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Thomas H. W. Ziesemer

Maastricht Economic and social Research institute on Innovation and Technology (UNU-MERIT)
email: info@merit.unu.edu | website: http://www.merit.unu.edu

Boschstraat 24, 6211 AX Maastricht, The Netherlands
Tel: (31) (43) 388 44 00

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Semi-endogenous growth models with domestic and foreign private and public R&D linked to VECMs

THOMAS H.W. ZIESEMER Department of Economics, Maastricht University, and UNU-MERIT. P.O. Box 616, 6200 MD Maastricht, The Netherlands. Phone: +31 43 38-83872/84402. Orcid.org/0000-0002-5571-2238. Email: T.Ziesemer@maastrichtuniversity.nl.

Abstract We present semi-endogenous growth models with total-factor productivity as functions of domestic and foreign private and public R&D. In a small country case with a Cobb-Douglas TFP production function, foreign R&D drives steady-state growth and the production function can be a long-term relation in a vector-error-correction model. Marginal productivity conditions can be long-term relations for a vector-error-correction model if the functional form is of a Cobb-Douglas type or a CES function generalised to a VES function. In case of a VES function, steady states exist only for special cases of parameter restrictions.

Keywords: Productivity, endogenous (un)balanced growth, public R&D expenditure, foreign spillover. JEL code: O38, O40, O41, H54, H87.

1. Introduction

In recent work, Soete et al. (2019) estimate the link between GDP, TFP, domestic and foreign private and public R&D using a vector-error-correction (VEC) approach. As this approach is a method letting the data speak (Juselius 2006) it is desirable to have a theoretical underpinning for it. Therefore, we develop (semi-) endogenous growth models that have steady state relationships that we could estimate in principle as long-term cointegrating equations of a VEC model. We explore the long-term relations for two alternative TFP production functions with domestic and foreign public and private R&D as inputs: The Cobb-Douglas function as a special case of the CES function and Mukerji's (1963) VES function generalising the CES function by way of replacing the CES parameter by several different parameters.

2. A Basic Model

2.1 Basics

The output production functions for the home and foreign (indicated by a '*') country, ignoring capital and labour, are

$$Q = A, Q^* = A^* \tag{1}$$

Total factor productivity function is

$$A = F(A_{-1}, R, t) \qquad \text{or } A^* = F^*(A^*_{-1}, R^*, t) \tag{2}$$

*R, R** are (1, 4) vectors of elements $R_j, R_j^*, j = b, g$ where *b* and *g* indicate business and non-business or (semi-) government R&D. Analogous to the ideas of 'learning by doing' (Arrow 1962), 'perspectives of experience' (Boston Consulting Group 1968), and 'learning from watching' (King and Robson 1993) we consider R&D capital stocks as research experience indicator. To keep the model simple, we assume that spillovers stem from R&D, for example through patenting including the publication of the innovation, and not from TFP, which includes complex phenomena such as organisation of the firms and other microeconomic and institutional issues, which are beyond the scope of this macroeconomic paper. R&D capital builds up according to the perpetual inventory method in the empirical literature (suppressing the index for current time),

$$R_{j,t+1} = R_j + I_j - \delta_j R_j \qquad j = b, g \tag{3}$$

Again, we assume a similar equation for a foreign country. Investment, I_j, includes the minimised costs for human capital, labs and low- or medium skill support of researchers. The advantage of this approach is that we do not have to model all of these factors separately.[1] The last term represents depreciation in the learning indicator. The disadvantage is that we cannot distinguish quantities and prices of factors in this simple version of the model. Doing so would lead us to larger models, which go beyond the limited number of variables typically included in vector-error-correction models, the starting point of our reasoning. A share *s* of output goes into R&D investment

$$I_j = s_j Q \qquad j = b, g \tag{4}$$

2.2 Dynamics and stability

With the production function (1) in the investment equation (4) that in the dynamic equations in (3) we get

$$R_{b,t+1} = R_b + s_b F(A_{t-1}, R_b, R_g; R_b^*, R_g^*) - \delta_b R_b,$$

$$R_{g,t+1} = R_g + s_g F(A_{t-1}, R_b, R_g; R_b^*, R_g^*) - \delta_g R_g \tag{3'}$$

Foreign R&D terms are exogenous for the home country. Subtracting current R-terms on both sides and dividing by them results in the growth rate version of these equations. Subtracting also $g \equiv \frac{A}{A_{-1}} - 1$ yields

$$g_b - g \equiv \frac{R_{b,t+1} - R_b}{R_b} - g = s_b F(A_{-1}, R_b, R_g; R_b^*, R_g^*)/R_b - \delta_b - g,$$

$$g_g - g \equiv \frac{R_{g,t+1} - R_g}{R_g} - g_A = s_g F(A_{-1}, R_b, R_g; R_b^*, R_g^*)/R_g - \delta_g - g \tag{3''}$$

Constant A/R_i ratios can insure constant growth rates. This requires that *A* and R_i have the same growth rate in the long run, and that this is compatible with the specification of the production functions *F* below. We define $y_g \equiv R_g/A$, $y_b \equiv R_b/A$. With these definitions, (3'') can be re-written as follows.

$$\hat{y}_b = \frac{s_b}{y_b} - \delta_b - g,$$

[1] It is possible though to model the underlying cost-minimisation in relation to the TFP production function of endogenous growth theory.

$$\hat{y}_g = \frac{s_g}{y_g} - \delta_g - g \qquad (3''')$$

Multiplying the first equation by y_b and the second by y_g we get

$$\dot{y} = s_b - (\delta_b + g)y_b,\ \dot{y}_g = s_g - \left(\delta_g + g\right)y_g \qquad (3^{iv})$$

The steady-state solution for y_g and y_b is

$$y_b^s = \frac{s_b}{(\delta_b+g)},\quad y_g^s = \frac{s_g}{(\delta_g+g)} \qquad (5)$$

For positive (negative) growth rates, both sides in (3^{iv}) are positive (negative) and therefore y-terms are increasing (decreasing) and thereby decrease (increase) the right-hand side. The process is therefore stable, going to the steady state solution (5) if a constant growth rate *g* exists (even if it varies in the adjustment process), which depends also on the production functions assumed below. As a result, R_g, A, and R_b have the same growth rate, if the production functions relating them do not contradict this.

2.3 *Extension to endogenous savings*

The country's problem in finding the optimal savings ratios is to maximise utility from consumption subject to the dynamic R&D equations:

$$Max_{s_b,s_g} \sum_{\tau=t}^{T} \rho^{\tau-t} U\left[\left(1 - s_{b,\tau} - s_{g,\tau}\right)F(A_{\tau-1}, R_\tau, \tau)\right]$$

$$\text{s.t.}\quad R_{b,\tau+1} = R_{b,\tau} + s_{b,\tau}F(A_{\tau-1}, R_\tau, \tau) - \delta_b R_{b,\tau}$$

$$R_{g,\tau+1} = R_{g,\tau} + s_{g,\tau}F(A_{\tau-1},\ R_\tau, \tau) - \delta_g R_{g,\tau},$$

given initial values for $A_{t-1} = A(t-1)$, $R_{b,t} = R_b(t)$ and $R_{g,t} = R_g(t)$, and $\rho < 1$.

T is the time horizon, τ is the index of future time periods with beginning value *t* as the present period. The constraints (3') include (1), (2) and (4) and use the definition $R_t = (R_{b,t}, R_{g,t}; R^*_{b,t}, R^*_{g,t})$. If *T* goes to infinity this is a standard dynamic optimisation problem. If *T-t = 2* we have a two period model. If *T-t = 1* this is a myopic one-period model with no reason to invest unless we specify a salvage function.

We solve the dynamic R&D equations (3') for the savings ratio and insert them into the objective function. The result with $g_{j,\tau} = (R_{j,\tau+1} - R_{j,\tau})/R_{j,\tau}$ is

$$Max_{R_{b,\tau+1},R_{g,\tau+1}} \sum_{\tau=t}^{T} \rho^{\tau-t} U\left[F(A_{\tau-1}, R_\tau, \tau) - (g_{b,\tau} + \delta_b)R_{b,\tau} - (g_{g,\tau} + \delta_g)R_{g,\tau}\right]$$

given initial values for $A_{t-1} = A(t-1)$, $R_{b,t} = R_b(t)$ and $R_{g,t} = R_g(t)$.

The first-order conditions for an interior solution are[2]

$$\rho U'_{\tau+1}\left[F_{2,\tau+1} - \delta_b\right] - U' = 0 \quad \text{or} \quad F_{2,\tau+1} - \delta_b = U'/(U'_{\tau+1}\rho) \qquad (6a)$$

[2] The (-1) before *U'* on the left-hand side comes from deriving $g_{j,\tau} = \left(R_{j,\tau+1} - R_{j,\tau}\right)/\ R_{j,\tau}$ with respect to $R_{j,\tau+1}$, which results in $1/R_{j,\tau}$, which is multiplied by the R-term multiplied to g in the objective function. Moreover, deriving *gR* for the next period makes *g*-terms drop out.

$$\rho U'_{\tau+1}\left[F_{3,\tau+1}-\delta_g\right]-U'=0 \quad \text{or} \quad F_{3,\tau+1}-\delta_b=U'/(U'_{\tau+1}\rho) \tag{6b}$$

The suffix 2 or 3 denotes derivations with respect to the second or third argument evaluated at period *τ+1*. As control variables are substituted, concavity of the objective function w.r.t. $R_{b,\tau+1,}, R_{g,\tau+1}$ and their changes is a sufficient condition for optimality (Kamien and Schwartz 2001). As the function under the utility function is linear in the changes, even under linear utility concavity of *F* with respect to R-terms is sufficient; under strictly concave utility even mildly increasing returns of *F* could be allow for.

The net marginal products should be equal to each other,[3] and, as usual, marginal products should equal capital costs. For an iso-elastic specification of the utility function

$$U(c)=\frac{c^{1-\omega}}{1-\omega}, \text{ we get } \frac{U'}{(U'_{\tau+1}\rho)}=\frac{(1+g_c)^{\omega}}{\rho}.$$

The right-hand sides of (6a) and (6b) are constant if the growth rate of *c* is constant. This is the case for constant growth rates of R-terms and *F*. Convergence of the integral to a constant value is necessary for a maximum to exist. It requires that the growth rate of discounted utility is negative, which is the case if the discount rate dominates the growth rate of utility in the long run, $\log\rho + (1-\omega)g < 0$, or $(1-\omega)g < -\log\rho$.[4]

3. Semi-endogenous growth with Cobb-Douglas function, and exogenous foreign R&D capital

3.1 Steady-state definition and growth rate in the Cobb-Douglas case

A special case of the function (2) for *A* is the Cobb-Douglas function with $1>\alpha,\beta>0; C\geq 1, b\geq 0;\ \varphi\gamma,\mu,\lesseqgtr 0$, and unspecified signs for the exponents of the foreign variables

$$A=(A_{-1})^{\varphi}R_b^{\alpha}R_g^{\beta}R_b^{*\gamma}R_g^{*\mu}Ce^{bt} \tag{2'}$$

C is a constant to adjust the level to the requirements of the data. As Kaldor and Mirrlees (1962), we add exogenous growth to the endogenous arguments. The empirical literature interprets $\gamma,\mu>0$ as dominance of positive spillovers, and $\gamma,\mu<0$ as dominance of negative spillovers and competition effects from abroad (Luintel and Khan 2004). Of course, in principle, γ and μ could have opposite signs, for example positive spillovers from public R&D and dominant competition effects from private foreign R&D. Taking growth rates, and using the assumption of equality of growth rates for *A* and domestic *R*-terms we get

$$\hat{A}=\frac{\gamma\widehat{R_b^*}+\mu\widehat{R_g^*}+b}{1-\varphi-\alpha-\beta}\equiv g \tag{2''}$$

This result shows semi-endogenous growth, as *A* is a function of endogenous variables, but the growth result has exogenous variables on its right-hand side. If *b > 0*, it adds an element of exogenous growth. These results so far assume $1-\varphi-\alpha-\beta>0$.

[3] If the sum of savings ratios is exogenous, we could maximise TFP through the manner of splitting the sum. Then, equal marginal products would also be a requirement for the maximisation.

[4] The growth rate of the discount factor $\rho^{\tau-t}$, calculated as log-difference, is $\log\rho<0$.

Whereas semi-endogenous growth models normally have the population growth rate on the right-hand side, here foreign R&D has this role. By implication, growth only stops if foreign R&D would stop growing in case $\gamma, \mu > 0$ and the exogenous growth rate *b* is zero. Even if *b* is positive, $\gamma, \mu < 0$ would imply that foreign R&D can have a competition effect, for example making the more productive domestic sectors smaller and thereby make TFP growth negative. It is therefore not so surprising that TFP growth has phases of being negative, not only in recessions (see Penn World Tables). Eaton and Kortum (1997) and NESTI (2017) emphasised this strong role for foreign R&D. However, the strong role of population growth may come back in future research if capital and labour would be re-introduced and foreign variables get endogenous.

3.2 Endogenous savings ratio for the Cobb-Douglas function

With a Cobb-Douglas production function like (2') we get from (6a) and (6b) $F_2 = \alpha A/R_b$, and $F_3 = \beta A/R_g$. Replacing the marginal products in (6a) and (6b) yields solutions of R&D-stock/TFP ratios as functions of the consumption growth rates or vice versa:

$$F_{2,\tau+1} = \alpha A/R_b = \frac{(1+g_c)^\omega}{\rho} + \delta_b \qquad \text{or} \qquad \frac{R_b}{A} = \frac{\alpha}{\frac{(1+g_c)^\omega}{\rho} + \delta_b} \tag{7a}$$

$$F_{3,\tau+1} = \beta A/R_g = \frac{(1+g_c)^\omega}{\rho} + \delta_g \qquad \text{or} \qquad \frac{R_g}{A} = \frac{\beta}{\frac{(1+g_c)^\omega}{\rho} + \delta_g} \tag{7b}$$

Using the definitions of $y_j \equiv R_j/A$ and these stock ratios in (5), yields endogenous savings ratios, with *g* from (2''):

$$s_b = y_b^s(\delta_b + g) = \frac{\alpha(\delta_b + g)}{\frac{(1+g_c)^\omega}{\rho} + \delta_b}, \quad s_g = y_g^s(\delta_g + g) = \frac{\beta(\delta_g + g)}{\frac{(1+g_c)^\omega}{\rho} + \delta_g} \tag{8}$$

As $(1+g)^\omega/\rho > 1$ for $\omega > 0$, and all other expressions are below unity, the savings ratios are below unity. In a steady state with equal growth rates for consumption and productivity, this determines the savings ratios. This result is only valid for TFP production functions such as the Cobb-Douglas function for which (6a,b) fixes the *A/R* ratios, and in the steady state for which consumption *x* has the same growth rate as *A*, R_b, and R_g.

3.3 Existence and stability of a steady state with Cobb-Douglas TFP function

The Cobb-Douglas function above is one possibility that may allow for a stable steady state and delivers the growth rate of *A*. We define $R^* \equiv R_b^{*\gamma} R_g^{*\mu} e^{bt}$, $y_g \equiv R_g/A$, $y_b \equiv R_b/A$. From (3'), we then get

$$\hat{y}_b = s_b (A_{-1})^\varphi R_b^{\alpha-1} R_g^\beta R^* C - \delta_b - g$$

$$\hat{y}_g = s_g (A_{-1})^\varphi R_b^\alpha R_g^{\beta-1} R^* C - \delta_g - g$$

with g from (2''). Multiplying through by y-terms then delivers (3^{iv}) again. Existence and stability are ensured.

For the TFP production function $A = (A_{-1})^{\varphi} R_b^{\alpha} R_g^{\beta} R^* C$, constancy of all terms requires $\varphi = -\alpha - \beta$, a strong fishing out effect simplifying (2''). Under this condition, a move towards constant y-terms results in

$$A = (A_{-1})^{\varphi} R_b^{\alpha} R_g^{\beta} R^* C = \left(\frac{A_{-1}}{A}\right)^{\varphi} \left(\frac{R_b}{A}\right)^{\alpha} \left(\frac{R_g}{A}\right)^{\beta} R^* C = \left(\frac{1}{1+g}\right)^{\varphi} (y_b)^{\alpha} (y_g)^{\beta} R^* C$$

Constant g and y-terms for a stable steady state ending with (5) requires that indeed A grows at the same rate as R^*. In the steady state, domestic R&D keeps the growth rate constant by just compensating fishing out in a way that the growth rate can be constant at the rate of foreign and exogenous R&D. Below we will consider other functions.

For the optimisation case, things are only slightly more complicated. In the optimisation, we have substituted the savings ratios from the dynamic equations into the objective function

$$s_{b,\tau} = \frac{(g_{b,\tau} + \delta_b) R_{b,\tau}}{F(A_{\tau-1}, R_\tau, \tau)} = (g_{b,\tau} + \delta_b) y_{b,\tau}$$

$$s_{g,\tau} = \frac{(g_{g,\tau} + \delta_g) R_{g,\tau}}{F(A_{\tau-1}, R_\tau, \tau)} = (g_{g,\tau} + \delta_g) y_{g,\tau}$$

This corresponds to (3^{iv}) in terms of non-steady-state values. Replacing the savings ratio there yields

$$\dot{y}_{b,\tau} = (g_{b,\tau} + \delta_b) y_{b,\tau} - (\delta_b + g) y_{b,\tau} = (g_{b,\tau} - g) y_{b,\tau}, \qquad (3^{v}a)$$

$$\dot{y}_{g,\tau} = (g_{g,\tau} + \delta_g) y_{g,\tau} - (\delta_g + g) y_{g,\tau} = (g_{g,\tau} - g) y_{g,\tau} \qquad (3^{v}b)$$

There is an additional effect on the savings ratios now. At first sight, these equations look tautological. However, the optimality conditions (7a) and (7b) are hard to fulfil with equality, because A/R-terms are given at any moment in time because productivity and R&D stocks move only slowly. This would suggest that the growth rate of consumption adjusts. However, it cannot balance two equations. Whenever the marginal product of private or public R&D is above (below) its optimum value, this is the case because its R&D level is below (above) optimum and therefore its saving ratio should be higher (lower)[5] than in the steady state. Therefore, temporarily higher (lower) growth rates of private or public R&D lead to a speed up of the respective *R/A* ratio. This the message of (3^{v}a, b). The growth rate difference is a policy reaction. The growth rate difference is set to zero if stability leads to a steady state where all growth rates are the same and the marginal productivity conditions hold.

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[5] Actually, it must be zero, because the problem requires 'a most rapid approach to a singular solution', where the latter is the equality of the two marginal products with not control variable in the equation, requires extending only the relative low variable (Kamien and Schwartz 2001).

3.4 A translation to the long-term relations of a VECM for the Cobb-Douglas function relations

The marginal productivity condition in equation (7a) suggests $logR_b = logA - log(F_2/\alpha)$, where the last term is a constant according to (6a). Equation (7b) suggests $logR_g = logA - log(F_3/\beta)$, where the last term is a constant according to (6b). Combining the last two equations by elimination of *logA* leads to $logR_b = logR_g + log(F_3/\beta) - log(F_2/\alpha)$. Two of these or the Cobb-Douglas production function, could be long-term relationships in a cointegrated VAR in line with theoretical modelling. This would lead to four possible cases with due adjustment of lags:

(i) The limiting case of a vector-error-correction model with full rank and estimation in levels with foreign terms as (weakly) exogenous, if confirmed by tests.

(ii) In case of no cointegration, the same model would be re-written in first differences.

(iii) In case of one cointegrating equation, we would have the following long-term relation from the TFP production function as expected value:

$$[(\varphi-1)\log(\frac{A}{1+g})+\alpha logR_b+\beta logR_g+\gamma logR_b^*+\mu logR_g^*+bt+c-g]_{-1}$$

We replace *A(-1)* by its steady-state value *A/(1+g)*. At the end of the bracket term, we have denoted that it enters with a lag in a VECM. Inside the brackets, it is necessary to include *g* in order to make the long-term relation identical to the log version of the production function divided by *A(-1)* on both sides. The long-term relation would have to be zero in a long-run equilibrium and therefore is written in its homogenous form. With terms for *b* and *R** determining the growth rate it can again be shown that we require $\varphi=-\alpha-\beta$ from an estimate.

(iv) In case of two long-term relations, we would imagine having (with *u* and *v* as residuals)

$$\left[E(u_{-1})=0=logA-logR_b-\log\left(\frac{F2}{\alpha}\right)\right]_{-1},$$

$$E(v_{-1})=0=[logR_b-\ logR_g-log\left(\tfrac{F_3}{\beta}\right)+log(F_2/\alpha)]_{-1}$$

Time trends do not appear in this case. Several terms here have unit coefficients for the case of a Cobb-Douglas function. This may seem unrealistic at first sight, but unit coefficients feature prominently in the long-term relations of textbook examples. The consumption-income-investment relation (Lütkepohl 2006) and the liquidity-interest relationships (Juselius 2006) also have unit coefficients, which estimations only obtain approximately in the more flexible VECM approaches. Pesaran (1997) points out that only under special cases of functions we can link the VECMs directly to theoretical models. VECM estimation in a log-log approach could test whether the relations are similar to those obtained here for Cobb-Douglas functions. As this may turn out to be dis-appointing, it may be interesting to look at other production functions as we do in the next section.

We may also use in addition output production function (1) suggesting *logQ = log (A)* as a long-term relation, but having dropped capital and labour from the model may lead to omitted variable or equation bias in the analysis of the VECM system of equations. Alternatively, this function could be used in one way or other to replace *log(A)*-terms above. Long-term relations in VECMs have two-way causality. The marginal productivity conditions would suggest that given *A*, R-terms are determined or explained. Conversely, the three variables determine each other and R-terms drive *A* according to the productions function requiring adjustments of R-terms again.

4. Long-term relations and Mukerji VES without constraints

Suppose now that we use a VES (variable elasticity of substitution) function of Mukerji (1963) for *A,* with *C* as a constant to determine the level of *A*:

$$A = e^{bt}\left[a(A_{-1})^{\varphi} + hR_b^{\alpha} + cR_g^{\beta} + d{R_b^*}^{\gamma} + f{R_g^*}^{\mu}\right]^{1/\epsilon} C \equiv e^{bt}B^{1/\epsilon}C \qquad (2^{iv})$$

Then $F_2 = \frac{1}{\epsilon}A^{1-\epsilon}e^{\epsilon bt}h\alpha {R_b}^{\alpha-1}C^{\epsilon}$ and $F_3 = \frac{1}{\epsilon}A^{1-\epsilon}e^{\epsilon bt}h\beta {R_g}^{\beta-1}C^{\epsilon}$ (9)

Mukerji (1963) considers it as a generalisation of the standard CES function. Arrow and Hurwicz (1958, p.550) and Houthakker (1960) use this function with $\epsilon = 1$ for utility. The terms in brackets, abbreviated as *B* here, are called 'addilog' in Houthakker (1960). If all the parameters in Greek letters have the same value, we have a CES function. If that value were zero, we would have a Cobb-Douglas function. Whereas these marginal productivity equations are log linear, the production function is not log-linear anymore. The version on the right-hand side of (2^{iv}) is log-linear in *B*. We could estimate *B* as a non-linear part of the production function. For a constant growth rate of consumption, which exists under conditions shown below, the constancy of the marginal products in (6a,b) yields long-term relations for *A* and R_b as well as for R_b and R_g from (9):

$$logA = \frac{(1-\alpha)}{(1-\epsilon)}logR_b - \frac{\epsilon b}{(1-\epsilon)}t + \frac{1}{(1-\epsilon)}\log\left(\frac{\epsilon\left(\delta_b + \frac{(1+g)^{\omega}}{\rho}\right)}{h\alpha C^{\epsilon}}\right) \qquad (9a)$$

$$logR_b = \frac{1}{(\alpha-1)}log\frac{\delta_b + \frac{(1+g)^{\omega}}{\rho}}{\delta_g + \frac{(1+g)^{\omega}}{\rho}} - \frac{1}{(\alpha-1)}log\frac{\alpha}{\beta} + \frac{(\beta-1)}{(\alpha-1)}logR_g \qquad (9b)$$

From (6a,b) we obtain the first equation by taking logs of the first marginal productivity condition, and the second by equalising the marginal productivity conditions first. The first equation is a long-term relation between *A* and R_b; the second term there is a time trend and the third a constant, both consisting of the parameters in (7a). The second equation is a long-term relation between business R&D stock R_b and public R&D stock R_g without a time trend.[6] In the model and in VECMs all relations may have two-way causality.

Looking at the equations in detail, we see that for α < 1, $\epsilon < 1$ implies that private R&D has a positive effect on TFP. The existence of the log expression in the last term requires $\epsilon > 0$. If $1 > \epsilon > 0$, the time trend has a negative effect.[7] If $\epsilon > 0$, a positive growth rate requires the assumption $\alpha < \epsilon$ (see equation (10) below). Under this assumption, the elasticity of TFP with respect to private R&D is larger than unity and smaller than unity for the reversed causality. If, in (9b), depreciation rates are

[6] However, empirical econometric work would first add a time trend in order to avoid finding a significant regression coefficient just because both variables have a time trend (Wooldridge 2013, chapter 10.5), and perhaps take it out later if this does not change parameter estimates. The significance test for the time trend could also be a crosscheck for our model specification using a VES function.
[7] Soete et al. (2019) find it for a similar VEC model for the Netherlands.

equal, the first part of the constant is zero. If 1 > α > β, the second part of the constant is positive, and public R&D has a coefficient larger than unity. If 1 > β > α, the second part of the constant is negative and public R&D has a coefficient smaller than unity. If both parts of the intercept were zero, private R&D would be zero (positive) in the hypothetical case of evaluating a regression where the log of public R&D would be zero (positive). In other words, only with a sufficiently low rate of depreciation and a high elasticity of production of private relative to public R&D, private R&D could exist without public R&D. However, the marginal product of public R&D would go to infinity if public R&D goes towards zero, provided $\beta > 0$. Equations (9a, b) have five coefficients consisting of the steady state growth rate g and nine parameters: $b, \alpha, \beta, \delta_b, \delta_g, C, \epsilon, \rho, h$. The estimated coefficients then generate five equations from which we could try to solve for five parameters with calibrating assumptions for the others. For example, we could impose the depreciation rates used in the data-making procedure, the discount rate in line with the literature, and one of the others. Assuming equal rates of depreciation, the coefficients of (9b) would allow solving for α and β. Using this in (9a) could give results for b and ϵ. A VECM thus could test first whether we have these two long-term relations, or perhaps more or less. If we obtain these two, we could try to back out the parameters and compare the result to those of the Cobb-Douglas function. Whether or not the result comes close to the non-linear VES, we could think of extending the model to a non-linear simultaneous equation system including the production function and try to obtain parameter estimates. So far, we have not imposed balanced growth assumptions in the form of parameter constraints for the productivity function.

5. *Steady state growth for two countries*

5.1 Two-country steady state for a Mukerji VES function with constraints

Narrowing down the consideration to explore the possibility of steady state results of the growth model leads to the following. Taking differences of equation (9a) and assuming again that TFP and private R&D have the same growth rate, we get semi-endogenous growth, ultimately driven by exogenous technical change:

$$g = dlogA = dlogR_b = \frac{\epsilon b}{\epsilon - \alpha}. \quad (10)$$

The second equation, (9b), shows a positive relation between private and public R&D as long as $\alpha, \beta < 1$ as assumed above. The stability analysis above suggests that for any constant savings ratio, also R_g would have the same growth rate as TFP, A. Differentiating equation (9b) above, balanced growth of public and private R&D would require $\alpha = \beta$, a step back towards a CES function for the given VES function. Only if public and private R&D are equally productive in the production of A can we have a steady state.

For a set of sufficient conditions for the existence of a steady state, suppose

(i) for the foreign country there is a VES function like (2^{iv}) with variables and parameters having a '*',

(ii) there is a relation symmetric to (10), $g^* = dlogA^* = dlogR_b^* = \frac{\epsilon^* b^*}{\epsilon^* - \alpha^*}$ (11)

(iii) $\alpha = \beta = \varphi$, and $\gamma = \mu$, and for the foreign country $\alpha^* = \beta^* = \varphi^*$, and $\gamma^* = \mu^*$.

(iv) $\alpha g = \gamma g^*$, $\alpha^* g^* = \gamma^* g$ or $g/g^* = \gamma/\alpha = \alpha^*/\gamma^*$.

As all other parameters have already been used, assumption (iv) can be seen as putting a constraint on γ^* as necessary for balanced productivity growth of a two-country model. With these assumptions, arguments in the VES function (2^{iv}) have exponential growth functions with identical exponents multiplied to *t*, and we can re-write the production function for the home country as

$$A = e^{(b+\frac{\alpha g}{\epsilon})t}\left[\underline{B}\right]^{1/\epsilon}C = e^{(b+\frac{\gamma g^*}{\epsilon})t}\left[\underline{B}\right]^{1/\epsilon}C \qquad (12)$$

Underlining of *B* expresses that all arguments in *B* multiplied to the growth rate term $e^{(\frac{\alpha g}{\epsilon})t}$ or identical ones according to (iv) above, are fixed to a certain value when hypothetically arriving at the steady state. Logical consistency requires that the value of *g* in (10) is equal to the first growth rate in (12), which we can confirm as follows:

$$g = b + \frac{\alpha g}{\epsilon} = b + \frac{\alpha\frac{\epsilon b}{\epsilon-\alpha}}{\epsilon} = b + \frac{\alpha b}{\epsilon-\alpha} = \frac{b(\epsilon-\alpha)+\alpha b}{\epsilon-\alpha} = \frac{\epsilon b}{\epsilon-\alpha} \bullet$$

Combining this with (i)-(iv), yields $g^* = \frac{\alpha g}{\gamma} = \frac{\alpha}{\gamma}\frac{\epsilon b}{\epsilon-\alpha} = \frac{\gamma^*}{\alpha^*}\frac{\epsilon b}{\epsilon-\alpha} = \frac{\epsilon^* b^*}{\epsilon^*-\alpha^*}$; by implication, if *b* and *b** differ, the signs of $\frac{\epsilon b}{\epsilon-\alpha}$ and $\frac{\epsilon^* b^*}{\epsilon^*-\alpha^*}$ must be the same for a steady state to exist.

We define again $y_g \equiv R_g/A$, $y \equiv R_b/A$. From (3’) with Mukerji-VES function, we then get the accumulation dynamics of the VES growth model:

$$\hat{y}_b = s_b e^{bt}\left[a(A_{-1})^{\varphi} + hR_b^{\alpha} + cR_g^{\beta} + dR_b^{*\gamma} + fR_g^{*\mu}\right]^{1/\epsilon} C/R_b - \delta_b - g$$

$$\hat{y}_g = s_g e^{bt}\left[a(A_{-1})^{\varphi} + hR_b^{\alpha} + cR_g^{\beta} + dR_b^{*\gamma} + fR_g^{*\mu}\right]^{1/\epsilon} C/R_g - \delta_g - g$$

Multiplying through by y-terms yields

$$\dot{y}_b = \frac{s_b e^{bt}\left[a(A_{-1})^{\varphi} + hR_b^{\alpha} + cR_g^{\beta} + dR_b^{*\gamma} + fR_g^{*\mu}\right]^{1/\epsilon} C}{A} - (\delta_b + g)y_b$$

$$\dot{y}_g = \frac{s_g e^{bt}\left[a(A_{-1})^{\varphi} + hR_b^{\alpha} + cR_g^{\beta} + dR_b^{*\gamma} + fR_g^{*\mu}\right]^{1/\epsilon} C}{A} - (\delta_g + g)y_g$$

After some manipulation, using the assumptions imposed above in this section, the domestic and, by symmetry, the foreign system are as follows (see appendix):

$$\dot{y}_b = s_b\left[a\left(\frac{1}{1+g}\right)^{\alpha} + h(y_b)^{\alpha} + c\left(y_g\right)^{\alpha} + d(y_b^*)^{\gamma} + f\left(y_g^*\right)^{\gamma}\right]^{1/\epsilon} C - (\delta_b + g)y_b$$

$$\dot{y}_g = s_g\left[a\left(\frac{1}{1+g}\right)^{\alpha} + h(y_b)^{\alpha} + c\left(y_g\right)^{\alpha} + d(y_b^*)^{\gamma} + f\left(y_g^*\right)^{\gamma}\right]^{1/\epsilon} C - (\delta_g + g)y_g$$

$$\dot{y^*}_b = s^*{}_b\left[a^*\left(\frac{1}{1+g^*}\right)^{\alpha^*} + h^*(y_b^*)^{\alpha^*} + c^*\left(y_g^*\right)^{\alpha^*} + d^*(y_b)^{\gamma^*} + f\left(y_g\right)^{\gamma^*}\right]^{1/\epsilon^*} C^* - (\delta^*{}_b + g^*)y_b{}^*$$

$$\dot{y^*}_g = s^*{}_g\left[a^*\left(\frac{1}{1+g^*}\right)^{\alpha^*} + h^*(y_b^*)^{\alpha^*} + c^*\left(y_g^*\right)^{\alpha^*} + d^*(y_b)^{\gamma^*} + f\left(y_g\right)^{\gamma^*}\right]^{1/\epsilon^*} C^* - (\delta^*{}_g + g^*)y_g{}^*$$

These are four equations in $y_g \equiv R_g/A$, $y_b \equiv R_b/A$, $y_g^* \equiv R^*{}_g/A^*$, $y_b^* \equiv R_b^*/A^*$. As we have divided the production function by A, the terms multiplied to the savings ratios equal unity. Then, in steady state, the solution is analogous to (5):

$$y_g = \frac{s_g}{\delta_g+g}, \; y = \frac{s_b}{\delta_b+g}; \; y_g{}^* = \frac{s^*{}_g}{\delta^*{}_g+g^*}, \; y_b{}^* = \frac{s^*{}_b}{\delta^*{}_b+g^*} \qquad (5')$$

This proves the existence of a steady-state solution in the presence of a restricted Mukerji VES function and given savings ratios, optimal or not. Stability also works analogous to the analyses above. Domestic and foreign growth rates can differ in the steady state of this model, but they must be proportional to each other as formulated in requirement (iv) above. Otherwise, countries will have non-proportional growth rates and steady states do not exist, because the VES without restrictions contradicts the part of the system without production function.

In general, a VECM with a linear time trend is more flexible than a balanced growth model, and the long-run solution can have different growth rates for each variable. For our model, this might require introducing trends in discount rates or depreciation rate or adding factor-specific technical change to the TFP VES function.

Linking back to the long-term relations (9a) and (9b), we see that (9a) would be unchanged under constraints (i)-(iv), but the slope of (9b) would become unity as in the Cobb-Douglas case. In the case of two cointegrating relations, a growth model with a VES function is slightly more general than the Cob-Douglas case. Including the VES function in the estimation model would require giving up the linear VEC framework.

6. Summary and conclusion

We have provided semi-endogenous growth models for TFP production with domestic and foreign public and private R&D and exogenous or endogenous savings ratios. Foreign R&D and exogenous technical change in the Cobb-Douglas case of a small country ultimately drive the long-run growth rate. If foreign R&D, which is driving the growth, is more damaging than exogenous technical change is helpful, the growth rate can be low and even negative. Our growth model in combination with a CD or restricted VES function can generate special cases of one or two long-term relations as in empirical VECM models.

Overall, the log-log regression approach for R&D in Soete et al. (2019) is similar to the semi-endogenous growth models of this paper. This is less so when moving away from CD to VES, where the marginal productivity functions are still linear, but the production function is not. The models of this paper provide the theoretical causality underlying the VECMs. The numerical results of the two approaches are likely to differ. The VECMs have more flexibility in the number of lags, but less in the functional forms. This raises the question which of these is better.

For future research, we think of modifying the log linear framework to include non-linear production functions. To the extent that production functions are straightjackets, we will have serial correlation through mis-specification. This may happen to occur for all production functions, the CD or VES of this paper or other functions not discussed here. The standard remedy for mis-specification when no alternative specification is available is adding lags. The lags in the VECMs therefore can be seen as a response to mis-specification when using Cobb-Douglas functions. The question then will be whether functions that are more general show less mis-specification.

Our model has suppressed the potential endogeneity of foreign R&D variables. In future work we will extend the theoretical model to have mutually optimal reactions of both countries to endogenous foreign R&D variables in differential games and perhaps to a broader model including biased technical change in the TFP production function, as well as capital and labour.

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Appendix: The dynamic system with Mukerji VES function

Bringing (A/e^{bt}) into the brackets yields

$$\dot{y}_b = s_b\left[a\frac{(A_{-1})^{\varphi}}{(A/e^{bt})^{\epsilon}} + h\frac{R_b^{\alpha}}{(A/e^{bt})^{\epsilon}} + c\frac{R_g^{\beta}}{(A/e^{bt})^{\epsilon}} + d\frac{{R_b^*}^{\gamma}}{(A/e^{bt})^{\epsilon}} + f\frac{{R_g^*}^{\mu}}{(A/e^{bt})^{\epsilon}}\right]^{1/\epsilon} C - (\delta_b + g)y_b$$

$$\dot{y}_g = s_g\left[a\frac{(A_{-1})^{\varphi}}{(A/e^{bt})^{\epsilon}} + h\frac{R_b^{\alpha}}{(A/e^{bt})^{\epsilon}} + c\frac{R_g^{\beta}}{(A/e^{bt})^{\epsilon}} + d\frac{{R_b^*}^{\gamma}}{(A/e^{bt})^{\epsilon}} + f\frac{{R_g^*}^{\mu}}{(A/e^{bt})^{\epsilon}}\right]^{1/\epsilon} C - (\delta_g + g)y_g$$

Pulling terms with (A/e^{bt}) under the exponents of the inputs results in

$$\dot{y}_b = s_b\left[a\left(\frac{A_{-1}}{(A/e^{bt})^{\epsilon/\varphi}}\right)^{\varphi} + h\left(\frac{R_b}{(A/e^{bt})^{\epsilon/\alpha}}\right)^{\alpha} + c\left(\frac{R_g}{(A/e^{bt})^{\epsilon/\beta}}\right)^{\beta} + d\left(\frac{R_b^*}{(A/e^{bt})^{\epsilon/\gamma}}\right)^{\gamma} + f\left(\frac{R_g^*}{(A/e^{bt})^{\epsilon/\mu}}\right)^{\mu}\right]^{1/\epsilon} C - (\delta_b + g)y_b$$

$$\dot{y}_g = s_g\left[a\left(\frac{A_{-1}}{(A/e^{bt})^{\epsilon/\varphi}}\right)^{\varphi} + h\left(\frac{R_b}{(A/e^{bt})^{\epsilon/\alpha}}\right)^{\alpha} + c\left(\frac{R_g}{(A/e^{bt})^{\epsilon/\beta}}\right)^{\beta} + d\left(\frac{R_b^*}{(A/e^{bt})^{\epsilon/\gamma}}\right)^{\gamma} + f\left(\frac{R_g^*}{(A/e^{bt})^{\epsilon/\mu}}\right)^{\mu}\right]^{1/\epsilon} C - (\delta_g + g)y_g$$

Next, we divide numerator and denominator by *A*:

$$\dot{y}_b = s_b\left[a\left(\frac{A_{-1}/A}{(A/e^{bt})^{\epsilon/\varphi}/A}\right)^{\varphi} + h\left(\frac{R_b/A}{(A/e^{bt})^{\epsilon/\alpha}/A}\right)^{\alpha} + c\left(\frac{R_g/A}{(A/e^{bt})^{\epsilon/\beta}/A}\right)^{\beta} + d\left(\frac{R_b^*/A^*}{(A/e^{bt})^{\epsilon/\gamma}/A^*}\right)^{\gamma} + f\left(\frac{R_g^*/A^*}{(A/e^{bt})^{\epsilon/\mu}/A^*}\right)^{\mu}\right]^{1/\epsilon} C - (\delta_b + g)y_b$$

$$\dot{y}_g = s_g \left[a \left(\frac{A_{-1}/A}{(A/e^{bt})^{\epsilon/\varphi}/A} \right)^{\varphi} + h \left(\frac{R_b/A}{(A/e^{bt})^{\epsilon/\alpha}/A} \right)^{\alpha} + c \left(\frac{R_g/A}{(A/e^{bt})^{\epsilon/\beta}/A} \right)^{\beta} + d \left(\frac{R_b^*/A^*}{(A/e^{bt})^{\epsilon/\gamma}/A^*} \right)^{\gamma} + f \left(\frac{R_g^*/A^*}{(A/e^{bt})^{\epsilon/\mu}/A^*} \right)^{\mu} \right]^{1/\epsilon} C - (\delta_g + g) y_g$$

In the neighbourhood of a steady state, we have $A/A(-1) = 1 + g$, and $\frac{\left(\frac{A(0)e^{gt}}{e^{bt}}\right)^{\frac{\epsilon}{\alpha}}}{[A(0)e^{gt}]} = e^{(g-b)t(\epsilon/\alpha)-gt}$ for A(0) =1. For $g = \frac{\epsilon b}{\epsilon-\alpha}$ found above, we have $(g-b)(\epsilon/\alpha) - g = \left(\frac{\epsilon b}{\epsilon-\alpha} - b\right)\left(\frac{\epsilon}{\alpha}\right) - \frac{\epsilon b}{\epsilon-\alpha} = \left(\frac{\epsilon b}{\epsilon-\alpha} - \frac{b(\epsilon-\alpha)}{\epsilon-\alpha}\right)\left(\frac{\epsilon}{\alpha}\right) - \frac{\epsilon b}{\epsilon-\alpha} = \frac{\alpha b}{\epsilon-\alpha}\left(\frac{\epsilon}{\alpha}\right) - \frac{\epsilon b}{\epsilon-\alpha} = \frac{b}{\epsilon-\alpha}\epsilon - \frac{\epsilon b}{\epsilon-\alpha} = 0$. The domestic relative *A*-terms in the denominators all have zero growth rates if $\alpha = \beta = \varphi$ as assumed in (iii) in the main text. For domestic-foreign relative terms like $\left(\frac{A(0)e^{gt}}{e^{bt}}\right)^{\frac{\epsilon}{\gamma}} / (A^*(0)e^{g^*t}$ with A*(0) = 1 we have the growth rate $(g-b)\left(\frac{\epsilon}{\gamma}\right) - g^*$. For $\alpha g/\gamma = g^*$ and $g = \frac{\epsilon b}{\epsilon-\alpha}$, we get $(g-b)\left(\frac{\epsilon}{\gamma}\right) - g^* = \left(\frac{\epsilon b}{\epsilon-\alpha} - \mathrm{b}\right)\left(\frac{\epsilon}{\gamma}\right) - \alpha\frac{\epsilon b}{\epsilon-\alpha}/\gamma$; cancelling *b*, $\epsilon, 1/\gamma$, we get $\left(\frac{\epsilon}{\epsilon-\alpha} - 1\right) - \alpha\frac{1}{\epsilon-\alpha}$ = $\left(\frac{\alpha}{\epsilon-\alpha}\right) - \alpha\frac{1}{\epsilon-\alpha} = 0$. By implication, relative domestic-foreign A-terms are also constant provided $\gamma = \mu$. With zero growth rate and initial values equal to unity at the moment of arrival in the steady state, we can set them to unity.

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