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STRUCTURAL TRANSFORMATION IN GENERAL EQUILIBRIUM

Alessio Moro[#] and Carlo Valdes[*]

Abstract

Models of structural change in general equilibrium are commonly used to address a number of questions regarding the behaviour of the macro-economy. In this paper, we first revise the main mechanisms at work in generating structural change in a multi-sector environment. These effects emerge due to both an interaction between consumers' preferences and technological change and to different income elasticities of the various goods and services entering the utility function. Next, we address the issue of measurement of these models when comparing them to the data. The typical assumption in multi-sector models is to define GDP as aggregate output in units of a numeraire good, often chosen to be the investment good. However, this procedure is equivalent to deriving nominal GDP in the data (i.e. total output of the economy in units of one particular good), and not to deriving a measure of real GDP. We then discuss how GDP in the model should be measured to provide a statistic that is comparable with the data in national accounts. The last part of the paper is devoted to show how structural transformation from manufacturing to services, when appropriately compared to the data, generates a decline in GDP growth and volatility along the growth path of an economy.

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1. Introduction

In recent decades, multi-sector general equilibrium models have become popular as a means of addressing a number of issues in macroeconomics and have provided insights into how the process of structural transformation affects several dimensions of the macroeconomy. Two of these dimensions are GDP growth and GDP volatility. Moro (2015), for example, studies how different sectoral compositions of GDP can account for differences in GDP growth across countries. He finds that a rise in the importance of the services sector can explain most of the differences in GDP growth measured in cross-country data. Leon-Ledesma & Moro (2019) show that the process of structural change can explain the increasing real investment rates observed across countries as income grows. Moro (2012) shows that an increase in the services sector share causes a reduction in the volatility of aggregate TFP, GDP, manufacturing and services consumption, investment and employment. Similarly, Carvalho & Gabaix (2013) show that the Great Moderation in the US is due to a decline in the manufacturing sector, while the recent increase of macroeconomic volatility is due to the growth of the financial sector. More recently, work by Duernecker et al. (2017a) studies what will be the effect of structural change on growth in the future of the US economy, finding that this process will reduce aggregate productivity growth by half as much as in the past. General equilibrium models of structural change are also employed to study issues related to monetary policy (Galesi & Rachedi 2019), international trade (Kehoe & Ruhl 2013), gender issues (Ngai & Petrongolo 2017; Cerina et al. 2017), aggregate productivity (Duarte & Restuccia 2010; Herrendorf & Valentinyi 2012), home production (Ngai & Pissarides 2008; Moro et al. 2017), and the skill-premium (Buera & Kaboski 2012).

This paper discusses three main issues related to structural change in general equilibrium models. The first issue relates to the mechanisms at work in generating structural change in this class of models. The second is concerned with how we should relate these models to the data. In particular, we will consider the appropriate concept of GDP in these models. The third issue we discuss is an application of a model of structural transformation to the growth and volatility experience of middle- and high-income countries. In addressing these issues, we will aim at maximising intuition by keeping the amount of algebra to a minimum.

Regarding the first issue, there are two main mechanisms generating structural change in general equilibrium in a closed economy. The first mechanism emerges from the interaction between *consumers preferences* and *technological change* that is differential across sectors. To fix intuition, consider the extreme case of an economy with only two firms, one producing coffee and the other sugar, and a representative consumer with Leontief preferences over the two goods and providing its unit of time to the two firms in exchange for a wage. Assume that markets are under perfect competition so that in equilibrium the price of output is always equal to the marginal cost. In this environment, regardless of the wage received, the consumer demands every cup of coffee together with a spoon of sugar. If the technologies of the two firms are equally productive the consumer will sell half of its time to the coffee firm and half to the sugar firm. We thus have a general equilibrium in which in all markets (for goods and labour) demand is equal to supply. Assume now that for some reason the technology to produce sugar improves, so that the amount of labour time needed to produce one unit of sugar declines, and as a result the price of sugar relative to coffee declines. To understand the effect of this change on the general equilibrium we should ask how the consumer reacts to the change in the relative price. Due to Leontief preferences, after a change in the relative price of goods the substitution effect is zero, and only the income effect plays a role. As the consumer is willing to consume the two goods in a given proportion, the effect is that labour time flows out of the sugar sector and into to the coffee sector. Note the apparent counter intuitive result: labour flows towards the least productive sector! This is the well-known Baumol's cost disease, suggesting that if the elasticity of substitution across goods is sufficiently small, the least productive sectors attract most of the labour force in an economy. The second mechanism at work is the different *income elasticity* of the various goods and services in the economy. Going back to the coffee and sugar example above, consider now a utility function in which the income elasticity of coffee is larger than one and that of sugar smaller than one. In this case, as the consumer's income grows, she is going to increase the share of income devoted to purchasing coffee and decrease that used to purchase sugar. Note that this happens for given prices. In the case when prices are also changing, there will be two effects working contemporaneously; firstly, that due to the interaction between preferences and technology, and secondly, that working through the different income elasticities of consumption. In Section 2 we formalise these two effects in a two-sector model.

The second issue addressed in the paper regards measurement, and so the interpretation of the structural change model. This is not only a technical issue, but has important theoretical implications, because there are several potential measures of GDP in the model, and depending on the chosen one, the predicted effect of structural change on GDP could be dramatically different. A common feature of most applications of structural change is that they define aggregate output, which is the model's counterpart of real GDP in the data, as total production expressed in units of a numeraire good. This procedure is equivalent to what is done in the data when expressing total production in units of a currency, that is, when nominal GDP is constructed. However, this procedure delivers a variable that is not directly comparable with what is measured as real GDP in the data. In this paper, we show how to appropriately match the equilibrium of a structural change model to the data. This involves using equilibrium allocations and prices in the same way quantities and prices are used in the data to construct quantity and price indices. To do this we discuss the Fisher chain weighted quantity index that is used in national accounts and show that it is independent of the numeraire chosen (in the data and in the model).

Finally, we devote the last part of the paper to an application of structural transformation to GDP growth and volatility in which GDP is appropriately measured with a Fisher chain weighted index. We show that the process of structural transformation is tightly linked to the process of economic growth. This interaction works as follows: As income grows, structural transformation is generated through the mechanisms described above. In particular, as the consumer becomes richer, a higher income elasticity of services with respect to manufacturing and more rapid technological change in manufacturing induces the consumer to increase the share of services in consumption expenditure. As structural change occurs, capital and labour flow to the least productive sector (i.e. services) and this reduces the growth rate of GDP. These effects emerge only if the model is appropriately measured with a Fisher index. Otherwise, we would only observe in the model that growth affects structural transformation, but not that structural transformation affects growth.

The reminder of the paper is organised as follows: Section 2 presents a two-sector model of structural transformation and describes the main mechanisms at work; Section 3 discusses how to measure output in the model to allow a comparison with the data; Section 4 describes an application of the model to GDP growth and volatility; and finally, Section 5 concludes.

2. A Growth Model of Structural Transformation

In this section we present a two-sector general equilibrium growth model of structural transformation. Over time, the economy experiences growth due to exogenous technological change in the production functions of the two sectors. In the general equilibrium, these improvements in technology also generate structural transformation over time.

2.1. The Production Side

There are two sectors in the economy, manufacturing (m) and services (s). In each sector there is a representative firm producing under perfect competition using a Cobb–Douglas production function. Thus, technology in each sector is given by:

$$y_{it} = k_{it}^{\theta}(A_{it}n_{it})^{1-\theta}$$

where θ is a parameter defining the capital intensity in production, k_{it} is the amount of capital, n_{it} is the amount of labour in sector i, with $i = m, s$, and A_{it} is the technological level in sector i, which is exogenously given and growing at a rate $\gamma_i > 0$ over time. The manufacturing sector produces a good that can be used for consumption or investment. The output of services is used only for consumption.

Firms maximise profits (Π) at each point in time, t:

$$\max_{n_{it},k_{it}} \Pi_i = \max_{n_{it},k_{it}}\left[p_{it}k_{it}^{\theta}(A_{it}n_{it})^{1-\theta} - W_t n_{it} - R_t k_{it}\right]$$

where p_{it} is the price of output, R_t is the rental rate of capital and W_t is the wage rate. Given the form of the production functions two results emerge from profit maximisation that are particularly useful in our context. First, in general equilibrium the capital-labour ratio in each sector is equal to the aggregate one:

$$\frac{k_{it}}{n_{it}} = \frac{K_t}{N_t}, i = m, s$$

where K_t is the total amount of capital and N_t is the total amount of labour in the economy.

Second, if we choose manufacturing as the numeraire, $p_{mt} = 1$, the general equilibrium price of services relative to manufacturing is given by:

$$p_{st} = \left(\frac{A_{st}}{A_{mt}}\right)^{1-\theta} \quad (1)$$

Equation (1) implies that regardless of the allocation of capital and labour across sectors, the relative prices are technologically determined. Equation (1) also suggests that the production side of the economy is summarised by the evolution of technology across sectors. This further implies that the aggregate production possibility frontier of the economy is linear and so, regardless of the actual amount of capital and labour used in the two sectors, it is always possible to give up production of one unit of services in the economy to obtain p_{st} units of manufacturing in exchange. The optimal response of the consumer to the change in p_{st} determines structural change in the economy.

Finally, in each period, the feasibility conditions are such that in general equilibrium capital and labour demanded in the two sectors must be equal to their aggregate supply:

$$k_{mt} + k_{st} = K_t$$

$$n_{mt} + n_{st} = N_t = 1$$

and aggregate labour is normalised to one.

2.2. The Demand Side

The demand side of the model is given by an intertemporal maximisation problem in which the consumer chooses both consumption and savings over time, and how to allocate consumption in each period between manufacturing and services.[1]

The maximisation problem is:

[1] The reader is referred to Herrendorf et al. (2014) for a similar exposition in a three-sector model.

$$\max_{\{c_{mt}, c_{st}, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log C_t$$

Subject to: $$c_{mt} + p_{st}c_{st} + K_{t+1} = K_t(1 - \delta + R_t) + W_t$$

And to: $$C_t = \left(\omega_m^{1/\varepsilon}(c_{mt})^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{1/\varepsilon}(c_{st} + \bar{c}_s)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

$$c_{mt} + p_{st}c_{st} = P_t C_t - p_{st}\bar{c}_s$$

Here C_t is the aggregate consumption index, which depends on the amount of manufacturing consumption c_{mt} and services consumption c_{st}. The parameter $\bar{c}_s$ is positive and is interpreted as the home production of services. Technically, this parameter implies that the income elasticity of services is larger than one, and that of manufacturing smaller than one. This non-homothetic parameter implies that the consumption index C_t multiplied by the price index P_t does not equate consumption expenditure $c_{mt} + p_{st}c_{st}$, a detail clarified in the last constraint. The parameter β is the subjective discount factor, δ is the depreciation rate of capital, ω_m and ω_s are weights of the two types of consumption in preferences and ε governs the elasticity of substitution between manufacturing and services in preferences. [2] In what follows we show that $\bar{c}_s$ and ε are the key parameters to generate structural transformation as income grows.

2.3. The Intertemporal Problem

The above problem can be split into two separate problems, an intertemporal one in which the household chooses how to allocate resources over time and a static one in which the household decides how to split resources into the two types of consumption. The solution of the intertemporal problem provides a *Euler equation* stating the relationship between nominal consumption spending in two periods of time. We can express the Euler equation as:

[2] Note that the consumer provides N_t amount of time in the labour market so the total wage received is $W_t N_t$. However, we normalised $N_t = 1$.

$$\frac{1}{\beta}\frac{P_tC_t}{P_{t-1}C_{t-1}} = R_t + 1 - \delta \qquad (2)$$

This equation states that the ratio of consumption expenditure in two different periods depends on the subjective discount factor β, the return on capital R_t and the depreciation rate δ.

2.4. The Static Problem

The static problem is the one that generates structural transformation over time. This is:

$$\max_{c_{mt}, c_{st}} \left(\omega_m^{1/\varepsilon}(c_{mt})^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{1/\varepsilon}(c_{st} + \bar{c}_s)^{\frac{\varepsilon-1}{\varepsilon}}\right)$$

(3)

Subject to:

$$c_{mt} + p_{st}c_{st} = \bar{w}_t$$

In this problem, total expenditure at time t, $\bar{w}_t$, is taken as given, as it is derived in the intertemporal problem and it is equal to $P_tC_t - p_{st}\bar{c}_s$. Thus, in Equation (3), the consumer takes as given the total amount of consumption she needs to attain, C_t, and looks for the cheapest combination of the two goods that allow her to attain such an amount.

Consider now the first order conditions for Equation (3) in the case $\bar{c}_s = 0$. These conditions deliver:

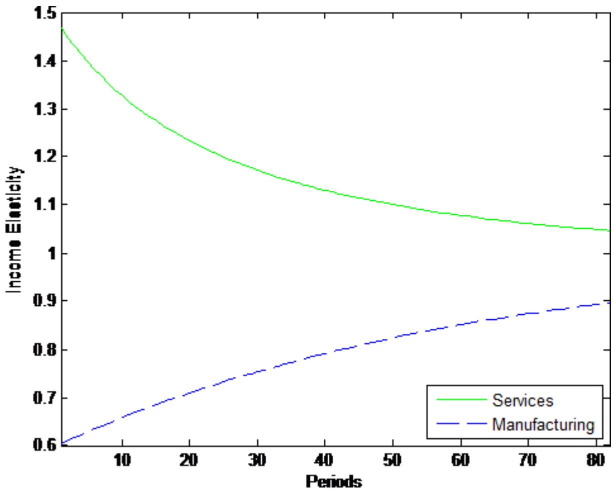
$$\frac{p_{st}c_{st}}{c_{mt}} = \frac{\omega_s}{\omega_m}(p_{st})^{1-\varepsilon}$$

The left-hand side of this equation is the ratio of the share of services $p_{st}c_{st}/(c_{mt} + p_{st}c_{st})$ to the share of manufacturing $c_{mt}/(c_{mt} + p_{st}c_{st})$ in consumption. By defining the variable $x = p_{st}c_{st}/c_{mt}$ we can analyse how this variable changes with an increase in the relative price, p_{st}.

$$\frac{dx}{dp_{st}} = (1-\varepsilon)\frac{\omega_s}{\omega_m}(p_{st})^{-\varepsilon} \qquad (4)$$

The above derivative is positive only when $0 < \varepsilon < 1$. Thus, an increase in the relative price of services, p_{st}, increases the share of services relative to the share of manufacturing only if the elasticity of substitution, ε, is between zero and one, that is when goods and services are imperfect complements. [3]

Figure 1: Income Elasticity of Manufacturing and Services Consumption



Source: Moro (2015).

Consider now the case in which $\bar{c}_s > 0$. In this case, the solution of the model is given by:

$$c_{mt} = \frac{\bar{w}_t + p_{st}\bar{c}_s}{p_{st}^{1-\varepsilon}\frac{\omega_s}{\omega_m} + 1}$$

$$c_{st} = \frac{\bar{w}_t + p_{st}\bar{c}_s}{p_{st}^{\varepsilon}\frac{\omega_m}{\omega_s} + p_{st}}$$

Note that in the static problem given by Equation (3), $\bar{w}_t$ can be also defined as the level of income of the consumer which is spent on the two types of consumption goods. Endowed with

[3] The reader is referred to Ngai & Pissarides (2007) for a detailed description of this mechanism.

this definition, we can compute the income elasticity of c_{mt} and c_{st} as:

$$\xi_{mt} = \frac{\bar{w}_t}{\bar{w}_t + p_{st}\bar{c}_s} \tag{5}$$

$$\xi_{st} = \frac{\bar{w}_t}{\bar{w}_t - p_{st}^{\varepsilon}\frac{\omega_m}{\omega_s}\bar{c}_s} \tag{6}$$

These are also reported in Figure 1, which uses the parametrisation in Moro (2015). There are two things to note about equations (5) and (6). First, the income elasticity of manufacturing is smaller than one while the income elasticity of services is larger than one. This is due to the fact that the non-homothetic term $\bar{c}_s$ is larger than zero. Second, the two elasticities are time-varying and they both converge to one asymptotically. Thus, *ceteris paribus*, as income grows the consumer increases the share of services and decreases that of manufacturing. However, this process slows down over time and when income is sufficiently large, both elasticities converge to one, and the shares of the two sectors stabilise at a constant value.[4]

To sum up, there are two mechanisms generating structural change in the general equilibrium of the economy. The first emerges, which can be defined as a *relative price effect* from the interaction between consumers preferences and technological change that is differential across sectors. The second, which can be labelled as *income effect*, is due to the different income elasticity of the various goods and services in the economy. The works of Boppart (2014) and Comin et al. (2019) quantify, using different general equilibrium models, the amount of structural transformation generated by the two effects in the US, finding that the income effect can account for a proportion between 50% (Boppart 2014) and 75% (Comin et al. 2019), with the price effect accounting for the remaining part.

We conclude this section by noting that structural change in the model can be measured using one of the following three concepts: consumption shares, value added shares, or employment

[4] The reader is referred to Kongsamut et al. (2001) for a detailed description of this mechanism.

shares. The income and relative price effects described above induce not only a change in the shares of consumption in total consumption expenditure, but also an equivalent change in value added and employment shares of the various sectors. This is a key aspect of the labour market in the general equilibrium of the structural change model: labour is reallocated across sectors in the same manner as consumption is reallocated across sectors. If the consumption share of services increases, the employment share of the services sector in total employment also increases accordingly.[5]

3. From the Model to the Data

The general equilibrium model of structural transformation displays rich predictions regarding the behaviour of the economy both at the aggregate and at the sectoral level. As discussed above, there is both a theoretical effect going from growth to structural transformation, and one going from structural transformation to growth. However, in a multi-sector model, the concept of aggregate output is not well defined, as there are several ways to measure such a variable. It turns out that, depending on the way GDP is measured in the model, the effect of structural transformation on growth can be observed or not.

A common practice in the literature, in fact, is to assume that GDP in the model is given by aggregate output expressed in units of a numeraire good, usually chosen to be the price of the investment good. While this practice is easy to handle, it is also misleading because it involves comparing GDP in the data with something different from GDP in the model. Leon-Ledesma & Moro (2019) and Duernecker et al. (2017b), are the first to show how to construct GDP in a multi-sector model in a way that is consistent with the National Income and Production Accounts

[5] Note that the *level* of consumption shares differs from that of value added and employment shares due to the presence of investment, which is produced in the manufacturing sector. That is, in each period the consumption share of manufacturing is smaller than the value added and employment shares of manufacturing.

(NIPA) methodology. The procedure involves taking the prices and quantities of each sector which result from the general equilibrium of the model and using them to construct a chain-weighted Fisher index for GDP. Thus, in the same way statistical agencies construct GDP in the data by using observed prices and quantities of individual products, it is possible to construct a model derived measure of GDP. In what follows we closely follow the appendix in Leon-Ledesma & Moro (2019) to describe the methodology.

Consider the general equilibrium of the two-sector model of structural transformation described in Section 2. For two periods, t and $t-1$, we have the following objects: the price of services and manufacturing in the two periods ($p_{s,t}$, $p_{s,t-1}$, $p_{m,t}$, $p_{m,t-1}$) and the quantities of services and manufacturing in the two periods ($q_{s,t}$, $q_{s,t-1}$, $q_{m,t}$, $q_{m,t-1}$). Using these, we can construct the Laspeyres and Paasche quantity indices as computed by NIPA:

$$Q_t^L = \frac{p_{m,t-1}q_{m,t}+p_{s,t-1}q_{s,t}}{p_{m,t-1}q_{m,t-1}+p_{s,t-1}q_{s,t-1}}$$

$$Q_t^P = \frac{p_{m,t}q_{m,t}+p_{s,t}q_{s,t}}{p_{m,t}q_{m,t-1}+p_{s,t}q_{s,t-1}}$$

The Fisher quantity index is then given by a weighted average of the Laspeyres and Paasche indices

$$Q_t^F = \sqrt{Q_t^L Q_t^P} \tag{7}$$

Note that the Laspeyres quantity index is *independent* of the numeraire chosen. This is because it is a function of *relative prices*. To see this, divide the numerator and denominator by the same price at $t-1$ to give:

$$Q_t^L = \frac{q_{m,t}+\frac{p_{s,t-1}}{p_{m,t-1}}q_{s,t}}{q_{m,t-1}+\frac{p_{s,t-1}}{p_{m,t-1}}q_{s,t-1}}$$

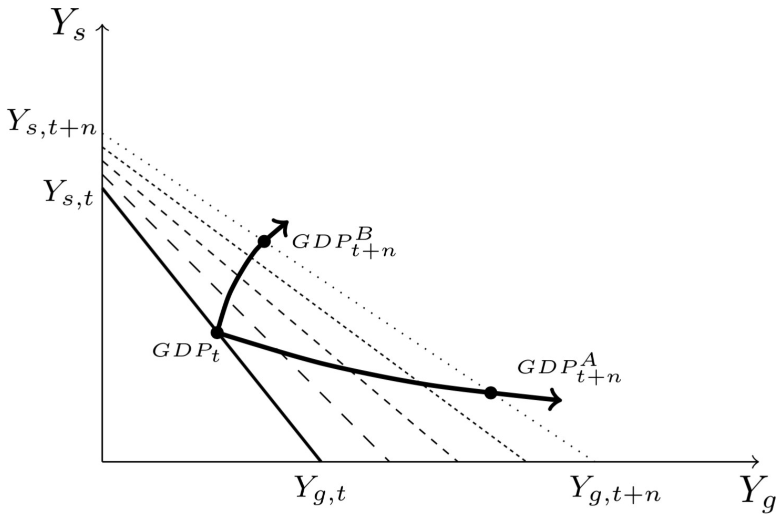
thus implicitly choosing manufacturing as the numeraire, or:

$$Q_t^L = \frac{\frac{p_{m,t-1}}{p_{s,t-1}} q_{m,t} + q_{s,t}}{\frac{p_{m,t-1}}{p_{s,t-1}} q_{m,t-1} + q_{s,t-1}}$$

implicitly choosing services as the numeraire. In a similar vein, it is possible to show that the Paasche index is independent of the numeraire. It follows that also the *Fisher quantity index*, which is a weighted average of the two indices Laspeyres and Paasche, *is independent of the numeraire*. This result has a powerful implication, because the same property holds both in the model and in the data. This means that regardless of the numeraire chosen, and even if the numeraire is different in the data to that in the model, the chain-weighted Fisher GDP index is comparable between data and model. This is clearly not the case for GDP expressed in units of the numeraire in the model since as long as the relative price $p_{m,t}/p_{s,t}$ varies over time, the behaviour of aggregate output in units of manufacturing is different from that of aggregate output in units of services.

To see the implications of appropriately measuring GDP in the model, consider Figure 2. This reports the evolution of the production possibility frontier of the economy described in Section 2 under the assumption that technological change grows faster in manufacturing than services, $\gamma_m > \gamma_s$. Thus, at each point in time, given the current endowment of capital and labour, the economy can place itself on any point of the frontier. For instance, at time t, the frontier is identified by the two points $(Y_{g,t}, 0)$ and $(0, Y_{s,t})$, where $Y_{g,t}$ is the total amount of manufacturing that can be produced in the economy if only manufacturing is produced, and $Y_{s,t}$ is the total amount of services that can be produced in the economy if only services is produced. Due to the assumption that $\gamma_m > \gamma_s$ the frontier changes slope as time passes, since if the economy produces only manufacturing, its total output can grow faster than if the economy produces only services. Thus, if the economy displays structural change, transitioning from a production more intensive in manufacturing to one more intensive in services as time passes, the growth rate of GDP slows down. However, this slowdown can be appreciated only if GDP is appropriately measured. As mentioned above, a common practice is to measure GDP in the model as total output in units of manufacturing. This amounts to measuring the growth rate of $Y_{g,t}$ along the x-axis in Figure 2. To put it differently, the growth rate of Y_g from t to t+n is given by the Euclidean distance between $Y_{g,t}$ and $Y_{g,t+n}$ which is, in turn, given by the segment on the x-axis that connects the two points. However, the growth rate of aggregate output expressed in units of manufacturing can be very different from the actual growth rate of GDP. To see this, consider the path in Figure 2 going from GDP_t in period *t* to GDP_{t+n}^A in period $t + n$. In these two cases GDP at each *t* is an index that combines manufacturing and services in different proportions. What is the growth rate of this index between the two periods? By the same logic used for $Y_{g,t}$ and $Y_{g,t+n}$ we can use the Euclidean distance between the two points. In this case, the distance between the two points takes into account that i) the relative price between the two goods is changing and ii) the quantities of the two products are changing. From Figure 2, it is evident that the distance between GDP_t and GDP_{t+n}^A is larger than the distance between GDP_t and GDP_{t+n}^B. This is because in the first case the economy transitions towards manufacturing, while in the second case it transitions towards services. Such transition has the effect of accelerating (if towards manufacturing) or decelerating (if towards services) GDP growth when GDP is appropriately measured. The Fisher formula is the analogue for the data of the Euclidean distance used in the example above. Thus, only when GDP is measured with the Fisher index does the theoretical prediction that structural change affects growth in general equilibrium emerge. To conclude this section, note that the reasoning made here extends to all applications that study the relationship between structural change and aggregate GDP.

Figure 2: Evolution of the Production Possibility Frontier and Two Possible GDP Trajectories



Source: Leon-Ledesma & Moro (2019)

4. An Application of Structural Change in General Equilibrium: GDP Growth and Volatility

Moro (2015) provides an example in which structural transformation in a general equilibrium setting, once brought to the data with a proper NIPA methodology, can deliver rich predictions on the behaviour of GDP. He uses a two-sector general equilibrium model of structural transformation between manufacturing and services to show that a large fraction of the differences in GDP growth and volatility between middle- and high-income economies can be accounted for by structural change alone. In this section we present a simplified version of Moro (2015) and the main quantitative results of that work. To do this, we modify the model presented in Section 2 by dropping capital from the model. This simplification implies that the household no longer has to solve the intertemporal problem, but only the static one in each period t. In turn, the only difference between two different periods is given by the technological level, which evolves exogenously in the two sectors. Thus, the general equilibrium over time becomes a sequence of static general equilibrium problems, one for each period t.

4.1. Model and General Equilibrium

Technology in each sector at time t is now given by:

$$y_{it} = A_{it} n_{it}, \tag{8}$$

with $i = m, s$. Using this production function, firms maximise profits as in Section 2.1 by choosing now simply the optimal amount of labour.

Using the same notation as in Section 2 for variables, the household solves, at each t:

$$\max_{\{c_{mt}, c_{st}\}} \left(\omega_m^{1/\varepsilon} (c_{mt})^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{1/\varepsilon} (c_{st} + \bar{c}_s)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

subject to:

$$c_{mt} + p_{st} c_{st} = W_t$$

To fix ideas, we define the concept of general equilibrium in each period t in this model:

Definition: In this economy there exists a competitive general equilibrium in each period t consisting of: a set of prices $\{p_{st}, p_{mt} = 1, W_t\}$, allocations for the representative household $\{c_{st}, c_{mt}, N_t\}$, manufacturing firm $\{n_{mt}\}$, and services firm $\{n_{st}\}$ such that, given prices: (a) the allocations $\{c_{st}, c_{mt}, n_t\}$ solve the household problem; (b) the amount of labour $\{n_{mt}\}$ solves the manufacturing firm problem and the amount of labour $\{n_{st}\}$ solves the services firm problem; and (c) all markets clear.

Moro (2015) discusses that in this environment it is possible two derive two aggregate production functions, one in manufacturing units and one in services units. These are:

$$V_{mt} = A_{mt}N_t \tag{9}$$

and

$$V_{st} = A_{st}N_t \tag{10}$$

In equilibrium, total labour in the economy is $N_t = 1$ at each t, so the difference between the two cases is given by the aggregate TFP term, which is:

$$TFP_{mt} = A_{mt}, \tag{11}$$

in the first case and

$$TFP_{st} = A_{st} \tag{12}$$

in the second. This specification is convenient because the behaviour of the production possibility frontier (PPF) evolves along the two axes in Figure 2 and is summarised uniquely by the evolution of technology in the two sectors: A_{mt} determines how the PPF moves along the x-axis and A_{st} determines how the frontier moves along the y-axis.

Consider now:

Assumption 1: *Technology in manufacturing and services evolves at time* t *according to a growth factor given by:*

$$\frac{A_{mt}}{A_{m,t-1}} = (1+\gamma_m)e^{z_{mt}} \tag{13}$$

And

$$\frac{A_{st}}{A_{s,t-1}} = (1+\gamma_s)e^{z_{st}} \tag{14}$$

where $\gamma_m > \gamma_s$, *and* z_{mt} *and* z_{st} *are random components with zero mean, and finite variance and standard deviations such that* $\text{sd}(z_{mt}) > \text{sd}(z_{st})$.

Under Assumption 1, an economy that produces only manufacturing can grow faster but is also more volatile than an economy producing only services. This suggests that when structural change occurs from manufacturing to services in this environment, we should observe both a decline in the growth rate of GDP, and a decline in the volatility of GDP.

Assumption 2: $\bar{c}_s = 0$, $\omega_s = 1 - \omega_m$ and $\varepsilon = 1$.

Under Assumption 2, we have that the consumption index becomes $c = \log\left(c_m^{\omega_m} c_s^{1-\omega_m}\right)$ and sectors' size relative to output is fixed and given by ω_m for manufacturing and $1 - \omega_m$ for services. Also, with no investment in the economy, consumption coincides with GDP, $y = c$. We can now characterise the relationship between output growth and sectors' size:

Proposition 1. *Under assumptions 1 and 2, when the variance of z_{mt} and z_{st} is zero (deterministic growth), the GDP growth rate is given by:*

$$\gamma_y = \omega_m \gamma_m + (1 - \omega_m)\,\gamma_s,$$

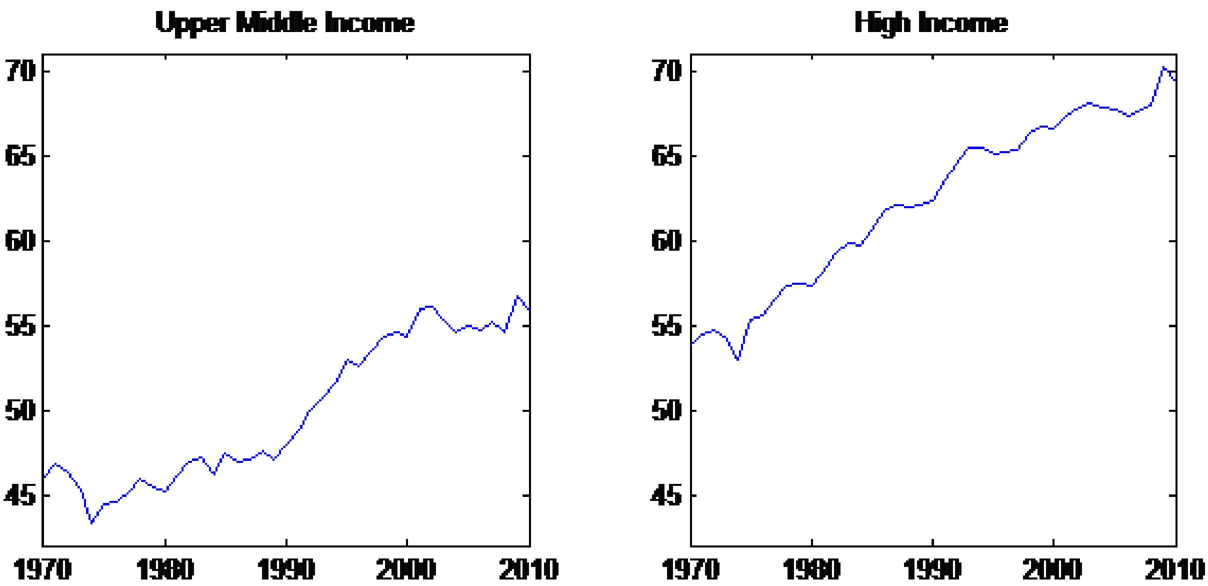
and the larger is ω_m, the larger is output growth and aggregate TFP growth. A similar result can be obtained for output volatility.

Proposition 2. *Under assumptions 1 and 2, the standard deviation of GDP growth is given by:*

$$\text{sd}\left(\gamma_y\right) = \omega_m \text{sd}(z_{mt}) + (1 - \omega_m)\text{sd}(z_{st})$$

and the larger is ω_m, the larger is output volatility and aggregate TFP volatility.

Figure 3: Share of Services in GDP in Upper Middle-Income Countries and High-Income Countries from 1970 to 2010



Source: Moro (2015) based on World Bank data.

Propositions 1 and 2 state that the larger the relative size of manufacturing (which displays higher TFP growth and volatility), the higher is the growth rate and volatility of output and aggregate TFP. Thus, we can expect that in a model in which the relative size of sectors changes with structural transformation, this process affects GDP growth and volatility. We can then use the model to quantitatively assess the importance of structural composition for the observed differences in GDP growth and volatility between upper-middle and high-income economies. To do this we drop Assumption 2 to study a model economy in which the relative size of services endogenously increases over time as TFP grows in the two sectors. In this case, the effect on aggregate output of TFP movements depends both on the structure of the economy and on the speed of structural change (which in turn depends on the parameters ε and $\bar{c}_s$), and the equivalent to propositions 1 and 2 cannot be derived. However, an appropriately calibrated version of the model can provide quantitative results suggesting that the negative relationship between both aggregate output growth and volatility and the relative size of the services sector still holds with non-homotheticity of preferences and an elasticity of substitution between goods in consumption different from one. We devote the next subsection to this application.

4.2. Using the Model to make Quantitative Predictions

In this subsection we describe an application in which a general equilibrium model of structural transformation model is used to study GDP growth and volatility. The exercise was first proposed in Moro (2015). To motivate the exercise, we begin from the empirical observation that across countries the share of services increases with income levels. Figure 3 shows the average (across countries) share of services between 1970 and 2010 in two groups of countries defined by the World Bank as middle- and high-income.[6] Middle-income countries display in 2010 a level of the share of services similar to that of high-income countries in 1970. At the same time, per-capita GDP growth and volatility for the 1970-2010 period is larger for the group of middle-income countries than for the group of high-income countries. This is reported in the second and fourth columns of Table 1, which show that growth is 42% larger and volatility 30% larger in middle-income countries relative to high-income countries.

Given the above differences in the data, one might ask to what extent the difference in the sectoral composition between the two groups of countries plays a role in shaping differences in GDP growth and volatility. To answer this question, we can use the general equilibrium model described in Section 4.1 and study how GDP growth and volatility change as the share of

[6] See Moro (2015) for details of the dataset used.

services increases. We retain Assumption 1 from that section, while we drop Assumption 2. Assumption 1 is motivated by the fact that across countries a large body of empirical evidence suggests that TFP grows faster and is more volatile in manufacturing than in services. Thus, the underlying assumption of the exercise is that manufacturing and services display intrinsic differences in their technological change, both in the long- and the short-run. The intuition here is that if TFP grows faster and is more volatile in the manufacturing sector than in the services sector, structural change from manufacturing to services should reduce both aggregate GDP growth and volatility. We thus measure TFP processes for manufacturing and services in the US and use this parametrisation for our exercise.[7]

Next, we need to calibrate the rest of parameters of the model. While a detailed description of the calibration is beyond the scope of this paper, we describe here the main intuition. Given technological change, we want the model to reproduce a full transition of the share of services from the initial value of middle-income economies in 1970 to the final value of high-income economies in 2010. If the model economy can reproduce this entire pattern of the share of services, we can then compare GDP growth and volatility in the first part of the transition of the model with middle-income economies and growth and volatility in the second part of the transition with high-income economies. To put it differently, given technological change we want the model to generate an increase in the share of services in GDP from 0.46 (the value in middle-income countries in 1970) to 0.69 (the value in high-income countries in 2010) in 82 periods. These 82 periods correspond to twice the number of years in the 1970-2010 period. We then take the ratio of GDP growth and volatility generated by the model in the first 41 periods, which correspond to middle-income countries in the data, with those generated in the last 41 periods, which correspond to high-income economies, and compare these figures with those in

[7] Thus, implicit in this procedure is that the US provides representative measures for all countries.

the data.

Table 1: Calibrated model versus data of upper-middle income (UMI) and hi-income (HI) countries

Counterfactual with US TFP processes				
	Average per capita GDP growth rate		Average per capita GDP growth volatility	
	Model	Data	Model	Data
Upper-middle income (Model first part)	2.61%	2.57%	2.14%	3.82%
High income (Model second part)	2.17%	1.81%	1.73%	2.93%
Ratio UMI/HI	1.20	1.42	1.24	1.30

Source: Moro (2015).

The results are reported in Table 1. In the data, middle-income countries display a growth rate of GDP which is 42% larger than high-income countries and a volatility that is 30% larger. By comparing the first and the second part of the transition of the model, we obtain a GDP growth rate 20% larger in the first part, and a volatility 24% larger in the second part. Thus, structural change in the model accounts for roughly half of the difference in GDP growth and 80% of volatility differences between middle- and high-income countries.

To conclude, we note that the results reported in Table 1 would be completely washed out were GDP to be measured in units of a numeraire good. This can easily be seen in the simplified model of Section 4.1. In this case, GDP measured in units of manufacturing (i.e. the numeraire) is given by:

$$V_{mt} = A_{mt}N_t$$

and with total labour in the economy always equal to $n_t = 1$, we have that GDP growth is always given by:

$$\frac{A_{mt}}{A_{m,t-1}} = (1 + \gamma_m)e^{z_{mt}}$$

regardless of the dimension of the share of services. Thus, measuring GDP in units of a numeraire good does not allow the model to display the property that structural transformation affects GDP.

5. Conclusions

In this paper we revised the main mechanisms driving structural change in general equilibrium. Structural change is generated either by an interaction between heterogeneous technological change at the sector level and consumer preferences, or by a non-homothetic component in the utility function. We then discussed how the outcome of a general equilibrium model should be compared to the data. The way GDP is constructed from the general equilibrium becomes key in the interpretation of the results, and so becomes an important part of the theory. We concluded the paper by reporting an application of a general equilibrium theory of structural change to GDP growth and volatility and showed that structural transformation can account for a large fraction of changes in GDP growth and volatility along the growth path.

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