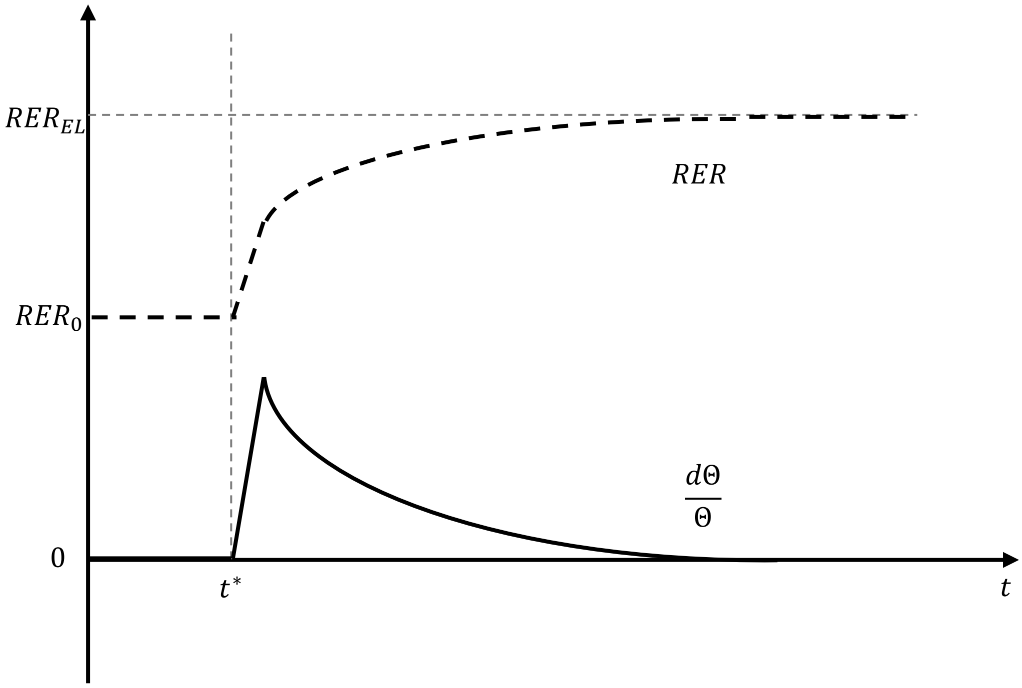
$$\frac{d\Theta}{\Theta} = e + p_f - p_d = (\varphi/2)\big(e + p_f - w + \hat{a}\big) \tag{39}$$

$$\frac{d(d\Theta/\Theta)}{de} = \frac{1}{2}\frac{\partial\varphi}{\partial e} + \frac{\varphi}{2} + \frac{\varphi}{2}\frac{\partial\hat{a}}{\partial e} > 0 \tag{40}$$

The total effect of a nominal devaluation on the growth of the real exchange rate is ambiguous. We expect that $\partial\varphi/\partial e < 0$, since a currency devaluation increases the imported raw materials share of unit variable cost. On the other hand, if the Marshall-Lerner condition holds, that is, $(1 + \eta + \psi) < 0$, and the Verdoorn coefficient is positive $(\lambda > 0)$, then we expect that a devaluation increases growth and, consequently, the labour productivity $\partial\hat{a}/\partial e > 0$. Since nominal wages are rigidy in the short run, its partial derivative is negligible. It will be assume that the gains from trade caused by a devaluation outweight the increased unit production cost $(|\partial\varphi/\partial e| < \varphi + \varphi\,\partial\hat{a}/\partial e)$, and hence and increase in the rate of devaluation of the nominal exchange rate also increases the rate of devaluation of the RER. In other words, if the monetary authority decides to increase permanently the rate of change of the nominal exchange rate, say from 2% to 4% per period of time, then we have an increase in the rate of change of real exchange rate. In the long run, however, since $e + p_f = p_d$ and $w - \hat{a} = p_d$, equation (39) reduces to $e + p_f - p_d = 0$. It means that relative prices do not change in the long run, and gains in the home country's price competitiveness are completely offset over time.

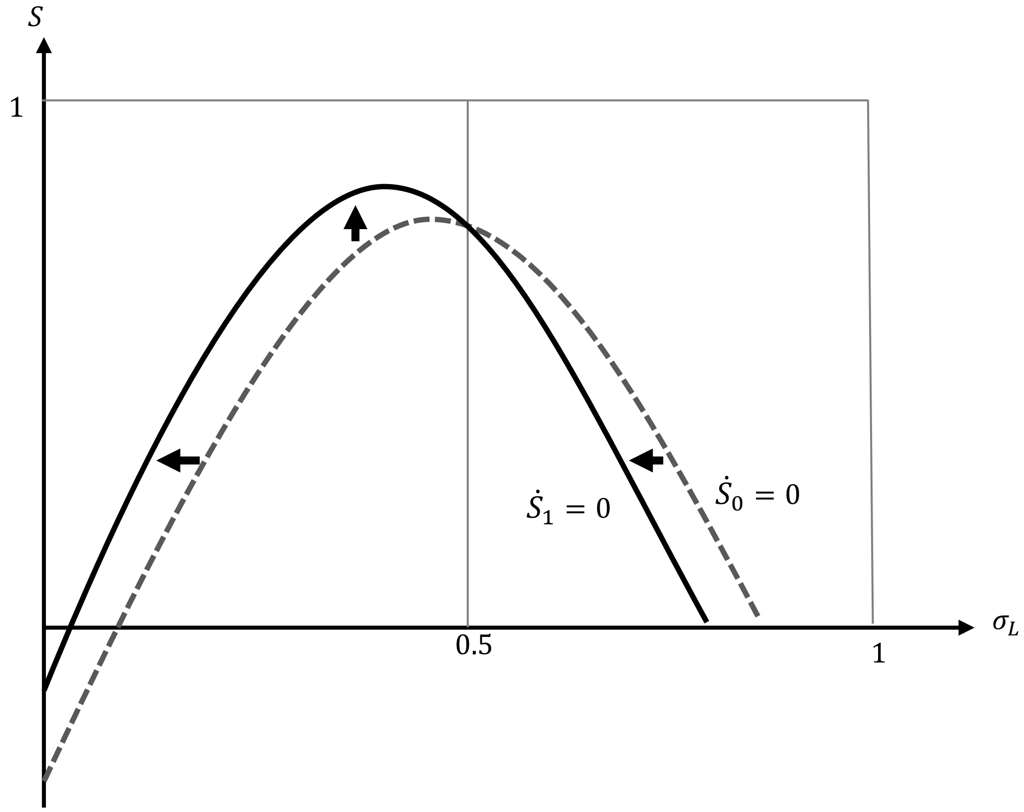
Even though the rate of change of RER converges to zero at diminishing positive rates through time, the same is not true if we consider the RER in level. Given a sustained rise in the rate of change of nominal exchange rate, the rate of change of domestic prices will be lagging behind at an increasing rate of change. The domestic prices adjustment process will continue until the growth rate of domestic prices matches the increased rate of change of nominal exchange rate. As a result, assuming the domestic price level does not overshoot the level of the nominal exchange rate, the RER will be higher in the long run than its initial value. This is a crucial remark, for it means that every increase in the level of RER is permanent in the model. Figure 6 below illustrates this mechanism laid out above and shows how permanent, exogenous shocks in the nominal exchange rate cause permanent shifts in the equilibrium RER value. Figure 6 shows how a positive, permanent shock in the growth rate of the nominal exchange rate in a given time t^* increases the level of the RER from its initial value RER_0 to its long-term equilibrium level RER_{EL}:

FIGURE 6 – The permanent impact of a nominal devaluation on the RER



That said, now we must see how a currency devaluation impacts on both *loci* $\dot{S} = 0$ and $\dot{\sigma}_L = 0$. As for the *locus* $\dot{S} = 0$ described by equation (25), let us see first how the parabola vertex $\left(\sigma_L^*, S(\sigma_L^*)\right)$ shifts after a currency depreciation. From equation (26) we can say that a currency devaluation reduces the point of maximum σ_L^*, since $d\sigma_L^*/d\Theta = -\mu/2$. On the other hand, as $dS(\sigma_L^*)/d\Theta = (\alpha_2/\alpha_3)[(1 - 3\Theta\mu)/2] > 0$, the vertex coordinate $S(\sigma_L^*)$ will augment, once we expect $\Theta\mu < 1/2$. With respect to the real roots σ_{L1} and σ_{L2} (appendix IV), we can say both will be reduced, as $d\sigma_{L1}/d\Theta \cong d\sigma_{L2}/d\Theta \cong d\sigma_L^*/d\Theta = -\mu/2$ and the variation of $\sqrt{\rho_1^2 + 4\rho_0\alpha_2}/2\,\alpha_2$ is negligible. Figure 7 illustrates such a dynamic.

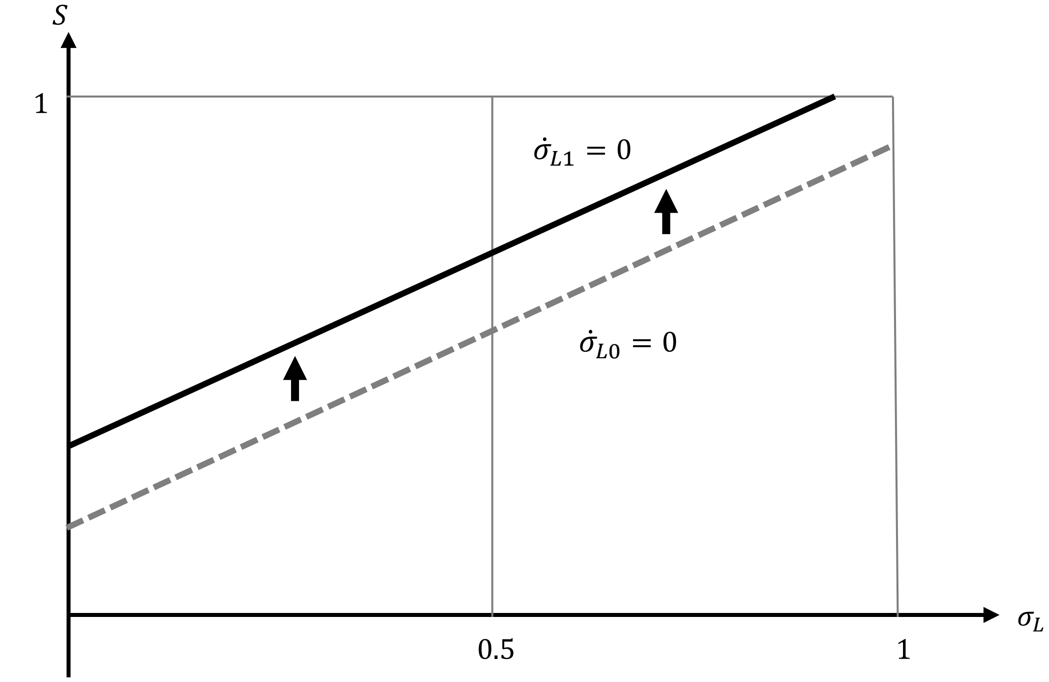
FIGURE 7 – The impact of an undervaluation in *locus* $\dot{S} = 0$



The grey dotted line representing the *locus* $\dot{S}_0 = 0$ is the initial curve whereas the black line representing the *locus* $\dot{S}_1 = 0$ is the new curve after a currency depreciation.

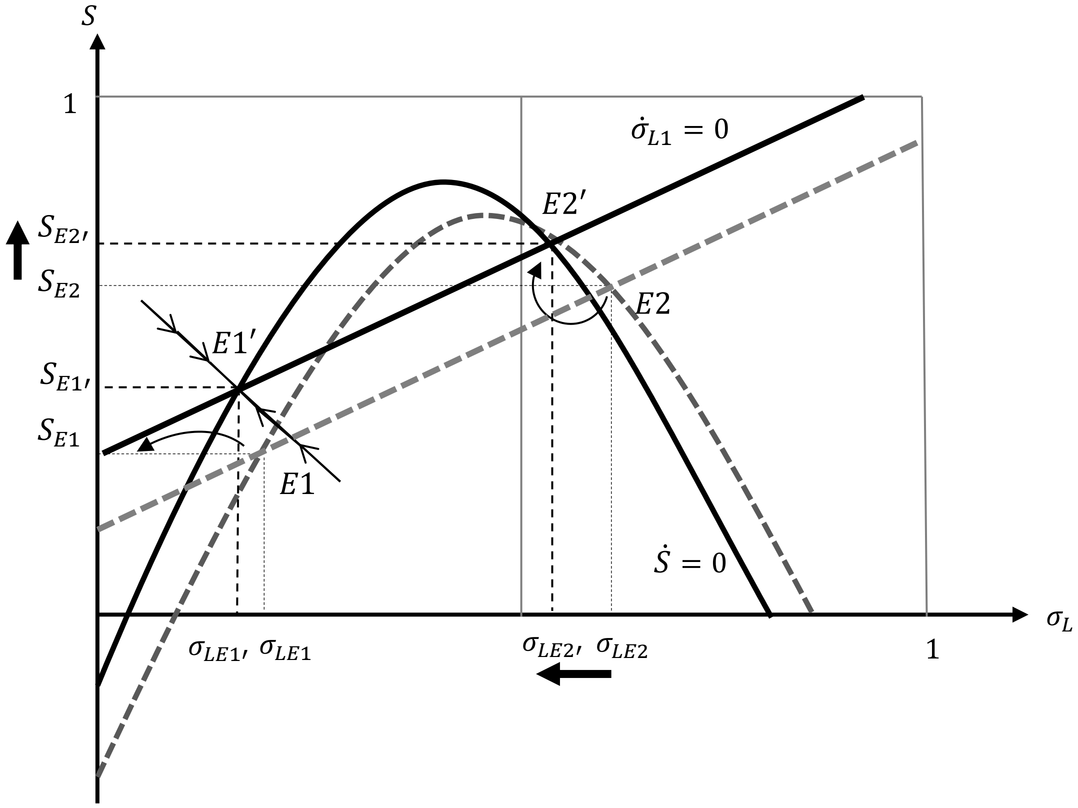
As for the *locus* $\dot{\sigma}_L = 0$ described in the equation (35), since the level of the RER is positively related to nothing but its intercept, a currency devaluation will shift the curve upwards without affecting its slope. More formally, from equation (35), we have $dS/d\Theta = \mu/\omega_1(1-\lambda)\beta_2 z > 0$. Figure 8 shows what happens with the locus $\dot{\sigma}_L = 0$.

FIGURE 8 – The impact of an undervaluation on *locus* $\dot{\sigma}_L = 0$



Once again, the grey dotted line $\dot{\sigma}_{L0} = 0$ is the initial curve and the black one $\dot{\sigma}_{L1} = 0$ is the new locus after a devaluation. Figure 9 bellow shows the impact of an increase in Θ on the dynamics between technological progress and income distribution in the long run.

FIGURE 9 – The impact of a devaluation on the dynamics between S and σ_L



Before we analyse the dynamics in Figure 9, we must first see how the wage and the profit shares respond to changes in the RER. If we divide the mark-up price from equation (9) by domestic price, substitute equation (12) in the mark-up factor, and solve for σ_L, then we obtain:

$$1 = \delta\Theta(\sigma_L + \Theta\mu) \Rightarrow \sigma_L = \frac{1}{\delta\Theta} - \Theta\mu \tag{41}$$

$$\frac{d\sigma_L}{d\Theta} = -\frac{1}{\delta\Theta^2} - \mu < 0 \tag{42}$$

The derivative (42) shows that a currency devaluation reduces the wage share. The same analysis can be extended for the profit share:

$$\frac{1}{\delta\Theta} = \sigma_L + \Theta\mu \Rightarrow \sigma_K = 1 - \frac{1}{\delta\Theta} \tag{43}$$

$$\frac{d\sigma_K}{d\Theta} = \frac{1}{\delta\Theta^2} > 0 \tag{44}$$

It can be seen in the derivative (44) that the profit share and the RER are positively related. Bearing these results in mind, we can now proceed to the analysis of Figure 9.

Let us start with the LWS region around the unstable solution $E1'$. A devaluation shifts the initial short-term solution to the left of $E1$, as a devaluation reduces the wage-share, and the long-term equilibrium solution from $E1$ to $E1'$. Since $E1'$ is a saddle point, only displacements along the separatrix make the system to move towards the new equilibrium point $E1'$ after a devaluation. However, only by a fluke a devaluation would place the short-term solution on the separatrix, which makes this scenario highly unlikely. Perturbations in any other direction will be amplified, as the system veers off the new equilibrium point. Taking the scenario represented in Figure 9, if the initial short-term solution lies below (and to the left of) the separatrix crossing the new long-term equilibrium $E1'$, then a devaluation reduces both σ_L and S, thus initiating a vicious circle. It happens because a devaluation increases the profit share to the detriment of the wage share. Since in the LWS region technological progress is wage-led, an increase in the profit share discourage capitalists to invest in technological improvements; such a reduction in the wage share and in the pace of technological progress also reduces growth, which undermines the workers' bargaining power, thereby bringing down once again the wage share and so on. Conversely, if the initial short-term solution is placed above (and to the right of) the new separatrix, then both σ_L and S

increase until the system solution falls within the HWS region. This later scenario seems to be the most desirable by policymakers, as it yields a domestic income more evenly distributed and a higher relative level of technological capabilities for the home country. Nonetheless, given the inverse relationship between wage share and RER, only under very restrictive conditions a positive RER shock will place the short-term solution to the right of the separatrix crossing $E1'$. That is to say that, in a LWS region, it is more likely for a currency devaluation to worsen the income distribution and impair the country's technological catching-up process. In this case, a currency appreciation or a wage increase seem to be the most recommended policy measures for a country to boost technological catching-up and reach a more equal income distribution. In a LWS region, a currency appreciation increases the wage share, which intensifies technological changes and propels growth; fast growth rates favour workers during the bargaining process, thus augmenting the wage share anew and so forth, generating a virtuous circle of technological progress, reduction of income inequality, and growth.

As for the analysis of the HWS region around the stable point $E2'$, it can be seen that a devaluation shifts the equilibrium solution from $E2$ to $E2'$. Since in this region the technological progress is profit-led, a devaluation accelerates the pace of technical change, due to an increase in the profit rate, and hence worsens the income inequality as a consequence. Alternatively, a currency appreciation slows down the technological catching-up process and redistributes domestic income in favour of workers. In short, perturbations in the stable solution induce S and σ_L to move in opposite directions. This rases an important question: if we assume that the long-term growth of output is positively related to the relative technological capabilities (S) and the wage share (σ_L), as described in equation (33), and given the fact that the RER is positively related to S in the HWS region and negatively related to σ_L, then what is the net impact of a devaluation on long-term growth? In the case of the LWS region, apart from the unlikely scenario where the system solution is moving along the separatrix, it is easy to see that if both S and σ_L increase (decrease) simultaneously in a virtuous (vicious) circle, then the long-term growth rate must also increase (decrease). However, in the case of the HWS region, the impact of a currency devaluation on long-term growth is ambiguous. The next section seeks to shed some light on this issue.

IV. THE NET IMPACT OF A CURRENCY DEVALUATION ON LONG-TERM GROWTH

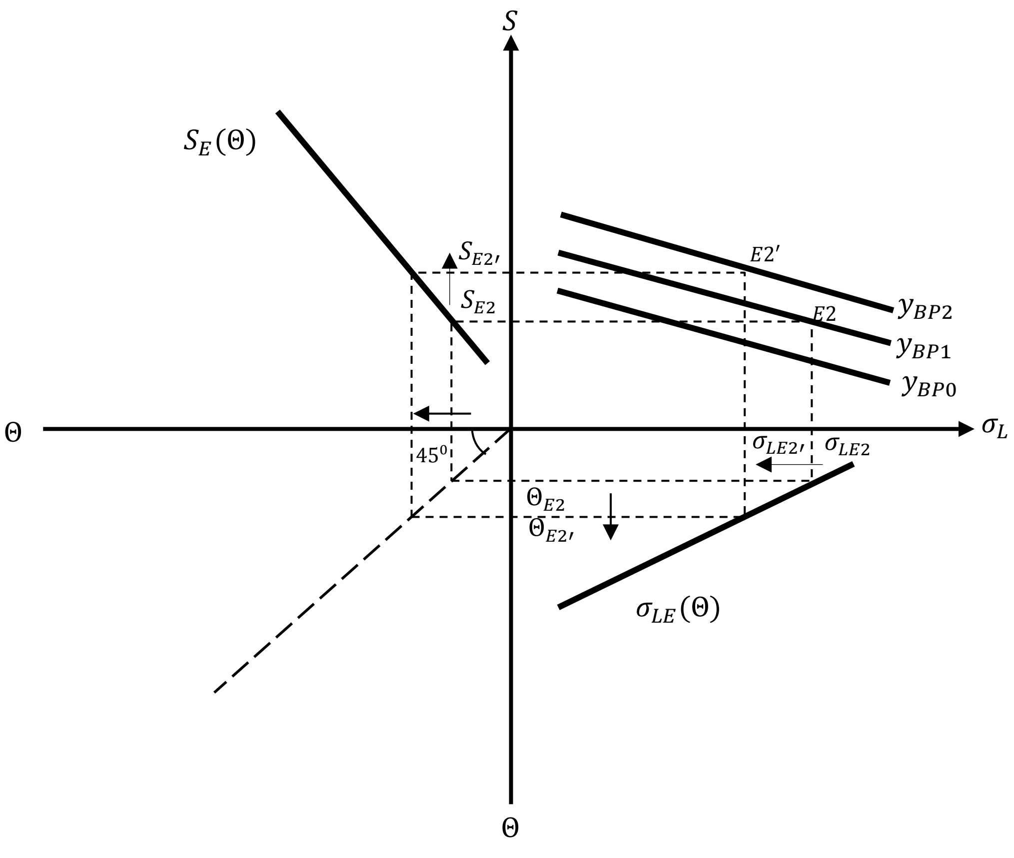
In order to evaluate the net impact of a devaluation on long-term growth, let us return to equation (33). This equation represents the extended Thirwall's Law with endogenous elasticities, as follows:

$$y_{BP} = \frac{\varepsilon}{\pi} z = (\beta_0 + \beta_1 \sigma_L + \beta_2 S) z \tag{33}$$

The assessment of an impact of a currency depreciation on long-term growth must consider, not only how changes in the RER affect relative technological capabilities (S) and wage share (σ_L), but also the values of the parameters β_0, β_1 and β_2.

That said, let us now illustrate in Figure 10 below the net impact of a currency devaluation on long-term growth in the HWS region.

FIGURE 10 – The impact of a currency devaluation on long-term growth



In Figure 10, equation (33) is represented in the first quadrant as a set of iso-growth curves, where each of which consists of a constant equilibrium growth rate in the space (σ_L, S). The slope of these curves is given by $-\beta_1/\beta_2$.

As for the fourth quadrant, we must analyse the relationship between RER and the equilibrium wage share in the HWS region (σ_{LE2}). Let us rewrite σ_{LE2} expressed in (37), as follows

$$\sigma_{LE2} = \frac{\gamma_2\rho_1 - \alpha_3\gamma_1}{2\alpha_2\gamma_2} + \frac{\sqrt{\Delta}}{2\alpha_2\gamma_2} \tag{37*}$$

where $\Delta = (\gamma_2\rho_1 - \alpha_3\gamma_1)^2 + 4\alpha_2\gamma_2(\gamma_2\rho_0 - \alpha_3\gamma_0)$, which is assumed to be positive, as we have two distinct real roots. In order to write σ_{LE2} as a function of the RER we have to substitute $\rho_0 = \alpha_0 + \alpha_2(1-\Theta\mu)\Theta\mu$, $\rho_1 = \alpha_2(1-\Theta\mu)$, and $\gamma_0 = \Theta\mu - 1 + \omega_0 - \omega_1[(1-\lambda)\beta_0 z - a_0 - n]$ in (37*). The RER effect on σ_{LE2} is given by the differential

$$\frac{d\sigma_{LE2}}{d\Theta} = \frac{\mu}{2}\left[\frac{\alpha_2\gamma_2(1-3\Theta\mu) + \alpha_3(\gamma_2 - 2)}{\sqrt{\Delta}} - 1\right] < 0 \tag{45}$$

The term $[\alpha_2\gamma_2(1-3\Theta\mu) + \alpha_3(\gamma_2-2)]$ is ambiguous. The difference $(1-3\Theta\mu)$ is expected to be positive, once the imported raw material share of income $(\Theta\mu)$ is also assumed to be sufficiently small, say less than 30%. On the other hand, the difference $(\gamma_2 - 2)$ is positive, since γ_2 is assumed to be positive and less than unity. Therefore, given $\alpha_2, \gamma_2, \alpha_3 \in (0,1)$, $(1-3\Theta\mu) > 0$, $(\gamma_2 - 2) < 0$, and $\sqrt{\Delta} > 0$, it is plausible to assume that the term $[\alpha_2\gamma_2(1-3\Theta\mu) + \alpha_3(\gamma_2-2)](\Delta^{-1/2})$ is less than unity in absolute value, or even negligible, and hence the differential $d\sigma_{LE2}/d\Theta < 0$. In short, it can be said that the RER and the equilibrium wage share in the HWS region (σ_{LE2}) are inversely related. This falls in line with the scenario represented in Figure 9, where a currency devaluation reduces the wage share $(\sigma_{LE2'} - \sigma_{LE2} < 0)$. In Figure 10 the relationship between RER and σ_{LE2} is assumed to be linear for convenience. One can also think of it as a linear approximation of the actual function around the initial equilibrium point (Θ, σ_{LE2}).

In order to explore the relationship between RER and the country's long-term relative technological capabilities level in the HWS region (S_{E2}) presented in the second quadrant of Figure 10, we must first define S as a function of the equilibrium wage share, such as $S_{E2} = S(\sigma_{LE2})$. Accordingly, by plugging the equilibrium wage share σ_{LE2} from the equation (37*) into equation (25) – it could be also in equation (35) without loss of generality, since

S_{E2} is determined by the intersection between equations (25) and (35) – and differentiating with respect to the RER, we have:

$$\frac{dS(\sigma_{LE2})}{d\Theta} = \frac{\mu}{\gamma_2}\left\{1 + \frac{\gamma_1}{2}\left[\frac{\alpha_2\gamma_2(1 - 3\Theta\mu) + \alpha_3(\gamma_2 - 2)}{\sqrt{\Delta}} - 1\right]\right\} > 0 \qquad (46)$$

It follows from (45) that the term $[\alpha_2\gamma_2(1 - 3\Theta\mu) + \alpha_3(\gamma_2 - 2)](\Delta^{-1/2}) - 1$ is negative and less than or equal to unity in absolute value, that is $[\alpha_2\gamma_2(1 - 3\Theta\mu) + \alpha_3(\gamma_2 - 2)](\Delta^{-1/2}) - 1 \in [-1,0)$. Accordingly, since $\gamma_1/2 \in (0,1)$, we can say that the term $\{1 + (\gamma_1/2)(\Delta^{-1/2})[\alpha_2\gamma_2(1 - 3\Theta\mu) + \alpha_3(\gamma_2 - 2) - \sqrt{\Delta}]\}$ is positive, thus implying that $dS(\sigma_{LE2})/d\Theta > 0$. That is to say, the higher the RER, the higher the country's relative equilibrium technological capabilities. The function in the second quadrant is also assumed to be a linear approximation around the initial equilibrium point (Θ, S_{E2}). This result agrees with the scenario illustrated in Figure 9, where a currency devaluation in the home country reduces the technology gap $(S_{E2\prime} - S_{E2} > 0)$.

Having defined the relations between the endogenous variables, we can now analyse the scenario represented in Figure 10. It starts in the third quadrant with a currency devaluation $(\Theta_{E2'} - \Theta_{E2} > 0)$. In the fourth quadrant, we can see that the devaluation reduces the wage share in income $(\sigma_{LE2\prime} - \sigma_{LE2} < 0)$. On the other hand, the same devaluation increases the technological progress in the home country $(S_{E2\prime} - S_{E2} > 0)$, as shown in the second quadrant. Finally, in the first quadrant, since the iso-growth curves are sufficiently elastic – that is, the slope of the iso-growth curves is sufficiently low in absolute value $|\beta_1/\beta_2|$ – we can say that a currency devaluation spurs growth in the long run $(y_{BP2} - y_{BP1} > 0)$.

To sum up, this model shows that the net effect of a currency devaluation on long-term growth is ambiguous. It depends not only on the parameters of the model, but also on the magnitude in absolute values of changes in its endogenous variables, namely the technology gap and the wage share, and on the type of technological progress regime, that is wage-led or profit-led. Therefore, policymakers should take into account the idiosyncrasies concerning all these issues with respect to the country under consideration before they decide to promote economic recovery or simply boost growth by the use of currency devaluations.

V. SUMMARY

This paper contributes to the post-Keynesian literature on balance-of-payments constrained growth, income distribution and technological innovation by developing a theoretical framework in which the non-price competitiveness of the economy is determined by the dynamics between relative technological capabilities and income inequality. An increase in the pace of technological innovation induces to improvements in the non-price competitiveness of domestic goods. On the other hand, an increase in the wage share of income changes the consumption pattern of both capitalists and workers, and hence also affects non-price competition factors of the economy.

In terms of policy, although the literature has been emphasizing the importance of a devalued RER for output growth, exchange rate impacts on the non-price competitiveness of the economy via simultaneous changes in income distribution and technological change have been neglected. This paper also contributes to the balance-of-payments constrained growth literature by exploring at length the links between a currency devaluation and improvements in non-price competitiveness of domestic goods. It establishes a set of conditions under which a currency devaluation can boost or harm growth in the long run. In a LWS region, given the constraints imposed on the parameters in this work, without loss of generality, a currency appreciation seems to be more effective to boost technical change, wage share and growth. On the other hand, in a HWS region, a devaluation will cause technological progress and wage share to move in opposite directions, which means that the net impact of a real devaluation on long-term growth is ambiguous and depends on the parameters of the model.

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Appendix I

$$\varphi = \frac{(W/a)}{\left(\frac{W}{a} + P_f E\mu\right)}$$

In rates of change:

$$\frac{d\varphi}{\varphi} = \frac{dln}{dt}\left[\frac{T(W/a)}{T\left(\frac{W}{a} + P_f E\mu\right)}\right] = \frac{d}{dt}\left[\ln(W) - \ln(a) - \ln\left(\frac{W}{a} + P_f E\mu\right)\right]$$

$$\frac{d\varphi}{\varphi} = w - \hat{a} - \left\{\frac{1}{\frac{W}{a} + P_f E\mu}\left[\left(\frac{\dot{W}a - W\dot{a}}{a^2}\right) + \left(\dot{P}_f E + P_f \dot{E}\right)\mu\right]\right\}$$

$$\frac{d\varphi}{\varphi} = w - \hat{a} - \left\{\frac{1}{\frac{W}{a} + P_f E\mu}\left[\left(\frac{\dot{W}}{a} - \frac{W}{a}\hat{a}\right) + \left(\dot{P}_f E + P_f \dot{E}\right)\mu\right]\right\}$$

$$\frac{d\varphi}{\varphi} = w - \hat{a} - \left\{\frac{1}{\frac{W}{a} + P_f E\mu}\left[\left(\frac{W}{W}\frac{\dot{W}}{a} - \frac{W}{a}\hat{a}\right) + \left(\frac{P_f}{P_f}\dot{P}_f E + \frac{E}{E}P_f \dot{E}\right)\mu\right]\right\}$$

$$\frac{d\varphi}{\varphi} = w - \hat{a} - \left\{\frac{1}{\frac{W}{a} + P_f E\mu}\left[\frac{W}{a}(w - \hat{a}) + P_f E\mu\left(p_f + e\right)\right]\right\}$$

$$\frac{d\varphi}{\varphi} = w - \hat{a} - \left[\varphi(w - \hat{a}) + (1 - \varphi)\left(p_f + e\right)\right]$$

$$\frac{d\varphi}{dt} = \varphi(1 - \varphi)\left(w - \hat{a} - p_f + e\right)$$

Appendix II

Proposition: given $\tau = 0$, in the long term $\varphi = \bar{\varphi} \;\Rightarrow\; p_d = p_f + e$.

<u>Proof:</u>

$$\varphi = \frac{P^W}{P^E + P^W} \Rightarrow \frac{1}{\varphi} = 1 + \frac{P^E}{P^W}$$

where $P^W = W/a$ and $P^E = EP_f\mu$. If P^E/P^W is constant, then φ is constant as well. Therefore:

$$\frac{P^E}{P^W} = k \text{ or } ln\left(\frac{P^E}{P^W}\right) = ln(c)$$

where $c > 0$ is a constant. Taking the differential of the logarithm with respect to time:

$p^E = p^W$

Until now it is proven that $\varphi = \bar{\varphi} \Rightarrow p^E = p^W$. The next step is to prove the following:

$p^E = p^W \Rightarrow p_d = p_f + e$

Rewriting equation (10) and considering $\tau = 0$ in the long term, we obtain:

$$p_d = \varphi p^W + (1 - \varphi)p^E \qquad (*)$$

where $p^W = w - \hat{a}$ and $p^E = p_f + e$.

This way, considering that $\varphi = \bar{\varphi} \Rightarrow p^E = p^W$ in $(*)$, we have:

$p_d = p^W = p^E = p_f + e$, which had to be demonstrated.

APPENDIX III

Equation (23):

$$\dot{S} = \alpha_0 + \alpha_1 \sigma_K - \alpha_2 \sigma_K^2 - \alpha_3 S$$

If $\sigma_L = 1 - \sigma_L - \Theta\mu$, then:

$$\dot{S} = \alpha_0 + \alpha_1(1 - \sigma_L - \Theta\mu) - \alpha_2(1 - \sigma_L - \Theta\mu)^2 - \alpha_3 S$$

$$\dot{S} = (\alpha_0 + \alpha_1 - \alpha_2) + (-\alpha_1 + 2\alpha_2)\sigma_L - \alpha_2\sigma_L^2 + (-\alpha_1 + 2\alpha_2)\Theta\mu - \alpha_2(\Theta\mu)^2 - \alpha_2\sigma_L\Theta\mu - \alpha_3 S$$

Assuming $\alpha_1 = \alpha_2$:

$$\dot{S} = [\alpha_0 + \alpha_2(1 - \Theta\mu)\Theta\mu] + \alpha_2\sigma_L - \alpha_2\sigma_L^2 - \alpha_2\sigma_L\Theta\mu - \alpha_3 S$$

where $\rho_0 = \alpha_0 + \alpha_2(1 - \Theta\mu)\Theta\mu$. Then:

$$\dot{S} = \rho_0 + \alpha_2[(1 - \Theta\mu)\sigma_L - \sigma_L^2] - \alpha_3 S$$

By making $\rho_1 = \alpha_2(1 - \Theta\mu)$, we have equation (24):

$$\dot{S} = \rho_0 + \rho_1\sigma_L - \alpha_2\sigma_L^2 - \alpha_3 S$$

APPENDIX IV

Relation between the technological gap and income distribution within the locus $s = 0$:

$$S = \frac{\rho_0}{\alpha_3} + \frac{\rho_1}{\alpha_3}\sigma_L - \frac{\alpha_2}{\alpha_3}\sigma_L^2$$

This appendix is twofold. First, we will analyse the conditions with respect to the domain of σ_L. Later, we ought to do the same regarding $S(\sigma_L^*)$.

A) Domain of σ_L

Condition to obtain two distinct real roots ($\Delta > 0$):

$\rho_0 + \rho_1\sigma_L - \alpha_2\sigma_L^2 = 0$

Given $\rho_0 < 0$ and $\alpha_2 > 0$, the following condition must be fulfilled:

If $\Delta > 0$, then $\rho_1^2 > -4\rho_0\alpha_2$.

A.1) First root: $\sigma_{L1} \in (0,1)$

Condition I: $\sigma_{L1} > 0$

$$\sigma_{L1} = \frac{-\rho_1 + \sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{-2\alpha_2} = \sigma_L^* - \frac{\sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{2\alpha_2} > 0 \Rightarrow$$

$$\alpha_2(1 - \Theta\mu) > \sqrt{\rho_1^2 + 4\rho_0\alpha_2}$$

$[\alpha_2(1 - \Theta\mu)]^2 = \rho_1^2 > \rho_1^2 + 4\rho_0\alpha_2$. TRUE, once we are assuming $\rho_0 < 0$.

Condition II: $\sigma_{L1} < 1$

TRUE, once $0 < \sigma_{L1} < \sigma_L^* < 1/2$

A.2) Second root: $\sigma_{L2} \in (0,1)$

Condition I: $\sigma_{L2} > 0$

$$\sigma_{L2} = \frac{-\rho_1 - \sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{-2\alpha_2} = \sigma_L^* + \frac{\sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{2\alpha_2} > 0$$

Since all the parameters are positives, the statement is TRUE. Thus $\sigma_{L2} > 0$.

Condition II: $\sigma_{L2} < 1$

$$\sigma_{L2} = \frac{-\rho_1 - \sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{-2\alpha_2} < 1$$

If $\sigma_{L2} \in (\sigma_L^*, 1)$, then $\sigma_{L2} - \sigma_L^* < 1 - \sigma_L^*$. Therefore:

$$\sigma_L^* + \frac{\sqrt{\rho_1^2 + 4\rho_0\alpha_2}}{2\alpha_2} - \sigma_L^* < 1 - \frac{(1-\Theta\mu)}{2}$$

$$\sqrt{\rho_1^2 + 4\rho_0\alpha_2} < \alpha_2(\Theta\mu + 1)$$

$$\alpha_2^2(1-\Theta\mu)^2 + 4\rho_0\alpha_2 < \alpha_2^2(\Theta\mu + 1)^2$$

$$4\rho_0 < \alpha_2[(\Theta\mu + 1)^2 - (1-\Theta\mu)^2]$$

By considering:

$$[(\Theta\mu + 1)^2 - (1-\Theta\mu)^2] = [(\Theta\mu + 1) + (1-\Theta\mu)][(\Theta\mu + 1) - (1-\Theta\mu)]$$

and then rearranging the terms, we have:

$$\rho_0 < \alpha_2\Theta\mu$$

From appendix III we know $\rho_0 = \alpha_0 + \alpha_2(1-\Theta\mu)\Theta\mu$. Then:

$$\alpha_0 + \alpha_2(1-\Theta\mu)\Theta\mu < \alpha_2\Theta\mu$$

$$\alpha_0 - \alpha_2(\Theta\mu)^2 < 0$$

Therefore, once $\alpha_0 < 0$, then this condition is TRUE. In this work we will assume such a condition for the sake of the graphic exposition.

B) Domain of $S(\sigma_L^*)$

$$S(\sigma_L^*) = \frac{\rho_0}{\alpha_3} + \frac{\rho_1}{\alpha_3}\frac{(1-\Theta\mu)}{2} - \frac{\alpha_2}{\alpha_3}\frac{(1-\Theta\mu)^2}{4}$$

Substituting $\rho_1 = \alpha_2(1-\Theta\mu)$ in the equation above and rearranging the terms, we obtain:

$$S(\sigma_L^*) = \frac{\rho_0}{\alpha_3} + \frac{\alpha_2}{\alpha_3}(\sigma_L^*)^2$$

B.1) $S(\sigma_L^*) > 0$

The condition is

$$\frac{\rho_0}{\alpha_3} + \frac{\alpha_2}{\alpha_3}(\sigma_L^*)^2 > 0 \Rightarrow \rho_0 > -\alpha_2(\sigma_L^*)^2$$

As $\rho_0 = \alpha_0 + \alpha_2(1-\Theta\mu)\Theta\mu$, we have:

$$\alpha_0 + \alpha_2(1-\Theta\mu)\Theta\mu < \alpha_3 - \alpha_2\frac{(1-\Theta\mu)^2}{4}$$

Rearranging the terms:

$$\alpha_2\Theta\mu(2 - 3\Theta\mu) > (-4\alpha_0 - \alpha_2)$$

If this inequality holds, then $S(\sigma_L^*) > 0$.

B.2) $S(\sigma_L^*) < 1$

The condition is

$$\frac{\rho_0}{\alpha_3} + \frac{\alpha_2}{\alpha_3}(\sigma_L^*)^2 < 1 \Rightarrow \rho_0 < \alpha_3 - \alpha_2(\sigma_L^*)^2$$

As $\rho_0 = \alpha_0 + \alpha_2(1 - \Theta\mu)\Theta\mu$, we have:

$$\alpha_0 + \alpha_2(1 - \Theta\mu)\Theta\mu < \alpha_3 - \alpha_2\frac{(1 - \Theta\mu)^2}{4}$$

Rearranging the terms:

$$\alpha_2\Theta\mu(2 - 3\Theta\mu) < (4\alpha_3 - 4\alpha_0 - \alpha_2)$$

We expect the term $\alpha_2\Theta\mu(2 - 3\Theta\mu)$ must be positive but quite small, once we also expect $\Theta\mu < 2/3$. It means the condition under discussion only holds if the term $(4\alpha_3 - 4\alpha_0 - \alpha_2)$ is sufficiently positively high. If we assume $\alpha_0 < 0$, then the condition can be reasonably fulfilled. Therefore, $S(\sigma_L^*) < 1$.

APPENDIX V

The terms of the Jacobian matrix are:

$$J_{11} = -\alpha_3 < 0$$

$$J_{12} = \rho_1 - 2\alpha_2 = \alpha_2(1 - \Theta\mu - 2\sigma_L) = 2\alpha_2(\sigma_L^* - \sigma_L) \gtreqless 0$$

$$J_{21} = \theta\gamma_2 > 0$$

$$J_{22} = -\theta\gamma_1 < 0$$