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The La Marca Model Revisited: Structuralist Goodwin Cycles with Evolutionary Supply Side and Balance of Payments Constraints

Danilo Spinola*

Abstract

This research investigates the causes of endogenous volatility in Latin America by expanding the La Marca (2010) model. The expansion consists in study: (I) Price-neutrality, (II) stability of the external sector, and (III) fixed income distribution. We also add (IV) an evolutionary supply-side in which productivity is at the centre of the economic dynamic through international technology transfer and the Kaldor-Verdoorn effect. The results show that (1) Latin American parameter values increase the endogenous oscillatory adjustment. (2) In all cases the model converges. (3) The price-neutrality assumption and external sector stability depend on specific parameter values to show either a cyclical or a monotonic convergence pattern. (4) Fixed income distribution lead to a monotonic trajectory, reducing oscillations. (5) The inclusion of the productivity dynamics generates new sources of volatility in the relationship between productivity, capacity utilization, and net external assets.

Keywords: Economic Cycles, Structuralism, Macroeconomic Dynamics.

JEL: E32; F44; O11; O30

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1. Introduction

The recurrence of a boom and bust dynamic in key economic variables, such as GDP growth, is a persistent problem for many economies, especially developing countries (Koren & Tenreyro, 2007). This volatility has strong impacts in the economic structures, raising uncertainty, fostering productive specialization, and increasing the fragility of the economic system (Lavopa & Szirmai, 2014). Stylized facts show that economic volatility has an important regular component, in which the literature explains using (I) concept of growth episodes (Szirmai, 2008) and (II) cycle theories (Korotayev & Tsirel, 2010).

The study of Cliometrics is classical in the economic theory. Despite its long tradition, the existence and determinants of cycles on many economic variables are still open to the academic debate. Econometric evidence is substantial and points to the existence of these cycles in key macroeconomic variables (Korotayev & Tsirel, 2010). The debate surrounding growth cycles is especially important for the group of countries in which volatility is in overall higher, which includes most part of low- and medium- income countries. In poor countries, we observe the repetition of short periods of growth succeeded by strong crisis and followed by adjustment periods that <u>weaken the structure of the economy</u> (Foster-McGregor et al., 2015).

There are many economic traditions focused on discussing, modelling and explaining economic cycles. Business cycle theory has always been a hot topic in economics. Schumpeter (1939) with the technology cycle theory, and Goodwin (1967), with the growth and distribution cycle model, proposed to answer these questions using their own theoretical framework. Since the 1980's, real business cycle theorists have also been leading the discussion about cycles in a neoclassical perspective (Hartley, Hoover, & Salyer, 1998).

In the Structuralist theory, the role of economic structures is the main underlying aspect that defines the behaviour of an economic system (Prebisch, 1950). Productive structure, labour markets, external sector, and institutions define the real economic and development possibilities of a country (Cimoli & Porcile, 2014). The presence of weak economic structures is a central problem in the Latin American Structuralist tradition (Taylor, 1983). This weakness leads to the emergence of high amplitude short-growth cycles, and inherent instability. Latin America is a continent in which macroeconomic volatility has been a constant issue, but few efforts were made to model it endogenously. For this reason, there is still a, open gap to model the emergence of an endogenous[1] cyclical dynamics for Latin America in a centre-periphery framework.

This paper follows the growth model tradition started by Harrod and Goodwin. Goodwin (1967) designed a model in which the structure behind the economy defines a pattern in which growth and distribution runs in a cyclical predator-prey dynamics. This pattern was later developed in a Structuralist version by Barbosa-Filho & Taylor (2006). Finally, La Marca (2010) gave a further contribution to the model, combining the Structuralist Goodwin model with a stock-flow framework by Foley & Taylor (2004). The La Marca model consists in a three dimensional dynamic system in which external sector dynamics is added to the growth and distributional interaction through accumulation of foreign assets/liabilities. This specification is able to produce dampened cycles in the trajectory to the steady state. The La Marca (2010) model is originally thought to address fast-growing export-led economies such as the ones in East Asia. As our objective is to study Latin American economies, we apply to this model some adjustments and expansions.

The La Marca model cares for its internal consistency. It merges the Goodwin cyclical dynamics with the Balance of Payments Constrains Model (BPCM), aka Thirlwall model (Thirlwall, 1979), starting from a Stock-Flow consistent framework. It is an interesting and complex model despite the fact that no other work in the literature has until now proposed to explore it further, opening a research methodological gap. In this paper we propose to analyse and expand the La Marca model, observing the growth-cyclical pattern of countries who find themselves in the middle income trap (Lavopa & Szirmai, 2014). The main focus is on Latin America. The flexibility of this model, allowing us to relate economic cycles, external sector and structural elements in an oligopolistic economy - in which income distribution plays a central role - justifies our selection for this model. The La Marca (2010) model can be easily related to the Structuralist framework, raising important insights to debate about the structural elements/failures behind developing countries' high economic volatility.

[1] The idea behind deterministic endogenous cycles is that the trajectory of the dynamic system oscillates indefinitely, neither converging to the steady state nor having an explosive trajectory. Cycles emerge as a central characteristic of the economic structure, and not by the presence of exogenous shocks (such as in the Real Business Cycle theory).

In the La Marca (2010) model, economic actors interact in an open market economy. The dynamic behaviour and interaction between capacity utilization, income distribution (wages/profits), external sector (current account), productivity and nominal exchange rate defines the trajectory and the equilibrium conditions of the system. Under specific conditions, we test if the model can generate closed orbits, reproducing a Lotka-Volterra cyclical mechanism. The proposed model consists in a Structuralist Goodwin model with an evolutionary supply side and endogenous nominal exchange rate with Balance of Payments constrains.

In order to further study this model, we impose some assumptions based on the main discussion of the Balance of Payments Constrained Model (BPCM), aka Thirlwall Model. This way we expand the La Marca (2010) model in the directions stated above. The study goes in the following direction:

I) <u>Study the structure of the La Marca original system:</u>

1) Impose the classic assumption of price-neutrality found on the BPCM (Thirlwall, 1979), defining the transition dynamics between short- to long-run. In this case the real exchange rate is constant, growing at a zero rate.
2) Zero growth rate in the current account, avoiding the existence of structural deficits, another central assumption of the BPCM (Blecker, 2016).
3) Assume the case in which income distribution is constant. Profit share follows a fixed mark-up level, and the wage share is fixed. In this sense, we analyse the relationship between the evolution of the external account, and capacity utilization.

II) <u>Model a Supply Side structure</u>

4) Define *productivity dynamics* by removing the assumption of constant technology, adding the Kaldor-Verdoorn effect (Kaldor, 1975). Through learning processes, investments lead to quantitative improvements in the productive structure, raising productivity.
5) Consider the world North-South technology gap dynamics in a centre-periphery framework. The North is technologically dynamic while the south lags behind. Firms with external assets have more learning opportunities, adopting technology from abroad, which raises their domestic productivity.

Our La Marca (2010) model expansion observes the interaction between exchange rate, wage share, output growth, capacity accumulation, net foreign asset accumulation, productivity, balance of payments constrains, and technology transfer.

In terms of research gap, we raise the question: how do cycles emerge from a dynamic endogenous pattern between productivity and foreign sector? We observe the absence of models that reproduce endogenous cycles (deterministic) for Latin American economies. This is a topic frequently discussed in the theory, and a matter of many econometric studies (Erten & Ocampo, 2013). However, the literature does not offer well formalized models to address this specific topic. In this sense **this paper aims at contributing to the Structuralist theory by studying and expanding a formal framework in search for deterministic cycles on the La Marca model, emerging from the external sector and the productive structure**.

2. Literature Review:

2.1. Demand side cycles: Richard Goodwin and the Lotka Volterra Cycles

The Lotka Volterra (LV) is a specific type of dynamic model in which its peculiar specification results in the formation of closed orbit solutions. In two dimensions this model creates a Predator-Prey dynamics, very much used in Biology and Ecological studies. It follows a specification in which one variable is the predator and the other is the prey. Both interact generating a cyclical dynamic. The LV model can be generalized for more than two dimensions (Kolgomorov model) (Gandolfo, 1971). In this case it is possible to have deterministic cycles that fluctuate around the long-run equilibria (steady state).

The use of the Lotka-Volterra system in economics can be traced back to the work of Goodwin (1967), who established a model in which economic activity and income distribution interact dynamically. The Goodwin model creates endogenous cycle in the relationship between wage share (predator) and employment (prey), reproducing a predator-prey dynamics in a closed saving-determined growing economy. This framework gave origin to a large amount of models in which economic activity (effective demand) and income distribution interact. The original model, thought for the US economy, finds profit squeeze cycles slightly damped and repetitive.

The Structuralist tradition, which accounts for the role of structural elements underlying the economic system, has made efforts to expand the Goodwin model. This tradition developed itself from Raul Prebisch, Celso Furtado and the whole Latin American tradition of thinkers that put structural condition in a centre-periphery framework in the centre of the analysis. Since the 1980's, Lance Taylor has formalized many of the concepts in the Structuralist theory (Taylor, 1983). In an attempt to expand the Goodwin Model with Structuralist features, Barbosa-Filho & Taylor (2006) developed a dynamic system relationship between wage share and capacity utilization in a demand-driven economy. This model is based on the classical works of Kalecki (1971) and Steindl (1952) and puts the distributive conflict in the centre of the analysis. The Structuralist Goodwin was also computed to the US economy, describing its *profit led*[2] characteristics.

More recently, La Marca (2010) developed a model that merges the Structuralist Goodwin system with the Foley & Taylor (2004) model. The latter suggests that heterodox models should use social account matrices in order to derive its causal relations, being stock-flow consistent. Foley & Taylor (2004) also works for an open economy and also adds a financial elements (equities) to the model. La Marca (2010) used then the framework developed by Foley & Taylor (2004) to developed an open economy version of the Goodwin model. Economic growth and fluctuations in output, capacity utilization, distribution and real exchange rate interact in an open economy generating damped cycles. La Marca (2010) extends the Structuralist Goodwin to an open economy, which results in a more complex structure to the Goodwin model, expanded it to three dimensions.

2.2. Supply-side cycles: productivity, Schumpeterian cycles, structural change and catching up.

Productivity and technological change are important aspects usually neglected in the Goodwin tradition. The Goodwin (1967) model traces back to the relationship between income distribution and economic activity, but there is a passive role of the supply-side. It assumes a Phillips curve to define the relationship between employment and real wages, and a Leontief production function, with fixed coefficients. Demand determines the adjustments on employment and

[2] A pro-profit distribution has net positive effects on investment and growth.

economic activity through the income distribution effects. The role of the supply side is ignored. Nonetheless, there are many supply side theories that address economic cycles. Some are presented here in this section, being part of the theoretical foundations of the La Marca (2010) model expansion.

The motivation behind comparing and merging the demand- and supply-side theories of economic cycles goes hand in hand with the recent efforts to reconcile the demand-led Keynesian macroeconomic framework with the Schumpeterian evolutionary microeconomic supply-side theory. This research follows in this sense the “Keynes meets Schumpeter” tradition (Dosi, Fagiolo, & Roventini, 2010).

In the mainstream growth theories, there is a central role for productivity (Total Factor Productivity) in a supply-side driven economic system. The Total Factor Productivity is associated to technological change. This is the idea behind real business cycles that considers technological shocks as exogenous, in which the system itself reacts readjusting itself towards the equilibrium point (Plosser, 1989). Business cycles are created by stochastic shocks that converge monotonically (or with damped cycles) to a long run equilibrium trajectory. There are no endogenous deterministic persistent cycles in the model. The system dynamics gets a cyclical behaviour only from these stochastic shocks.

The evolutionary tradition offers a supply-side perspective focused on the presence of deterministic endogenous cycles. Schumpeter (1939) gave an explanation to the existence of endogenous business cycles intrinsic to the economic structure. The central argument is focused on the role of technology. It changes the industrial paradigm through an endogenous innovative mechanism inherent of the capitalist system. This leads to a constant need of the economic structures to adjust to its new conditions because of its own competitive characteristics (Nelson & Winter, 1977). The constant flow of innovation (some that are successful and others not) gives rise to a cyclical behaviour (Silverberg & Verspagen, 1995). This competitive process constantly changes the whole characteristics of the economic system, resulting in big productivity change. The Neo-Schumpeterian models usually deal with individual or sectoral innovations and are currently using tools such as the Agent-Based models – e.g. Ciarli et al (2010), Gaffeo et al (2008). The cyclical aggregate behaviour in this framework results from the interaction of individual heterogeneous agent's behaviour – following a complex dynamic with many non-linearities. This is an interesting way to deal with cycles, but the complexity involved in such simulation models turn these models into overly sensitive to the parameters, being not the scope of this research.

Structural Change also plays a central role in terms of the supply side cyclical behaviour. Krugman & Venables (1995) show that multi-sector models in open economies defines the specialization patterns and fragility of an economic structure, and Koren & Tenreyro (2007) link this fragility to volatility. Each sector has a different productivity level and the reallocation of resources plays a central role in defining the productivity and competitiveness of an economy. In terms of the evolutionary theory, the discussion about the emergence of new sectors and reallocation between sectors is also a relevant source of volatility (Silverberg & Verspagen, 1995). The Structuralist Theory has some important contributions in this regard. Cimoli & Porcile (2014) develop a toolbox to link a North-South framework, Structural Change, Balance of Payments Constraints and Technological Gap. The Kaldor-Verdoorn effect (Kaldor, 1975) is a central concept in this discussion, linking economic activity and productivity. The economy is externally constrained and has its fragility patter related to how it absorbs external shocks from terms of trade.

The Cimoli & Porcile (2014) model offers a theoretical basis to understand the supply side constrains to Latin America. This model, however, does not deal with cycles, opening a theoretical gap in the new-structuralist theory to be addressed in this research.

Finally, we discuss the international technology transfer leading a catching up process (Verspagen, 1991). Countries lagged technologically have in principle more potential to catch-up to the technology frontier. Nevertheless the learning process is not natural. Lagged countries need to build a certain level of domestic capabilities that allow them to learn from abroad. This way, they are only able to reach a virtuous catch-up process after creating some baseline conditions to learn and develop.

There are many reconciliation challenges to the academic research in defining endogenous model for middle income countries. In this paper we address the following topics:

I. Integrate a productivity dynamics to the Goodwin model through (a) adding a Kaldor-Verdoorn effect, and (b) Modelling firms' learning in an international environment.

II. Expand the open Structuralist Goodwin Model in a two-sector model following Krugman & Venables (1995) (future work based on the Dutt (2002) model).

These models focus on the role of the supply-side in generating the cyclical dynamics in an open economy in which growth and income distribution dynamically interact.

2.3.Critiques to the Thirlwall model

Another important source of volatility is related to the role of the external sector in the economic system. In this sense the nominal exchange rate plays a central role. For this discussion we bring the Thirlwall framework of the Balance of Payments Constrained Model (BPCM).

The BPCM, which is also known as the Thirlwall model, offers reconciliation between the supply-side and the demand-side perspectives. The Thirlwall (1979) model is a demand-led one as it ultimately depends on the current account to define the long-run growth possibilities (Thirlwall's law). On the other hand, as the dynamic behaviour of the current account depends on income and price elasticities of imports and exports. These are directly related to the condition of the productive structure and technology. The elasticities have been endogenized many authors such as Cimoli (1988), and Cimoli & Porcile (2014).

The BPCM has two main underlying assumptions. First, it assumes that the growth rate of net foreign income is equal to zero in the long-run. In this sense, there cannot be structural explosive deficits in the external sector, and the current account is stable. Second, for the short-run, the model considers terms of trade and financial flows. In the long-run, those are assumed constant. This raises the price-neutrality assumption – the real exchange rate is constant and grows at zero rate. Considering these assumptions, the short-run model shows a stable exchange rate and a stable external position, which leads the BPCM to the Thirlwall Law (in which domestic growth adjust to the income elasticity ratio).

There is a more radical version of the BPCM called the Balance of Payments Dominance theory (Ocampo, 2011). In this version Latin American countries have their economic growth adjusted by the external sector conditions even in the short-run. The nominal exchange rate adjusts to the level that turns net exports equal to zero (so growth of exports and imports can be the same after that).

From the Structuralist theory, financial flows and terms of trade are important elements that create volatility. We aim to reconcile this with the La Marca (2010) model adding these elements to the system. We use a flexible nominal exchange rate that adjusts itself in order to guarantee that the growth of imports and exports grows at the same rate (zero growth in the current account for the long-run). In this way the real exchange rate would have a constant moving equation. This aspect is further explained when discussing the expansions of the model.

3. The original model

The La Marca (2010) model is derived by a Social Accounting matrix with a stock-flow consistent set of accounts. The model implicitly starts from a Goodwin production function (Leontief). It is a demand-led model in which aggregate demand and income distribution plays a central role. The La Marca (2010) model has the following assumptions (mathematics in the annex):

1) There are four sectors: households, firms, government and external sector (ROW). Firms consist of a productive sector including industrial enterprises and the domestic financial sector. Firms can invest abroad with portfolio investments, loans, FDI or liquid assets (deposits and any kind of foreign currency reserves). Their net liabilities are denominated in foreign currency. These firms finance new investments through retained earnings and issuing new equities. They can pay back loans and equities. The central point is the transaction with the foreign sector, in which misalignments between national expenditure and income generate and increase or reduction in net foreign assets.
2) Government debt is negligible. The model abstracts from monetary policies.
3) There are three types of assets: productive capital, equities and net foreign assets (net foreign liabilities).

4) There are specific characteristics of the labour- and product- market equilibrium, the determinants of savings, investment and current account.
5) Labour discipline defines the real wage setting (Bowles & Boyer, 1988).
6) There is a Keynesian Investment function that is autonomous from savings.
7) External sector is fundamental to define investment and demand expansion. There is a low substitutability between foreign and domestic investment.
8) There is a non-linear dynamics between real exchange rate and capacity utilization, and between distribution and net asset position.
9) Emerging economies have assets and liabilities denominated in foreign currency.
10) Consumers and investors do not seem to borrow against future income and retained earnings are a crucial source of finance for firms.
11) This is a pure demand-led model.
12) Distribution: interaction between workers, firms and government with trait of confliction. Conflictive natures lead to emergency of cycles. Kaleckian and Kaldorian tradition in Wage/Profit led economies (Bhaduri & Marglin, 1990). Real exchange rate here becomes a distributive variable.
13) Cycles: non-clearing labour market, distributional conflict, non-marginal productivity pricing.

The basic blocks of the model are described in the annex and in the original La Marca (2010) paper. The next subsections present the dynamic aspects of the model, which consists in a relationship between a distribution wage share motion equation ($\dot{\psi}$), an economic activity capacity utilization motion equation ($\dot{u}$), and an external sector adjustment external asset accumulation equation ($\dot{b}$).

3.1.Wage Share distributive equation

Output (X) in this model is divided in profit share π, wage share ψ, and the share of imported intermediate inputs (a) in domestic currency – conversion done using the real exchange rate (ξ). In this sense, the sum of the shares is equal to one. $\psi + \pi + \xi a = 1$.

The motion equation of the Wage Share ($\dot{\psi}$) has the following specification (derivation in the annex):

$$\dot{\psi} = \tau(\psi^* - \psi) \tag{1}$$

In eq. (1) τ represents the speed of adjustment between the equilibrium value of the wage share (ψ^*) and the effective wage share (ψ). The model assumes a linear adjustment process to the equilibrium point. The equilibrium value of the wage share is defined by a labor discipline real wage (Bowles & Boyer, 1988) the motion equation of the wage share result as (more information in the annex):

$$\dot{\psi} = \tau[l \exp(1 + ulk) - \psi] \tag{2}$$

In which $l = L/X$, being l the fixed amount of effective labor (L) per unit of product (X). $u = X/K$, u is the output (X) to capital (K) ratio, used as an index of capacity utilization. $k = K/N$ being k a constant for the relationship between capital (K) and employable working population (N).

The equilibrium value of the wage share follows a Phillips curve, in which employment rate h has a positive relationship to wages. $h = \frac{ulk}{\varepsilon}$ in which ε is the degree of effort exerted by workers. As mentioned, the equilibrium value comes from a labor discipline real wage Phillips curve theory. It links the employment and the capacity utilization rate consistent with a labour market equilibrium wage share. The profit share and the share of intermediate inputs adjust to the wage share in the following way: $\xi = \frac{1-\psi}{a\left(1+\frac{1}{\eta}\right)}$ and $\pi = \frac{1-\psi}{\eta\left(1+\frac{1}{\eta}\right)}$ in which η is the price elasticity of domestic output in world market.

3.2. Capacity utilization equation

The capacity utilization moving equation adjusts the goods market through the identity between domestic investment (g), domestic savings (σ) and net foreign investments (z) – derivation in the annex.

$$\dot{u} = \lambda(g + z - \sigma) \quad (3)$$

In this eq. (3) λ is the speed of adjustment. g is the domestic investment. It follows a Keynesian investment function in which $g = \alpha\pi u + \gamma$. α is the sensitivity of investment to profitability and γ in the exogenous investment component that represent the "animal spirit" of the capitalists. The variable z is the sum of all the components of the current account that depend on the exchange rate and σ is the total national savings. The values of z and σ are the exchange rate sensitive elements of the external sector and the total savings, respectively[3]. A substitution of the variables that define the values of g, z and s in eq. (3), as originally in La Marca (2010) results in the following equation:

$$\dot{u} = \lambda\left[\left(\alpha - s_p\right)\pi - s_h\psi - \xi a\right]u + \gamma + \xi^{\eta}x + \left(1 - s_p\right)j\xi b \quad (4)$$

In this equation s_h is the propensity to save of households and s_p is the aggregated propensity to save. x is the export-capital ratio, j is the interest rate and $b = B/K$ is the real value of foreign assets (B) per unit of capital (K). Capacity utilization evolves consistently with the level of savings, investment, interest payments and net exports. Capacity utilization then adjusts itself to the value that balances the product market.

3.3.External asset equation

The third dynamic equation relates to the movements in the net external position (b). It relates the internal and external changes in net asset accumulation. It comes from the relationship between current account surplus and an increase of claims on the foreign sector. The relationship can be described as follows (more in the annex):

$$\dot{b} = \frac{1}{\xi}(\sigma - g) - gb \quad (5)$$

[3] We have $\sigma = s_h[(1 - s_b)(\pi u + j\xi b) + \psi u] - \upsilon(\pi u + j\xi b) + s_b(\pi u + j\xi b)$ and $z = \xi^{\eta}x + j\xi b - \xi a u$. In which υ is the propensity to consume out of capital gains.

In this sense the growth (reduction) of net assets depends positively on internal savings but negatively on the internal and external investments. When substituting σ and g we end up with:

$$\dot{b} = \frac{(s_p - \alpha)\pi u + s_h \psi u - \gamma}{\xi} - (g - s_p j)b \tag{6}$$

The three motion equations (2), (4) and (6) form a system of dynamic equations, basis of the La Marca model. The trajectory and stability conditions of the system depend then on the assumptions regarding the parameter values. La Marca extensively discuss each of the values one by one. The stability conditions are possible to observe through an analysis of the Jacobian (J) of the system in its steady state.

$$J = \begin{bmatrix} \partial\dot{\psi}/\partial\psi & \partial\dot{\psi}/\partial u & 0 \\ \partial\dot{u}/\partial\psi & \partial\dot{u}/\partial u & \partial\dot{u}/\partial b \\ \partial\dot{b}/\partial\psi & \partial\dot{b}/\partial u & \partial\dot{b}/\partial b \end{bmatrix} \tag{7}$$

The motion equation of the wage share does not depend of the net external assets, so $\partial\dot{\psi}/\partial b$. A stable system must negative eigenvalues in their real part.

3.4. Consolidated model, Steady State and Jacobian.

As above discussed, the La Marca model consists in a system of three dynamic equations. It has the following system:

$$\dot{\psi} = \tau[(l \exp(1 + ulk) - \psi)]$$

$$\dot{u} = \lambda\{[(\alpha - s_p)\pi - s_h\psi - \xi a]u + \gamma + \xi^{\eta}x + (1 - s_p)j\xi b\}$$

$$\dot{b} = \frac{(s_p - \alpha)\pi u + s_h \psi u - \gamma}{\xi} - (g - s_p j)b$$

We calculate then the steady state and the jacobian to understand the structural characteristics of the model.

In the steady state we have that $\dot{\psi} = \dot{u} = \dot{b} = 0$. In this sense, the steady state conditions are:

$$\psi^* = l\exp(1 + u^* lk)$$

$$u^* = -\frac{\left[\gamma + \xi^{*\eta}x + \left(1 - s_p\right)j\xi^* b^*\right]}{\left[\left(\alpha - s_p\right)\pi^* - s_h\psi^* - \xi^* a\right]}$$

$$b^* = \frac{\xi^{*\eta}x - \xi^* a u^*}{g^* - j}$$

Being $\xi^* = \frac{1-\psi^*}{a\left(1+\frac{1}{\eta}\right)}$ and $\pi^* = \frac{1-\psi^*}{\eta\left(1+\frac{1}{\eta}\right)}$.

In order to calculate the Jacobian matrix (eq.7) we take the partial derivatives of each of the motion equations in relation to another (annex for the math):

$$J = \begin{bmatrix} -\tau & \tau l^2 k\psi & 0 \\ \lambda\left[\frac{\partial\xi}{\partial\psi}\left(\eta\xi^{\eta-1}x - au\right) - \left(s_p - \alpha\right)\left(\frac{\partial\pi}{\partial\psi}\right)u + \frac{\partial\xi}{\partial\psi}\left(1 - s_p\right)jb - s_h u\right] & -\lambda\left[\left(s_p - \alpha\right)\pi + s_h\psi + \xi a\right] & \lambda\left(1 - s_p\right)j\xi \\ \frac{1}{\xi}\left(\frac{\partial\sigma}{\partial\psi} - \frac{\partial g}{\partial\psi}\right) - \frac{\partial\xi}{\partial\psi}\frac{(\sigma - g)}{\xi^2} - \frac{\partial g}{\partial\psi}b & \frac{1}{\xi}\left[\frac{\partial\sigma}{\partial u} - \frac{\partial g}{\partial u}(1 + \xi b)\right] & -\left(g - s_p j\right) \end{bmatrix}$$

From the study of the variable signs done in the La Marca (2010) model we end up with the jacobian results[4], which would give us the following signs:

$$J = \begin{bmatrix} - & + & 0 \\ - & - & + \\ + & + & - \end{bmatrix}$$

This type of jacobian gives a pair of conjugate complex numbers for two eigenvalues, and a third real negative eigenvalue, showing a cyclical dampened pattern around a stable point. The result is a dynamic adjustment in which there are dampened cycles between capacity utilization and income distribution, as we can see in the simulation in the next session.

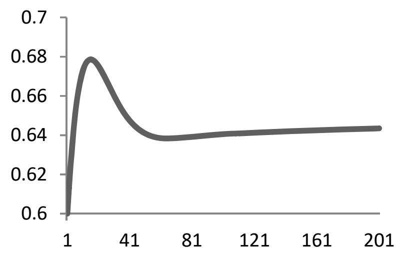
[4] In which $\frac{\partial\xi}{\partial\psi} = \left[-\frac{1}{a\left(1+\frac{1}{\eta}\right)}\right]$; $\frac{\partial\pi}{\partial\psi} = \left[-\frac{1}{\eta\left(1+\frac{1}{\eta}\right)}\right]$; $\frac{\partial\sigma}{\partial\psi} = \left[s_h u - s_p u\frac{1}{\eta\left(1+\frac{1}{\eta}\right)} - s_p jb\frac{1}{a\left(1+\frac{1}{\eta}\right)}\right]$; $\frac{\partial\sigma}{\partial u} = \left[s_p\frac{1-\psi}{\eta\left(1+\frac{1}{\eta}\right)} + s_h\psi\right]$; $\frac{\partial g}{\partial\psi} = \left[\frac{-\alpha u}{\eta\left(1+\frac{1}{\eta}\right)}\right]$

3.5.Calibrations of the La Marca (2010) model

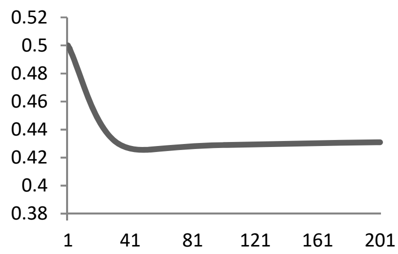
In this sub-section we see two distinct simulations taken from the original La Marca model. The first one shows the original results of the paper, in which the author calibrates the model using reasonable values. The model shows the presence of dampened cycles, as observed analysing the structure of the model. The second simulation reinforces the cyclical adjustment mechanism with higher cycles. It shows an interesting pattern in which the variables oscillate many times before going to its equilibrium value.

Figure 1. Reproduction of the La Marca (2010) original results.

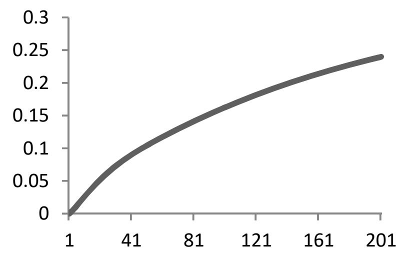
Wage share (ψ)



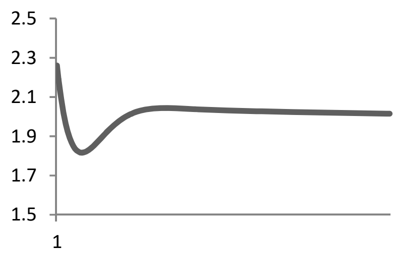
Capacity utilization (u)



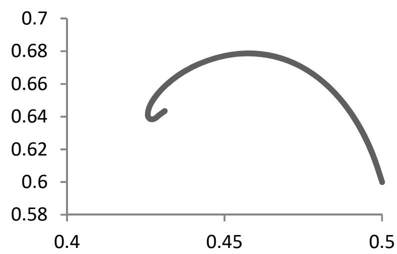
Net foreign assets (b)



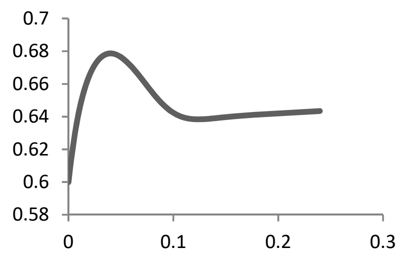
Real exchange rate (ξ)



ψ x u



ψ x b



Parameter Values: $\tau = 1$, $\lambda = 1$, $k = 20$, $l = 0.1$, $\gamma = 0.05$, $\alpha = 0.5$, $\eta = 1.3$, $a = 0.1$, $x = 0.05$, $j = 0.03$, $s_b = 0.5$, $s_h = 0.3$.
Initial conditions: $\psi_0 = 0.6$, $u_0 = 0.5$, $b_0 = 0$.
Steady state: $\psi = 0.65$, $u = 0.43$, $b = 0.34$.
Eigenvalues: $\lambda_1 = -0.71 + 0.59i$, $\lambda_2 = -0.7 - 0.59i$, $\lambda_3 = -0.06$.

$$J = \begin{bmatrix} -1 & 1.29 & 0 \\ -0.33 & -0.41 & 0.02 \\ 0.14 & 0.08 & -0.06 \end{bmatrix}$$

Figure 1. reproduces the *Figure 5* results taken from the original La Marca (2010) paper. The upper left figure shows the evolution of wage share on time. On upper middle we have the capacity utilization. The top right graph shows the net external assets. Bottom left shows the real exchange rate dynamics. On bottom middle we see the cyclical relation between growth and distribution. Finally the bottom right figure shows the relation between wage share and net financial assets.

The original model results in a small oscillation dampened cycle configuration. The Jacobian of the dynamic model under these assumptions results with a negative trace and positive determinant – which results in a stable dynamics. Moreover, the eigenvalues of the Jacobian consists in a pair of conjugate of complex numbers. This results in the generation of cycles. The final result of the model is the emergence of a dampened cycle trajectory with a small number of oscillations.

The adjustment process can be understood in the following terms (La Marca, 2010):

<u>Stagflationary phase</u>: An initial shock displays the variables from its equilibrium value (e.g. fiscal contraction). There is an excess supply which results in "forced exports". Output and capacity utilization fall to balance supply and demand (domestic plus foreign) at the current real exchange rate. Wage share grows, squeezing profits and appreciating the real exchange rate. The economy slows down and domestic prices increases relative to foreign prices. Competitive exports and net assets revenue reduces, which lead demand to reduce more than supply. Employment then reduces more than its equilibrium value – starting a reversal of the wage dynamics.

<u>Stagnationary phase:</u> Prices slow down, the real exchange rate depreciates and profits and competitive exports start recovering. Output and wage contraction balances production and demand. Further wage reduction brings the economy to a recovery phase.

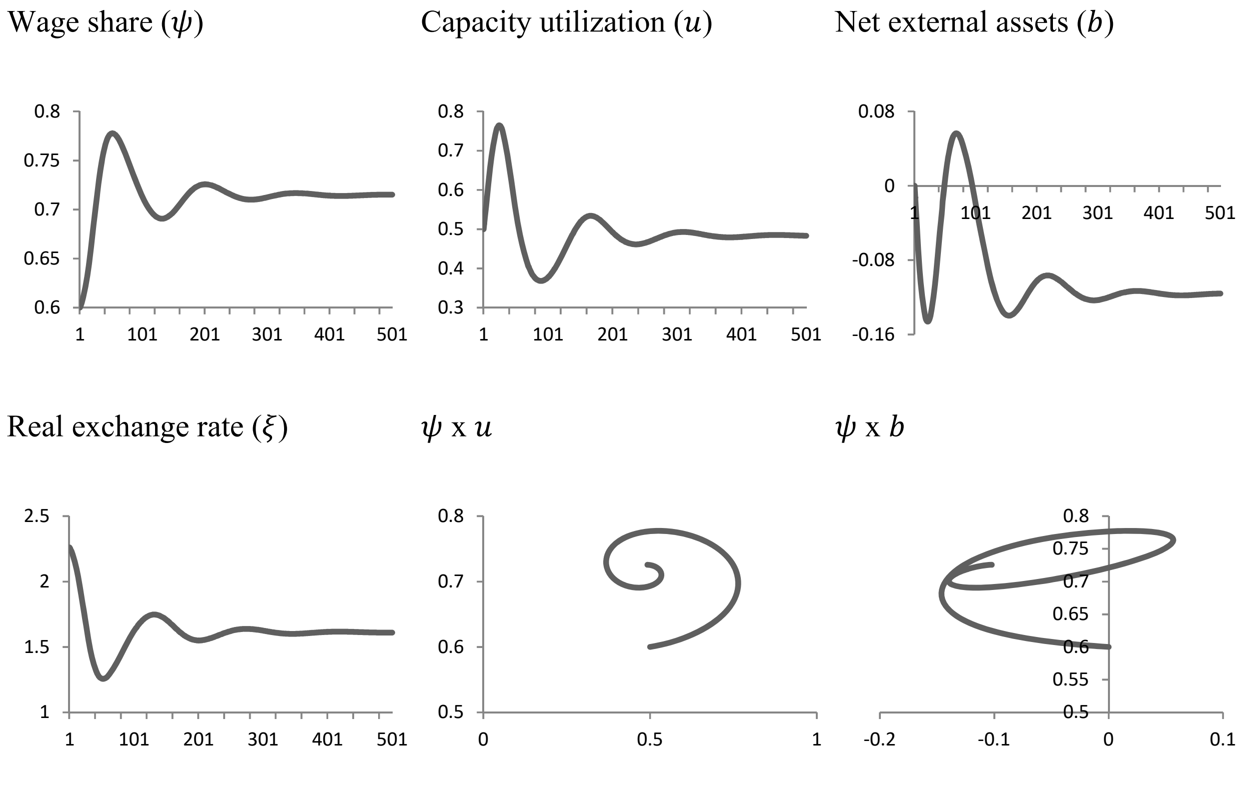
<u>Recovery Phase:</u> There is output and capacity utilization growth. There is "forced imports" which fill the gap between fast-growth demand and lagged supply. There is an inflationary boom (costs raising wages and prices) that leads to a reduction in profits. The role of the net asset-

capital ratio (b), as a response to the output-distribution dynamic feeds back into the aggregate demand equilibrium.

This cyclical dynamics arise from the complex social relationship in the model. The growth-distribution dynamics feeds into the evolution of the real exchange rate, international competitiveness and factor payments that combine to generate oscillations in the current account and trade balance.

Using the same model, it is possible to test different parameters to observe how that would affect the evolution of the distinct variables. In a closer calibration to what could represent a Latin American economy, we see that an interesting pattern emerges:

Figure 2. Alternative calibration of the La Marca model (Latin America)



Parameter Values: $\tau = 0.1$, $\lambda = 1$, $k = 20$, $l = 0.01$, $\gamma = 0.05$, $\alpha = 5$, $\eta = 1.3$, $a = 0.1$, $x = 0.01$, $j = 0.1$, $s_b = 0.5$, $s_h = 0.6$.
Initial conditions: $\psi_0 = 0.6$, $u_0 = 0.5$, $b_0 = 0$.
Steady state: $\psi = 0.71$, $u = 0.48$, $b = -0.12$.
Eigenvalues: $\lambda_1 = -0.12 + 0.44i$, $\lambda_2 = -0.12 - 0.44i$, $\lambda_3 = -0.34$

$$J = \begin{bmatrix} -0.1 & 0.14 & 0 \\ -1.34 & -0.12 & 0.01 \\ 0.91 & 0.09 & -0.33 \end{bmatrix}$$

In Figure we have on the top left the behaviour of the wage share on time. Top centre is the capacity utilization rate on time, while top right is the net external assets on time. Bottom left is the dynamic behaviour of the real exchange rate on time. In the bottom centre we see the between wage share and capacity utilization. Finally in the bottom right is the relationship between net foreign assets and wage share.

The results show a much stronger oscillation with dampened cycles in wage share and capacity utilization, as well as between the wage share and net external assets. The net assets show a peak in the first cycle but then smooth towards its steady state (negative one in this case), also showing a cyclical dynamics with the wage share. The real exchange rate follows the opposite of the wage share, with an interesting cyclical behaviour. Considering the current specification of the model it is possible to define a dynamic pattern with dampened cycles.

The rationality behind this case (Latin American country) is similar to the previous case (Southeast Asian growing country). The main difference is that the adjustment mechanism happens in many rounds, being every round weaker than the previous one. The economy under these conditions suffers higher volatility and has less capacity to adjust itself from shocks (e.g. external price changes), which is a much similar pattern to the behaviour of a Latin American economy. It is relevant to mention that La Marca initially thought the model for a rapid developing Asian country such as Korea.

4. Model Analysis

4.1. BPCM Assumptions - price neutrality and Fixed Real Exchange Rate

The La Marca (2010) model assumes a constant nominal exchange rate (fixed e). The real exchange rate fluctuates according to the behavior of terms of trade. In the La Marca model, there is the possibility that in the long run the economy operates with chronic external deficit. For the case of Latin America, the external constrain historically played a central role in its economic process. If we accept the Balance of Payments Constrained Model (BPCM), initially developed by Thirlwall (1979), the author highlights that in the long run an economy cannot have explosive deficits. Considering the price-neutrality assumption as well, the Thirlwall Law highlights that no economy can grow above the rate of growth compatible with the balance of payments – growth given in its simpler version by the ratio between income elasticities of exports and imports, times foreign growth.

In the original La Marca model, the real exchange rate is defined as $\xi = \frac{e\bar{P}}{P}$. P is the domestic good price, $\bar{P}$ the foreign good price and e the nominal exchange rate, which is considered a constant. Considering the BPCM, the long-run real exchange rate dynamics adjusts itself to stabilize the external sector. In the Balance of Payments dominance of Ocampo (2011), this is also valid for the short-run. In this way, the real exchange rate is constant, and the nominal one fluctuates in order to create a trajectory in which the net external assets (B) tend to a steady state equilibrium in which the growth of exports is equal to the growth of imports. This discussion can be summarized in the assumption that the real exchange rate is constant.

$$\xi = \iota \tag{8}$$

ι is a given constant. When considering the Balance of Payments Dominance of Ocampo (2011), in the case of middle income trapped countries, the balance of payments needs to be balanced even in the short-run. In order to do so, the nominal exchange rate must adjust itself to move towards this goal. In this case, the real exchange rate should have zero growth, being always in its steady state. This implies that $\dot{\xi} = 0$.

This assumption goes in line with the first of the two main assumptions of the BPCM, in which prices are neutral in the long-run, do not affecting the growth rate compatible with balance of payments (equilibrium growth rate). This assumption has its roots in the very low price elasticity of imports and exports, by a stability of financial flows, and by the assumption that the terms of trade (real exchange rate) grows at zero rate in the long-run (Blecker, 2016).

The second main assumption concerns the fact that the growth of exports cannot de bigger or smaller than the growth of imports in the long-run (McCombie, 2012). This would result in an explosive behaviour of the current account, creating major external imbalances that are not sustainable according to the BPCM. In order to follow this second assumption we set that the net foreign asset growth is constant, growing at zero rate ($\dot{B} = 0$), which implies that the net foreign asset growth per unit of capital grows also at zero rate ($\dot{b} = 0$).

When adding these BPCM element to the La Marca model, we reach a simpler 2-dimensional system in which growth and distribution variates with a constant real exchange rate. We then have:

$\dot{\xi} = 0 \implies$ BPCM Assumption 1

$\dot{b} = 0 \implies$ BPCM Assumtpion 2

When applying these assumptions to the model, the system results the following:

$$\dot{\psi} = \tau[(l \exp(1 + ulk) - \psi)]$$

$$\dot{u} = \lambda\{[(\alpha - s_p)\pi - s_h\psi - \iota a]u + \gamma + \iota^\eta x + (1 - s_p)j\iota b\}$$

With $\dot{b} = 0$. This completely changes the characteristics and the structure of the model and result in a relationship that is very similar to the one described by Goodwin (1967) between growth and distribution. We can analytically check the impacts on the trajectory by looking at the steady state and the Jacobian:

Steady State ($\dot{\psi} = 0$ and $\dot{u} = 0$):

$$\psi^* = \tau[(l \exp(1 + u^* lk))]$$

$$u^* = \frac{-[\gamma + \iota^\eta x + (1 - s_p)j\iota b]}{\lambda[(\alpha - s_p)\pi^* - s_h\psi^* - \iota a]}$$

For the jacobian of partial derivatives, as $\frac{\partial \xi}{\partial \psi} = 0$, we have:

$$J = \begin{bmatrix} -\tau & \tau l^2 k \psi \\ \lambda\left[-(s_p - \alpha)\left(\frac{\partial \pi}{\partial \psi}\right)u - s_h u\right] & -\lambda[(s_p - \alpha)\pi + s_h\psi + \iota a] \end{bmatrix}$$

Considering the positive values of τ, l, k, ψ, λ, s_p, s_h, α, u, π, a, ι, and $\frac{\partial \pi}{\partial \psi}$, we have the following jacobian:

$$J = \begin{bmatrix} - & + \\ - & - \end{bmatrix}$$

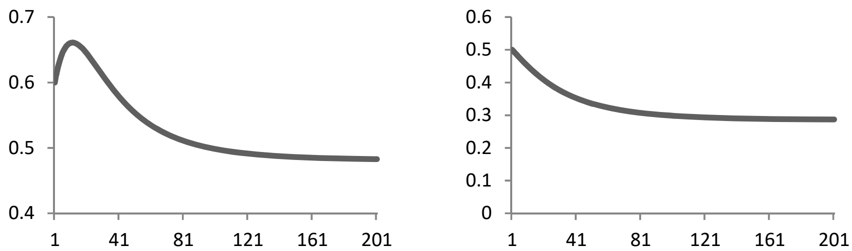
The matrix shows a negative trade and a positive determinant: $Tr(J_{\bar{b}}) < 0$ and $Det(J_{\bar{b}}) > 0$. This results in a pair of conjugate eigenvalues with positive real part and imaginary part. The result depends on the magnitude of the values, which could result in is a dampened cyclical adjustment between ψ and u (Goodwin dynamics), or in a monotonic convergence.

The results then show that the cyclical dynamics hold with the two assumptions of the BPCM, which opens the discussion about the real need of accepting these two strong assumptions to explain the cyclical economic adjustment in developing countries.

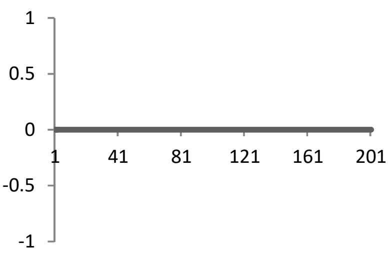
We can simulate the new system using similar values to the original La Marca model in order to show the differences in the adjustment dynamics when adopting both assumptions (price-neutrality and constant net asset accumulation), so the net foreign assets grow at zero rates in the long-run.

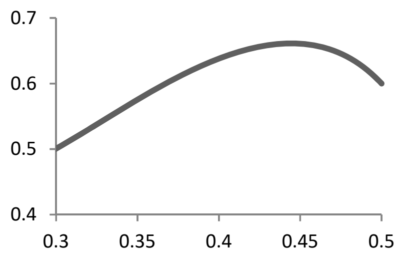
Figure 3. Modified La Marca model with $\dot{\xi} = 0$ and $\dot{b} = 0$

Wage share (ψ) Capacity Utilization (u)



Net Foreign Assets (b) ψ x u





Parameter values: $\tau = 1$, $\lambda = 1$, $k = 20$, $l = 0.1$, $\gamma = 0.05$, $\alpha = 0.5$, $\eta = 1.3$, $a = 0.1$, $x = 0.05$, $j = 0.03$, $s_b = 0.5$, $s_h = 0.3$.
Initial conditions: $\psi_0 = 0.6$, $u_0 = 0.5$, $b_0 = 0$, $\xi_0 = 0.01$.
Steady state: $\psi = 0.47$, $u = 0.28$, $b = 0$, $\xi = 0.01$.
Eigenvalues: $\lambda_1 = -0.91$, $\lambda_2 = -0.26$, $\lambda_3 = 0$,.
$J = \begin{bmatrix} -1.00 & 0.96 \\ -0.06 & -0.18 \end{bmatrix}$

In Figure , the top figures represent the evolution of wage share, and capacity utilization, while the bottom left the net exports/capital respectively on time. The bottom right figure shows the real the relationship between capacity utilization and wage share.

This case followed the same calibration values of Figure 1.. The system converges, showing a small oscillation, but it does not generate cycles (there is no complex eigenvalues). They follow a monotonic convergence to a stable equilibrium point. Variables such as the wage share and the net assets initially increase. Then they start converging to the steady state. The capacity utilization converges monotonically to the steady state. By definition, the real exchange rate keeps itself in its same initial value.

4.2.Fixed income distribution.

Distribution is a central aspect in the harrodian tradition of growth models. In this case we fix the income distribution between profits and wages, and study the relationship between capacity utilization (economic activity) and external sector. Considering a monopolistic economy, we have that the profit share of the economy is a function of the mark-up (z), so $\pi = \frac{z}{z+1}$. In this sense, we may fix the profit share as $\pi = \bar{\pi}$ and analyze the case in which the wage share is also constant ($\psi = \bar{\psi}$), growing at a zero rate $\dot{\psi} = 0$.

That would imply that the equations for ψ, π, and ξ would completely change, and the fixed income distribution would help us understand the relationship between capacity utilization (u) and balance of payments (b), a central discussion in the BPCM framework.

Considering a fixed income distribution, which implies that the wage share is constant, we have that its growth rate is equal to zero: $\dot{\psi} = 0$. That would also imply that $\pi = \bar{\pi}$ and $\xi = \bar{\xi}$. This results in the following system:

$$\dot{u} = \lambda\{[(\alpha - s_p)\pi - s_h\psi - \xi a]u + \gamma + \xi^\eta x + (1 - s_p)j\xi b\}$$

$$\dot{b} = \frac{(s_p - \alpha)\pi u + s_h\psi u - \gamma}{\xi} - (g - s_p j)b$$

By defining $\dot{u} = 0$ and $\dot{b} = 0$, we get the steady state.

$$u^* = -\frac{[\gamma + \xi^\eta x + (1 - s_p)j\xi b^*]}{[(\alpha - s_p)\pi - s_h\psi - \xi a]}$$

$$b^* = \frac{(s_p - \alpha)\pi u + s_h\psi u^* - \gamma}{\xi(g - s_p j)}$$

The Jacobian of partial derivatives is the following:

$$J = \begin{bmatrix} \lambda[(\alpha - s_p)\pi - s_h\psi - \xi a] & \lambda(1 - s_p)j\xi \\ \frac{(s_p - \alpha)\pi + s_h\psi}{\xi} - b\alpha\pi & s_p j - \alpha\pi u - \gamma \end{bmatrix}$$

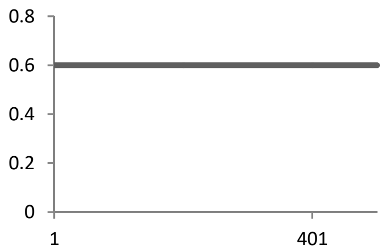
Considering the sign value study of the La Marca model, this would result in:

$$J = \begin{bmatrix} - & - \\ + & + \end{bmatrix}$$

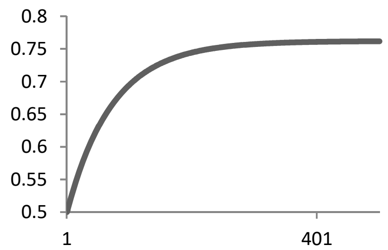
What we observe is that the eigenvalues of this model will depend on the magnitude of the values of the jacobian. The magnitude of the absolute values would define if the model converges or diverges from the steady state (possibility of divergence), and if there is the presence of cycles. In our simulation we see that the model is likely to operate with real numbers, without an imaginary part.

Figure 4. Simulation for fixed income distribution ($\dot{\psi} = 0$)

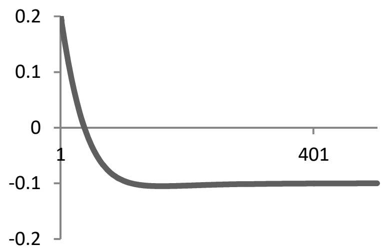
Wage share (ψ)



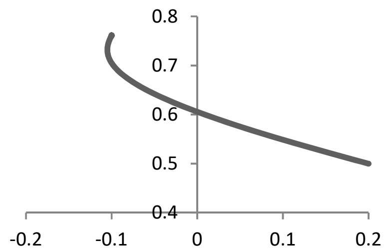
Capacity Utilization (u)



Net external assets (b)



u x b



Parameter values: $\tau = 1$, $\lambda = 1$, $k = 20$, $l = 0.1$, $\gamma = 0.05$, $\alpha = 1.2$, $\eta = 1.3$, $a = 0.1$, $x = 0.05$, $j = 0.01$, $s_b = 0.5$, $s_h = 0.3$.
Initial conditions: $\psi_0 = 0.6$, $u_0 = 0.5$, $b_0 = 0.2$, $\xi_0 = 0.01$.
Steady state: $\psi = 0.60$, $u = 0.76$, $b = -0.1$, $\xi = 0.01$.
Eigenvalues: $\lambda_1 = -0.31$, $\lambda_2 = -0.13$, $\lambda_3 = 0$, , $\lambda_4 = 0$.
$J = \begin{bmatrix} -0.38 & -0.13 \\ 0.32 & 0.05 \end{bmatrix}$

From the original La Marca values, we see that the adjustment converges but it loses the cyclical dynamics it had before. Wage share is constant to its initial value, and the growth of net external asset reduces then increases with changes in the capacity utilization. We then see a monotonic convergence between growth and balance of payments, in which an increase in growth shows a negative correlation with accumulation of external assets. A rise in economic growth have negative effects in the Balance of Payments, being a potential source of external constrains, following then the BPCM framework.

The presence of fixed income distribution ends up removing the cyclical component from the model. The source of endogenous oscillation disappears when we fix income distribution, which is an overly strong assumption. Without any expansion to the system, income distribution changes shows as a central element causing structural volatility.

5. Model expansion:

The scope of this model is to focus on the behaviour of economies trapped in the middle income, especially in the case of Latin America. The middle income trap (Lavopa & Szirmai, 2014) is a concept that highlights how countries may not advance economically above certain level as its competitiveness in manufactured export goods is reduced by rising wages (Glawe & Wagner, 2016).

The cyclical aspect is fundamental as it reproduces a dynamic that may not monotonically converge to the steady state, but remains fluctuating in an endogenous mechanism that raises volatility and keeps an economy trapped. The cycles with neutral stability ($RE(\lambda_i) = 0$ and $I(\lambda_i) \neq 0$) are only possible when we see that competitiveness in terms of productivity is also affected by the distribution/output behaviour in the exact value of a hopf bifurcation value. In this sense, we search to further develop the Structuralist literature, starting its central idea in which a chronic volatility is generated by structural failures, in terms of a weak and fragile productive structure, and a low capacity to innovate (Porcile & Spinola, 2018). We briefly develop this idea in the next session.

5.1. Productivity dynamics

Following the theoretical debate in section 2, in this section we expand the La Marca (2010) model by adding a productivity dynamics that incorporates the role of the supply side as a relevant determinant of the model. We implement it through the use of the Kaldor-Verdoorn effect (Kaldor, 1975). This effect incorporates learning by doing, which allows the occurrence of increasing returns to scale. In this sense, an increase in demand, either by a growth in output ($\dot{X}$) or in investment (g), results in a rise in productivity.

As can be observed in the Annex, Productivity is given by ε/l. The work effort (ε) will be kept considered stable, but the effective labor per unit of product (l) is endogenized. The Kaldor-Verdoorn effect relates investment and productivity, so: $l = f(g)$, $dl/dg < 0$. In this sense, investment growth leads to a reduction of the effective labor per product (L/X) and increases productivity.

In addition to that, the accumulation of external assets can generate positive technological spillovers in terms of productivity to the internal firms. The technology transfer is a fundamental aspect to understand the relationship between North-South, marked by the presence of a technology gap (Verspagen, 1991). The domestic firms become more competitive as they learn with the activities of their subsidiaries located abroad. Technology transfer to the domestic economy has the positive effect on domestic firms' productivity. This raises the average productivity in the country. The effects of technology transfer in productivity ($\theta < 0$) is given by:

$$\dot{l} = \rho g + \theta b + \phi l \tag{9}$$

In the productivity dynamics ρ is the learning-by-doing Kaldor-Verdoorn effect and θ captures the technological transfer from foreign firms to domestic firms. ϕ is a decreasing effect of the level of labor-output. Because l is seen as inverse of productivity, $\rho = d\dot{l}/dg >$; $\theta = d\dot{l}/db < 0$ and $\phi = d\dot{l}/dl < 0$. The option for a linear equation is a matter of pure simplification.

After implementing this addition, the new dynamic system can be presented as the following:

$$\dot{\psi} = \tau[(l \exp(1 + ulk) - \psi)]$$

$$\dot{u} = \lambda\{[(\alpha - s_p)\pi - s_h\psi - \xi a]u + \gamma + \xi^\eta x + (1 - s_p)j\xi b\}$$

$$\dot{b} = \frac{(s_p - \alpha)\pi u + s_h\psi u - \gamma}{\xi} - (g - s_p j)b$$

$$\dot{l} = \rho g + \theta b + \phi l$$

Steady State ($\dot{\psi} = \dot{u} = \dot{b} = \dot{l} = 0$):

$$l^* = -\frac{\theta b^* + \rho g^*}{\phi} =$$

$$b^* = \frac{(s_p - \alpha)\pi u^* + s_h \psi u^* - \gamma}{\xi^*(g^* - s_p j)}$$

$$\psi^* = l^* \exp(1 + u^* l^* k)$$

$$u^* = -\frac{[\gamma + \xi^{*\eta} x + (1 - s_p) j \xi^* b^*]}{[(\alpha - s_p)\pi^* - s_h \psi^* - \xi^* a]} = 0$$

Considering the derivatives in the original La Marca defining the first three rows and three columns, the expansion of the jacobian adds the partial derivatives for l:

$$J = \begin{bmatrix} \partial\dot{\psi}/\partial\psi & \partial\dot{\psi}/\partial u & \partial\dot{\psi}/\partial b & \partial\dot{\psi}/\partial l \\ \partial\dot{u}/\partial\psi & \partial\dot{u}/\partial u & \partial\dot{u}/\partial b & \partial\dot{u}/\partial l \\ \partial\dot{b}/\partial\psi & \partial\dot{b}/\partial u & \partial\dot{b}/\partial b & \partial\dot{b}/\partial l \\ \partial\dot{l}/\partial\psi & \partial\dot{l}/\partial u & \partial\dot{l}/\partial b & \partial\dot{l}/\partial l \end{bmatrix}$$

Observing the Signs:

$$J = \begin{bmatrix} - & + & 0 & + \\ - & - & + & 0 \\ + & + & - & 0 \\ + & + & - & - \end{bmatrix}$$

From the Jacobian signs, we observe the interaction between each of the four equations, which lets us say about the stability conditions of the model for each pair of equations. We decompose the system in pairwise effects, resulting in the following structure:

$J_{\psi,u} = \begin{bmatrix} - & + \\ - & - \end{bmatrix}$ – Cyclical convergence

$J_{\psi b} = \begin{bmatrix} - & 0 \\ + & - \end{bmatrix}$ - Cyclical Convergence

$J_{\psi l} = \begin{bmatrix} - & + \\ + & - \end{bmatrix}$ - Monotonic Convergence

$J_{u,b} = \begin{bmatrix} - & + \\ + & - \end{bmatrix}$ – Monotonic convergence/divergence

$J_{u,l} = \begin{bmatrix} - & 0 \\ + & - \end{bmatrix}$ - Conditional Cyclical convergence

$J_{b,l} = \begin{bmatrix} - & 0 \\ - & - \end{bmatrix}$ - Conditional Cyclical convergence

We observe the possibility of cycles emerging from the following relations: (1) wage share and capacity utilization (Goodwin). (2) Wage share and net foreign assets. (3) Capacity utilization and productivity. (4) Net foreign assets and productivity. The first two effects have been already discussed, so we focus on the last two.

Capacity utilization and productivity

Initially this relationship is thought in terms of increasing returns brought by the Kaldor-Verdoorn law. Increases in economic activity have positive effects on productivity. What we observe in our dynamic system is that the relationship between capacity utilization and productivity has a cyclical aspect.

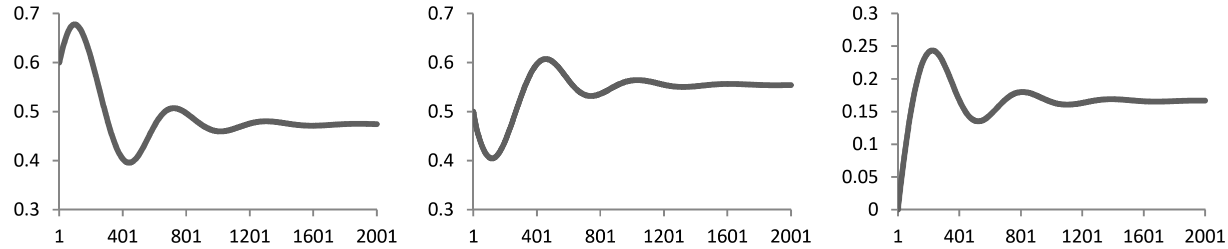
A higher productivity counter-balances the effects of capacity utilization, in the sense that the same output can be produced with a smaller rate of capacity. So we observe initially an increase in output, raising productivity. This rise in productivity pressures for a virtuous reduction of capacity, as output increases given higher labour productivity. This behaviour ends ups resulting in a cyclical adjustment towards the equilibrium value.

Net foreign assets and productivity

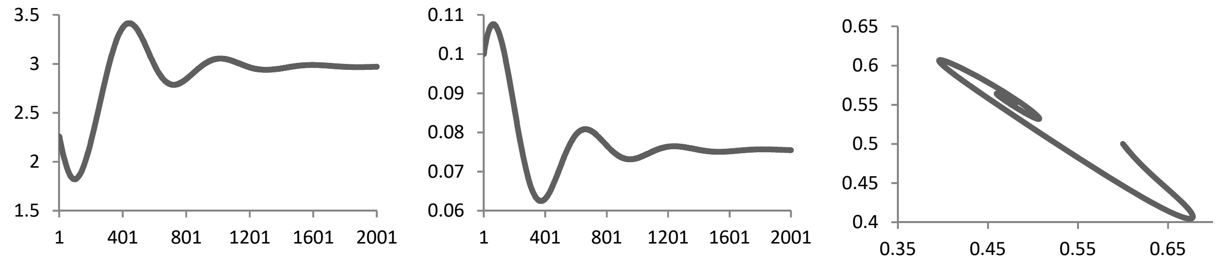
A second element of the expansion is the relationship between the accumulation of net foreign assets and productivity. The accumulation of external assets let domestic firms learn from foreign activities, which raises their productivity. In dynamical terms, we see that a cyclical aspect emerges in this relationship. A way to explain that is related to two possible effects: First, more productive firms from developing countries become more attractive to be acquired by big companies in the developed world (denationalization, a very common feature during the 1990's in Latin America). Second, Firms in developing countries find a barrier to their growth in productivity from technology transfer. As this transfer is not an automatic movement, these companies manage to catch-up until a certain point in which they stagnate, as companies in the developed world continue with their innovative virtuous strategies in the world level. In this sense, this counterbalances dynamically the effects of increases in productivity, generating this oscillatory behaviour.

Figure 5. Simulation 1 - Modified La Marca results with productivity dynamics, original calibration

Wage Share (ψ) Capacity Utilization (u) Net foreign Assets (b)



Real Exchange Rate (ξ) Labour-Output Ratio (l) ψ x u



Parameter values: $\tau = 0.1$, $\lambda = 1$, $k = 20$, $l = 0.1$, $\gamma = 0.05$, $\alpha = 0.5$, $\eta = 1.3$, $a = 0.1$, $x = 0.05$, $j = 0.03$, $s_b = 0.5$, $s_h = 0.3$, $\rho = 0.03$, $\theta = -0.02$, $\phi = -0.001$
Initial conditions: $\psi_0 = 0.6$, $u_0 = 0.5$, $b_0 = 0$, $l = 0.1$.
Steady state: $\psi = 0.47$, $u = 0.55$, $b = 0.16$, $l = 0.09$
Eigenvalues: $\lambda_1 = -0.029 + 0.108i$, $\lambda_2 = -0.029 - 0.108i$, $\lambda_3 = -0.413$, $\lambda_4 = -0.029 - 0108i$.

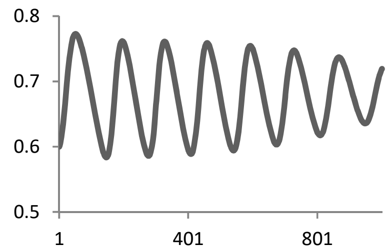
$$J = \begin{bmatrix} -0.100 & 0.071 & 0 & 1.153 \\ -0.338 & -0.471 & 0.032 & 0 \\ 0.094 & 0.04 & -0.094 & 0 \\ -0.003 & 0.003 & -0.02 & -0.001 \end{bmatrix}$$

The first five figures from top to bottom, and left to right, represent the evolution of the wage share, capacity utilization, net external asset/capital, real exchange rate and effective labour per unit of product in time. The last figure shows how growth and distribution evolve between themselves. We see that the oscillatory pattern with dampened cycles exists for all variables. All of them stabilize in an equilibrium point.

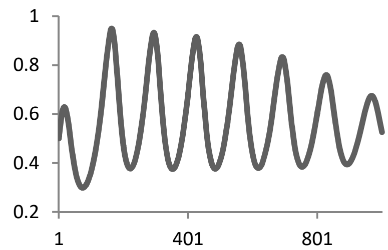
We observe that the addition of a productivity dynamics, a structural elements of an economic system, result in other sources of oscillatory behaviour. All variables in the system now oscillate, not only growth and distribution. This brings another elements to discuss the role of foreign sector adjustments. Balance of payments adjustments then cyclically also converge to a stable point. It is interesting to notice that still all results converge to a stable point. This expansion opens the floor to start discussing the cyclical aspects related to economic structures, advancing what Taylor (1983) started developing.

Figure 6. Simulation 2 - Modified La Marca results with productivity dynamics, Hopf calibration value

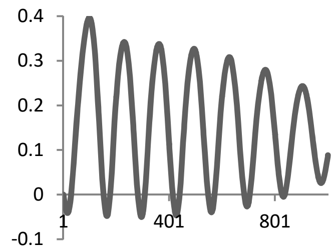
Wage Share (ψ)



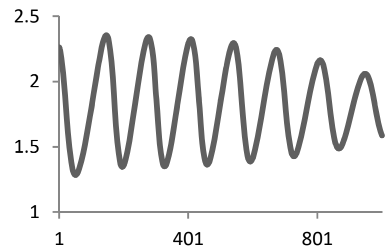
Capacity Utilization (u)



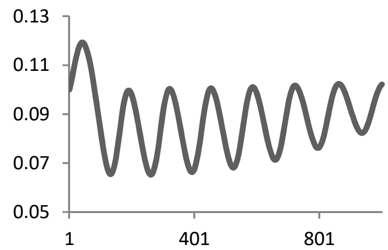
Net foreign Assets (b)



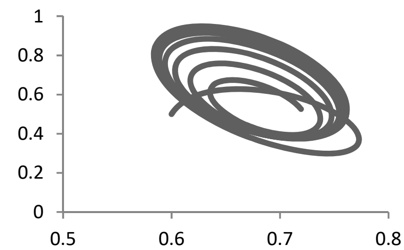
Real Exchange Rate (ξ)



Labour-Output Ratio (l)



ψ x u



Parameter values: $\tau = 0.1$, $\lambda = 1$, $k = 20$, $l = 0.01$, $\gamma = 0.05$, $\alpha = 5$, $\eta = 1.3$, $a = 0.1$, $x = 0.05$, $j = 0.3$, $s_b = 0.4$, $s_h = 1$. $\rho = 0.01$, $\theta = -0.03$.
Initial conditions: $\psi_0 = 0.6$, $u_0 = 0.5$, $b_0 = 0$, $l = 0.1$.
Steady state: $\psi = 0.69$, $u = 0.49$, $b = 013$, $l = 0.1$
Eigenvalues: $\lambda_1 = -0.037 + 0.41i$, $\lambda_2 = -0.037 - 0.41i$, $\lambda_3 = -0.214 + 0.198i$, $\lambda_4 = -0.214 - 0.198i$.

$$J = \begin{bmatrix} -0.1 & 0.13 & 0 & 1.38 \\ -1.50 & -0.32 & 0.01 & 0 \\ 0.92 & -0.01 & -0.09 & 0 \\ -0.01 & 0.01 & -0.03 & 0 \end{bmatrix}$$

If we change the parameters of the expanded model, we can reach a situation in which the cyclical dynamics repeats itself indefinitely. Using the variable θ as the adjustment variable, we calibrate it to a value that gets closer to a situation in which there is neither convergence nor divergence – so cycles can repeat themselves indefinitely (The hopf bifurcation point).

This specification generates a very interesting pattern. The initial value when in the hopf bifurcation value, converges to a limit cycle. However, it may not stay in the cycle indefinitely unless in the exact value that splits the convergence and the divergence areas. When observing the value of the eigenvalues, we see that the model is still stable, but it generates a chaotic behaviour in which it keeps oscillating for a big number of runs and then stabilizes itself. The same pattern was observed under other initial values. We reinforce that the variable value $\theta = -0.03$ was simply calibrated to be closer to the hopf bifurcation point (Lorenz, 1989). Any small change in the system will result in a change in the trajectory, leading to either an explosive behaviour or a regular cyclical stability. If we change the parameters of the system, then The hopf bifurcation parameter must then be adjusted to a different calibration.

What these results tell us is that the economy under these conditions may enter in a cyclical pattern in which it will never reach a stable equilibrium. The convergence pattern will always pull the economy to a volatility pattern, even with the absence of external shocks. In these sense, an endogenous pattern of volatility emerges in the economy that pushes to a trapped region.

6. Discussion of the results

6.1. Accepting the BPCM assumptions

Figure 3 shows that, when we change the exchange rate regime to a flexible nominal exchange rate, this affects in part the dynamics of the system.

The cyclical dynamics can hold when accepting the two assumptions of the BPCM, which opens the discussion about the real need of accepting these two strong assumptions to explain the cyclical economic adjustment in developing countries. There is a whole literature tradition that question the assumptions of the BPCM (Blecker, 2016; McCombie, 2012).

If accepting these assumptions do not affect the convergence behaviour, we have an additional argument to defend the BPCM and the Thirlwall Law (Blecker, 2016). That occurs in the sense that an economy can have non-price neutrality and an explosive behaviour in its external accounts but still reach an equilibrium point after an oscillatory period.

Depending on specific parameter values, the trajectory to equilibrium can show a monotonic convergence, being much less volatile than a cyclical adjustment. In this sense, we can raise the argument behind the classic Dornbusch Latin Triangle (Dornbusch, 1992) discussion. In this theory, the exchange rate regime affects the endogenous pattern of volatility. Considering that an adoption of fixed real exchange rates (fluctuating nominal rates) changes the dynamics, we can observe in the La Marca (2010) model that this results in a change from a dampened cyclical adjustment to a monotonic convergence. The fact that we do not observe explosive behaviour in any case indicates that the two assumptions are not necessary to keep the basic characteristics of the model.

The trajectory to the steady state changes from dampened cycles to a monotonic trajectory. This implies that changes in the exchange rate regime of the economy reduce endogenous oscillations. An external sector policy aimed at avoiding external debt reduces the endogenous pattern of volatility. This result is very much in line with the BPCM and the Balance of Payment Dominance theories. In which a middle-income economy, that cannot hold foreign debt in its own currency, has its growth directed constrained by the behaviour of its external sector. A direct adjustment to the external sector then solves the volatility. The cost is high though in terms of economic activity and distribution. As it can be seen in the model, a change in the currency regime reduces the economic activity and the part of income that go to wages when compared to the fixed exchange rate regime.

This situation is very similar to what is observed to many Latin American countries in the 1990's decade. Taking the case of Brazil as an example, the transition to the fixed nominal exchange rate regime in 1994 resulted in increases in real wages, in the wage share and in the utilization capacity rate. The country started suffering from pressures in its Balance of Payments with the fixed nominal exchange rate. After the crisis of 1998, and return to the flexible nominal exchange rate, economic activity was strongly reduced as well as real wages (rise in internal prices and major nominal exchange rate devaluation).

Another interesting observation concerns the fact that when we change the parameter to values that are closer to the Latin American reality, the model itself results in a higher volatility (as we can see in our alternative simulations). In this sense, the own characteristic of the economy raises its fragility pattern that endogenously generates volatility, being that not only a matter of external shocks, as defended by the Real Business Cycle tradition, but also because of the structural elements of these economies.

6.2. Fixed income distribution

When we fix the income distribution we can directly observe the relationship between capacity utilization and net external assets. The result shows a monotonic convergence pattern to an equilibrium point.

From the original values, the adjustment still converges but it loses the cyclical dynamics it had before. Wage share is constant to its initial value, and the growth of net external asset initially reduces followed by an increase, as capacity utilization changes. We then see a monotonic convergence between growth and balance of payments, in which an increase in growth shows a negative correlation with accumulation of external assets. A rise in economic growth have negative effects in the Balance of Payments, being a potential source of external constrains, following then results that are similar to the BPCM framework ideas.

6.3. Productivity dynamics

In

Figure , the inclusion of a productivity dynamics is part of a search to find deterministic stable cycles. This type of cycle happens when, in the presence of no shocks, the system in inherently unstable. We check if productivity interacts with income distribution and economic activity, generating stable cycles. With our current specification, which considers the effects of the Kaldor-Verdoorn effect and technology transfer, deterministic cycles only appear under very specific conditions. This leads us to the study of the hopf bifurcation parameters. The model itself, however, shows an interesting cyclical relationship between productivity and economic activity, and between productivity and the accumulation of net external assets.

These interesting relationships can be explained by (1) a higher productivity that counter-balances the effects of a rise in capacity utilization. The rise in productivity pressures for a virtuous reduction of capacity, as output increases given higher labour productivity. This behaviour ends ups resulting in a cyclical adjustment towards the equilibrium value. (2) More productive firms from developing countries are more interesting to be acquired by big companies in the developed world; and Firms in developing countries find a barrier to their growth in productivity from technology transfer. In this sense, these elements dynamically counterbalance the effects of increases in productivity, generating an oscillatory behaviour.

The calibration we used was aimed to reproduce a Latin American middle-income economy. One in which the behaviour of the real exchange rate is endogenously unstable. The endogeneity of productivity is central to explain this behaviour. An increase in the wage share has negative effects on productivity itself, but in a wage-led economy it boosts growth, which through an increase in the capacity utilization affects investment. The rise in productivity occurs through the Kaldor-Verdoorn effect. This compensatory dynamics gives rise to the cycles.

The central contribution resides in the fact that the economic cycles are explained as a pure endogenous mechanism in these economies. It is not a result of exogenous shocks, such as the policy shocks of the Real Business Cycle, technological shocks of the Schumpeterian theory or the Terms of Trade shocks as in the traditional Structuralist perspective. We argue that the cyclicality is a pattern generated by DNA of these economies.

7. Conclusion

This paper proposed to study and expand the La Marca (2010) model in 3 different fronts: (1) considering the BPCM assumption of price-neutrality and stable behaviour of the balance of payments; (2) accepting a constant income distribution between wages and profits, and (3) adding a productivity dynamics and technology transfer to the model. This work offers a small contribution, adding a jigsaw, to a puzzle that is still open in the Structuralist literature: what is behind the "chicken flights" growth pattern, in which countries have their growth processes interrupted after a small virtuous period. This is one of the biggest challenges for the low- and middle- income countries to sustain growth and overcome the medium-income trap. In other terms: how to think about endogenous deterministic cycles that are characteristic of middle-income countries in a Centre-Periphery framework?

The results show (1) Latin American parameters increase the endogenous oscillatory adjustment. (2) In all cases the model converges. (3) The price-neutrality assumption and external sector stability depend on parameters to have a cyclical or monotonic convergence. This reinforces the BPCM argument, and may imply a reduction in endogenous volatility. (4) Fixed income distribution lead to a monotonic trajectory, reducing volatility. (5) The inclusion of the productivity dynamics generates new sources of volatility in the relationship between productivity, capacity utilization, and net external assets, being in line with the Structuralist argument of structural fragility.

We observed that when we change the La Marca (2010) original parameters (originally thought to a growing Asian economy), to values that represent a Latin American Economy, this result in much higher oscillatory behaviour. Endogenously these Latin American economies end up creating higher oscillations because of their fragile structural characteristics. Economic activity, income distribution, and net foreign assets then interact with higher sensibility in the context of more vulnerable economies.

The inclusion of the BPCM assumptions does not change the convergence pattern of the model. This is relevant in the sense that the absence of an explosive divergent pattern guarantees that the BPCM leads to a stable equilibrium both in the short- and the long-run. The empirical critiques to the Thirlwall Law's heroic assumptions remain relevant, but without meaning that the BPCM do not hold in terms of its stability and brings another element to defended it in the debate about the critiques to the BPCM (Blecker, 2016; McCombie, 2012).

The currency regime a la Dornbusch (1992)'s Latin Triangle is also briefly discussed. A flexible nominal exchange rate focused on balancing the external sector changes the dynamic properties of the model. The convergence trajectory to the steady state may not generate cycles depending on the parameters. This is an important finding. The Thirlwall model and the Balance of Payments Dominance of Ocampo (2011) state the relevance of the external constrains the long- and short-run respectively. An exchange rate mechanism that is able to adjust the external sector automatically (no debt accumulation) results - in the La Marca (2010) model - in a pattern that reduces the endogenous instability in the adjustment mechanism. Despite the flexibility that the nominal exchange rate offers, when it keeps the real exchange rate constant, it reduces volatility. But there are costs in terms of the steady state. It reduces the equilibrium values of the wage share and the capacity utilization. Achieving smaller volatility involves a trade-off, a reduction in the economic activity and income concentration on profits.

The perspective embedded in this paper clearly states that the answers to our research questions are related to the supply-side of the economy. Schumpeter and the evolutionary school offer some central contributions to understand the complex dynamic that emerges in a world in which technological change is at the centre of the development debate. The paper offer a simple solution to the inclusion of productivity in the model, taking into account a learning by doing Kaldor-Verdoorn element and a technology transfer/learning from domestic firms that have assets abroad (increasing the average productivity of the economy). The results indicate that even the simple inclusion of a productivity dynamics is able to, under very specific conditions, generate deterministic stable cycles. This is of fundamental importance, because it shows that even in the presence of no shocks, the system in inherently not stable, endogenously cyclical.

Finally, this paper offers an invitation to expand new contributions in the Structuralist Kaldor-Thirlwall framework of La Marca (2010) exploring even further the Schumpeterian aspects of the economic cycles. The cyclicality aspect of middle income trapped economies could be further analysed with other model expansions: adding a multi-sector model, exploring further the technological dynamic, increasing the heterogeneity of agents. The Structuralist literature on cycles is still scarce on modelling techniques and the additions of new theoretical approaches and new models to observe specific aspects related to low- and middle- income countries offers an open space for a whole new road of research opportunities.

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Annex 1. Variable list

X	Aggregate Production	r	Profit Rate
K_i	Capital Stock	g	Domestic investment rate
x	Output-capital ratio	ξb	Foreign investments
L	Labour	m	Mark-up over costs
l	Labour-output ratio	P	Prices
B	Value of Foreign Assets	p_e	Equities unit price
b	Net foreign assets rate	σ	Domestic Savings
D_i	Dividends	γ	Animal Spirit
N	Employable working population	α	Investment sensitivity to profitability
H	Amount of hours worked	ξ	Real Exchange Rate
h	employment rate (H/N)	κ	Current Account (χK)
k	Capital-population ratio	η	Price-elasticity of domestic output in world market.
ε	effort exerted by workers	z	Non-exchange rate dependent current account (Net foreign savings)
a	Share of imported intermediate inputs	τ	Wage Share speed of adjustment.
u	Capacity Utilization	λ	speed of adjustment capital utilization
ψ	Wage Share	s	Propensity to save
π	Profit Share		SUBSCRIPTS:
j	Real return on net foreign assets	b	firms
E	Equities	h	Households
e	Nominal Exchange rate	p	Weighted rate
ι	Constant for RER	g	Government
ρ	Learning-by-doing Kaldor-Verdoorn effect	f	Foreign Sector
θ	Technological transfer from foreign firms to domestic firms	$*$	Equilibrium value
ϕ	Decreasing effect of the level of labour-output	J	Jacobian
Λ	Balance of payments result	λ_i	Eigenvalues
T_i	Taxes	Y_i	Income per social class
t_i	Tax rate	m	Mark-up
S	Savings	Ω	Liabilities
C_o	Costs	w	Wage per worker

Annex 2.1. La Marca (2010) Balance Sheets and main equations

Table 1. Balance Sheets

Households		Firms		Foreign (RoW)	
$p_E E$	Ω_h	K	$p_E E$	$-\xi B$	Ω_f
		ξB	Ω_b		

Table 2. Social Accounts Matrix

	Output Cost	HH	Firms	Gov	ROW	Cap	Equit	Bonds	Tot
Output		C_h		G	χK	gK			X
Incomes									
Household (HH)	ψX		D_b						Y_h
Firms	πX				$j\xi B$				Y_b
Government		T_h	T_b						Y_g
RoW	$\xi a X$								Y_f
Funds									
HH		S_h					$-p_E\dot{E}$		0
Firms			S_b			$-gK$	$p_E\dot{E}$	$-\xi\dot{B}$	0
Govvernment				(S_g)					0
ROW					(S_f)			$\xi\dot{B}$	0
Total	X	Y_h	Y_b	Y_g	Y_f	0	0	0	

Output: $X = \psi X + \pi X + \xi a X = C_h + G + \chi K + gK$ (A.1)

$$X = Y_h + Y_b + Y_g + Y_f$$

Incomes: (A.2)

Households: $Y_h = C_h + T_h + S_h = \psi X + D_b$

Firms: $D_b + T_b + S_b = \pi X + j\xi B = Y_b$ (A.3)

Government: $G + (S_g) = T_h + T_b = Y_g$ (A.4)

RoW: $\chi K + j\xi B + (S_f) = \xi a X = Y_f$ (A.5)

χK – Current Account
$\xi a X$ – Share of intermediate import goods
$j\xi B$ – Revenue from foreign assets
$p_E\dot{E}$ – New Equities
$\xi\dot{B}$ – New Foregein Investment

Flow of funds:

Househoulds: $S_h = p_E\dot{E}$ (A.6)

Firms Savings: $S_b = gK + \xi\dot{B} - p_E\dot{E}$ (A.7)

Government: $S_g = 0$ (A.8)

RoW: $S_f = \xi\dot{B}$ (A.9)

Adding Flows to Income:

Households Flows: $C_h + T_h + p_E\dot{E} = \psi X + D_b$ (A.10)

Firms Flows: $D_b + T_b + gK + \xi\dot{B} - p_E\dot{E} = \pi X + j\xi B$ (A.11)

Government Flows: $G = T_h + T_b$ (A.12)

$$\text{RoW Flows: } \chi K + j\xi B - \xi \dot{B} = \xi a X \tag{A.13}$$

External sector:

$$-S_f = \xi \dot{B}$$

As:

$$\chi K + j\xi B - \xi \dot{B} = \xi a X \Rightarrow \ \xi \dot{B} = \chi K + j\xi B - \xi a X$$

From (A.13):

$$\xi \dot{B} = (\xi x + j\xi b - \xi a u)K - \Lambda \Rightarrow \xi \dot{B} = \xi x K + j\xi B - \xi a X - \Lambda \Rightarrow$$
$$\Rightarrow \xi a X = \ \xi x K - \Lambda + \ j\xi B - \ \xi \dot{B} \Rightarrow \chi K = \xi^{\eta} x K - \Lambda$$
$$\Lambda = \xi^{\eta} x K - \chi K \tag{A.14}$$

<u>Prices and Income Distribution:</u>

$l = L/X$ $\quad$ $u = X/K$ $\quad$ $k = K/N$

$h = H/N$ $\quad$ $\varepsilon = L/H$

$$\text{Employments Rate: } h = \frac{H}{N} = \frac{H}{L}\frac{X}{K}\frac{L}{X}\frac{K}{N} = \frac{ulk}{\varepsilon} \tag{A.15}$$

$$\text{Wage share: } \psi = \frac{wl}{P\varepsilon} \tag{A.16}$$

$$\text{Real Exchange Rate: } \xi = \frac{e\bar{P}}{P} \tag{A.17}$$

Profit Share, Costs: $C_o = (wL + aP\xi) = \left(\frac{wl}{\varepsilon} + e\bar{P}a\right)$

P = Mark-up * Costs = $(1+m)(wL + aP\xi) \Rightarrow P = (1+m)\left(\frac{wl}{\varepsilon} + e\bar{P}a\right)$

Profit = sales - costs = $PX - C_o$=> Profit Rate = Profit/Capital Value => $r = \frac{(PX - C_o)}{PK}$

$$r = \frac{PX - \left(\frac{wl}{\varepsilon} + e\bar{P}a\right)}{PK} \tag{A.18}$$

Real value of total profits :$rK = \pi X$ or $r = \pi u$

$$\psi X + \pi X + \xi a X = X \ \Rightarrow \pi + \psi + \xi a = 1 \Rightarrow \ \pi = 1 - \psi - \xi a \Rightarrow \pi = 1 - \frac{wl}{P\varepsilon} - \frac{e\bar{P}}{P}a \Rightarrow$$

$$\Rightarrow \ \pi = \frac{1}{P}\left(P - \frac{el}{\varepsilon} - e\bar{P}a\right) \Rightarrow \pi = \frac{1}{P}(P - C_o) \Rightarrow \ \pi = \frac{\left[(1+m)\left(\frac{wl}{\varepsilon}+e\bar{P}a\right) - \left(\frac{wl}{\varepsilon}+e\bar{P}a\right)\right]}{(1+m)\left(\frac{wl}{\varepsilon}+e\bar{P}a\right)} \Rightarrow \ \pi = \frac{m}{(1+m)}$$

$$\pi = \frac{m}{(1+m)} = m(\psi + \xi a) \tag{A.19}$$

$\eta -$ elasticity of the price elasticity of domestic output in the world market

$$\pi = \frac{1-\psi}{\eta\left(1+\frac{1}{\eta}\right)} \tag{A.20}$$

$$\xi = \frac{1-\psi}{a\left(1+\frac{1}{\eta}\right)} \tag{A.21}$$

Annex 2.2.Labor Discipline Theory - From Bowles & Boyer (1988, 1989)

$\omega_f -$ single firm's wage rate per hour worked, $\omega_a -$ wage offered by any other firm

$h -$ employment rate as a proxy of the probability of reemployment.
Employment rent - $\omega_f - h\omega_a$

$$\varepsilon = \ln(\omega_f - h\omega_a) \tag{A.22}$$

$\omega^o -$ Labor cost-minimizing wage rate offered by a single competitive firm (optimal wage rate)
Firm's profit Maximization per effective work unit (ω/ε) at eq. wage:

$$\omega^o = \varepsilon\left(\frac{\partial\varepsilon}{\partial\omega_f}\right)^{-1}$$

$$\omega^o = (\omega_f - h\omega_a)\ \ln(\omega_f - h\omega_a) \tag{A.23}$$

ω_a and h are given for individual firm, but vary in the aggregate

$$\frac{d\omega^o}{dh} = \left(2 - h\frac{\omega_a}{\omega^o}\right)\omega_a$$

Aggregate effect:

$$\frac{d\omega}{dh} = \frac{d\omega^o}{dh}\left(1 - \frac{h}{\omega_a}\frac{d\omega^o}{dh}\right)^{-1} = \frac{\left(2 - h\frac{\omega_a}{\omega^o}\right)\omega_a}{1 - h\left(2 - h\frac{\omega_a}{\omega^o}\right)}$$

Equilibrium as a uniform wage rate $\omega = \omega_f = \omega_a$

$$\frac{d\omega}{dh} = \frac{(2-h)\omega}{(1-h)^2}$$

Integrating, we have the market real wage as function of employment rate $\omega^* = \omega^*[h]$

$$\omega^* = c\,\frac{\exp\left(\frac{1}{1-h}\right)}{1-h} \tag{A.24}$$

c – integration constant normalized to 1

$\omega^* = \frac{\exp\left(\frac{1}{1-h}\right)}{1-h} = (1 + ulk)\exp(1 + ulk)$ and $\varepsilon^* = \frac{1}{1-h} = 1 + ulk \Rightarrow h = \frac{ulk}{\varepsilon} = \frac{ulk}{1+ulk}$

The equilibrium wage share: $\psi^* = \omega^*/\varepsilon^* \Rightarrow \psi^* = l\exp\left(\frac{1}{1-h}\right) = l\exp(1 + ulk)$

$$\dot{\psi} = \tau(\psi^* - \psi)$$

$$\dot{\psi} = \tau[(l\exp(1 + ulk) - \psi)] \tag{A.25}$$

Partial derivatives:

$\frac{\partial\dot{\psi}}{\partial u} = \tau l^2 k\psi$	$\frac{\partial\dot{\psi}}{\partial\psi} = -\tau$

Annex 2.3. Model derivation:
Effective demand
Profit rate: r, Revenue from foreign investments: $j\xi b$
Net foreign asset rate (b): $b = \frac{B}{K}$
The capitalized value of net profits gives the asset value of invested capital:
$$q = \frac{(1 - t_b)(\pi u + j\xi b)}{j} = \frac{p_E E}{K}$$
$t_b -$ tax rate of profits. g – Investment rate (capital accumulation).
$$g = \alpha\pi u + \gamma$$
$$g = \frac{\dot{K}}{K} = \widehat{K}$$

$$\alpha = \alpha^o(1 - t_b)$$
Trade account:
$$\chi K + j\xi B + \left(S_f\right) = \xi a X = Y_f$$
$\xi a X$ – Technologically fixed component of imported inputs
$\xi^\eta x K$ - a component of "exchange rate-sensitive competitive net exports"
Λ – Component of net imports that respondes elastically to excess supply
$$\chi K = \xi^\eta x K - \Lambda \quad \text{(A.26)}$$

Equilibrium between total aggregate demand and total supply:
$$C_h + G + \chi K + gK = \psi X + \pi X + \xi a X \quad \text{(A.27)}$$
$$\chi K = \psi X + \pi X + \xi a X - C_h - G - gK$$
Or
$$\Lambda = \xi^\eta x K - \chi K$$
$$\Lambda = \xi^\eta x K - (\psi X + \pi X + \xi a X - C_h - G - gK)$$
$$\Lambda = \xi^\eta x K - \psi X - \pi X - \xi a X + C_h + G + gK$$
$$\Lambda = gK - (\psi X - C_h) - (\pi X) + (\xi^\eta x K - \xi a X) + G$$
Being $G = T_h + T_b$
$$\Lambda = gK - (\psi X - C_h) - (\pi X) + (\xi^\eta x K - \xi a X) + T_h + T_b$$
$$\Lambda = gK - (\psi X - C_h - T_h) - (\pi X - T_b) + (\xi^\eta x K - \xi a X)$$
Adding to both sides $j\xi B$ and D_b, then:

$$\Lambda = gK - (\psi X + D_b - C_h - T_h) - (\pi X + j\xi B - D_b - T_b) + (\xi^\eta x K + j\xi B - \xi a X) \quad \text{(A.28)}$$

Consumption demand depends on a fraction of their wealth ($p_E E$). As:
$$p_E E = qK \Rightarrow cP_E E = cqK$$
Wealth Effect*:* cqK
$$v^o = \frac{c}{j}$$
s_h^o – propensity to save of households out of net income:
s_b^o – propensity to save of firms our of net income

$$s_h = s_h^o(1 - t_h)$$
$$s_b = s_b^o(1 - t_b)$$
υ − Propensity to save out of capital gains
$$\upsilon = \upsilon^o(1 - t_b)$$
Household and firms's total savings in units of capital:
$$\sigma^h = \frac{(\psi X + D_b - C_h - T_h)}{K} = s_h[(1 - s_b)(\pi u + j\xi b) + \psi u] - \upsilon(\pi u + j\xi b)$$
$$\sigma^b = \frac{(\pi X + j\xi B - D_b - T_b)}{K} = s_b(\pi u + j\xi b)$$
Total Savings:
$$\sigma = \sigma^h + \sigma^b = s_h[(1 - s_b)(\pi u + j\xi b) + \psi u] - \upsilon(\pi u + j\xi b) + s_b(\pi u + j\xi b)$$

Defining: $s_p = (1 - t_b)[s_h^o(1 - t_h)(1 - s_b^o) - \upsilon^o + s_b^o]$
Then: $\sigma = s_p(\pi u + j\xi b) + s_h\psi u$
z – sum of current account components that respond to the RER
$$z = \xi^\eta x + j\xi b - \xi a u \tag{A.29}$$
<u>Production adjustment</u>
λ – fraction of excess demand that is filled by imports
$$\lambda\Lambda = \dot{u}K \Rightarrow \dot{u} = \frac{\lambda\Lambda}{K}$$
From the excess demand function:
$$\Lambda = gK - (\psi X + D_b - C_h - T_h) - (\pi X + j\xi B - D_b - T_b) + (\xi^\eta xK + j\xi B - \xi aX)$$
$$\dot{u} = \frac{\lambda\big(gK - (\psi X + D_b - C_h - T_h) - (\pi X + j\xi B - D_b - T_b) + (\xi^\eta xK + j\xi B - \xi aX)\big)}{K}$$
$$\dot{u} = \frac{\lambda[g - (\psi u + D_b - C_h - T_h) - (\pi u + j\xi b - D_b - T_b) + (\xi^\eta x + j\xi b - \xi a u)]}{K}$$
$$\sigma = s_p(\pi u + j\xi b) + s_h\psi u$$
$$\sigma = (\psi u + \pi u + j\xi b - C_h - T_h - T_b)$$
$$z = \xi^\eta x + j\xi b - \xi a u$$
Tough passage. Saving-investment condition becomes the law of motion of the capacity utilization change in the long-run:
$$\dot{u} = \lambda(g + z - \sigma)$$
Being:
$$z = \xi^\eta x + j\xi b - \xi a u$$
$$\sigma = s_p(\pi u + j\xi b) + s_h\psi u$$
$$g = \alpha\pi u + \gamma$$

Then:
$$\dot{u} = \lambda\{[(\alpha - s_p)\pi - s_h\psi - \xi a]u + \gamma + \xi^\eta x + (1 - s_p)j\xi b\} \tag{A.30}$$

Partial Derivatives:
$$\frac{\partial\dot{u}}{\partial u} = -\lambda[(s_p - \alpha)\pi + s_h\psi + \xi a]$$
$$\frac{\partial\dot{u}}{\partial\psi} = \lambda\left[\frac{\partial\xi}{\partial\psi}(\eta\xi^{\eta-1}x - a u) - (s_p - \alpha)\left(\frac{\partial\pi}{\partial\psi}\right)u + \frac{\partial\xi}{\partial\psi}(1 - s_p)jb - s_h u\right]$$

$$\frac{\partial \dot{u}}{\partial b} = \lambda\left(1 - s_p\right)j\xi$$

<u>External Balance</u>:
Current account surplus is an increase of claims of the foreign sector:

$$-S_f = \xi\dot{B}$$

$$\xi\dot{B} = (\xi x + j\xi b - \xi a u)K - \Lambda$$

Dynamic equation of the share of foreign good priced debt as a function of capacity utilization and growth rate:

$$\dot{b} = x - au - \dot{u}(\xi\lambda)^{-1} + (j - g)b$$

Using $\dot{u} = \lambda(g + z - \sigma)$

$$\dot{b} = \frac{(\sigma - g)}{\xi} - gb$$

$$\dot{b} = \frac{\left(s_p - \alpha\right)\pi u + s_h\psi u - \gamma}{\xi} - \left(g - s_p j\right)b \qquad \text{(A.31)}$$

Partial derivatives:

$$\frac{\partial \dot{b}}{\partial u} = \frac{1}{\xi}\left[\frac{\partial \sigma}{\partial u} - \frac{\partial g}{\partial u}(1 + \xi b)\right] \quad \Bigg| \quad \frac{\partial \dot{b}}{\partial \psi} = \frac{1}{\xi}\left(\frac{\partial \sigma}{\partial \psi} - \frac{\partial g}{\partial \psi}\right) - \frac{\partial \xi}{\partial \psi}\frac{(\sigma - g)}{\xi^2} - \frac{\partial g}{\partial \psi}b \quad \Bigg| \quad \frac{\partial \dot{b}}{\partial b} = -\left(g - s_p j\right)$$

Asset equilibrium:

$$b = \frac{s_p\pi + s_h\psi u - g}{g - s_p j}$$

Steady state $\dot{u} = \dot{\psi} = \dot{b} = 0$

$$b^* = \frac{\xi^{*\eta}x - \xi^* a u^*}{g^* - j}$$

Annex 2.4. Dynamic Models

$$\dot{\psi} = \tau[(l\exp(1+ulk)-\psi)]$$

$$\dot{u} = \lambda\{[(\alpha-s_p)\pi - s_h\psi - \xi a]u + \gamma + \xi^{\eta}x + (1-s_p)j\xi b\}$$

$$\dot{b} = \frac{(s_p-\alpha)\pi u + s_h\psi u - \gamma}{\xi} - (g-s_pj)b$$

Jacobian (ψ, u, b):

$$J = \begin{bmatrix} -\tau & \tau l^2 k\psi & 0 \\ \lambda\left[\frac{\partial\xi}{\partial\psi}(\eta\xi^{\eta-1}x - au) - (s_p-\alpha)\left(\frac{\partial\pi}{\partial\psi}\right)u + \frac{\partial\xi}{\partial\psi}(1-s_p)jb - s_h u\right] & -\lambda[(s_p-\alpha)\pi + s_h\psi + \xi a] & \lambda(1-s_p)j\xi \\ \frac{1}{\xi}\left(\frac{\partial\sigma}{\partial\psi} - \frac{\partial g}{\partial\psi}\right) - \frac{\partial\xi}{\partial\psi}\frac{(\sigma-g)}{\xi^2} - \frac{\partial g}{\partial\psi}b & \frac{1}{\xi}\left[\frac{\partial\sigma}{\partial u} - \frac{\partial g}{\partial u}(1+\xi b)\right] & -(g-s_pj) \end{bmatrix}$$

$$\frac{\partial\dot{\psi}}{\partial\psi} = -\tau \qquad \frac{\partial\dot{\psi}}{\partial u} = \tau l^2 k\psi$$

$$\frac{\partial\dot{u}}{\partial\psi} = \lambda\left[\frac{\partial\xi}{\partial\psi}(\eta\xi^{\eta-1}x - au) - (s_p-\alpha)\left(\frac{\partial\pi}{\partial\psi}\right)u + \frac{\partial\xi}{\partial\psi}(1-s_p)jb - s_h u\right] \qquad \frac{\partial\dot{u}}{\partial u} = -\lambda[(s_p-\alpha)\pi + s_h\psi + \xi a]$$

$$\frac{\partial\dot{u}}{\partial b} = \lambda(1-s_p)j\xi \qquad \frac{\partial\dot{b}}{\partial u} = \frac{1}{\xi}\left[\frac{\partial\sigma}{\partial u} - \frac{\partial g}{\partial u}(1+\xi b)\right]$$

$$\frac{\partial\dot{b}}{\partial\psi} = \frac{1}{\xi}\left(\frac{\partial\sigma}{\partial\psi} - \frac{\partial g}{\partial\psi}\right) - \frac{\partial\xi}{\partial\psi}\frac{(\sigma-g)}{\xi^2} - \frac{\partial g}{\partial\psi}b \qquad \frac{\partial\dot{b}}{\partial b} = -(g-s_pj)$$

<u>Expanding the derivatives</u>

$$\dot{\psi} = \tau[(l\exp(1+ulk)-\psi)]$$

$$\dot{u} = \lambda\{[(\alpha-s_p)\pi - s_h\psi - \xi a]u + \gamma + \xi^{\eta}x + (1-s_p)j\xi b\}$$

$$\dot{b} = \frac{(s_p-\alpha)\pi u + s_h\psi u - \gamma}{\xi} - (g-s_pj)b$$

$$\xi = \frac{1-\psi}{a\left(1+\frac{1}{\eta}\right)} \qquad \pi = \frac{1-\psi}{\eta\left(1+\frac{1}{\eta}\right)}$$

$$\frac{\partial\xi}{\partial\psi} = \left[-\frac{1}{a\left(1+\frac{1}{\eta}\right)}\right] \qquad \frac{\partial\pi}{\partial\psi} = \left[-\frac{1}{\eta\left(1+\frac{1}{\eta}\right)}\right]$$

$$\sigma = s_p(\pi u + j\xi b) + s_h\psi u$$

$$\frac{\partial\sigma}{\partial\psi} = s_p u\frac{\partial\pi}{\partial\psi} + s_p jb\frac{\partial\xi}{\partial\psi} + s_h u \Rightarrow \qquad \frac{\partial\sigma}{\partial u} = s_p\pi + s_h\psi$$

$$\frac{\partial\sigma}{\partial\psi} = \left[s_h u - s_p u\frac{1}{\eta\left(1+\frac{1}{\eta}\right)} - s_p jb\frac{1}{a\left(1+\frac{1}{\eta}\right)}\right] \qquad \frac{\partial\sigma}{\partial u} = \left[s_p\frac{1-\psi}{\eta\left(1+\frac{1}{\eta}\right)} + s_h\psi\right]$$

$$g = \alpha\pi u + \gamma = \alpha u \frac{1-\psi}{\eta\left(1+\frac{1}{\eta}\right)} + \gamma$$

$$\frac{\partial g}{\partial \psi} = \left[\frac{-\alpha u}{\eta\left(1+\frac{1}{\eta}\right)}\right] \qquad \frac{\partial g}{\partial u} = \left[\alpha \frac{1-\psi}{\eta\left(1+\frac{1}{\eta}\right)}\right]$$

<u>Model expansion:</u>

$$\dot{\psi} = \tau[(l \exp(1+ulk) - \psi)]$$
$$\dot{u} = \lambda\{[(\alpha - s_p)\pi - s_h\psi - \xi a]u + \gamma + \xi^\eta x + (1-s_p)j\xi b\}$$
$$\dot{b} = \frac{(s_p - \alpha)\pi u + s_h\psi u - \gamma}{\xi} - (g - s_p j)b$$
$$\dot{l} = \rho g + \theta b + \phi l$$

Steady State

$$\dot{l} = 0 \Rightarrow \rho g + \theta b + \phi l = 0 \Rightarrow l^* = -\frac{\theta b^* + \rho g^*}{\phi}$$

$$\dot{b} = 0 \Rightarrow \frac{(s_p - \alpha)\pi u + s_h\psi u - \gamma}{\xi} - (g - s_p j)b = 0 \Rightarrow b^* = \frac{(s_p - \alpha)\pi u^* + s_h\psi u^* - \gamma}{\xi^*(g^* - s_p j)}$$

$$\dot{\psi} = 0 \Rightarrow \tau[(l \exp(1+ulk) - \psi)] = 0 \Rightarrow \psi^* = l^* \exp(1+u^*l^*k)$$

$$\dot{u} = 0 \Rightarrow \lambda\{[(\alpha - s_p)\pi - s_h\psi - \xi a]u + \gamma + \xi^\eta x + (1-s_p)j\xi b\} = 0 \Rightarrow$$
$$u^* = -\frac{[\gamma + \xi^{*\eta} x + (1-s_p)j\xi^* b^*]}{[(\alpha - s_p)\pi^* - s_h\psi^* - \xi^* a]} = 0$$

Partial Derivatives and Jacobian (added to the original model):

$$\frac{\partial \dot{l}}{\partial \psi} = \rho\left[\frac{-\alpha u}{\eta\left(1+\frac{1}{\eta}\right)}\right] > 0 \qquad \frac{\partial \dot{\psi}}{\partial l} = \tau(kul+1)\exp(kul+1) > 0$$

$$\frac{\partial \dot{l}}{\partial u} = \alpha\pi > 0$$

$$\frac{\partial \dot{u}}{\partial l} = 0 \qquad \frac{\partial \dot{l}}{\partial b} = \theta < 0$$

$$\frac{\partial \dot{b}}{\partial l} = 0 \qquad \frac{\partial \dot{l}}{\partial l} = \phi < 0$$

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