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Uneven Development and the Balance of Payments Constrained Model: Terms of Trade, Economic Cycles, and Productivity Catching-up.

Danilo Spinola*

Abstract

This paper expands the Dutt (2002) version of the Balance of Payments Constrained Model (BPCM). We question the assumption of price-neutrality and the incompatibility between the BPCM and the Prebisch-Singer hypothesis (PSH) in terms of the long-run terms-of-trade dynamics. The research focuses on three main elements: (1) the long-run behaviour of the terms of trade in a Structuralist framework. (2) The cyclical endogenous dynamics in the relationship between economic activity and income distribution à la Goodwin. (3) Productivity gap and catching-up. This article adds to the Dutt(2002) model (a) a productivity gap dynamics in which the south has a catching-up element; (b) labour market by including a Phillips Curve for the relationship between employment rate and economic activity; (c) labour supply dynamics that considers the labour transfer issue between traditional and modern sectors. We find that the Structuralist/evolutionary arguments hold in the BPCM framework with these changes.

Keywords: Balance of Payments constrains, Terms of Trade, Economic Cycles, Latin American Structuralism.

JEL: E22, E32, O41.

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1 Introduction

Thirlwall's framework, aka the Balance of Payments Constrained Model (BPCM), is one of the most relevant contributions of the Post-Keynesian school of thought to economic theory (Davidson, 1990; Dutt, 2002). It states that the growth rate of an economic system must be compatible with the constraints imposed by the balance of payments. Assuming that terms of trade and financial flows are stable in the long run, Thirlwall derives a rule in which the growth rate of an economy depends directly on the income elasticity ratio between exports and imports. This in the literature became known as the Thirlwall Law (McCombie, 1989).

There is a large literature tradition focused on estimating the parameter of the Thirlwall Law, measuring the income elasticity of imports and exports for different countries (Alonso & Garcimartín, 1998). These measures show that the Thirlwall law offers a very good proxy to explain long-run growth rates, especially in developed countries. In developing economies, however, the correlation between observed growth rates and the ones predicted by the BPCM are not direct, as the short-term effects of terms of trade fluctuation and financial flows volatility systematically deviates the actual growth rates from the one predicted by the law (Thirlwall & Hussain, 1982).

Despite the strong relevance of the Thirlwall law in the development economics literature, the strict focus on the law itself neglects some important aspects of the BPCM (Dutt, 2002) such as uneven development and the short- to long-run transition dynamics. This article tackle these two important aspects usually ignored in the broad debate about the Thirlwall framework.

A central assumption of the Thirlwall law is that price effects are stable in the long-run. This assumption, however, is subject to many critiques - as described by McCombie (2012). It contradicts the core of the Structuralist tradition, condensed in the Prebisch-Singer hypothesis, which defends the existence of a declining trend in the terms of trade for developing countries (Harvey, Kellard, Madsen, & Wohar, 2010). This trend is caused by a specialization of the productive structure in products with smaller income elasticity of demand (primary commodities). As income grows, the relative prices of primary commodities increase at a smaller rate than manufactured goods, resulting in a decline in the terms of trade. This reinforces the uneven development conditions between north and south (Dutt, 2002).

From the long-run framework of the Thirlwall Law, to the short-run one described in the Thirlwall model, we discuss the problem of transition dynamics. In the short-run both price and quantity effects serve as adjustment mechanisms: price- and income-elasticity of imports and exports, the terms of trade (real exchange rate), and financial flows adjustment the economy to its equilibrium. The transition between short- to long-run is observed from the explicit imports and exports equations. In this research we follow the Dutt (2002) model, that offers a possible solution to the transition dynamics. The Dutt model endogenizes the evolution of the terms of trade towards the long-run adjustment. It consists in a model with a north-south dynamics in which the north follows a monopolistic Keynesian-Kaleckian framework while the south is modelled in perfect competition with a Marx-Lewis perspective (Lewis, 1954). Despite critiques to this model related to the way the south is modelled, we see it as an important contribution to start the discussion between uneven development and transition dynamics.

Economic volatility is another important element in discussing economic dynamics, being a source of uneven development. Countries with bigger GDP oscillations, usually developing countries, face higher challenges in achieving a stable development process. Regular patterns of oscillation (cycles) may emerge in the transitory dynamics between short-run and long-run. A canonical contribution goes back to the Goodwin (1967) model, that discussed the emergence of endogenous cycles from the relationship between economic activity and income distribution. When considering the BPCM, the volatility sources come from terms of trade and financial flows, which are ignored in the Thirlwall law. This concept of volatility in the Thirlwall model is directly related to the concept of fragility, the resilience of an economic system to external shocks. We bring the Goodwin model to the Thirlwall framework by expanding the Dutt (2002) model.

The role of the productive structure is brought to the model by the Structuralist theory. In Prebisch (1950), the behaviour of the terms of trade is defined by the productive structure of a country in the context of an international division of labour. A specialized structure with smaller labour productivity level has a higher productivity gap between laggard south and advanced north. This impacts in a decline in the terms of trade, as the southern economy is specialized in products with smaller income elasticity, which reduces the growth possibilities of the south. On the other hand, the productivity gap may follow its own dynamics. Laggard economies may have higher opportunities for learning, getting closer to the productivity levels of the north. So the gap may create leapfrogging opportunities (Lee, 2013).

The objective of this research is to observe the transition dynamics in the BPCM capturing economic cycles, the productivity gap, and the behaviour of terms of trade. We initially follow the baseline model defined by Dutt (2002) and expand it by: (a) adding a productivity gap dynamics in which the south has a catching-up element; (b) model the labour market of the south economy by including a *Phillips Curve* to discuss the relationship between employment rate and economic activity; (3) add a labour supply dynamics that considers the *Lewisham* problem of the labour transfer between traditional and modern sectors (Lewis, 1954). The inclusion of these elements changes the structure of the Dutt model, resulting in a 4-dimensional expanded dynamic system that is able to generate interesting patterns in the trajectory between short- and long-run. We here study the thoroughly the model and present some scenario possibilities.

We raise the following points: How can we describe the way price dynamics affect the results of the BPCM when we assume price non-neutrality. In this sense, can we see if technology efforts and structural change are related to these price effects or not. In terms of fragility and volatility, what can we state about countries that are away from the technological frontier, and what are the causes that determine the magnitude of the cycles. On the other hand, we discuss what are the feedback loops caused by this higher volatility. Finally, under which conditions can we reach a virtuous development process in the context of non-neutrality of price effects, focused on the effects on economic growth?

After this introduction, in section 2 we develop a brief literature review discussing the BPCM. In section 3 we raise the research questions related to this research. In section 4 we show the development of the baseline model of this research, based on Dutt (2002). In section 5 we add the expansions to the model. In section 6 we discuss the properties of the expanded model. In section 7 we study the signs of the model to discuss stability and cycles. In section 8 we develop some simulations and analyse the results. In section 9 we discuss the main results. Finally, in section 10 we conclude this paper.

2 Literature review

2.1 Thirlwall's Law and the Thirlwall Model

The Thirlwall model (Thirlwall, 1979) is a growth model that links the economic growth possibilities with the constrains imposed by the balance of payments. The system is derived from the behaviour of the external sector. The model can be explicitly derived from export and import functions. Using the variables defined in Dutt (2002) we have:

$$M = \theta_M (1/P)^{-\mu} Y^{\varepsilon} \quad (1)$$

$$X = \theta_X (P)^{-\nu} Y_f^{\delta} \quad (2)$$

In which M and X represent total import and total export respectively. θ_M and θ_X are constants. Y is the domestic income and Y_f the foreign income. The relative price P represents the price ratio between domestic prices (P_d) and foreign prices (P_f), in domestic currency – multiplied by the real exchange rate (E). $P = P_d/EP_f$. μ and ν are the price elasticities of imports and exports, respectively. Finally, ε and δ are the income elasticity of imports and exports. Imports increase with higher domestic income while exports grow with higher foreign income. Import falls with increases in the relative price while exports rise. Price elasticities define the growth sensitivity to price changes and income elasticities the growth sensitivity to output/income changes.

The variable F represents the financial flows. The equilibrium of the balance-of-payments occurs when we have balance between net exports plus net financial flows and net imports:

$$PX + F = M \quad (3)$$

Writing eq.(3) in terms of growth rates we have:

$$[1 - (F/M)]\left[\hat{P} + \hat{X}\right] + (F/M)\hat{F} = \hat{M} \quad (4)$$

In which the hat above the letters implies growth rates. When we replace M, X and P by eq. (1), (2) and 3, we end up with the Thirlwall growth equation, which is given by:

$$\hat{Y} = (1/\varepsilon)\left\{(1 - \mu - \nu)\hat{P} + [1 - (F/M)]\delta\widehat{Y_f} + (F/X_N)\left[\hat{F} - (1 - \nu)\hat{P}\right]\right\} \quad (5)$$

In the long-run, Thirlwall considers that there is no change in the Terms of Trade ($\hat{P} = 0$), and capital flows are constant ($\hat{F} = 0$). These assumptions result in the **Thirlwall Law**. Net capital flows are stable in the long run, and the growth rate will depend on the ratio between income elasticities of exports and income elasticity of imports, multiplied by the rate of growth of foreign GDP.

$$\hat{Y} = (\delta/\varepsilon)\,\widehat{Y_f} \quad (6)$$

or

$$\hat{Y} = (1/\varepsilon)\,\hat{X} \tag{7}$$

Income growth rate depends on the income elasticities, which is usually considered exogenous[1]. The usual simplification ($\hat{P} = \hat{F} = 0$) results in some theoretical and empirical problems. (1) As pointed by Dutt (2002), the simplification ignores the transitional dynamics. We cannot observe the trajectory between short- and long-run, which neglects some possible effects in the adjustment process that may affect the final outcome (steady state). (2) There are also a high number of articles discussed in Blecker (2016) and McCombie (2012), that question the empirical validity of the Thirlwall law, especially for developing regions (effective rate diverging from the income elasticity ratio) (Thirlwall & Hussain, 1982). (3) The BPCM assumes that terms of trade are do not affect the long-run, contradicting the decline in the terms of trade theory ($\hat{P} < 0$) (Prebisch, 1950).

2.2 Uneven Development and the Transitional Dynamics

The questions posed in this research concern the dynamics between the "short-run" version of the Thirlwall model and its "long-run" version (Thirlwall Law). Dutt (2002) proposes a transition dynamics in an open north-south framework. In this model he focus on uneven development, an important but usually neglected matter raised by Thirwall (2012).

Uneven development is a central aspect to study the behaviour of developing economies. This discussion contraposes itself to the Solow-type neoclassical growth models, and approaches itself to the old Structuralist ideas of Prebisch and Singer (Prebisch, 1950; Singer, 1950).

In the old structuralism, the position of an economy in the international division of labour defines its role in the system and its development possibilities. A country specialized in the production and export of raw materials tends to progressively lag behind than those that produce and export manufactured goods. Products are heterogeneous in terms of their price and income elasticity of demand. Products with higher income elasticity of demands (manufactured goods) have a rise in their demand as international economy grows, resulting in higher relative prices than raw materials. This ends up in the emergence of an uneven development in which the core countries of the system advance in their productive structure while the periphery remains trapped in lower levels of economic development. According to the old Structuralist ideas, the way out of this

[1] There are however articles focused on endogenizing the income elasticities in the structuralist theory, relating income elasticities to the behavior of the productive structure (Cimoli & Porcile, 2014; Porcile & Spinola, 2018).

vicious system is through increases in public intervention to foster the creation and development of modern manufacture sectors – through a process of import substitution industrialization (ISI).

The Thirlwall framework does not focus directly on the productive heterogeneity (as the old Structuralist ideas), but on the role of changes in terms of trade (prices of exports divided by prices of imports) and financial flows, to define the growth rate considering the structural aspects of the economy (captured by price and income elasticity of imports and exports). There is no inherent trend of decline of the terms of trade as in the classical structuralism, but price behaviour play a role in stablishing the growth possibilities. It is important to say that in Thirlwall's model, autonomous demand[2] is endogenous to the behaviour of the external sector, so investments are then endogenous to the balance of payments possibilities.

2.3 Assumptions and Empirical validity of the Thirlwall Law.

As observed, the Thirlwall model depends on two assumptions to keep its validity, that the terms of trade and financial flows grow at zero rate in the long-run. This can be condensed in what Blecker (2016) highlights as the main assumption in the Thirlwall model: price effects are neutral in the long-run. In order to have that, we must assume that either price-elasticity is too low "Elasticity Pessimism" ($\mu + \nu \approx 1$), or that the real exchange rate grows at zero rate in the long run ($\hat{P} = 0$), or that financial flows are balance itself close to zero in the long-run ($\hat{F} = 0$). Accepting these three assumptions led to the fact that economic adjustments to the equilibrium occur in quantities rather than prices. As price effects are neutral, the domestic growth rate then adjust the system to the conditions imposed by the balance of payments constrains.

These three assumptions, however, have been passive to critiques from the BPCM researchers. Empirically we still have an open debate if actually price effects are neutral in the long-run.

2.4 The role of price effects and the incompatibility between Prebisch and Thirlwall

In the new Structuralist ideas (Cimoli & Porcile, 2014), the theories by Raul Prebisch and Anthony Thirlwall are usually seen as complementary. However, there is a very important element that is usually neglected in the discussion and that raises a big contradiction in the theory. In the theory of decline in the terms of trade (Prebisch-Singer hypothesis), there is a long-run tendency to a reduction in prices ($\hat{P} < 0$). When accepting Prebisch-Singer and that the

[2] Consumption, Investment and Government Spending.

Marshall-Lerner condition holds (price elasticity imports is bigger than 1), price effects are not neutral in the long-run, but declining in developing countries. This perspective contradicts the main assumptions that lead to the Thirlwall Law ($\hat{P} = 0$).

With the Prebisch-Singer hypothesis, adjustments to the equilibrium may not occur only in quantities, but also on prices (terms of trade), defining the long-run economic growth rate. In this sense, price effects impact on long-run economic growth, which opens space to discuss how this happens. Is it through the real exchange rate, as defended by the New Developmentalist School?

2.5 Cycles: Labour market and the Goodwin dynamics

Economic Volatility is an important element that creates constrains to economic development. In this sense, merging the BPCM with the Goodwin system is a way to capture the relationship between economic constrains from the external sector, and economic constrains from oscillations in the economic system. We add this by adding the Relationship between income distribution and unemployment rate (economic activity) through the definition of a modified version of the Phillips Curve.

2.6 Catching-up and productivity dynamics

On the other hand, the productivity gap may follow its own dynamics, following an evolutionary approach. Laggard economies may have higher opportunities for learning, getting closer to the productivity levels of the north. So the Gap may create opportunities.

3 Research Considerations:

This research proposes to relate (1) the behaviour of the terms of trade with (2) cyclical endogenous volatility, and (3) the productive sector through the discussion of Productivity Gap. It consists in a Thirlwall-Goodwin model with elements of productivity gap.

Why are we using the Dutt (2002) model? The seminal Dutt (2002) model offers a starting point to answer the research questions. It criticizes the Thirlwall Law offering a theoretical proposal inside the BPCM framework to challenge the idea that prices are neutral in the long-run. This model allows us to approach the Prebisch-Singer hypothesis, in which prices decline in the long-run, while observing transition dynamics and uneven development.

Why do we want to expand the model?

The model restricts itself to the topic of the Terms of Trade, and how it evolves from capital accumulation. We add elements that deal with the matter of volatility in developing countries, discussing its determinants and its effects. Following a Structuralist perspective, we associate the regular volatility (cycles) to the fragile structural conditions in the south. An element of productivity and productivity gap, associated to the topics of structural change and technological dynamics closes the model. We then manage to link three Structural central issues of developing economies: Balance of payments constrains, high growth volatility and a fragile productive structure.

In this research we raise some specific discussions - some theoretically-related and others development-related. The theoretical one focuses on the assumptions and derivations of the BPCM: the critique of the Thirlwall Law, the incorporation of Goodwin Cycles, and the theories of catching-up. The second group is related to development issues, trying to raise conditions for a virtuous development strategy.

An important discussion focuses on how price dynamics affect the results of the BPCM when assuming $\hat{P}$ endogenous and $\hat{P} \neq 0$. In this sense we may discuss under which conditions we can observe a Prebisch-Singer behaviour$(\hat{P} < 0)$. In terms of the structural elements, how can technology efforts and structural change are related to price effects. We debate the conditions that lead to a virtuous catching-up process, discussing how countries away from the technological frontier show themselves as more fragile and volatile.

Considering the effects of volatility in the process of economic development we discuss what determines the magnitude of the cycles and the impacts of a higher volatility on development. Under which conditions can we reach a virtuous development process in the context of non-neutrality of price effects and the effects on economic growth.

4 The baseline Dutt (2002) model

A basic North-South model based on the Thirlwall law states that the relationship between growth rates in North and South depends on the ration between income elasticity of imports in the North and South.

$$\widehat{Y_S}/\widehat{Y_N} = \varepsilon_N/\varepsilon_S \tag{8}$$

Dutt (2002) discusses two economies that interact through their external sector: (i) a South economy, marked by perfect competition with fixed real wage and unemployment labour

(following a Marx-Lewis structure), and (ii) a North economy that has imperfect competition, in which firms practice mark-up pricing and excess capacity, with a Kalecki-Keynes structure.
The monopolistic north has its price level defined by a mark-up function over costs, with price-making firms:

$$P_N = (1 + z)W_N b_N \tag{9}$$

In which P_N is the price level in the north; z consists in the mark-up ($z \geq 1$); W_N is the wage level in the north and b_N is the fixed unit labor requirement for the northern good (also understood as the inverse of labor productivity). An increase in markup and/or on costs (unitary wages) raises price levels, as well as a reduction in labour productivity (increases in productivity have a negative impact on prices).
The south follows a perfect competition specification. Southern GDP (Y_S) operates at full capacity. It follows a fixed the relationship between capital stock in the south (K_S) and the fixed capital-output ratio in the south (a_S).

$$Y_S = K_S/a_S \tag{10}$$

Real wages in the south (V_S) is defined as the ratio between nominal wages (W_S) and price index in the south (P_S):

$$V_S = W_S/P_S \tag{11}$$

Consumers in the north consume all their income, while capitalists save a fraction (s_N) of their income. The north spends a fraction α of their consumption expenditure on southern goods (and the rest on the northern goods). This fraction is equal to:

$$\alpha = \alpha_0 Y_N^{\varepsilon_N - 1} P^{1-\mu_N} \tag{12}$$

In which α_0 is the autonomous part of the northern expenditure in southern goods; Y_N is the GDP in the north. Considering $E = 1$, The terms of trade (P) is given by the ratio between prices in the south (P_S) and prices in the north (P_N):

$$P = P_S/P_N \tag{13}$$

In the south, workers spend all their income on southern goods. Southern capitalists save a fraction (s_S) and consume the rest. Part of the consumption (β) is spent on the northern good. Analogous to α; β can be described as:

$$\beta = \beta_0(\sigma_S Y_S)^{\varepsilon_S - 1}(1/P)^{1-\mu_S} \tag{14}$$

β_0 is the autonomous part of the southern expenditure in northern goods. σ_S is the profit share of southern total income. This profit share is the residual from the wage share (ω_S) on total output: $\omega_S = b_S V_S$. The profit share can be specified as the part of total income that does not go to wages:

$$\sigma_S = (1 - b_S V_S) \tag{15}$$

In which b_S is the labor-output ratio in the south ($b_S = L_S/Y_S$).

The investment function follows a Kaleckian specification based on Bhaduri & Marglin (1990), in which capacity utilization affects the capitalists perception of economic activity. When capacity utilization increases, capitalists perceive it as an increase in effective demand, which stimulates them to increase total capacity by immobilizing capital in order to sustain the increases in the demand. Investments in the north are then given by:

$$g_n = I_N/K_N = \gamma_0 + \gamma_1(u) \tag{16}$$

In which I_N is total investment in the north, γ_0 and γ_1 are positive constants. u consists on the rate of capacity utilization, which is given by $u = Y_N/K_N$. The next step is to find explicit equations for northern and southern exports. Considering the equations for P_S and X_S, the total value of southern exports is given by:

$$P_S X_S = \alpha\{[1 + (1 - s_N)z]/(1 + z)\}P_N Y_N \tag{17}$$

Applying eq.(12) on eq.(17), we end up with the equation for southern exports, which can be given in its reduced form as:

$$X_S = \theta_S P^{-\mu_N} Y_N^{\varepsilon_N} \tag{18}$$

In which $\theta_S = \alpha_0[1 + (1 - s_N)z]/(1 + z)$. θ_S is a constant.

The northern total exports to the south are equal to the southern imports from the north, being the southern total imports $M_S = \beta\sigma_S Y_S$ we have:

$$P_N X_N = \beta\sigma_S P_S Y_S \tag{19}$$

Using eq.(14) on eq.(19), the equation for northern exports is given by:

$$X_N = \theta_N(1/P)^{-\mu_S} Y_S^{\varepsilon_S} \tag{20}$$

In which we have the constant $\theta_N = \beta_0 \sigma_S^{\varepsilon_S}$.

This simple static model highlights the properties of the north-south interaction. Southern and Northern exports are explicitly addressed in order to reach equilibrium in current account, balancing the values of exports in north and south.

4.1 Dynamics properties of the Thirlwall model

The dynamic properties of the Dutt (2002) model are derived from the excess demand (ED) functions in the north and south. In the south, excess demand (ED_S) is given by:

$$ED_S = C_{SS} + I_{SS} + X_S - Y_S \tag{21}$$

And as $M_S = C_{SS} + I_{SS} - Y_S$ and $M_S = \left(\frac{1}{P}\right) X_N$:

$$ED_S = X_S - \left(\frac{1}{P}\right) X_N \tag{22}$$

While, analogously, excess demand in the north (ED_N) is given by:

$$ED_N = C_{NN} + I_N + X_N - Y_N \tag{23}$$

$$ED_N = I_N - S_N + X_N - PX_S \tag{24}$$

Following a market clearing equilibrium, there is no excess demand in the long-run:

$$ED_i = 0 \tag{25}$$

The equilibrium condition can be used in eq. (21) and (23). When substituting all variables and applying the equilibrium in eq. (24), the results give us the following static equations for terms of trade and capacity utilization $ED_S = X_S - \left(\frac{1}{P}\right) X_N = 0$:

$$X_N = PX_S \tag{26}$$

Then substituting eq.(26), we have:

$$P = [(\theta_S/\theta_N)(uK_N)^{\varepsilon_N}(a_S/K_S)^{\varepsilon_S}]^{1/(\mu_N+\mu_S-1)} \tag{27}$$

From the Saving-Investment balance condition in the north ($I_N = S_N$):

$$u = \gamma_0/[s_N\sigma_N - \gamma_1] \tag{28}$$

In the long-run the **capital stock grows according to the rates of capital accumulation** in the two regions ($g_i = I_i/K_i$). The short-run conditions are always satisfied ($ED_i = 0$). In this sense accumulation in the north is given by:

$$g_N = \gamma_0 + \gamma_0\gamma_1/[s_N\sigma_N - \gamma_1] \quad (29)$$

In the south, savings determine investments. Southern workers do not save, only southern capitalists do save. The savings function is then given by the propensity to save times the profit share, times output:

$$S_S = s_S\sigma_S\, K_S/a_S \quad (30)$$

The investment function is then given by the value of total savings in domestic currency:

$$I_S = P^{\xi} S_S \quad (31)$$

ξ is a constant with positive value. The next step is to define the savings – investment conditions for the south. Combining eq. (28) and (29) to the south we have the following equation for capital accumulation:

$$g_S = s_S P^{\xi}\, \sigma_S/a_S \quad (32)$$

When deriving eq.(27) we get the dynamic properties of the model. Terms of trade then fluctuate following the relationship between capital accumulation in the north and the south:

$$\hat{P} = [1/(\mu_N + \mu_S - 1)](\varepsilon_N g_N - \varepsilon_S g_S) \quad (33)$$

Which means that terms of trade (P) will fluctuate depending on the gap between investment in north and south weighted by their respective income elasticity of imports. So terms of trade here are not neutral in the long-run, changing an important assumption of the Thirlwall model.

The model just described is the canonical Dutt (2002) model. In the next session we expand this model by adding a productivity gap dynamics and modelling the labour market. These modifications completely change the characteristics of the dynamic model, resulting in the emergence of interesting new patterns.

5 Expansion of the Dutt-Thirlwall Model

The original Dutt model results in a one dynamic equation for the dynamics of the terms of trade. This depends on the gap between capital accumulation in the north and in the south. In this expansion we focus on creating a productivity dynamics that is able to define other patterns, rather than a monotonic convergence and/or divergence.

5.1 Productivity Dynamics

In eq.(9), northern price levels are defined as function of mark-up over costs $P_N = (1 + zWNbN$. From this equation we introduce an initial productivity and wage rate dynamics to the north, which follows a constant rate of growth:

$$\widehat{b_N} = -\beta_N \tag{34}$$

$$\widehat{W_N} = \beta_N \tag{35}$$

β_N is a constant. Labour productivity in the north ($\lambda_N = \frac{1}{N}$) grows exogenously, and wages track productivity, growing both at the same rate. $Y_S = K_S / a_S$ remains for the south, considering that there is no idle capacity, and that the capital-output ratio is constant. $W_S/P_s = V_S$ defines the value of real wages. In this sense, productivity and real wages follow the same path, growing according to technological progress. Technological progress in the north is assumed constant and stable[3].

In Dutt (2002), the real wages in the south (V_S) are fixed. Capitalist's income ($Y_{S,K}$) in south is equal to $Y_{S,K} = (1 - b_s V_s) P_S Y_S$ and the share of that in total income is $\sigma_s = (1 - b_s V_s)$. We now endogenize b_s and V_s developing a labor productivity dynamics. There is also the need to specify $b_s V_s$, so they stay within bounds (wage share cannot be smaller than zero or higher than one).

Labour productivity (λ_i) is the inverse of the unit labor requirement for the production of a good:

$$\lambda_i = \frac{1}{b_i} \tag{36}$$

Using this this definition, the productivity gap[4] (G) between north and south is:

$$G = \ln\left(\frac{\lambda_N}{\lambda_S}\right) = \ln\left(\frac{b_S}{b_N}\right) \tag{37}$$

In eq.(32), productivity grows in the north at a constant rate $\widehat{\lambda_N} = \beta_N$. We consider in the south an extra effect which rises on this economy for being a laggard one, a gap dynamic effect.

[3] We understand, on the other hand, the central aspect of evolutionary major technological change happening in waves. (Schumpeter, 1939)

[4] Cimoli & Porcile (2014)

Labour productivity growth in south can be written as a constant rate (β_S) plus the effect of the productivity gap, which we put as a catching-up effect:

$$\widehat{\lambda_S} = \beta_S + \rho G \tag{38}$$

Being $\beta_S < \beta_N$, and considering the definition of productivity gap on eq.(37), we can work out the dynamics of the technology gap ($\hat{G}$) as:

$$\hat{G} = (\beta_N - \beta_S) - \rho G \tag{39}$$

5.2 Labour market and the Phillips curve

A second addition to the model consists in developing a labour market dynamics. In order to do so we endogenize real wages. We use, in the southern economy, a modified version of the famous *Phillips curve*, relating real wages to the employment rate:

$$\widehat{V_S} = -m + n\left(\frac{L_S}{\Lambda_S}\right) = -m + nl_s \tag{40}$$

m and *n* are constants. L_S consists on total employment, and Λ_S the total workforce. In this sense, the employment rate (l_s) can be defined as:

$$l_s = \frac{L_S}{\Lambda_S} \tag{41}$$

Here we observe that this addition changes the characteristics of the model. equations (12) - (16) stay the same, but the profit share $\sigma = (1 - b_s V_s)$ is no longer a constant. Defining the wage share in the south as $\omega_S = b_S V_S$, we have that its growth rate is given by:

$$\widehat{\omega_S} = \widehat{b_S} + \widehat{V_S} = -\beta_S - \rho G - m + nl_S \tag{42}$$

And the growth rate of the profit share in the south (σ_S) follows then the opposite variation of the wage share (ω_S), which results in the following equation:

$$\widehat{\sigma_S} = -\widehat{\omega_S} = \beta_S + \rho G + m - nl_S \tag{43}$$

Equation (27) does not change. But for equations (30) and (31) σ_S becomes a variable, not a constant parameter. From equation (31) we then have that $\hat{P} = \frac{1}{\mu_N + \mu_S - 1}(\varepsilon_N g_N - \varepsilon_S g_S)$. If we expand g_N and g_S from equations (27) and (30) respectivelly, we end up with the following equation for the evolution of the Terms of Trade:

$$\hat{P} = \frac{1}{\mu_N + \mu_S - 1}\left[\varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S}\right] \tag{44}$$

5.3 Employment rate, population growth and the Lewis dynamics

We may consider two different situations that lead to the same result. The first occurs in an economy divided by a traditional and a modern sector. When there is a higher percentage of income going to labour, this creates an incentive to workers move to formal activities of the economy. The second situation is related to migration. A higher incentive may lead workers to move from one country to the other looking for better life conditions, increasing the labour supply in the receiver country.

Considering a linear specification relating the increases in wage share (and reduction in profit share) in the labour supply we have:

$$\widehat{\Lambda_S} = \varphi - \psi \sigma_S \tag{45}$$

φ is the constant autonomous population growth and ψ is a constant that measures the elasticity to move to from the traditional to the formal sector (or the migration cost). As we defined in eq.(41) that $l_S = L_S/\Lambda_S$, we have then in growth rates that:

$$\widehat{l_S} = \widehat{L_S} - \widehat{\Lambda_S} \tag{46}$$

Total employment dynamics: $b_S = \frac{L_S}{Y_S} \Rightarrow L_S = b_S Y_S \Rightarrow \widehat{L_S} = \widehat{b_S} + \widehat{Y_S} \Rightarrow$

Employment rate dynamics: $\widehat{l_S} = \widehat{L_S} + \widehat{\Lambda_S} = \widehat{b_S} + \widehat{Y_S} - \widehat{\Lambda_S}$

As we have the capital-output labour $b_S = L_S/Y_S$, the total labor growth in the south is $L_S = b_S Y_S$. In growth rates:

$$\widehat{L_S} = \widehat{b_S} + \widehat{Y_S} \tag{47}$$

From Eq. (10) $Y_S = K_S/a_S$. As there is no depreciation, therefore $\widehat{K_S} K_S = I_S$. From the Savings-Investment southern condition ($I_S = P^{\xi} S_S$) and being $S_S = s_S \sigma_S Y_S$, we have that $I_S = P^{\xi} s_S \sigma_S \frac{K_S}{a_S}$. This result takes takes us to:

$$\widehat{K_S} = \frac{I_S}{K_S} = P^{\xi} s_S \sigma_S \frac{1}{a_S} \tag{48}$$

As $\widehat{a_S} = 0$, $\widehat{K_S} = \widehat{Y_S}$ Which gives us finally the dynamic equation for the employment rate, from $\widehat{l_S} = \widehat{b_S} + \widehat{Y_S} - \widehat{\Lambda_S}$:

$$\widehat{l_S} = -\beta_S - \rho G + P^{\xi} s_S \sigma_S \frac{1}{a_S} - \varphi + \psi \sigma_S \tag{49}$$

In summary, we end up with a system of four differential equations:

$$\hat{P} = \frac{1}{\mu_N + \mu_S - 1}\left[\varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S}\right]$$

$$\widehat{\sigma_S} = \beta_S + \rho G + m - n l_S$$

$$\widehat{l_S} = -\beta_S - \rho G + P^{\xi} s_S \sigma_S \frac{1}{a_S} - \varphi + \psi \sigma_S$$

$$\hat{G} = (\beta_N - \beta_S) - \rho G$$

This system defines the north-south dynamics between terms of trade, distribution in the south, southern employment rate in the south and the productivity gap. The trajectory defines the relationship between the short- and the long-run in the model. The next step is to analyse the dynamic properties of this system.

6 Dynamic properties of the expanded model

In this section we discuss the dynamic properties of the expanded model. From the four equations of our system we analytically calculate the steady state conditions and the Jacobian studying the trajectory and stability conditions.

In order to calculate the Steady State we set our dynamic variables equal to zero. $\hat{P} = \widehat{\sigma_S} = \widehat{l_S} = \hat{G} = 0$. The Jacobian is a matrix of the partial derivatives of all pair of dynamic variables involved in the system. After computing the Jacobian we study the signs of all its elements one by one.

Setting the dynamic variables equal to zero we have the Steady State, which is (for the mathematical passages, see the annex):

$$P^* = \left[\frac{\gamma_0 a_S \psi}{\left[\varphi + \beta_N - \frac{\varepsilon_N \gamma_0}{\varepsilon_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right)\right]} \frac{\varepsilon_N}{\varepsilon_S s_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right)\right]^{1/\xi} \tag{51}$$

$$\sigma_S^* = \frac{1}{\psi}\left[\varphi + \beta_N - \frac{\varepsilon_N \gamma_0}{\varepsilon_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right)\right] \tag{52}$$

$$l_S{}^* = \frac{1}{n}(\beta_N + m) \tag{53}$$

$$G^* = \frac{(\beta_N - \beta_S)}{\rho} \tag{54}$$

And after taking the partial derivatives of our system, we have the Lagrangean of the model, which is, considering the row/column order as P, σ_S, l_S and G respectively:

$$J = \begin{bmatrix} -\frac{1}{\mu_N + \mu_S - 1}\frac{\varepsilon_S s_S \sigma_S}{a_S}\xi P^{\xi-1} & -\frac{1}{\mu_N + \mu_S - 1}\frac{\varepsilon_S s_S P^{\xi}}{a_S} & 0 & 0 \\ 0 & 0 & -n & \rho \\ \xi P^{\xi-1} s_S \sigma_S \frac{1}{a_S} & P^{\xi} s_S \frac{1}{a_S} - \psi & 0 & -\rho \\ 0 & 0 & 0 & -\rho \end{bmatrix} \tag{55}$$

To have the steady state conditions we check the signs of the lagrangean on the steady state.

6.1 Values for the Lagrangean in the Steady State (Stability conditions)

Considering our jacobian as:

$$J = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

We have:

$$a_{11} = -\frac{1}{\mu_N + \mu_S - 1}\frac{\varepsilon_S s_S \sigma_S}{a_S}\xi P^{*\xi-1}$$

a. From the Marshall-Lerner condition, $\mu_N + \mu_S > 1$. This implies $\frac{1}{\mu_N + \mu_S - 1} > 0$

b. The relationship $\frac{\varepsilon_S s_S \sigma_S}{a_S}$ has that $0 < s_S \sigma_S < 1$. As it is possible that $a_S > \varepsilon_S$, and being both higher than 0, we will will most likely have $0 < \frac{\varepsilon_S s_S \sigma_S}{a_S} < 1$.

c. $\xi > 0$, so the sign of $\xi P^{*\xi-1}$ depends on the sign of P on the Steady State.

d. A $sign(a_{11}) = -sign(P^*)$. If we only consider the positive value of P^*, then $sign(a_{11}) < 0$

$$a_{12} = -\frac{1}{\mu_N + \mu_S - 1}\frac{\varepsilon_S s_S {P^*}^{\xi}}{a_S}$$

Analogous to a_{11}, $\text{sign}(a_{12}) = -\text{sign}(P^*)$. Being $\text{sign}(P^*) > 0$, then $\text{sign}(a_{12}) < 0$.

$$a_{13} = a_{14} = a_{21} = a_{22} = 0$$

$$a_{23} = -n$$

n is always a positive number, so $\text{sign}(a_{23}) < 0$

$$a_{24} = \rho$$

$\rho > 0$, then $\text{sign}(a_{24}) > 0$

$$a_{31} = \xi {P^*}^{\xi-1} s_S \sigma_S \frac{1}{a_S}$$

Analogously to the previous cases, $\text{sign}(a_{31}) = \text{sign}(P^*)$. Being $\text{sign}(P^*) > 0$, then $\text{sign}(a_{31}) > 0$.

$$a_{32} = {P^*}^{\xi} s_S \frac{1}{a_S} - \psi$$

The sign of a_{32} depends on the relationship between ${P^*}^{\xi} s_S \frac{1}{a_S}$ and ψ. If ${P^*}^{\xi} s_S \frac{1}{a_S} > \psi$ then $sign(a_{32}) > 0$

$$\mathrm{a}_{34} = \mathrm{a}_{44} = -\rho$$

As $\rho > 0, Sign(\mathrm{a}_{34}) = \text{Sign}(\mathrm{a}_{44}) < 0$

As $a_{33} = a_{41} = a_{42} = a_{43} = 0$:

$$Sign(J) = \begin{bmatrix} -Sign(P) & -Sign(P) & 0 & 0 \\ 0 & 0 & - & + \\ Sign(P) & Sign\left(P^{\xi} s_S \frac{1}{a_S} - \psi\right) & 0 & - \\ 0 & 0 & 0 & - \end{bmatrix}$$

We can then analyse the value of P in the steady state (P^*)

$$P^* = \left[\frac{\gamma_0 a_S \psi}{\left[\varphi + \beta_N - \frac{\varepsilon_N \gamma_0}{\varepsilon_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right)\right]}\frac{\varepsilon_N}{\varepsilon_S s_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right)\right]^{1/\xi}$$

a. $\gamma_0 a_S \psi$ is a product of three positive numbers, as well as $\frac{\varepsilon_N}{\varepsilon_S s_S}$

b. As defined by Dutt (2002), the relationship between $s_N\sigma_N - \gamma_1$ is a central condition that needs to be positive in order to have a converging trajectory dynamics, so $s_N\sigma_N > \gamma_1$. So $\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right) > 0$.

c. The relationship between $(\varphi + \beta_N)$ and $\left[\frac{\varepsilon_N\gamma_0}{\varepsilon_S}\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)\right]$ defines the final value of the sign. If $(\varphi + \beta_N) > \left[\frac{\varepsilon_N\gamma_0}{\varepsilon_S}\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)\right]$ then $Sign(P) > 0$.

Considering $\mathrm{Sign}(P) > 0$ and that usually, $\mathrm{P}^{\xi}\mathrm{s_S}\frac{1}{\mathrm{a_S}} < \psi$, then we have as the signs of the Lagrangean of our system:

$$\mathrm{Sign}(J) = \begin{bmatrix} - & - & 0 & 0 \\ 0 & 0 & - & + \\ + & - & 0 & - \\ 0 & 0 & 0 & - \end{bmatrix}$$

From the Jacobian signs, we observe the interaction between each of the four equations, which lets us say about the stability conditions of the model:

$J_{P\sigma_S} = \begin{bmatrix} - & - \\ 0 & 0 \end{bmatrix}$ – Converging to a curve

$J_{\sigma_S l_S} = \begin{bmatrix} 0 & - \\ - & 0 \end{bmatrix}$ – Closed Orbit

$J_{Pl_S} = \begin{bmatrix} - & - \\ + & - \end{bmatrix}$ - Cyclical Stability

$J_{\sigma_S G} = \begin{bmatrix} 0 & + \\ 0 & - \end{bmatrix}$ - Converging to a curve

$J_{PG} = \begin{bmatrix} - & 0 \\ 0 & - \end{bmatrix}$ - Convergence to a point

$J_{l_S G} = \begin{bmatrix} 0 & - \\ 0 & - \end{bmatrix}$ - Converging to a curve

Following the specifications and the conditions imposed in our system we have a stable system. Cycles emerge from the relationship between σ_S and l_S. The cyclical convergence aspect of P comes from the relationship between P and l_S.

7 Defining values for each of the parameters.

In this section we discuss the expected values for each parameter. In this way we can define the possible values for the Jacobian and the stability conditions of the model.

Table 1. Variable list to observe signals

I.	a_s	VIII.	s_S	XV.	σ_N
II.	ξ	IX.	b_S	XVI.	β_S
III.	μ_N	X.	b_N	XVII.	β_N
IV.	μ_S	XI.	γ_0	XVIII.	ρ
V.	ε_S	XII.	γ_1	XIX.	ψ
VI.	ε_N	XIII.	m	XX.	φ
VII.	s_N	XIV.	n		

I. Estimations for the Capital-Output ratio (a_s). When estimating the values of a_s using the Penn World Tables we observe that a_s has a value between 2 and 5.

Table 2. Capital-output estimation average for the period 2000-2014. Selected countries.

Argentina	2.8	Germany	4.0
Brazil	3.7	France	4.0
Colombia	3.7	UK	3.7
Mexico	3.0	Italy	5.1
Canada	3.6	China	3.3
USA	3.3	India	3.0

Source: Penn World Tables 9.0

II. Dutt (2002) states that the variable ξ is smaller than 1 and bigger than 1.

III. Price elasticity of imports of the North and the South are usually considered small in the BPCM theory, what Blecker ((2016)) calls "elasticity pessimism". Considering them n their absolute value $\mu_N > 0$ and $\mu_S > 0$. We follow the Marshall-Lerner condition, in which $\mu_N + \mu_S > 1$, as argued by Dutt (2002).

IV. The same for point III. We consider here $\mu_N \approx 1$ and $\mu_S \approx 1$

V. According to Dutt (2002), when $\varepsilon_N < 1$, increases in northern income results in a lower proportion of expenditure in the Southern good, so the southern good is income-inelastic. As increases in southern income will result in a higher proportion of expenditure in the Northern good, we consider $0 < \varepsilon_s < 1$.

VI. The value of the income elasticity of import demand is always higher than zero ($\varepsilon_N > 0$). It is possible to consider two cases, the one in which a higher income results in higher expenditure in the Northern good ($0 < \varepsilon_N < 1$). And the case in which a higher income results in higher expenditure in the Southern good ($\varepsilon_N > 1$) – this is the one we decide.

VII. s_N consists in the fraction of income saved by capitalists. As defined before $s_N = \frac{S_N}{\sigma_S Y_S}$, so we consider $0 < s_N < 1$.

VIII. Idem for point VII. $0 < s_S < 1$.

IX. From the model, $b_S = L_S/Y_S$. Using the PWT we can calculate these values from many countries using as L_S the number of persons engaged, and as Y_S the output-sided real GDP at chained PPP.

Table.3. number of persons engaged (in dec.) divided by the output-sided real GDP at chained PPP. Selected countries

Argentina	0.28	Germany	0.12
Brazil	0.34	France	0.11
Colombia	0.37	UK	0.13
Mexico	0.25	Italy	0.12
Canada	0.11	China	0.48
USA	0.09	India	0.73

X. As observed in point IX, developing countries have higher values for the labour-output ratio, so $0 < b_N < b_S < 1$.

XI. γ_0 and γ_1 are positive constants, which shows that the Northern investment rate depends on the rate of capacity utilization measured by Y_N/K_N, because higher capacity utilization implies more buoyant markets and higher profits. We consider γ_0 the animal spirit. It does not have an economic meaning in itself, but we consider its value between 0 (low capitalists confidence) and 1 (high capitalists confidence). $0 < \gamma_0 < 1$.

XII. $\gamma_1 > 0$ is the sensitivity of capacity utilization changes on the investment-capital ratio. We consider this value close to 0 but positive. $\gamma_1 \approx 0$.

XIII. m and n are the effects on the southern real wage dynamics. m is the constant rate of decrease in real wage growth. In this work we consider a small value (about 1% decrease maximum). So $0 < m \leq 1$.

XIV. n is the elasticity of real wage to changes in the employment rate. It also has a value between 0 and 1.

XV. The constant profit rate of the north follows the range between zero (all income goes to wages) and one (all income goes to profits). So $0 < \sigma_N < 1$.

XVI. β_S and β_N are the exogenous parameters in the growth rate of labor productivity. We consider that the exogenous rate of technical change improvements is higher in the North than in the South: $\beta_N > \beta_S$. Usually this value is very close to zero.

XVII. According to the previous β_N – Exogenous growth of labor productivity – between 0 and 0.1. So we have $0 < \beta_S < \beta_N < 0.1$.

XVIII. ρ is the effect of changes in the gap on the evolution of labour productivity in the south. Increases in G also increases the productivity gap and have negative effects on the southern productivity. We then consider ρ having a positive value (so $-\rho$ is negative). We consider it usually has a value between 0 and 1. So $0 < \rho < 1$.

XIX. φ is the autonomous population growth. In developed countries it is very often negative while still positive in many developing countries. We will consider it having a positive value very close to zero $\varphi > 0$ and $\varphi \approx 0$.

XX. ψ is the elasticity of the labor supply to increases in the wage share. It has a positive number between zero and 1.

Table 4. Results for the parameter values

$3 < a_s < 5$	$0 < s_n < 1$	$0 < n < 1$
$0 < \xi < 1$	$0 < s_s < 1$	$0 < \sigma_N < 1$
$\mu_N \approx 1$	$0 < b_N < b_S < 1$	$0 < \beta_S < \beta_N < 0.1$
$\mu_N + \mu_S > 1$	$0 < \gamma_0 < 1$	$0 < \rho < 1$
$0 < \varepsilon_s < 1$	$\gamma_1 \approx 0$	$\varphi \approx 0$
$0 < \varepsilon_N < 1$ or $\varepsilon_N > 1$	$0 < m \leq 1$	$0 < \psi < 1$

Based on these parameter values we are able to build some scenarios for our model. We then define a baseline situation for the Southern and the Northern countries.

8 Scenarios

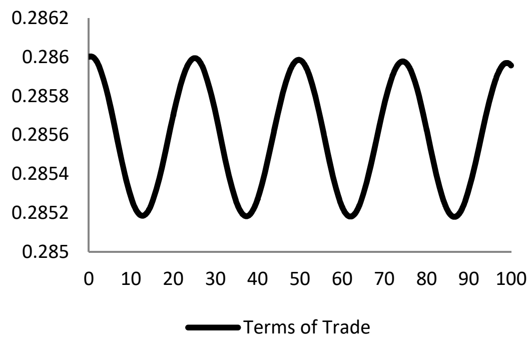
8.1 Baseline

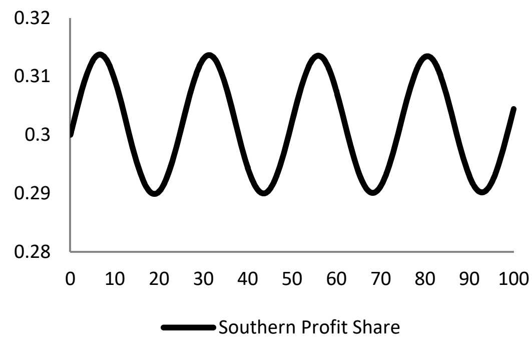
The baseline model is built as part of an effort to select credible parameters values for a developing economy in the South (Latin America) and for a developed country in the North. The developing economy is marked by a low industrialization (capital-output ratio) close to 3 (levels close to Mexico and Argentina). It follows the Marshall-Lerner condition ($\mu_N + \mu_S > 1$). The income elasticities of imports are higher in the north than in the south. Savings rate in the north is around 35% while in the south it is around 18%. Initial value for the profit rate in the south is 30%, being 40% in the north. Exogenous increase in labour force is about 4% while the country shows higher labour force elasticity to increases in the wage share. The parameters used for our baseline are the following:

Table 1. Baseline Model: Parameter Values and initial values

$a_s = 3$	$\varepsilon_N = 1.05$	$m = 0.9$	$\rho = 0.3$
$\xi = 0.5$	$s_N = 0.35$	$n = 1$	$\varphi = 0.04$
$\mu_N = 1$	$s_s = 0.18$	$\sigma_N = 0.4$	$\psi = 0.2$
$\mu_S = 1$	$\gamma_0 = 0.005$	$\beta_S = 0.005$	
$\varepsilon_S = 0.95$	$\gamma_1 = 0.06$	$\beta_N = 0.03$	
Initial Values			
$P_0 = 0.286$	$\sigma_{S_0} = 0.3$	$l_{S_0} = 0.92$	$G_0 = 0.5$

Figure 1. Baseline Results





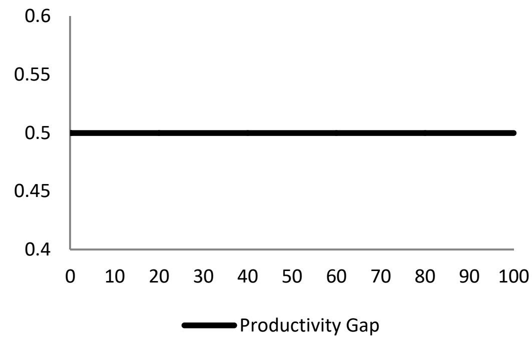


Table 2. Baseline: Steady state and Eigenvalues

Steady State	$P^* = 0.285$	$\sigma_S^* = 0.301$	$l_S^* = 0.929$	$G^* = 0.5$
Eigenvalues	$e_1 = -0.0003 + 0.255\text{i}$		$e_2 = -0.0003 - 0.255\text{i}$	
	$e_3 = -0.0250 + 0.000\text{i}$		$e_4 = -0.0039 + 0.000\text{i}$	

The baseline results show a situation in which the technology gap is constant at its initial level of 0.5. Terms of trade oscillate around a stable trend. Southern profit shares oscillate between the values of 29% and 31%. And the Employment rate between 92% and 93.5%. These are stable cycles that repeat themselves indefinitely (do not reaching the equilibrium value).

8.2 <u>Scenario 1</u>: Declining terms of trade in a lagging behind scenario

From the baseline calibration we define a case in which the economy is under a more fragile situation. We can compare this case with a Latin American country. It consists in a less industrialized economy (smaller a_s), in which the learning process occurs at a slower pace (reduced ρ) and the autonomous productivity growth is smaller (reduction in β_S). It is also an economy in which there is a smaller elasticity to move from the traditional to the modern sector.

Table 3. Values for the lagging behind scenario

Initial Values

$P_0 = 0.286$	$\sigma_{S_0} = 0.3$	$l_{S_0} = 0.92$	$G_0 = 0.5$

Modified Values

$a_s = 2$	$\beta_S = 0.001$	$\rho = 0.05$	$\varphi = 0.03$

Figure 2. Results for Scenario 1

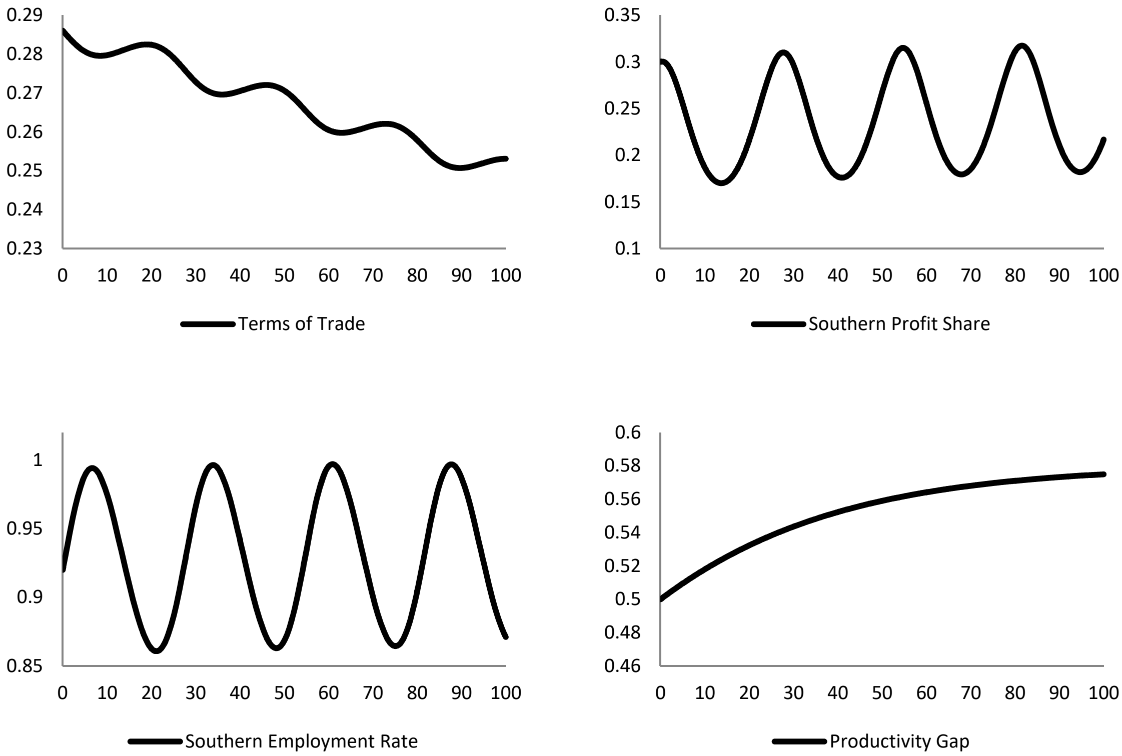


Table 4. Scenario 1: Steady State and Eigenvalues

Steady State	$P^* = 0.182$	$\sigma_S^* = 0.251$	$l_S^* = 0.930$	$G^* = 0.58$

Eigenvalues	$e_1 = -0.0003 + 0.236\text{i}$	$e_2 = -0.0003 - 0.236\text{i}$
	$e_3 = -0.0290 + 0.000\text{i}$	$e_4 = -0.0038 + 0.000\text{i}$

The structure of a less industrialized, more fragile economy, consist in the specialization on the exports of products with smaller technology intensiveness. In this economy there are smaller learning efforts and the structural change goes towards less productive sectors. The results can be seen on Figure 2. This more fragile economy reaches an equilibrium hit a higher productive gap than the baseline model.

The Prebish-Singer hypothesis is valid in this scenario. The southern, peripheral county has a decline in its terms of trade. This decline follows a cyclical adjustment toward a new equilibrium value ($P^* = 0.182$) that would be reached after many time periods. Capital accumulation and domestic growth would follow the same trajectory as the terms of trade ($\hat{Y}_S = g_S = \overline{s_S} P^{\xi}\, \sigma_S / \overline{a_S}$), showing a reduction in the growth rate.

This more fragile economy also shows the presence of higher oscillations. When calculating the standard deviation in the baseline we have that $SD_b(l_s) = 0.007$ and $SD_b(\sigma_s) = 0.008$. For scenario 1 we have $SD_{s1}(l_S) = 0.046$ and $SD_{s1}(\sigma_S) = 0.061$. We see a high increase in the amplitude of the oscillations. The steady state values are equal. However, a more specialized economy shows a higher endogenous pattern of volatility, increasing the amplitude of the oscillations.

8.3 Scenario 2: Increases in the Terms of Trade and Catching-Up.

In this scenario we emulate an economy which, starting from the baseline, focus on two main aspects: structural change and catching-up. In this scenario we increase the capital-output ratio, the exogenous productivity rate, the catching-up and the rate to which workers from the traditional sector can move to the modern one.

Initial Values

$P_0 = 0.286$	$\sigma_{S_0} = 0.3$	$l_{S_0} = 0.92$	$G_0 = 0.5$

Modified Values

$a_s = 4$	$\beta_S = 0.01$	$\rho = 0.07$	$\varphi = 0.04$

Figure 8.3. Results for Scenario 2

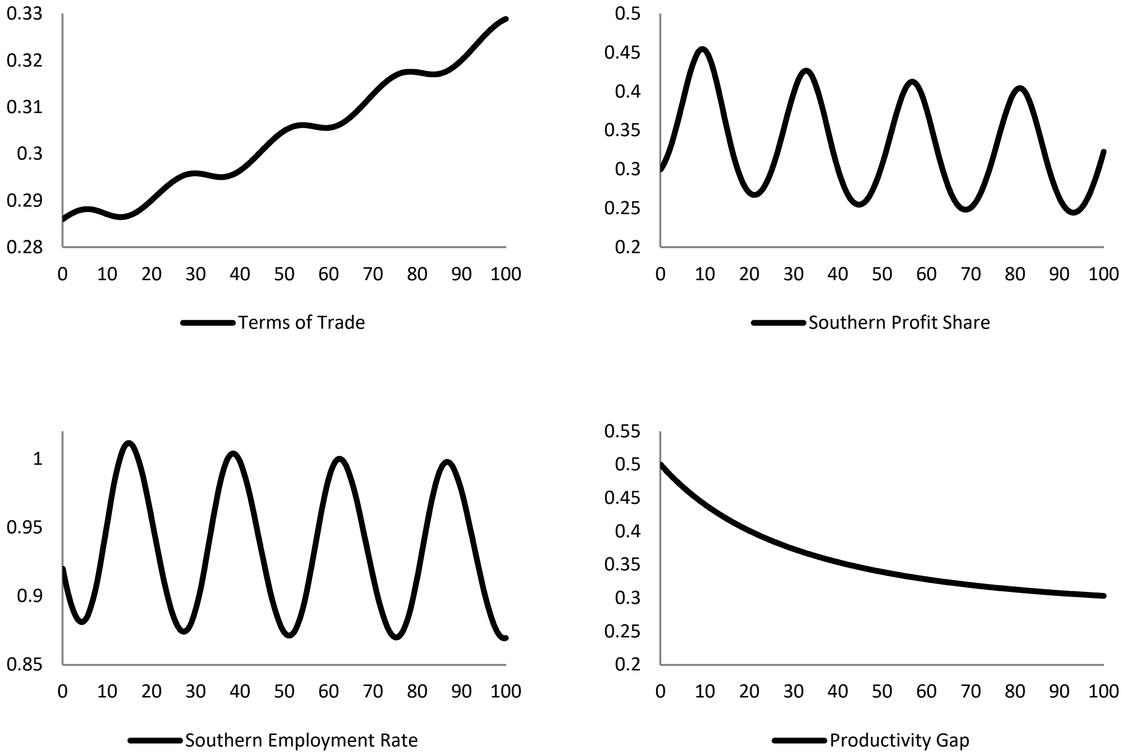


Table 5. Scenario 2: Steady State and Eigenvalues

Steady State	$P^* = 0.507$	$\sigma_S^* = 0.301$	$l_S^* = 0.930$	$G^* = 0.28$
Eigenvalues	$e_1 = -0.0003 + 0.255\text{i}$		$e_2 = -0.0003 - 0.255\text{i}$	
	$e_3 = -0.020 + 0.000\text{i}$		$e_4 = -0.003 + 0.000\text{i}$	

This scenario 2 show the opposite situation compared to the previous scenario 1. The southern economy reduces the productivity gap with the north. There is economic diversification and structural change towards more productive sector. In this scenario, the country reverses the tendency to reduce its terms of trade, and increase its growth rate, rising the terms of trade.

This growth and structural change process creates oscillations. However, the increases the amplitude of oscillations is milder when compared to the baseline, resulting in a much smaller volatility than the one compared with Scenario 1: $SD_{s2}(\sigma_S) = 0.048$ and $SD_{s2}(l_S) = 0.048$.

9 Discussion of the results

The price dynamics directly affect the behaviour of the effective economic growth, as we have: $\hat{Y}_S = g_S = \overline{s_S} P^{\xi}\, \sigma_S / \overline{a_S}$. The effective growth rate will then depend on the dynamic behavior of prices (P) (terms of trade/real exchange rate) and income distribution (σ_S). This changes the structure of the BPCM in the sense that the in this scenario economic growth is dependent on price effects. Quantity effects also play a role, defined by the behavior of income distribution. The final result defining long-run economic growth is then much more complex than the more simple result highlighted in the Thirlwall model.

The Prebisch-Singer hypothesis can be observed under some specific conditions, when $\varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S} > \varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right)$. Considering the northern characteristics as fixed, this would mean that a decrease in industrialization (reduction of a_S), a smaller in the wage share (rise in σ_S), rise in the propensity to save (s_S) and a rise in the income elasticity of imports (ε_S) – reducing productive capacity and specialization result in a pattern in which there is a trend to a decrease in the terms of trade – and on economic growth.

A southern country that does not advance with structural change and learning opportunities, and does not include workers from the traditional to the modern sector, has the tendency to follow the cyclical decline in its terms of trade. Specialization in low technology intensive sectors and a lack of learning opportunities will results in a decline in terms of trade (and economic growth). In terms of technology efforts this is an indirect effect, as the technology gap (G) does not directly affect terms of trade (P), but productivity (l_S) and income distribution (σ_S). Increases in the technology gap reduce the growth rate of the employment rate, but increases the growth rate of the profit rate.

Following the ideas of the Goodwin model, cycles emerge from the relationship between economic activity and income distribution. Oscillations are endogenous to all economies (we can check the Jacobian of the model), as we can see from the baseline model. We observe that both a catching-up and a falling behind pattern raise volatility compared to the initial stable condition, however, the falling behind scenario is much more volatile and unstable than the catching-up scenario

From the analysis of the model, an increase in volatility rises when the absolute values of $\frac{\partial \hat{\sigma}_S}{\partial l_S}$ and $\frac{\partial \hat{l}_S}{\partial \sigma_S}$ increase without changing their sign. As $\frac{\partial \hat{l}_S}{\partial \sigma_S} = -n$ and $n > 0$, when n increases we observe more intense oscillations. The same for when $\frac{\partial \hat{\sigma}_S}{\partial l_S} = P^{\xi} s_S \frac{1}{a_S} - \psi$ rises its value. In this sense, industrialization reduces volatility (rise in a_S), as well as a reduction in the elasticity between employment rate and wages (a flexible labour market rises volatility compared to a more rigid market). Increases in the autonomous propensity to save increases volatility and economic growth (rise in the terms of trade). In this sense, growth brings an increase in volatility, but it can be compensated by increases in structural change.

Higher growth happens when $P > 0$, so the south can grow at a higher rate than the north. This happens in a scenario in which a virtuous structural change, learning opportunities and higher quality in the employment are a priority in the economic development of an economy, as we can see in Scenario 2.

The results of this research are completely aligned with the structuralist perspective. We offer a perspective that deals with the critiques to the BPCM assumptions, and at the same time manages to harmonize it with the Prebisch-Singer Hypothesis and the Center-Periphery framework. The model is able to capture some macroeconomic effects that trap developing economies in underdevelopment, discussing structures, volatility, external constrains and development traps.

10 **Conclusion**

This article expands the canonical Dutt (2002) model. Amitava Dutt, in a critique to an excessive focus of the literature on the Thirlwall law, highlighting some usually neglected central aspects of the Thirlwall framework: uneven development and the transition dynamics between short- to long-run. The author endogenizes, from a North-South model, the behaviour of the terms of trade in a balance of payments constrained model (BPCM) framework.

Our paper adds to the Dutt (2002) model a Goodwin cyclical dynamics (using a Phillips curve), a Lewisian labour market transition between traditional and modern sectors, and a productivity (and technological) catching-up dynamics for the southern economy in relationship to the northern one. In this sense we extend to the BPCM a post-keynesian distributive dynamics and a structuralist-evolutionary aspect of structural change and technological catching-up.

In large part of the paper we focus on the development and the consistency of our expanded model, which results in a complex 4-dymensional dynamic system. The value range defined for our parameters show, from the Jacobian, a convergence pattern with cycles. Cycles initially emerge from the relationship between distribution (profit share) and employment rate (economic activity). This cyclical relationship affects the terms of trade dynamics, which adjusts cyclically to its equilibrium rate of growth. Productivity and technological catching-up occur when a southern country presents structural change towards more productive sectors and when there are conditions to benefit from learning opportunities. This catching-up virtuous patters has the effect of reducing endogenous volatility.

The expanded model is able to reproduce cycles with neutral stability, showing a pair of conjugate eigenvalues from the Jacobian. In this sense, we are able to reproduce cycles that oscillate around the steady state without ever reaching it. This condition represents our central search for endogenous volatility, from the moment in which cycles are reproducing in this way, our question comes to how to reduce the volatility.

From the scenarios we see that increases in industrialization and learning efforts generate a pattern with higher growth, with increases in the terms of trade, reduction in the technology catching-up and smaller volatility then compared to the falling behind scenario. A virtuous catching-up strategy raises volatility when compared to a stagnated economy, however, this rise in volatility is much smaller than the observed in the case of a falling behind scenario (when a country specializes its productive structure).

Under the Thirlwall Law, price effects are neutral. We challenge this assumption. The main question in this article concerns what happens in the Thirlwall framework when we criticize the assumption of price neutrality, offering a solution to the transition dynamics. We observe that it is possible to have structuralist/evolutionary arguments emerging from the Thirlwall framework even when the Thirlwall law is under siege. For the BPCM theorists, accepting price non-neutrality is a return to the neoclassical world in which prices adjust the model to the equilibrium. We offer an alternative to question the price assumption without entering the neoclassical world.

Growth reduces or increases in the long run depending on income distribution and on the terms of trade, both depending also on technology catching-up and on the employment rate. This relationship results in endogenous oscillatory cycles, part of the DNA of these economies. In this sense, from the Thirlwall framework we can develop a system that relates how a southern country depends on terms of trade – which depends on the quality of the structure.

Finally, In the Thirlwall law, learning and structural change are brought to the model by endogenizing income elasticities of imports and exports (Cimoli & Porcile, 2014). In our expanded version of the BPCM, we do not need to endogenize them to get the same result. The behaviour of the productive structure comes from the evolution of productivity catching-up and terms of trade. The Prebisch-singer hypothesis can be reproduced from the expanded Thirlwall framework. We offer then a proposal to conciliate the theory of decline in the terms of trade to the balance of payments constrained model when prices are not neutral. Prebisch and Thirlwall are compatible, but the Thirlwall law needs to be though in different terms for this to hold.

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Annex 1. Variable List

Y_i	Total income (f – foreign)	γ_0	Autonomous Investment parameter
K_i	Capital Stock	γ_1	Sensitivity of Investment to Capacity Utilization
C_i	Total Consumption	α	Share of northern expenditure on southern goods
S_i	Total Savings	α_0	Autonomous part of the share of northern expenditure on southern goods
I_i	Total Investment	β	Share of southern expenditure on northern goods
M_i	Total Imports	β_0	Autonomous part of the share of southern expenditure on northern goods
X_i	Total Exports	g_i	Capital accumulation rate
P_i	Prices	θ_i	Constants to the import/export functions
P	Terms of Trade	λ_i	Labour Productivity
L_i	Total Employment	G	Productivity Gap
V_i	Real Wages	l_i	Employment rate
W_i	Nominal Wages	ρ	Sensitivity of Gap growth to Gap Level (Catching-Up)
F	Financial Flows	β_i	Autonomous productivity growth
Λ_i	Total Workforce	m	Constant growth of real wages
b_i	Fixed unit labour requirement	n	Real wage sensitivity to South labour
ε_i	Income Elasticity of Imports	ξ	Exponential to the Terms of Trade
μ_i	Price Elasticity of Imports	δ	Income Elasticity Exports
a_i	Capital-Output Ratio	ν	Price Elasticity Exports
s_i	Propensity to Save	φ	intercept of the Lewis part
ω_i	Wage Share	ψ	slope of the Lewis part
σ_i	Profit Share		Subscripts:
u	Capacity Utilization	N	Northern country
z	Markup	S	Southern country
ED	Excess Demand		

Annex 2. Mathematical Appendix

$\hat{Y}_S/\hat{Y}_N = \varepsilon_N/\varepsilon_S$	(A.1)	$u = Y_N/K_N$	(A.2)	$b_S = L_S/Y_S$	(A.3)
$b_N = L_N/Y_N$	(A.4)	$a_N = K_N/Y_N$	(A.5)	$Y_S = K_S/a_S$	(A.6)
$V_S = W_S/P_S$	(A.7)	$P = P_S/P_N$	(A.8)	$\omega_S = b_S V_S$	(A.9)
$\sigma_S = (1 - b_S V_S)$	(A.10)	$s_S \sigma_S = \frac{S_S}{Y_S}$	(A.11)		

Wage Share (South): $\omega_S = \frac{W_S L_S}{P_S Y_S} \Rightarrow \omega_S = b_S V_S$

Profit Share (South): $\sigma_S = 1 - \omega_S \Rightarrow \sigma_S = 1 - b_S V_S$

Prices (North): $P_N = (1+z)\frac{W_N L_N}{Y_N} \Rightarrow P_N = (1+z) W_N b_N$

Wage Share (North): $\omega_N = \frac{W_N L_N}{P_N Y_N} \Rightarrow \omega_N = \frac{W_N L_N}{(1+z)\frac{W_N L_N}{Y_N} Y_N} \Rightarrow \omega_N = \frac{1}{(1+z)}$

Profit Share (North): $\sigma_N = 1 - \omega_N$

Investment rate (North): $g_N = I_N/K_N = \gamma_0 + \gamma_1(u)$

North expenditure in Southern goods: $\alpha = \alpha_0 Y_N^{\varepsilon_N - 1} P^{1-\mu_N}$

South expenditure in Northern goods[5]: $\beta = \beta_0 (\sigma_S Y_S)^{\varepsilon_S - 1} (1/P)^{1-\mu_S}$

Total exports of the South: $P_S X_S = \alpha[\omega_N + (1 - s_N)\sigma_N] P_N Y_N \Rightarrow$

$$\Rightarrow P_S X_S = \alpha \left\{ \frac{[1 + (1 - s_N)z]}{(1+z)} \right\} P_N Y_N \Rightarrow P_S X_S = \alpha_0 Y_N^{\varepsilon_N - 1} P^{1-\mu_N} \left\{ \frac{[1 + (1 - s_N)z]}{(1+z)} \right\} P_N Y_N \Rightarrow$$

$$\Rightarrow P_S X_S = \alpha_0 Y_N^{\varepsilon_N - 1} (P_S/P_N)^{1-\mu_N} \left\{ \frac{[1 + (1 - s_N)z]}{(1+z)} \right\} P_N Y_N$$

$$\Rightarrow P_S X_S = \alpha_0 Y_N^{\varepsilon_N - 1} (P_N/P_S)^{\mu_N - 1} \left\{ \frac{[1 + (1 - s_N)z]}{(1+z)} \right\} P_N Y_N \Rightarrow$$

[5] We corrected the notation error of the Dutt (2002) paper (P is actually 1/P).

$$\Rightarrow X_S = \alpha_0 Y_N^{\varepsilon_N - 1}(P_N/P_S)^{\mu_N - 1}\left\{\frac{[1+(1-s_N)z]}{(1+z)}\right\}(P_N/P_S)Y_N \Rightarrow$$

$$\Rightarrow X_S = Y_N^{\varepsilon_N}(P_N/P_S)^{\mu_N}\alpha_0\left\{\frac{[1+(1-s_N)z]}{(1+z)}\right\} \Rightarrow given\ \theta_S = \alpha_0[1+(1-s_N)z]/(1+z) \Rightarrow$$

$$X_S = \theta_S P^{-\mu_N} Y_N^{\varepsilon_N} \quad \text{(A.12)}$$

Total exports of the North: $P_N X_N = \beta \sigma_S P_S Y_S \Rightarrow$

$$\Rightarrow P_N X_N = \beta_0(\sigma_S Y_S)^{\varepsilon_S - 1}(1/P)^{1-\mu_S}\sigma_S P_S Y_S \Rightarrow X_N = \beta_0(\sigma_S Y_S)^{\varepsilon_S - 1}(P)^{\mu_S - 1}\sigma_S(P_S/P_N)Y_S \Rightarrow$$

$$\Rightarrow X_N = \beta_0(\sigma_S Y_S)^{\varepsilon_S}(P)^{\mu_S} \Rightarrow given\ \theta_N = \beta_0 \sigma_S^{\varepsilon_S} \Rightarrow$$

$$X_N = \theta_N (1/P)^{-\mu_S} Y_S^{\varepsilon_S} \quad \text{(A.13)}$$

Excess Demand in the South: $ED_S = C_{SS} + I_{SS} + X_S - Y_S \Rightarrow$

$$\Rightarrow Considering\ Y_S = C_{SS} + I_{SS} + M_S\ and\ M_S = \frac{X_N}{P} \Rightarrow ED_S = X_S - \left(\frac{1}{P}\right)X_N$$

Excess Demand in the North: $ED_N = C_{NN} + I_N + X_N - Y_N \Rightarrow$

$$\Rightarrow Considering\ Y_N = C_{NN} + S_N + M_S\ and\ M_N = PX_N \Rightarrow ED_N = I_N - S_N + X_N - PX_S$$

Short-run equilibrium: $ED_i = 0$

Equilibrium Value: $ED_S = X_S - \left(\frac{1}{P}\right)X_N = 0 \Rightarrow X_N = PX_S$

$$\Rightarrow as\ X_S = \theta_S P^{-\mu_N} Y_N^{\varepsilon_N}\ and\ X_N = \theta_N(1/P)^{-\mu_S}Y_S^{\varepsilon_S} \Rightarrow \theta_N(1/P)^{-\mu_S}Y_S^{\varepsilon_S} = P\theta_S P^{-\mu_N}Y_N^{\varepsilon_N} \Rightarrow$$

$$\Rightarrow (1/P)^{-\mu_S}(1/P)^{-\mu_N}(1/P) = \frac{\theta_S Y_N^{\varepsilon_N}}{\theta_N Y_S^{\varepsilon_S}} \Rightarrow P^{\mu_N + \mu_S - 1} = \frac{\theta_S Y_N^{\varepsilon_N}}{\theta_N Y_S^{\varepsilon_S}} \Rightarrow$$

$$\Rightarrow as\ Y_S = K_S/a_S\ and\ u = Y_N/K_N \Rightarrow Y_N = uK_N \Rightarrow$$

$$\Rightarrow P^{\mu_N + \mu_S - 1} = \frac{\theta_S u K_N{}^{\varepsilon_N}}{\theta_N (K_S/a_S)^{\varepsilon_S}} \Rightarrow P^{\mu_N + \mu_S - 1} = \frac{\theta_S}{\theta_N} u K_N{}^{\varepsilon_N}\left(\frac{a_S}{K_S}\right)^{\varepsilon_S} \Rightarrow$$

$$P = [(\theta_S/\theta_N)(uK_N)^{\varepsilon_N}(a_S/K_S)^{\varepsilon_S}]^{1/(\mu_N+\mu_S-1)} \tag{A.14}$$

[6] $P = [(\theta_S/\theta_N)(uK_N)^{\varepsilon_S}(K_S/a_S)^{\varepsilon_N}]^{1/(\mu_N+\mu_S-1)}$(Dutt, error)

Capacity Utilization from Investment function (North): $I_N/K_N = \gamma_0 + \gamma_1(u) \Rightarrow$

$$\Rightarrow as\, u = Y_N/K_N \Rightarrow K_N = Y_N/u \Rightarrow uI_N/Y_N = \gamma_0 + \gamma_1(u) \Rightarrow u\left(\frac{I_N}{Y_N} - \gamma_1\right) = \gamma_0 \Rightarrow$$

$$\Rightarrow u = \frac{\gamma_0}{\left(\frac{I_N}{Y_N} - \gamma_1\right)} \Rightarrow assuming\ \frac{I_N}{Y_N} = s_N\sigma_N \Rightarrow I_N = S_N\sigma_N \Rightarrow$$

$$u = \frac{\gamma_0}{(s_N\sigma_N - \gamma_1)} \tag{A.15}$$

Capital accumulation rate (North): $g_N = \gamma_0 + \gamma_1(u) \Rightarrow g_N = \gamma_0 + \gamma_1 \frac{\gamma_0}{(s_N\sigma_N - \gamma_1)} \Rightarrow$

$$g_N = \gamma_0 + \frac{\gamma_1\gamma_0}{(s_N\sigma_N - \gamma_1)} \tag{A.16}$$

Total Savings (South): $s_s\sigma_s = \frac{S_s}{Y_s} \Rightarrow S_s = s_sY_s \Rightarrow$

$$S_s = s_s\sigma_s K_S/a_S \tag{A.17}$$

$$S_s = s_s\, \sigma_s K_S/a_S \text{ (Dutt)}$$

Investment (South): $I_s = P^{\xi}S_s$

Capital accumulation rate (South): $g_S = I_S/K_S \Rightarrow g_S = \frac{P^{\xi}S_S}{K_S} \Rightarrow g_S = \frac{P^{\xi}s_S\sigma_s K_S/a_S}{K_S} \Rightarrow$

$$\Rightarrow g_S = \frac{P^{\xi}s_S\sigma_s}{a_S} \Rightarrow$$

[6] Solving the error of the Dutt (2002) paper.

$$g_S = s_S P^{\xi}\, \sigma_s / a_S \tag{A.18}$$

Terms of trade variation: $p = \frac{d \ln P}{dt} \Rightarrow P = [(\theta_S/\theta_N)(uK_N)^{\varepsilon_N}(a_S/K_S)^{\varepsilon_S}]^{1/(\mu_N+\mu_S-1)} \Rightarrow$

$$P = [(\theta_S/\theta_N)(Y_N)^{\varepsilon_N}(Y_s)^{-\varepsilon_S}]^{1/(\mu_N+\mu_S-1)} \Rightarrow$$

$$\ln P = \frac{1}{(\mu_N + \mu_S - 1)}\left[\ln\left(\frac{\theta_S}{\theta_N}\right) + \varepsilon_N \ln Y_N - \varepsilon_S \ln Y_S\right] \Rightarrow \frac{d \ln P}{dt} = \frac{1}{(\mu_N + \mu_S - 1)}[\varepsilon_N g_N - \varepsilon_S g_S]$$

$$p = [1/(\mu_N + \mu_S - 1)](\varepsilon_N g_N - \varepsilon_S g_S) \tag{A.19}$$

<u>Add a productivity dynamics by endogenizing labour productivity.</u>

Labour Productivity (North): $\lambda_N = \frac{1}{b_N}$

Dynamics in the north: $\widehat{b_N} = -\beta_N, \widehat{\lambda_N} = \beta_N$ and $\widehat{W_N} = \beta_N \Rightarrow P_N = (1+z)W_N b_N \Rightarrow$

$$\Rightarrow \widehat{P_N} = \widehat{(1+z)} + \widehat{W_N} + \widehat{b_N} \Rightarrow \text{as } \widehat{(1+z)} = 0 \Rightarrow \widehat{P_N} = -\beta_N + \beta_N \Rightarrow \widehat{P_N} = 0$$

Labour Productivity (South): $\lambda_S = \frac{1}{b_S} \Rightarrow \widehat{\lambda_S} = -\widehat{b_S}$

Dynamics in the South: $\widehat{b_S} = -\beta_S - \rho G$ and $\widehat{\lambda_S} = \beta_S + \rho G$

Productivity Gap: $G = \ln\left(\frac{\lambda_N}{\lambda_S}\right) \Rightarrow G = \ln\left(\frac{b_S}{b_N}\right)$

Dynamics of the productivity Gap: $\Rightarrow \hat{G} = \widehat{b_S} - \widehat{b_N} \Rightarrow \hat{G} = -\beta_S - \rho G + \beta_N \Rightarrow$

$\hat{G} = (\beta_N - \beta_S) - \rho G$ with $\widehat{\lambda_N} = \beta_N$ and $\widehat{\lambda_S} = \beta_S + \rho G$

$$\hat{G} = (\beta_N - \beta_S) - \rho G \tag{A.20}$$

<u>Add a labour market dynamics by endogenizing real wages.</u>

Employment rate (South): $l_s = \frac{L_S}{\Lambda_S}$

Add Phillips curve to the real wage dynamics (South): $\widehat{V_S} = -m + nl_s \Rightarrow$

$$\widehat{V_S} = -m + n\left(\frac{L_S}{\Lambda_S}\right) \tag{A.21}$$

Wage Share dynamics: $\omega_S = b_S V_S \Rightarrow \widehat{\omega_S} = \widehat{b_S} + \widehat{V_S} \Rightarrow \widehat{\omega_S} = -\beta_S - \rho G - m + nl_S$

Profit Share Dynamics: $\widehat{\sigma_S} = -\widehat{\omega_S} \Rightarrow$

$$\widehat{\sigma_S} = \beta_S + \rho G + m - nl_S \tag{A.22}$$

Terms of trade dynamics

As σ_s is now endogenous, we have a new terms of trade dynamics:

$p = \hat{P} = \frac{1}{\mu_N + \mu_S - 1}(\varepsilon_N g_N - \varepsilon_S g_S) \Rightarrow$ as $g_N = \gamma_0 + \frac{\gamma_1 \gamma_0}{(s_N \sigma_N - \gamma_1)}$ and $g_S = s_S P^{\xi}\, \sigma_S / a_S \Rightarrow$

$$\Rightarrow \hat{P} = \frac{1}{\mu_N + \mu_S - 1}\left[\varepsilon_N\left(\gamma_0 + \frac{\gamma_1\gamma_0}{(s_N\sigma_N - \gamma_1)}\right) - \varepsilon_S s_S P^{\xi}\frac{\sigma_S}{a_S}\right] \Rightarrow$$

$$\hat{P} = \frac{1}{\mu_N + \mu_S - 1}\left[\varepsilon_N\gamma_0\left(1 + \frac{\gamma_1}{s_N\sigma_N - \gamma_1}\right) - \varepsilon_S\frac{s_S P^{\xi}\sigma_S}{a_S}\right] \tag{A.23}$$

$$P = P_S/P_N \;\Rightarrow\; \hat{P} = \widehat{P_S} - \widehat{P_N} \Rightarrow as\ \widehat{P_N} = 0 \Rightarrow \hat{P} = \widehat{P_S}$$

Employment rate Dynamics: $l_s = \frac{L_S}{\Lambda_S} \Rightarrow$ Let us consider $\widehat{\Lambda_S} = \varphi - \psi\sigma_S \Rightarrow \widehat{l_s} = \widehat{L_S} - \widehat{\Lambda_S} \Rightarrow$

Total employment dynamics: $b_S = \frac{L_S}{Y_S} \Rightarrow L_S = b_S Y_S \Rightarrow \widehat{L_S} = \widehat{b_S} + \widehat{Y_S} \Rightarrow$

Employment rate dynamics: $\widehat{l_s} = \widehat{L_S} + \widehat{\Lambda_S} = \widehat{b_S} + \widehat{Y_S} - \widehat{\Lambda_S}$

Growth dynamics (South): $Y_S = K_S/a_S \;\Rightarrow \widehat{Y_S} = \widehat{K_S} - \widehat{a_S} \Rightarrow$ as a_S is constant, $\widehat{a_S} = 0 \Rightarrow$

$$\widehat{Y_S} = \widehat{K_S}$$

Capital Acummulation (South): $I_S = P^{\xi} S_S \Rightarrow$

$\Rightarrow as\ S_s = s_s\sigma_s\, K_S/a_S \Rightarrow I_S = P^{\xi} s_S \sigma_S \frac{K_S}{a_S} \Rightarrow \Rightarrow as\ I_S = \widehat{K_S} K_S \Rightarrow \widehat{K_S} K_S = P^{\xi} s_S\sigma_S \frac{K_S}{a_S} \Rightarrow$

$$\widehat{K_S} = P^{\xi} s_S \sigma_S \frac{1}{a_S}$$

Employment dynamics (South): $\widehat{l_S} = \widehat{b_S} + \widehat{Y_S} + \widehat{\Lambda_S} \Rightarrow as\ \widehat{Y_S} = \widehat{K_S} \Rightarrow \widehat{L_S} = \widehat{b_S} + \widehat{K_S} - \widehat{\Lambda_S} \Rightarrow$

$$\widehat{l_S} = -\beta_S - \rho G + P^{\xi} s_S \sigma_S \frac{1}{a_S} - \varphi + \psi \sigma_S \tag{A.24}$$

Dynamic system:

$$\hat{P} = \frac{1}{\mu_N + \mu_S - 1}\left[\varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S}\right]$$

$$\widehat{\sigma_S} = \beta_S + \rho G + m - n l_S$$

$$\widehat{l_S} = -\beta_S - \rho G + P^{\xi} s_S \sigma_S \frac{1}{a_S} - \varphi + \psi \sigma_S$$

$$\hat{G} = (\beta_N - \beta_S) - \rho G$$

Steady State

Productivity Gap: $\hat{G} = 0 \Rightarrow (\beta_N - \beta_S) - \rho G = 0 \Rightarrow G^* = \frac{(\beta_N - \beta_S)}{\rho}$

Profit Share: $\widehat{\sigma_S} = 0 \Rightarrow \beta_S + \rho G + m - n l_S = 0 \Rightarrow \beta_S + \rho \frac{(\beta_N - \beta_S)}{\rho} + m - n l_S = 0 \Rightarrow$

$$n l_S = \beta_S + \rho \frac{(\beta_N - \beta_S)}{\rho} + m \Rightarrow l_S \Rightarrow \frac{1}{n}[\beta_S + (\beta_N - \beta_S) + m] \Rightarrow {l_S}^* = \frac{1}{n}(\beta_N + m)$$

Total prices: $\hat{P} = 0 \Rightarrow \frac{1}{\mu_N + \mu_S - 1}\left(\varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S}\right) = 0 \Rightarrow \varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S} = 0 \Rightarrow \varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) = \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S} \Rightarrow P^* = \left[\frac{1}{\sigma_S} \frac{a_S \varepsilon_N \gamma_0}{\varepsilon_S s_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right)\right]^{1/\xi}$

Total labour: $\widehat{l_S} = 0 \Rightarrow -\beta_S - \rho G + P^{\xi} s_S \sigma_S \frac{1}{a_S} - \varphi + \psi \sigma_S = 0 \Rightarrow$

$$\Rightarrow -\beta_S - \rho \frac{(\beta_N - \beta_S)}{\rho} + \frac{1}{\sigma_S} \frac{a_S \varepsilon_N \gamma_0}{\varepsilon_S s_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) s_S \sigma_S \frac{1}{a_S} - \varphi + \psi \sigma_S = 0 \Rightarrow$$

$\Rightarrow -\beta_N + \frac{\varepsilon_N \gamma_0}{\varepsilon_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varphi + \psi \sigma_S = 0 \Rightarrow -\beta_N + \frac{\varepsilon_N \gamma_0}{\varepsilon_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varphi = -\psi \sigma_S \Rightarrow$

$\Rightarrow \sigma_S = \frac{1}{\psi}\left[\beta_N - \frac{\varepsilon_N \gamma_0}{\varepsilon_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) + \varphi\right] \Rightarrow \Rightarrow \sigma_S^* = \frac{1}{\psi}\left[\varphi + \beta_N - \frac{\varepsilon_N \gamma_0}{\varepsilon_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right)\right]$

$$\text{ToT:}P^* = \left[\frac{\gamma_0 a_S \psi}{\left[\varphi+\beta_N-\frac{\varepsilon_N\gamma_0}{\varepsilon_S}\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)\right]}\frac{\varepsilon_N}{\varepsilon_S s_S}\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)\right]^{1/\xi}$$

Steady State

$$P^* = \left[\frac{1}{\sigma_S}\frac{a_S\varepsilon_N\gamma_0}{\varepsilon_S s_S}\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)\right]^{1/\xi} \tag{A.25}$$

$$l_S{}^* = \frac{1}{n}(\beta_N + m) \tag{A.26}$$

$$\sigma_S^* = \frac{1}{\psi}\left[\varphi+\beta_N-\frac{\varepsilon_N\gamma_0}{\varepsilon_S}\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)\right] \tag{A.27}$$

$$G^* = \frac{(\beta_N-\beta_S)}{\rho} \tag{A.28}$$

Calculating the partial derivatives:

$$\frac{\partial\hat{P}}{\partial P} = \left\{\frac{1}{\mu_N+\mu_S-1}\left[\varepsilon_N\gamma_0\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)-\varepsilon_S\frac{s_S P^{\xi}\sigma_S}{a_S}\right]\right\}' = -\frac{1}{\mu_N+\mu_S-1}\frac{\varepsilon_S s_S\sigma_S}{a_S}\xi P^{\xi-1}$$

$$\frac{\partial\hat{P}}{\partial\sigma_S} = -\frac{1}{\mu_N+\mu_S-1}\left(\varepsilon_S\frac{s_S P^{\xi}}{a_S}\right)$$

$$\frac{\partial\hat{P}}{\partial l_S} = 0; \frac{\partial\hat{P}}{\partial G} = 0; \frac{\partial\widehat{\sigma_S}}{\partial P} = 0; \frac{\partial\widehat{\sigma_S}}{\partial\sigma_S} = 0; \frac{\partial\widehat{\sigma_S}}{\partial l_S} = -n; \frac{\partial\widehat{\sigma_S}}{\partial G} = \rho$$

$$\frac{\partial\widehat{\sigma_S}}{\partial P} = \xi P^{\xi-1}s_S\sigma_S\frac{1}{a_S}$$

$$\frac{\partial\widehat{\sigma_S}}{\partial\sigma_S} = P^{\xi}s_S\frac{1}{a_S}-\psi; \frac{\partial\widehat{\sigma_S}}{\partial l_S} = 0; \frac{\partial\widehat{\sigma_S}}{\partial G} = -\rho; \frac{\partial\widehat{\sigma_S}}{\partial P} = 0; \frac{\partial\widehat{\sigma_S}}{\partial\sigma_S} = 0; \frac{\partial\widehat{\sigma_S}}{\partial l_S} = 0; \frac{\partial\widehat{\sigma_S}}{\partial G} = -\rho$$

Partial derivatives

	∂P	$\partial\sigma_S$	∂L_S	∂G
$\partial\hat{P}$	$-\frac{1}{\mu_N+\mu_S-1}\frac{\varepsilon_S s_S\sigma_S}{a_S}\xi P^{\xi-1}$	$-\frac{1}{\mu_N+\mu_S-1}\frac{\varepsilon_S s_S P^{\xi}}{a_S}$	0	0
$\partial\widehat{\sigma_S}$	0	0	$-n$	ρ
$\partial\widehat{l_S}$	$\xi P^{\xi-1}s_S\sigma_S\frac{1}{a_S}$	$P^{\xi}s_S\frac{1}{a_S}-\psi$	0	$-\rho$
$\partial\hat{G}$	0	0	0	$-\rho$

Signs of the Main equations of the model

$$\textbf{(1)}\ \hat{P} = \frac{1}{\mu_N+\mu_S-1}\left[\varepsilon_N\gamma_0\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)-\varepsilon_S\frac{s_S}{a_S}P^{\xi}\sigma_S\right]$$

1) Sign: Price Elast: $\frac{1}{\mu_N+\mu_S-1}$

As $\mu_N + \mu_S < 1$, we have that the price elasticity sign is higher than zero (+)

2) Sign: Capital Accumulation (North): $g_N = \gamma_0\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)$

According to Dutt, $s_N\sigma_N > \gamma_1$ should have a positive

For a meaningful equilibrium value of u we require $s_N\sigma_N > \gamma_1$ which is the standard condition in quantity adjustment models requiring that the responsiveness of saving to changes in output exceeds the responsiveness of investment for stability of output adjustment

$0 < \gamma_0 < 1$ $\gamma_1 \approx 1$ $0 < \sigma_N < 1$ $0 < s_n < 1$

3) Sign: Income Elastic of Exports (North): ε_N

$0 < \varepsilon_N < 1$ or $\varepsilon_N > 1$

Positive Sign

4) Sign: Product $\varepsilon_N g_N$

$0 < \varepsilon_N < 1$ or $\varepsilon_N > 1$

Positive Sign

g_N depends on point 2

5) Sign: Income Elastic of Exports (South): ε_S

$0 < \varepsilon_s < 1$

6) Sign: ratio $\frac{s_S}{a_S}$

$0 < s_s < 1$ $3 < a_s < 5$

7) Sign: Difference $\varepsilon_N\gamma_0\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)-\varepsilon_S\frac{s_S}{a_S}P^{\xi}\sigma_S$

8) Sign: $\frac{1}{\mu_N+\mu_S-1}\left[\varepsilon_N\gamma_0\left(1+\frac{\gamma_1}{s_N\sigma_N-\gamma_1}\right)-\varepsilon_S\frac{s_S}{a_S}P^{\xi}\sigma_S\right]$

Income Elastic of Exports (South): ε_S

$$\textbf{(2)}\ \widehat{\sigma_S} = \beta_S + \rho G + m - n\frac{L_S}{\Lambda_S} = \beta_S + m + \rho G - n\frac{L_S}{\Lambda_S}$$

9) Sign: $\beta_S + m$

$0 < \beta_S < \beta_N < 0.1$ $0 < m \leq 0.1$

10) Sign: ρ

$$0 < \rho < 1$$

11) Sign: $-\frac{n}{\Lambda_S}$

$0 < n < 0.1$ $\qquad$ $0 < \Lambda_S < 3$

12) Sign: $\beta_S + m + \rho G - n\frac{L_S}{\Lambda_S}$

Points 9, 10 and 11

$$\textbf{(3)}\ \widehat{\boldsymbol{L_S}} = -\boldsymbol{\beta_S} - \boldsymbol{\rho G} + \boldsymbol{P^{\xi}\sigma_S\frac{s_S}{a_S}}$$

13) Sign: $-\beta_S$

$$0 < \beta_S < \beta_N < 0.1$$

14) Sign: $-\rho$

$$0 < \rho < 1$$

15) Sign: $\frac{s_S}{a_S}$

$0 < s_s < 1$ $\qquad$ $3 < a_s < 5$

16) Sign: $-\beta_S - \rho G + P^{\xi}\sigma_S\frac{s_S}{a_S}$

$$\textbf{(4)}\ \widehat{\boldsymbol{G}} = (\boldsymbol{\beta_N} - \boldsymbol{\beta_S}) - \boldsymbol{\rho G}$$

17) Sign: $(\beta_N - \beta_S)$

$$0 < \beta_S < \beta_N < 0.1$$

18) Sign: $-\rho$

$$0 < \rho < 1$$

19) Sign: $(\beta_N - \beta_S) - \rho G$

Depends on points 17) and 18).

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