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UNU-MERIT Working Papers
ISSN 1871-9872

Maastricht Economic and social Research Institute on Innovation and Technology
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Maastricht Graduate School of Governance
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The Effects of Productivity and Benefits on Unemployment: Breaking the Link

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May 2016

In the standard macroeconomic search and matching model of the labour market, there is a tight link between the effects of (i) productivity on unemployment and (ii) unemployment benefits on unemployment. This tight link is at odds with the empirical literature. We present a two-sided model of labour market search where the household and firm decisions are decomposed into job offers, job acceptances, firing, and quits. In such a model, unemployment benefits affect households’ behaviour directly, without having to run via the bargained wage. In line with the evidence, productivity shocks may have quantitatively large effects on unemployment, while benefits only have moderate effects. Our analysis shows the importance of investigating the effects of policies on the households’ work incentives and the firms’ employment incentives within the search process.

Keywords: Unemployment benefits, search and matching, aggregate shocks, macro models of the labour market

JEL classification: E24, E32, J63, J64
1. Introduction

Labour markets are characterized by strong business cycle amplification and limited responsiveness of job findings to unemployment benefits. In particular, the volatility of the job-finding rate and unemployment are several times larger than the volatility of productivity or output. Thus, realistic macroeconomic models of the labour market require a strong amplification mechanism to be in line with aggregate time-series data. Furthermore, microeconomic studies only find a small elasticity of the job-finding rate with respect to changes of unemployment benefits (usually smaller than one). Costain and Reiter (2008) have shown that traditional search and matching models of the labour market cannot reconcile both strands of evidence.\(^1\) As the transmission of both aggregate shocks and changes of unemployment benefits in traditional search and matching models of the labour market occur via the bargained wage, strong amplification effects of productivity are tied to a high responsiveness of job findings to unemployment benefits.\(^2\) But this association is counterfactual.

In addition, the empirical literature shows no or ambiguous effects of unemployment benefits changes on the wage. However, in a standard search and matching model (Pissarides, 2000, chapter 1), unemployment benefits influence unemployment exclusively via the wage determination. Thus, if unemployment benefits have no effect on wages, they would thereby also have no effect on unemployment. This is clearly at odds with various cross-country studies or microeconomic evidence showing that more generous unemployment benefits increase unemployment.

Following Brown et al. (2015), this paper decomposes the matching process into its choice-theoretic components: (a) firms’ incentives to make job offers and to fire and (b) workers’ incentives to accept job offers and to quit. Productivity fluctuations and unemployment benefits have an impact on unemployment because they affect agents incentives and thereby their decisions to form matches.

Our modelling approach allows us to break the tight link between unemployment benefits and unemployment on the one hand and productivity and unemployment on the other hand. Consonant with the data, our model generates strong business cycle amplification effects at the same time a moderate reaction to unemployment benefits, which

\(^{1}\)Costain and Reiter (2008) furthermore show numerically that this result is robust to expanding the traditional search and matching model with endogenous search, endogenous separations, finite benefit duration, and efficiency wages.

\(^{2}\)To illustrate this point, imagine a simple wage formation rule \(w_t = \omega a_t + (1 - \omega) b\), where \(a\) is aggregate productivity and \(b\) are unemployment benefits. When \(\omega\) is large, the wage is very responsive to aggregate productivity and thus aggregate amplification effects are small. At the same time, with a large \(\omega\) wages do not react a lot to unemployment benefit changes, i.e. the responsiveness is also limited. For a formalization of this argument see Appendix C.
in combination the standard search and matching model fails to do.

If we accept the evidence that unemployment benefits have no or at most a limited effect on wages, we need to uncover further channels whereby unemployment benefits can have real labour market effects. In our model, unemployment benefits can have effects on unemployment, but these effects need not run through the wage. In the absence of wage effects, unemployment benefits nevertheless depress workers’ incentives to work.

We address these issues through a theoretical model of two-sided search among heterogeneous firms and workers and calibrating this model for the U.S. economy. We extend the model by Brown et al. (2015) by allowing workers who are fired or who quit their job to be immediately rehired. Matching and separation are viewed as analogous phenomena. The former is analysed in terms of the agents’ incentives to offer and accept jobs, the latter in terms of incentives to fire and quit. Match-specific shocks, specifically shocks to both firms’ costs and workers’ disutility of work, give rise to matching and separation.

The distinction between job offers and job acceptance, as well as between quitting and firing, is important from both a theoretical and a quantitative perspective. Job acceptance and quitting depend on households’ surpluses whereas job offers and firing depend on firms’ surpluses. In the search and matching model, with a standard matching function, no distinction is made between job offers and job acceptance or between quitting and firing; instead, the analysis is focused wholly on the job-finding rate and the separation rate. This is clearly at odds with legal procedures, in which quitting and firing are commonly distinguished from one another. It is also at odds with the empirical evidence, where quits and firing are also frequently distinguished (e.g. in surveys). By contrast, the distinction between job offers and job acceptance, while conceptually clear, has not played a major role in the empirical literature.

In our model, higher unemployment benefits lower a worker’s incentive to accept a job offer or to stay on the job. In principle, the first channel could also be captured in a model with endogenous search effort. However, in models with endogenous search effort both the functional form of the search function and the effects of more search effort on the job-finding probability are unobserved. In our model, the households’ quit and acceptance decisions and the firms’ offer and firing decisions depend, respectively, on the distributions of shocks to households’ disutility of work and firms’ costs. While these are also unobserved, we do observe business cycle dynamics of the U.S. quit rate (i.e. the outcome variable on the household side) and the U.S. firing rate in the JOLTS dataset. These allow us to uniquely pin down the household and firm distributions in our calibration.
From a macroeconomic perspective, quits are strongly procyclical and firings are strongly countercyclical, both in the data and in our model. In the aggregate, the cyclicality of the quit rate even seems to dominate the cyclicality of the firing rate, which is reflected in a procyclical separation rate. Most search and matching models with endogenous separations only consider the firing margin. It is well known from these models (e.g. Krause and Lubik, 2007) that excessively volatile separations lead to a collapse of the Beveridge curve. The combination of quit and firing decisions, both calibrated to meet the business cycle behaviour in the data, results in a strong Beveridge curve in our model. In applying our model to the influence of productivity and unemployment benefits on unemployment, we explicitly address the empirical evidence in terms of (i) business cycle regularities, (ii) the effects of unemployment benefits on reemployment wages, and (iii) the effects of unemployment benefits on unemployment.

Why are the results of our paper important? The inability of standard macroeconomic models of the labour market to reconcile large aggregate amplification effects and the small responsiveness to unemployment benefits casts doubt on the ability of these models to perform counterfactual policy exercises (e.g. the design of different unemployment insurance systems) and to analyze optimal labour market institutions or policy interventions. This is even more important as these questions often cannot be assessed by a pure applied econometric approach. Our paper offers a modelling framework that reduces the tension between microeconomic estimation results and macroeconomic amplification effects. Thus, it is a useful device for the analysis of different policies. Very importantly, our framework disentangles the effects of different policies on the firm and household side. This provides the opportunity to analyze the effects of targeted policies (e.g. targeted to increase the labour supply incentives) and offers new policy insights.3

The rest of the paper is organized as follows. Section 2 reviews the literature on the effects of unemployment benefits on (un)employment and wages. Section 3 derives a dynamic two-sided theoretical model, which decomposes labour market dynamics into the job-offer rate, the job-acceptance rate, the firing rate and the quit rate. Section 4 provides analytical results for the effects of productivity and unemployment benefits on the job-finding rate. Section 5 explains our calibration that is used for numerical simulations in Section 6. Section 7 briefly concludes.

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3Brown et al. (2014) show for example why sufficiently low minimum wages may not destroy jobs in the context of such a two-sided model. However, from a certain threshold onwards, minimum wages lead to job losses. Both observations are in line with the empirical literature.
2. Empirical Literature Review

The impact of unemployment benefit generosity on wages can result from various effects. First, it increases the value of unemployment and thereby the reservation wage. This may be expected to result in higher wages on reemployment. Second, it may enable unemployed workers to be more selective in finding a quality match, which in turn can yield higher wages. In line with the moral hazard argument, it could also extend unemployment duration with no significant increase in job quality and no wage effects. If benefits prolong unemployment duration they may even deteriorate job-match quality among others through skill depreciation. Third, near the end of the benefit period, the option of being unemployed becomes less attractive and reservation wages fall.

The evidence on the effects of unemployment benefit generosity (level and duration) on post-unemployment wages is mixed – on average they are non-existent or at best small. Early studies for the U.S., e.g. from Ehrenberg and Oaxaca (1976), Burgess and Kingston (1976), and Holen (1977) estimated the impact of an increase in unemployment benefits or benefit duration on reemployment wages and show positive significant effects for certain groups. But other studies for the U.S. as Classen (1977) and Blau and Robins (1986) find no significant effects or as Addison and Blackburn (2000) only small and marginally significant positive impacts of unemployment benefits on wages. Feldstein and Poterba (1984) and Blau and Robins (1986) for the U.S., Gorter and Gorter (1993) for the Netherlands, Prasad (2004) for Germany, and Addison et al. (2009) for European countries show that access to and higher levels of unemployment benefits increase reservation wages, though the elasticities are small. There is also little information on how the impacts on reservation wages actually affect reemployment wages. Schmieder et al. (2014) for example show that reservation wages are not binding for workers’ employment decisions. Recent studies exploiting reforms or discontinuities in unemployment benefit schemes show no significant effects on reemployment wages, e.g. the effect of benefit durations for Austria (Card et al., 2007; Lalive, 2007), for Slovenia (van Ours and Vodopivec, 2008), and for Germany (Caliendo et al., 2013). However, the latter highlight significant heterogeneities, whereby those exiting unemployment, while still insured, receive higher wages and those near the end of the benefit period lower wages. By contrast, Schmieder et al. (2014) also show sizeable and significantly negative effects of extended potential benefit duration on post-unemployment wages in Germany.

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4 See for example Tatsiramos (2014) and van Ours and Vodopivec (2008) for a discussion of these potential effects.

5 See Tatsiramos (2014). Effects can though be negative, large, and significant close to the end of the benefit period.
(via increased unemployment duration). Thus, the existing evidence does not reliably support the existence of a direct channel from unemployment benefits to reemployment wages. The clear evidence on positive significant impacts of benefit generosity on reservation wages may actually in turn support the mechanism presented in this paper, namely that it increases the value from unemployment and lowers workers’ incentive to accept a job offer.

Krueger and Meyer (2002) review important contributions on the empirical effect of unemployment benefit generosity on the duration of unemployment such as Mott (1985), Katz and Meyer (1990), Meyer (1990), and Card and Levine (2000). They report for these elasticities between 0.1 and 0.8. Similarly, Hornstein et al. (2005) report values of 0.1, 0.3, 0.6, and 1 for the elasticity of the hazard rate out of unemployment. The Hagedorn and Manovskii (2008) small surplus calibration of the search and matching model, in which the average unemployed worker is basically indifferent between working and not working, manages to generate labour market amplifications of comparable sizes as in the data, but implies an elasticity between six and sixty times larger than the available estimates (see Hornstein et al., 2005).

3. An Incentive Model of Two-Sided Search

Our analysis builds on the dynamic incentive model containing two-sided selection in the labour market of Brown et al. (2015). We investigate how unemployment benefits affect the incentives of workers and firms, thereby shaping their job offer and quit behaviour. We expand the model above by allowing workers separated at the beginning of the period to be rematched immediately. The sequence of decisions may be summarized as follows. First, the aggregate productivity shock and the idiosyncratic shocks for existing employment matches are revealed. Second, vacancies are posted. Third, firms make their firing decisions and the households make their job-quit decisions based on the realizations of the aggregate and idiosyncratic shocks and anticipating the results of wage bargaining. Fourth, unemployed workers (i.e. those unemployed in the previous period and those separated in the current period) make contact with vacancies. Fifth, the idiosyncratic shocks for new contacts are revealed. Sixth, the firms make their hiring decisions and the

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6 These results point at the high cost of long-term unemployment and can be the consequence of e.g. skill depreciation, stigmatization, or changes in job characteristics.

7 Hagedorn et al. (2013) argue that existing micro-studies only capture a small microeconomic effect, while they omit a large general equilibrium effect (that runs via the market tightness). Thus, they provide another way of reconciling large macroeconomic effects with small effects from microeconomic studies. The actual magnitude of the macroeconomic general equilibrium effect will remain a highly debated issue.

8 There are no direct job-to-job transitions via on-the-job search.
households make their job-acceptance decisions based on the realizations of the aggregate and idiosyncratic shocks and anticipating the bargaining results. Finally, the wage is determined.\footnote{The assumption that employment decisions are made before wage decisions parallels what is assumed in traditional search models. For example, in Pissarides (2000, chapter 1), vacancies are posted first, some workers are matched and then wages are determined. This assumption also permits us to distinguish between quit and firing decisions.}

### 3.1. Wage Determination

Section 2 shows that according to the empirical literature the effects of unemployment benefits on wages are very small or non-existent. The standard Nash-bargained model of wage formation often contains an effect of unemployment benefits on the wage, contrary to the relevant empirical literature.\footnote{It is, however, easy to think of various reasons why unemployment benefits are less important for wage formation in reality than in the wage formation of the canonical search and matching model. If the fall-back option of wage bargaining is a waiting period (instead of a destruction of the match), this would certainly be the case. In addition, wages may be negotiated at a more centralized level (e.g. at the firm-level) or wages may not be bargained at all (but for example posted by the firm).} Our model takes its queue from the empirical literature, demonstrating how unemployment benefits may have effects on employment without affecting the wage at all. For this purpose, we postulate a wage formation mechanism that (i) is simple but tractable, (ii) enables us to distinguish between job-offer decisions and job-acceptance decisions in the job-finding process and between firing decisions and quit decisions in the separation process, and (iii) omits the influence of benefits on the wage determination process.

Since we assumed that the wage is set after the employment decisions, the hiring and firing costs, as well as the match-specific random shocks, are already sunk.\footnote{This is the same assumption as in Pissarides (2009, p. 1364), and the corresponding footnote 30.} Thus, all workers obtain the same wage that, for simplicity, is assumed to be proportional to productivity $a$:

$$w_t = \omega a_t,$$

where $\omega$ ($0 < \omega < 1$) is a constant. The average aggregate productivity of each worker is $a$, a positive constant subject to random aggregate productivity shocks. Given the timing of economic decisions, wages are privately efficient because the idiosyncratic shock realizations are already sunk when the wage formation takes place.\footnote{This wage equation may be interpreted as the outcome of Nash bargaining between each employer and employee. In particular, this interpretation involves assuming that the fallback profit is zero (leaving future values unaffected) and that the fallback wage is either zero or an exogenous constant that is proportional to the negotiated wage. Such a bargaining game is in line with Hall and Milgrom (2008) and Binmore et al. (1986).}
Choosing this simple wage equation has three advantages. First, it ensures that our results are not generated by any kind of wage rigidity (since the wage moves proportionally with productivity). This is important since it is well-known that rigid wages imply that labour market shocks have larger amplification effects and thereby generate greater labour market volatilities (e.g. Hall, 2005). Second, as noted, it enables us to separate the decisions of workers and firms, thereby allowing us to distinguish firms’ firing from workers’ quit decisions. Third, our simple wage equation ensures analytical tractability.

This wage formation mechanism does not entail a loss of generality. We require a wage formation mechanism that does not contain unemployment benefits, in line with the empirical literature. However, our main result – that we break the tight link between productivity and unemployment on the one hand and unemployment benefits and unemployment on the other hand – does not depend on the specific functional form we have chosen. We could equally assume that the wage contains constants or some lagged terms. This would change the quantitative outcomes in our numerical simulation, but the main message of the paper would be unaffected.

3.2. The Firm’s Behaviour

We assume that the profit generated by a particular worker at a particular job is subject to a match-specific random cost shock $\varsigma_{it}$ in period $t$, which is meant to capture idiosyncratic variations in workers’ suitability for the available jobs. For example, workers in a particular skill group and sector may exhibit heterogeneous profitability due to random variations in their state of health, levels of concentration, and mobility costs, or to random variations in firms’ operating costs, screening, training, and monitoring costs, etc. The random shock $\varsigma_{it}$ is iid across workers-firm pairs, with a stable probability density function $G_{\varsigma}(\varsigma)$ that is publicly known. Let the corresponding cumulative distribution be $J_{\varsigma}(\varsigma)$. In each period of analysis a new value of $\varsigma_{it}$ is realized for each worker-firm pair. Since each pair draws from the same distribution of random shocks we omit the subscript $i$ for notational simplicity in the following. The unemployment benefits $b$, the hiring cost $hc$, and the firing cost $fc$ are all constant. The hiring cost includes the administrative costs, screening costs, retraining costs, and relocation costs, as well as the basic instruction, mentoring, and on-the-job training costs that are required to integrate the worker in the firm’s workforce. The firm maximizes the present value of its expected profit, with a time discount factor $\beta$. 
3.2.1. The Firing Decision

The expected present value of profit generated by an incumbent employee, after the random cost term $\zeta_t$ is observed, is

$$\pi^I_t (\zeta_t) = (a_t - w_t - \zeta_t) + \beta E_t \left[ (1 - \sigma_{t+1}) \pi^II_{t+1} - \phi_{t+1} fc \right],$$  

(2)

where $a_t$ is aggregate productivity, $w_t$ is the real wage, the superscript “I” stands for the incumbent employee, $\sigma_{t+1}$ is the separation rate, and $fc$ is the firing cost per worker paid with the firing probability $\phi_{t+1}$. $E_t [\pi^II_{t+1}]$ denotes the expected future average profit of an incumbent who will be retained:

$$E_t \left[ \pi^II_{t+1} \right] = E_t \left[ a_{t+1} - w_{t+1} - \left( \zeta_{t+1} \mid \zeta_{t+1} < \nu^I_{t+1} \right) + \beta \left( (1 - \sigma_{t+2}) \pi^II_{t+2} - \phi_{t+2} fc \right) \right],$$  

(3)

where

$$E_t \left[ \zeta_{t+1} \mid \zeta_{t+1} < \nu^I_{t+1} \right] = \int_{-\infty}^{\nu^I_{t+1}} G_\zeta(\zeta) d\zeta \over 1 - \phi_{t+1}$$

is the expectation of the random term $\zeta_{t+1}$ conditional on this random cost being sufficiently small to permit retention of the incumbent employee. We define the incumbent employee’s retention incentive $\nu^I_t$ as:

$$\nu^I_t = a_t - w_t + \beta E_t[(1 - \sigma_{t+1}) \pi^II_{t+1} - \phi_{t+1} fc] + fc.$$  

(4)

The firm’s incentive to keep an incumbent worker in employment is the difference between the gross expected profit from retaining the employed worker and the expected profit from firing her ($-fc$). Here and in the following, “gross” profit refers to the respective profit net of the idiosyncratic productivity component.

An incumbent worker is fired in period $t$ when the realized value of the random cost $\zeta_t$ is greater than the incumbent worker’s employment incentive: $\zeta_t > \nu^I_t$. Since the cumulative distribution of $\zeta_t$ is $J_\zeta(\nu^I_t)$, the employed worker’s firing rate is

$$\phi_t = 1 - J_\zeta(\nu^I_t).$$  

(5)

3.2.2. The Job-Offer Decision

The expected present value of profit generated by an entrant $\pi^E_t (\zeta_t)$, given that a contact has been made and the random cost $\zeta_t$ has been observed, is
\[ \pi_t^E (\varsigma_t) = a_t - w_t - \varsigma_t - hc + \beta E_t [(1 - \sigma_{t+1}) \pi_{t+1}^{II} - \phi_{t+1} fc], \]  \hspace{1cm} (6)

where \( hc \) is the constant hiring cost and the superscript “\( E \)” stands for “entrant”.

We define the firm’s expected job-offer incentive \( \nu_t^E \) as the difference between the gross expected profit from hiring a worker and the profit from not hiring her (i.e. zero):

\[ \nu_t^E = a_t - w_t - hc + \beta E_t [(1 - \sigma_{t+1}) \pi_{t+1}^{II} - \phi_{t+1} fc]. \]  \hspace{1cm} (7)

A job is offered when \( \nu_t^E > \varsigma_t \). Thus, the job-offer rate is

\[ \eta_t = J_{\varsigma} (\nu_t^E). \]  \hspace{1cm} (8)

Note that due to the hiring and firing costs, the retention incentive exceeds the job-offer incentive \( (\nu_t^I > \nu_t^E) \) and thus the retention rate exceeds the job-offer rate \( (1 - \phi_t > \eta_t) \).

### 3.3. The Worker’s Behaviour

The worker faces a discrete choice of whether or not to work. Her disutility of work effort at a given job is \( e_t \), a random variable, which is \( iid \) across worker-firm pairs, with a stable and publicly known probability density function \( G_e (e) \). The corresponding cumulative distribution is \( J_e (e) \). The random variable captures match-specific heterogeneities in the disagreeability of work, due to such factors as idiosyncratic reactions to particular workplaces or variations in the qualities of these workplaces. Due to the \( iid \) assumption and analogue to the firm’s problem, we omit the subscript \( i \) for notational simplicity in the following. The worker’s utility is linear in consumption and work effort. She consumes all her income and discounts the future with discount factor \( \beta \).

Observe that on the firm’s side, we distinguish between entrants \( (E) \) and incumbent workers \( (I) \), whereas on the worker’s side, we distinguish between employed \( (N) \) and unemployed \( (U) \) workers. The rationale for these two distinctions is that the firm can employ two types of workers (entrants and incumbents), whereas the worker can be in two states (employment and unemployment).

The worker’s expected present value of utility from working, \( \Omega_t^N (e_t) \), for a given realization of effort \( e \) is

\[ \Omega_t^N (e_t) = w_t - e_t + \beta E_t \left[ (1 - \sigma_{t+1}) \Omega_{t+1}^{NN} + \sigma_{t+1} \Omega_{t+1}^{NU} \right], \]  \hspace{1cm} (9)

where the superscript “\( N \)” stands for “employed”. \( E_t \left[ \Omega_{t+1}^{NN} \right] \) is the expected present
value of the future average utility of a worker who stays on the job (before the realized value of the shock $e_{t+1}$ is known), the superscript “$NN$” stands for “employed worker who stays employed”:

$$E_t \left[ \Omega_{t+1}^{NN} \right] = E_t \left[ w_{t+1} - (e_{t+1} | e_{t+1} < \iota_{t+1}^N) + \beta \left( (1 - \sigma_{t+2}) \Omega_{t+2}^{NN} + \sigma_{t+2} \Omega_{t+2}^{NU} \right) \right]. \quad (10)$$

The expectation of the future disutility of work conditional on not quitting is:

$$E_t \left[ e_{t+1} | e_{t+1} < \iota_{t+1}^N \right] = \iota_{t+1}^N - \int_{-\infty}^{\iota_{t+1}^N} eG_e(e) \, de, \quad (11)$$

where $\chi_{t+1}$ is next period’s quit rate.

$E_t \left[ \Omega_{t+1}^{NU} \right]$ is the expected present value of the future average utility of a worker who is separated from her job and can be immediately rehired, and the superscript “$NU$” stands for “formerly employed now unemployed”:

$$E_t \left[ \Omega_{t+1}^{NU} \right] = E_t \left[ \mu_{t+1} \Omega_{t+1}^{UN} + (1 - \mu_{t+1}) \Omega_{t+1}^{U} \right], \quad (11)$$

where $\mu_{t+1}$ is the next period’s job-finding rate. The expected present value utility from unemployment, $\Omega_t^{U}$, is

$$\Omega_t^{U} = b + \beta E_t \left[ \mu_{t+1} \Omega_{t+1}^{UN} + (1 - \mu_{t+1}) \Omega_{t+1}^{U} \right], \quad (12)$$

where the superscript “$U$” stands for “unemployed”. $E_t \left[ \Omega_{t+1}^{UN} \right]$ is the expected present value of the future average utility of a worker who finds a new job (before the realized value of the shock $e_{t+1}$ is known), the superscript “$UN$” stands for “formerly unemployed now employed”:

$$E_t \left[ \Omega_{t+1}^{UN} \right] = E_t \left[ w_{t+1} - (e_{t+1} | e_{t+1} < \iota_{t+1}^U) + \beta \left( (1 - \sigma_{t+2}) \Omega_{t+2}^{NN} + \sigma_{t+2} \Omega_{t+2}^{NU} \right) \right], \quad (13)$$

where

$$E_t \left[ e_{t+1} | e_{t+1} < \iota_{t+1}^U \right] = \frac{\int_{-\infty}^{\iota_{t+1}^U} eG_e(e) \, de}{\delta_{t+1}},$$

is the expectation of the future disutility of work conditional on accepting the job.

### 3.3.1. The Job-Acceptance Decision

An unemployed worker’s expected “job-acceptance incentive” $\iota_t^U$ is the expected difference between the gross utility from employment, $\Omega_t^N(e_t) + e$, and unemployment, $\Omega_t^U$:
\[ I_t^U = \Omega_t^N (e) + e - \Omega_t^U, \]  
which yields\(^\text{13}\)
\[ I_t^U = w_t - b + \beta E_t \left[ (1 - \sigma_{t+1}) \left( \Omega_{t+1}^{NN} - \left( \mu_{t+1} \Omega_{t+1}^{UN} + (1 - \mu_{t+1}) \Omega_{t+1}^U \right) \right) \right]. \]  
(15)

The unemployed accepts a job offer when \( e_t < I_t^U \). Consequently, the job-acceptance rate is
\[ \delta_t = J_e \left( I_t^U \right). \]  
(16)

3.3.2. The Quit Decision

An unemployed worker’s expected “non-quitting incentive” \( I_t^N \) is the expected difference between the gross value of employment, \( \Omega_t^N (e) + e \), and the value of being separated from employment into unemployment (with the option of being immediately rehired), \( \Omega_t^{NU} \):
\[ I_t^N = \Omega_t^N (e) + e - \Omega_t^{NU}, \]  
which yields\(^\text{14}\)
\[ I_t^N = (1 - \mu_t) (w_t - b) + \mu_t E_t \left[ e_t | e_t < I_t^U \right] + \beta (1 - \mu_t) E_t \left[ (1 - \sigma_{t+1}) \left( \Omega_{t+1}^{NN} - \left( \mu_{t+1} \Omega_{t+1}^{UN} + (1 - \mu_{t+1}) \Omega_{t+1}^U \right) \right) \right]. \]  
(18)

Note that the two worker incentives are distinct, since those who quit have the option to be immediately rehired. Consequently, the employee quits a job when \( e_t > I_t^N \). Thus the quit rate is\(^\text{15}\)
\[ \chi_t = 1 - J_e \left( I_t^N \right). \]  
(19)

3.4. Employment

An unemployed worker gets a job when three conditions are fulfilled: (i) she makes contact with an employer, (ii) she receives a job offer, and (iii) she accepts that offer.

\(^\text{13}\)For derivations see Appendix A.
\(^\text{14}\)For derivations see Appendix A.
\(^\text{15}\)Note that, by allowing workers who have been separated from their job to immediately reenter employment, the job-acceptance rate is not identical to the job staying rate (\( \delta_t \neq 1 - \chi_t \)).
For simplicity we set the probability $f$ that a worker makes contact with an employer in a given period equal to unity. It follows that the match probability $\mu_t$ is the product of job-offer probability $\eta_t$ and acceptance probability $\delta_t$:

$$
\mu_t = \eta_t \delta_t. \quad (20)
$$

An employee separates from her job when at least one of two conditions is satisfied: (i) she is fired or (ii) she quits, the separation probability is therefore

$$
\sigma_t = \phi_t + \chi_t - \phi_t \chi_t. \quad (21)
$$

The labour force is assumed to be constant and normalized to unity. The equilibrium employment rate, $n_t$, can therefore be described by the associated employment dynamics equation:

$$
n_t = \mu_t + (1 - (1 - \mu_t)\sigma_t - \mu_t)n_{t-1}, \quad (22)
$$

where the degree of employment persistence is given by $(1 - \mu_t - (1 - \mu_t)\sigma_t)$.

### 3.5. Free Entry of Firms

As in the conventional search literature, we assume free entry of firms, so that the number of vacancies $v_t$ is determined by a zero-profit condition.\(^{16}\) Let $\kappa$ be the cost of posting a vacancy and define market tightness as the ratio of vacancies and searching workers:

$$
\theta = \frac{v_t}{(u_{t-1} + \sigma n_{t-1})}. \quad (23)
$$

The probability that a vacancy is filled is $\mu_t/\theta_t = \mu_t((u_{t-1} + \sigma n_{t-1})/v_t)$, i.e. the match probability divided by the market tightness $\theta$. The expected profit per match is

$$(a_t - w_t - E_t [\varsigma_t | \varsigma_t < \nu^E] - hc + \beta E_t \left[ (1 - \sigma_{t+1}) \pi_{t+1}^{II} - \phi_{t+1}^f c \right]),$$

where

$$
E_t [\varsigma_t | \varsigma_t < \nu^E] = \frac{\int_{-\infty}^{\nu^E} G_\varsigma (\varsigma) d\varsigma}{\eta_t}
$$

is the expected value of the idiosyncratic productivity shock $\varsigma_t$ conditional on match formation.

\(^{16}\)We do not have to specify the number of firms as they face constant returns to scale (there is only an ex-post heterogeneity, once there are particular worker-firm pairs).
Thus, the zero-profit condition for posting vacancies is

$$\frac{\kappa}{(\mu_t/\theta_t)} = a_t - w_t - E_t \left[ \varsigma_t \varsigma_t < \nu_l^E \right] - hc + \beta E_t \left[ (1 - \sigma_{t+1}) \pi_{t+1}^I - \phi_{t+1} fc \right]. \quad (24)$$

3.6. The Labour Market Equilibrium and Aggregation

Given a process for aggregate productivity $a_t$, which we assume to be a first-order autocorrelated process, the labour market equilibrium is the solution of the system comprising the following equations:

- Incentives: the incumbent worker’s retention incentive $\nu_l^I$ (eq. (4)), the job-offer incentive $\nu_l^E$ (eq. (7)), the job-acceptance incentive $\iota_l^U$ (eq. (14)) and the job-quit incentive $\iota_l^N$ (eq. (17)).
- Employment decisions: the firing rate $\phi_t$ (eq. (5)) and the job-offer rate $\eta_t$ (eq. (8)).
- Work decisions: the job-acceptance rate $\delta_t$ (eq. (16)) and the quit rate $\chi_t$ (eq. (19)).
- Vacancies and market tightness: the number of vacancies $\nu_t$ (eq. (24)), given the definition of market tightness $\theta_t$ (eq. (23)).
- Match and separation probabilities: the match probability $\mu_t$ (eq. (20)) and the separation probability $\sigma_t$ (eq. (21)).
- Employment and wage: the employment rate $n_t$ (eq. (22)) and the wage $w_t$ (eq. (1)).

Output in this model economy is defined as aggregate production, $n_t a_t$, minus total operating costs for entrants and incumbent workers

$$\int_{-\infty}^{\mu_t^E} x G_x(s) \, ds \frac{k}{\eta_t} \mu_t \left[ (1 - n_{t-1}) + \sigma_t n_{t-1} \right],$$

$$\int_{-\infty}^{\mu_t^I} x G_x(s) \, ds \frac{k}{1 - \phi_t} (1 - \sigma_t) n_{t-1},$$
as well as firms’ hiring costs, $hc\mu_t [(1 - n_{t-1}) + \sigma_t n_{t-1}]$, firing costs, $f c\phi_t n_{t-1}$, and vacancy posting costs, $v_t \kappa$:

$$y_t = n_t \alpha_t - \left( \frac{f^E \varsigma G(\varsigma) d\varsigma}{\eta_t} + hc \right) \mu_t [(1 - n_{t-1}) + \sigma_t n_{t-1}]$$

$$- \frac{f^I \varsigma G(\varsigma) d\varsigma}{1 - \phi_t} (1 - \sigma_t) n_{t-1} - f c\phi_t n_{t-1} - v_t \kappa.$$ (25)

4. Analytical Results

To gain intuition for the key model mechanism, this section provides analytical results. For this purpose, we assume that separations, $\sigma$, are exogenous. Since this implies exogenous quit and firing rates, we can omit firing costs and firm and household shocks beyond the first period of production. We assume that hiring and firing takes place at the same time. Furthermore, we analyze the model in steady state (see Appendix B for details). Therefore, the employment equation is

$$n = \frac{\mu}{\mu + (1 - \mu) \sigma}. \quad (26)$$

Because separations are exogenous, the labour market is driven by the job-finding rate which is the product of job-offer and job-acceptance rate:

$$\mu = \delta \eta. \quad (27)$$

In order to understand the effects of unemployment benefits on the job-finding rate (and thus on unemployment), we have to disentangle the effects on the two rates (see Appendix B):

$$\frac{\partial \mu}{\partial b} = \frac{\partial \eta}{\partial b} \delta + \eta \frac{\partial \delta}{\partial b}. \quad (28)$$

The job-offer rate is

$$\eta = J_\varsigma(\nu^E), \quad (29)$$

with

$$\nu^E = \frac{a - w}{1 - \beta (1 - \sigma)} - hc \quad (30)$$

and the job-acceptance rate is

$$\delta = J_e(\nu^U), \quad (31)$$
with
\[
U = w - b + \beta \mu \int_{-\infty}^{U} eG(e)de \frac{1 - \beta(1 - \sigma - \mu)}{1 - \beta(1 - \sigma - \mu)}.
\] (32)

The reaction of the job-offer rate to unemployment benefit changes is
\[
\frac{\partial \eta}{\partial b} = -G(\nu) \left( \frac{\partial w}{\partial b} \frac{1}{1 - \beta(1 - \sigma)} \right).
\] (33)

This expression shows that the job-offer rate is only affected by unemployment benefit changes if there is a wage effect \(\frac{\partial w}{\partial b} > 0\). With the empirically plausible case \(\frac{\partial w}{\partial b} = 0\), the job-offer rate does not react to changes of \(b\) \(\left(\frac{\partial \eta}{\partial b} = 0\right)\).

The reaction of the job-acceptance rate with respect to unemployment benefit changes is a more complicated expression (see Appendix B). However, for the empirically plausible case in which wages do not react to unemployment benefits this expression simplifies to
\[
\frac{\partial \delta}{\partial b} = -G(\nu) \left( \frac{1}{1 - \beta(1 - \sigma - \mu)} \right).
\] (34)

This expression has an unambiguously negative sign (see Appendix B).

Equations (33) and (34) show that an increase of unemployment benefits depresses the job-finding rate (via the job-acceptance rate) and thereby raises unemployment even if unemployment benefits do not affect wages at all. Thus, our model can reconcile the following two empirical results. First, the effects of unemployment benefits on wages are non-existent or at best small (see Section 2). Second, there are moderate effects of unemployment benefits on the job-finding rate. Our incentive model of two-sided search offers a new mechanism for these phenomena. Larger unemployment benefits depress the incentives of workers to accept the job offer because their outside option becomes more attractive.

In addition, the incentive model of two-sided search allows us to decouple the effects of aggregate productivity shocks on the job-finding rate (which are known to be large) and of benefits on the job-finding rate (which are known to be moderate). We show in Appendix C that these two effects are tightly linked in a search and matching model (via the wage effect). In our model, the reaction of the job-offer rate to aggregate productivity changes is the combination of the reaction of job-offer and job-acceptance rate:
\[
\frac{\partial \mu}{\partial a} = \frac{\partial \eta}{\partial a} \delta + \eta \frac{\partial \delta}{\partial a},
\] (35)
\[ \frac{\partial \eta}{\partial a} = G_\varsigma(n^E) \frac{1 - \frac{\partial w}{\partial a}}{1 - \beta (1 - \sigma)}. \]  

(36)

\[ \frac{\partial \delta}{\partial a} = G_e(u^U) \left( \frac{\frac{\partial w}{\partial a} + \beta \frac{\partial \eta}{\partial a} \left( \int_{-\infty}^{u^U} e G_e(e) \, de - \delta U \right)}{1 - \beta (1 - \sigma - \mu)} \right). \]  

(37)

It is obvious that unemployment benefits and productivity may have very different effects on the job-finding rate. First, in the empirically plausible case \( \frac{\partial w}{\partial b} = 0 \), the job-offer rate does not react to changes of unemployment benefits (see equation 33), while the job-offer rate may show a strong reaction to productivity changes (see equation 36). Second, the reaction of the job-offer and job-acceptance rates depend on different distributions. The job-offer rate is a function of the density of the idiosyncratic operating cost distribution at the cutoff point \( G_\varsigma(n^E) \), while the job-acceptance rate is a function of the density of the idiosyncratic disutility distribution at the cutoff point \( G_e(u^U) \).

In order to assess the quantitative importance of these findings, we next simulate the full two-sided model. To limit our degrees of freedom regarding the shape of the idiosyncratic density functions, we use JOLTS business cycle data for quits and firings. We set the dispersion of idiosyncratic operating costs and idiosyncratic disutility in order to replicate the business cycle behaviour of these variables in the data. The calibrated model is then used for a counterfactual exercise where unemployment benefits are changed.

5. Calibration

We calibrate the full dynamic model to the U.S. labour market. We simulate monthly series that we subsequently aggregate to quarterly frequency.

As noted, we assume that each unemployed worker makes one contact in each time period. Steady state productivity is normalized to 1 and the wage share is \( \omega = 0.5 \), a value commonly used in the literature. The replacement rate is 0.4, such that unemployment compensation amounts to 40% of the steady state wage as also used by Shimer (2005) and Hall (2005). We set the firing cost to \( fc = 0.1 \) following the procedure in Brown et al. (2015). The discount factor is \( \beta = 1.04^{-1/12} \) which corresponds to a 4% annual interest rate. We target a steady state market tightness \( \theta \) of 1, which pins down the value of the vacancy posting cost.

We assume that the idiosyncratic shocks for the firm, \( \varsigma \), and household, \( e \), are drawn from a logistic distribution with scale factors \( s_\varsigma \) and \( s_e \) and expected values \( \bar{\varsigma} \) and \( \bar{e} \), respectively. As explained above, the calibration of these distributions is important for
the response of the model to aggregate shocks. We eliminate our degrees of freedom by making use of the information on quits and firings in the JOLTS data, namely their means and standard deviations relative to output. These four targets help us to uniquely pin down the parameters of the two distributions.

We use monthly time series from the first quarter of 2001 to the first quarter of 2014. The JOLTS data offers separate time series on both firings and quits. We target the mean values of these series in the data, namely a monthly private firing rate of 1.58% and a monthly private quit rate of 2.07%.\textsuperscript{17} The JOLTS quit series includes both quits into unemployment and job-to-job transitions (see Moscarini and Postel-Vinay, 2016). We account for job-to-job transitions by allowing for immediate rehiring of separated matches. We further use the standard deviations of the firing and quit rate relative to output in the data as targets in our model. For that purpose, we aggregate the monthly series to quarterly frequency, HP-filter with smoothing parameter $\lambda = 10^5$, and calculate the ratio of the standard deviations of the quit and firing rates and the standard deviation of output, $sd(\phi)/sd(y)$ and $sd(\chi)/sd(y)$, respectively.\textsuperscript{18} Finally, we target a steady state unemployment rate $u$ of 6.6%, which corresponds to the mean civilian unemployment rate (CPS) during our time span. We thus have five targets (the mean unemployment rate as well as the mean and relative volatility of the firing and quit rate) for five parameters: the mean and scale parameter of the firm and household distribution as well as the hiring cost. All targets and parameter values are displayed in Tables 1 and 2.

The variance of the idiosyncratic shocks is of significant importance for the aggregate dynamics. The lower it is, the stronger are the reactions to productivity and unemployment benefits in our model. Our baseline calibration yields a relatively large standard deviation of the idiosyncratic productivity shock of $sd(\varsigma) = 1.355$.\textsuperscript{19} Using this relatively more conservative value, we bias the dynamics against our model. This ensures that strong amplifications of productivity shocks are not driven by an unrealistically small standard deviation of the idiosyncratic cost shock in our calibration. At the same time, we have a relatively low standard deviation of the idiosyncratic utility shock of $sd(e) = 0.202$. Again, we bias the dynamics against our model since the reactions to unemployment benefits, which in our model work only through the household decision, are moderate in the data. In other words, our success in breaking the link between the

\textsuperscript{17}We do not consider “other separations” as these have no corresponding term in our model.

\textsuperscript{18}Output in the nonfarm business sector from BLS, Major Sector Productivity and Costs (MSPC) database.

\textsuperscript{19}This is much larger than values commonly used in the literature, e.g. between $sd(\varsigma) = 0.0375$ and $sd(\varsigma) = 0.12$, see Brown et al. (2015).
impact of productivity and unemployment benefits does not rest on unrealistic standard deviations of the idiosyncratic shocks.

Finally, we model productivity as a first-order autocorrelated process. We choose the autocorrelation parameter, $\rho$, and the standard deviation of the aggregate shock, $\sigma_a$, such that we match the autocorrelation and standard deviation of productivity in the data.\(^{20}\)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Productivity</td>
<td>1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Market tightness</td>
<td>1</td>
</tr>
<tr>
<td>$u$</td>
<td>Unemployment rate</td>
<td>6.6%</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Firing rate</td>
<td>1.58% JOLTS</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Quit rate</td>
<td>2.07% JOLTS</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Job-finding rate</td>
<td>33.9% Eq. (20)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Separation rate</td>
<td>3.6% Eq. (21)</td>
</tr>
</tbody>
</table>

\[
\frac{sd(\phi)}{sd(y)} \quad \text{Relative volatility firings} \quad 3.12 \quad \text{JOLTS/MSPC}
\]
\[
\frac{sd(\chi)}{sd(y)} \quad \text{Relative volatility quits} \quad 4.20 \quad \text{JOLTS/MSPC}
\]
\[
\frac{sd(a)}{sd(y)} \quad \text{Standard deviation productivity} \quad 0.0134 \quad \text{MSPC}
\]
\[
\text{autocorr}(a) \quad \text{Autocorrelation productivity} \quad 0.803 \quad \text{MSPC}
\]

Table 1: Targets and steady state values

6. Numerical Results

We simulate the model for 159 months corresponding to the time span in the data.\(^{21}\) For the business cycle statistics we aggregate the data to quarterly frequency and filter the data using a Hodrick-Prescott filter with smoothing parameter $\lambda = 10^5$. We repeat this exercise 1000 times and report means over these simulations. Keep in mind that in all these exercises our wage is only a function of aggregate productivity and – more in line with the empirical evidence – does not respond to changes in the level of benefits. This also implies that high unemployment benefits would not help to create high amplification in the model by implicitly making the wage rigid.\(^{22}\)

\(^{20}\)Productivity is output over employment in the nonfarm business sector from the BLS, Major Sector Productivity and Costs (MSPC) database. Moments are calculated from HP-filtered data ($\lambda = 10^5$) from 1Q2001-1Q2014.

\(^{21}\)We simulate longer time series, but discard the first 500 periods.

\(^{22}\)Although the rigid wage version of the search and matching model can also generate higher volatilities (see Hall, 2005), it implies the counterfactual prediction that wages are acyclical. Hagedorn and Manovskii (2008) use a high value for unemployment benefits, thereby a small-surplus calibration, in which the average unemployed worker is basically indifferent between working and not working. We do
### Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Autocorr. coeff productivity</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>SD of productivity shock</td>
</tr>
<tr>
<td>$f$</td>
<td>Contact rate</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Wage share</td>
</tr>
<tr>
<td>$b$</td>
<td>Unemployment benefits</td>
</tr>
<tr>
<td>$fc$</td>
<td>Firing cost</td>
</tr>
<tr>
<td>$hc$</td>
<td>Hiring cost</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Vacancy posting cost</td>
</tr>
<tr>
<td>$\bar{\varsigma}$</td>
<td>Mean of firm distr.</td>
</tr>
<tr>
<td>$s_{\varsigma}$</td>
<td>Scale param. of firm distr.</td>
</tr>
<tr>
<td>$sd(\varsigma)$</td>
<td>SD of firm distr.</td>
</tr>
<tr>
<td>$\bar{e}$</td>
<td>Mean of hh distr.</td>
</tr>
<tr>
<td>$s_{e}$</td>
<td>Scale param. of hh distr.</td>
</tr>
<tr>
<td>$sd(e)$</td>
<td>SD of hh distr.</td>
</tr>
</tbody>
</table>

| Table 2: Parameter values |

### 6.1. Business Cycle Dynamics

The volatilities of quits and firings relative to output are key to the calibration of the idiosyncratic shock distributions and hence to the response of the model to aggregate shocks. Table 3 shows that we hit the targets (i.e. the actual standard deviation of these variables).\(^{23}\) Interestingly, the model also produces correlations between quits, firings, separations, and output that are fairly close to the data (without having targeted them). As in the data, quits are countercyclical and firings are procyclical in our simulation (i.e. quits drop in a recession and firings rise in a recession). Furthermore, as in the data, the cyclicality of quits dominates the cyclicality of firings. Thus, total separations are procyclical. The procyclicality of separations distinguishes our model from most endogenous separation models that only consider a firing margin.

The business cycle dynamics of job-offer and acceptance rates are not targeted due to a lack of data availability. Thus, the standard deviations are determined endogenously via the other model parameters. The respective distributional shape is identified by quits/firings and the assumption that new matches and existing matches are hit by the same type of idiosyncratic shocks. The resulting standard deviation of unemployment relative to productivity is 2.35 and the relative standard deviation of the job-finding not rely on these assumptions.

\(^{23}\)Note that for consistency, separations in the data are calculated as in the model: $\sigma = \chi + \phi - \chi \phi$. Results are very similar if we take the separation series directly from JOLTS.
Table 3: Business cycle statistics of separation margin: Standard deviation relative to output and correlation with output.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{sd}(\phi)/\text{sd}(y)$</td>
<td>3.12</td>
<td>3.12</td>
</tr>
<tr>
<td>$\text{corr}(\phi,y)$</td>
<td>-0.99</td>
<td>-0.60</td>
</tr>
<tr>
<td>$\text{sd}(\chi)/\text{sd}(y)$</td>
<td>4.20</td>
<td>4.20</td>
</tr>
<tr>
<td>$\text{corr}(\chi,y)$</td>
<td>0.99</td>
<td>0.85</td>
</tr>
<tr>
<td>$\text{sd}(\sigma)/\text{sd}(y)$</td>
<td>1.26</td>
<td>1.69</td>
</tr>
<tr>
<td>$\text{corr}(\sigma,y)$</td>
<td>0.98</td>
<td>0.67</td>
</tr>
</tbody>
</table>

rate is 2.59 (see Table 4). These values are substantially higher than those typically found for a standard search and matching model as in Shimer (2005). In addition, the model produces a solid Beveridge curve with a correlation between vacancies and unemployment of -0.86. This is remarkable, given the problems of standard endogenous separation models to replicate the Beveridge curve (see e.g. Krause and Lubik, 2007). Of course, the relative volatilities of unemployment and the job-finding rate are lower than in the data.

Note, first, that we have assumed the wage to be completely flexible. A moderately rigid wage would lead to additional amplification. Second, the procyclicality of the separation rate works against a high volatility of unemployment and the job-finding rate. Most of the debate on the ability of the search and matching model to generate amplification is based on models with exogenous separations (e.g. Shimer, 2005; Hall, 2005; Hagedorn and Manovskii, 2008). To make our model comparable to this literature, we also run simulations with exogenous separations, which leads to even stronger amplification. Finally, it has to be kept in mind that we simulate our model with productivity shocks only, while in reality there are likely other driving shocks.

6.2. The Effects of a Rise in Unemployment Compensation

The previous section has shown that the model can generate substantial amplification. The relevant question is now how the job-finding rate and unemployment react to changes of unemployment benefits in the context of our calibration. For this purpose, we calculate the elasticity of unemployment and the job-finding rate to a 1% rise in unemployment

\[24\text{We take the exact same model and calibration as before but fix the firing and quit rate as well as the conditional expectations of the idiosyncratic shocks at their means. In this case, the model generates substantial amplification with the relative volatility of unemployment rising to 9.7 and the relative volatility of the job-finding rate rising to 7.}\]
compensation. The resulting elasticity of the unemployment rate is 1.32. The elasticity of the job-finding rate as well as its reciprocal, the value usually referred to in empirical studies, is (-)0.55. This value indeed lies exactly in the ballpark of empirical estimates as reported e.g. in Hornstein et al. (2005).

In addition, the elasticity of the job-finding rate with respect to productivity is 2.7 for our calibration. Therefore, the ratio between the elasticity of the job-finding rate with respect to productivity and the job-finding elasticity with respect to unemployment benefits is 4.9. Thus, our quantitative model makes a big step towards breaking the link between the effects of productivity and benefits on unemployment (for a comparison to the search and matching model see Appendix C).

### 7. Conclusion

In the standard macroeconomic model of the labour market, the job-finding rate is determined through a matching function that depends on the unemployment and vacancy rates. In this context, the only way in which unemployment benefits can affect unemployment is via the wage, vacancy creation, and hence the matching function. By implication, there is a tight link between the unemployment effects of productivity (where the responsiveness of the wage also plays a major role) and of unemployment benefits. But this tight link runs counter the available empirical evidence.

This paper overcomes this problem by decomposing the matching process into its choice-theoretic components, namely the profit-maximizing decisions of the firms (determining the job-offer rate and the firing rate) and the utility-maximizing decisions of the workers (determining the job-acceptance rate and the quit rate). The underlying idea is that macroeconomic shocks (such as productivity shocks) and labour market policy changes (such as changes in unemployment benefits) shift the incentives facing firms and workers, thereby endogenously changing the form of the matching process whereby unemployed people find the available jobs.
Since the standard matching function relates job matches mechanically to unemployment and vacancies, unemployment benefits can affect unemployment only via the same channel. In our analysis, by contrast, a rise in unemployment benefits reduces workers’ incentives to seek and keep jobs, thereby reduces their job-acceptance rate and raises their quit rate; this, in turn, has further influences on the firms’ incentives. Consequently, unemployment benefits can exert a direct effect on the job-offer and quit rate, rather than flowing through wage formation. This is consonant with the empirical evidence, which indicates that unemployment benefits have little if any influence on wages, but have an effect on unemployment.
References


A. Derivations of Worker’s Behaviour

An unemployed worker’s “job-acceptance incentive” \( i_t^U \) is the expected difference between the expected gross utility stream from employment, \( \Omega_t^N (e_t) + e_t \), and unemployment, \( \Omega_t^U \):

\[
i_t^U = \Omega_t^N (e_t) + e_t - \Omega_t^U
\]

(38)

Substituting eq. (11) and collecting terms yields

\[
i_t^U = w_t - b + \beta E_t \left[ (1 - \sigma_{t+1}) \Omega_{t+1}^{NN} + \sigma_{t+1} \Omega_{t+1}^{NU} - \mu_{t+1} \Omega_{t+1}^{U} - (1 - \mu_{t+1}) \Omega_{t+1}^U \right].
\]

(39)

An unemployed worker’s expected “non-quitting incentive” \( i_t^N \) is the difference between the expected gross value of employment, \( \Omega_t^N (e_t) + e_t \), and the value of being separated from employment into unemployment (with the option of being immediately rehired), \( \Omega_t^{NU} \):

\[
i_t^N = \Omega_t^N (e_t) + e_t - \Omega_t^{NU}
\]

(40)

Using eq. (11), (12), and (13) and collecting terms yields

\[
i_t^N = (1 - \mu_t) (w_t - b) + \mu_t E_t \left[ e_t | e_t < i_t^U \right] + \beta (1 - \mu_t) E_t \left[ (1 - \sigma_{t+1}) \left( \Omega_{t+1}^{NN} - (1 - \mu_{t+1}) \Omega_{t+1}^U \right) \right].
\]

(41)

B. Analytical Derivations

In order to make analytical statements, we make some simplifying assumptions. First, we assume that separations are exogenous. Second, we analyze the steady state version of the model. Third, we assume that hiring and firing take place at the same time. Thus, the employment equation becomes:
\[ n = \frac{\mu}{\mu + (1 - \mu)\sigma}. \] (42)

**B.1. Job-Offer Decision**

The firm’s job-offer incentive in the steady state is

\[ \nu^E = \frac{a - w}{1 - \beta (1 - \sigma)} - hc. \] (43)

The job-offer rate is thus:

\[ \eta = J_\varsigma (\nu^E). \] (44)

Differentiating with respect to the unemployment benefits, \( b \), yields

\[ \frac{\partial \eta}{\partial b} = J_\varsigma' (\nu^E) \frac{\partial \nu^E}{\partial b} \] (45)

\[ = G_\varsigma (\nu^E) \left( 1 - \frac{\partial w}{\partial a} \right) \frac{1 - \beta (1 - \sigma)}{1 - \beta (1 - \sigma)}. \] (46)

It is important to note that with exogenous separations, unemployment benefits \( b \) only affect the job-offer rate via the wage. There is no direct effect.

Likewise the reaction of the job-offer rate to a change in productivity is:

\[ \frac{\partial \eta}{\partial a} = G_\varsigma (\nu^E) \left( 1 - \frac{\partial w}{\partial a} \right) \frac{1 - \beta (1 - \sigma)}{1 - \beta (1 - \sigma)}. \] (47)

**B.2. The Job-Acceptance Decision**

To keep tractability, we assume that workers once separated can only be hired in the next period, thus, remain unemployed for one period in line with the original model by Brown et al. (2015). This simplifies the analytical expressions significantly.

The value of employment for an entrant, i.e. a newly hired worker, for a given shock \( e \) is:

\[ \Omega^N(e) = w - e + \beta \left( (1 - \sigma)\Omega^{NN} + \sigma \Omega^U \right). \] (49)

The value of employment for an incumbent worker is:
\( \Omega^{NN} = w + \beta \left( (1 - \sigma)\Omega^{NN} + \sigma\Omega^U \right). \)  

(50)

The value of unemployment is given by:

\( \Omega^U = b + \beta \left( \mu\Omega^{UN} + (1 - \mu)\Omega^U \right). \)  

(51)

Finally, the value of employment for an entrant before the shock is observed is:

\( \Omega^{UN} = w - \left( e|e < \iota^U \right) + \beta \left( (1 - \sigma)\Omega^{NN} + \sigma\Omega^U \right). \)  

(52)

The worker’s incentive to accept a job offer is:

\[
\iota^U = \Omega^N(e) + e - \Omega^U
= \Omega^{NN} - \Omega^U
= w - b + \beta \left[ \left( (1 - \sigma)\Omega^{NN} + \sigma\Omega^U \right) - \left( \mu\Omega^{UN} + (1 - \mu)\Omega^U \right) \right]
= w - b + \beta\mu \left( e|e < \iota^U \right) + \beta(1 - \sigma - \mu) \left( \Omega^{NN} - \Omega^U \right)
= \frac{w - b + \beta\mu \left( e|e < \iota^U \right)}{1 - \beta(1 - \sigma - \mu)}
\]  

(53)

with \( \left( e|e < \iota^U \right) = \int_{-\infty}^{\iota^U} eG_e(e) de \).

Taking into account \( \mu = \eta\delta \),

\[
\iota^U = \frac{w - b + \beta\eta \int_{-\infty}^{\iota^U} eG_e(e) de}{1 - \beta(1 - \sigma - \eta\delta)}
\]  

(54)

The job-acceptance rate is thus given by

\( \delta = J_e(\iota^U). \)  

(55)

**Changes of Unemployment Benefits**

The households react as follows to changes of unemployment benefits:
\[
\frac{\partial \delta}{\partial b} = \frac{\partial \delta}{\partial U} \frac{\partial U}{\partial b}
\]
\[= J_e(U) \frac{\partial U}{\partial b}
\]
\[= G_e(U) \frac{\partial U}{\partial b}.
\]

The derivative of the acceptance incentive with respect to unemployment benefits yields the following expression:

\[
\frac{\partial U}{\partial b} = \left( \frac{\partial w}{\partial b} - 1 + \beta \eta G_e(U) \frac{\partial U}{\partial b} \right) \left( 1 - \beta (1 - \sigma - \mu) \right) \left( w - b + \beta \eta \int_{-\infty}^{U} e G_e(e) \, de \right) - \left( \beta \eta \frac{\partial \delta}{\partial U} \frac{\partial U}{\partial b} \right) \left( w - b + \beta \eta \int_{-\infty}^{U} e G_e(e) \, de \right) = \left( 1 - \beta (1 - \sigma - \mu) \right)^2 \frac{-1}{1 - \beta (1 - \sigma - \mu)}.
\]

We focus on the empirically plausible case \( \frac{\partial w}{\partial b} = 0 \) and thus \( \frac{\partial \eta}{\partial b} = 0 \), which yields:

\[
\frac{\partial U}{\partial b} = \frac{- \beta \eta G_e(U) \frac{\partial U}{\partial b} \left( w - b + \beta \eta \int_{-\infty}^{U} e G_e(e) \, de \right) - \beta \eta G_e(U) \theta U}{\left( 1 - \beta (1 - \sigma - \mu) \right)^2}.
\]

By canceling \( 1 - \beta (1 - \sigma - \mu) \) on the right hand side and taking into account eq. (54) and \( \frac{\partial \delta}{\partial U} = G_e(U) \), we get:

\[
\frac{\partial U}{\partial b} = \frac{- \beta \eta G_e(U) \theta U}{\left( 1 - \beta (1 - \sigma - \mu) \right)}.
\]

The first derivative of the household’s job-acceptance rate with respect to unemployment benefits is thus given by the following expression:

\[
\frac{\partial \delta}{\partial b} = - \frac{G_e(U)}{\left( 1 - \beta (1 - \sigma - \mu) \right)} < 0.
\]

This expression has a negative sign. Both the numerator and the denominator are larger than zero.
Changes of Productivity

In addition, we derive the derivative of the acceptance rate with respect to productivity:

\[
\frac{\partial \delta}{\partial a} = \frac{\partial \delta}{\partial U} \frac{\partial U}{\partial a} = J_e(U) \frac{\partial U}{\partial a} = G_e(U) \frac{\partial U}{\partial a}. \tag{62}
\]

\[
\frac{\partial U}{\partial a} = \left( \frac{(1 - \beta (1 - \sigma - \mu)) \times \left( \frac{\partial w}{\partial a} + \beta \eta U G_e(U) \frac{\partial U}{\partial a} + \beta \frac{\partial \eta}{\partial a} \int_{-\infty}^{U} eG_e(e) \, de \right) - \beta \frac{\partial \eta}{\partial a} \left( w - b + \beta \eta \int_{-\infty}^{U} eG_e(e) \, de \right) \right)}{(1 - \beta (1 - \sigma - \mu))^2} \right)
\]

\[
\frac{\partial U}{\partial a} = \left( \frac{\frac{\partial w}{\partial a} + \beta \eta U G_e(U) \frac{\partial U}{\partial a} + \beta \frac{\partial \eta}{\partial a} \int_{-\infty}^{U} eG_e(e) \, de - \beta \frac{\partial \eta}{\partial a} \right)}{(1 - \beta (1 - \sigma - \mu))} \right) \tag{63}
\]

Using

\[
\frac{\partial \mu}{\partial a} = \frac{\partial \eta}{\partial a} \delta + \frac{\partial \delta}{\partial a} \tag{64}
\]

and

\[
\frac{\partial \delta}{\partial a} = G_e(U) \frac{\partial U}{\partial a}. \tag{65}
\]

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we get:

\[
\begin{align*}
\frac{\partial \delta}{\partial a} &= \left( \frac{\partial w}{\partial a} + \beta \eta U G_e(U) \frac{\partial w}{\partial a} + \beta \frac{\partial \eta}{\partial a} \int_{-\infty}^{U} eG_e(e) \, de \right) \left( 1 - \beta (1 - \sigma - \mu) \right) \\
&\quad - \beta \left( \frac{\partial w}{\partial a} \delta + \eta \frac{\partial \delta}{\partial a} \right) U G_e(U) \\
&= \left( \frac{\partial w}{\partial a} + \beta \frac{\partial \eta}{\partial a} \int_{-\infty}^{U} eG_e(e) \, de - \beta \frac{\partial \eta}{\partial a} \delta U \\n&\quad \left( 1 - \beta (1 - \sigma - \mu) \right) \right) \\
&= \left( \frac{\partial w}{\partial a} + \beta \frac{\partial \eta}{\partial a} \int_{-\infty}^{U} eG_e(e) \, de - \delta U \\n&\quad \left( 1 - \beta (1 - \sigma - \mu) \right) \right) .
\end{align*}
\] (66)

The derivative of the job-acceptance rate with respect to productivity is therefore:

\[
\frac{\partial \delta}{\partial a} = G_e(U) \left( \frac{\partial w}{\partial a} + \beta \frac{\partial \eta}{\partial a} \left( \int_{-\infty}^{U} eG_e(e) \, de - \delta U \right) \right) .
\] (67)

This expression shows two countervailing effects. An increase in productivity leads to higher wages (\(\frac{\partial w}{\partial a} > 0\)) and thereby higher incentives to accept job offers (first part in the numerator). By contrast, an increase of productivity raises the job-offer rate (\(\frac{\partial \eta}{\partial a} > 0\)) and thereby allows workers to be more selective. Note that the expression in parentheses in the numerator is clearly negative because the conditional expectation of the disutility of work must be smaller than the cutoff point:

\[
\int_{-\infty}^{U} eG_e(e) \, de < \delta U .
\]

**C. Tight Link in Search and Matching Model**

In a standard search and matching model, the dynamic job-creation condition is

\[
\frac{\kappa}{q(\theta_t)} = a_t - w_t(a_t, b_t) + (1 - \phi) \beta E_t \frac{\kappa}{q(\theta_{t+1})} .
\] (68)
With a Cobb-Douglas constant returns matching function, the worker-finding rate \( q \) is a function of market tightness \( \theta \):

\[
q(\theta_t) = \vartheta \theta_t^{1 - \xi},
\]

where \( \vartheta \) is the matching efficiency and \( \xi \) is the weight on vacancies in the matching function.

In steady state:

\[
\frac{\kappa}{q(\theta)} = \frac{a - w(a, b)}{1 - (1 - \phi) \beta}.
\]

Then, after some algebra, the job-finding rate \( \mu \), which in the search and matching model is equal to the contact rate \( f(\theta) \), can be described as:

\[
\mu = f(\theta) = \theta q(\theta) = \left( \frac{a - w(a, b)}{\kappa(1 - \beta (1 - \phi))} \right)^{\frac{\xi}{1 - \xi}}. \tag{71}
\]

To illustrate the tight link between unemployment benefits and wages on the one hand and productivity and wages on the other hand, let us assume the following illustrative wage formation equation:\textsuperscript{25}

\[
w(a, b) = \omega a + (1 - \omega) b. \tag{72}
\]

The intertemporal discounted surplus is decisive for the job-creation dynamics. Let us define the employer’s surplus as

\[
S = \frac{a - w(a, b)}{1 - \beta (1 - \phi)} = \frac{(1 - \omega) (a - b)}{1 - \beta (1 - \phi)}. \tag{73}
\]

The elasticity of the job-finding rate is

\[
\frac{\partial \ln \mu}{\partial \ln x} = \frac{\xi}{1 - \xi} \left( \frac{S}{\kappa} \right)^{\frac{\xi}{1 - \xi} - 1} \frac{1}{\kappa} \frac{\partial S}{\partial x} \frac{x}{\mu}, \tag{74}
\]

where \( x \) could either represent the unemployment benefits \( b \) or productivity \( a \). The reaction of the surplus is

\[
\frac{\partial S}{\partial a} = \frac{(1 - \omega)}{1 - \beta (1 - \phi)}, \tag{75}
\]

\textsuperscript{25}Usually, the outcome of the standard Nash bargaining solution is \( w(a, b) = \omega (a + \kappa \theta) + (1 - \omega) b \). We choose our simpler bargaining equation for several reasons. First, it allows us to derive analytical results. Second, our equation is closer to the wage that we employ in the main part.
\[
\frac{\partial S}{\partial b} = \frac{-(1 - \omega)}{1 - \beta (1 - \phi)}.
\] (76)

To see the tight link between the elasticity of the job-finding rate with respect to unemployment benefits and productivity, compare the two:

\[
\frac{\partial \ln \mu}{\partial \ln a} / \frac{\partial \ln \mu}{\partial \ln b} = \frac{a}{b}.
\] (77)

The ratio between benefits and aggregate productivity, \(b/a\), is set in the literature between 0.4 (Shimer, 2005) and close to 1 (Hagedorn and Manovskii, 2008). Thus, the elasticity of the job-finding rate \(\mu\) with respect to productivity \(a\) is (in absolute terms) between 1 and 2.5 larger than the elasticity of the job-finding rate \(\mu\) with respect to unemployment benefits \(b\). For the case with large amplifications (Hagedorn and Manovskii, 2008), \(a \approx b\). Thus, in this case, the two elasticities are roughly linked one to one.
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