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On the optimum timing of the global carbon-transition under conditions of extreme weather-related damages: further green paradoxical results

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by

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Summary: An optimal growth model is constructed in a context of irreversible investment in different concurrent production technologies in order to study the optimum timing of the various stages that can be identified in the transition from a carbon-based economy to a carbon-free economy. Such a transition is necessary to avoid runaway global warming. The model is based on the AK-model by Rebelo (1991) and uses multi-stage optimal control methods to illustrate the importance of the notion of capital goods as a produced means of production and as the physical carriers/manifestations of different technologies for the timing and intensity of the use of carbon-based and carbon-free technologies during a just-in-time transition. The transition needs to happen within a given cumulative emissions constraint in order to avoid runaway global warming and corresponding runaway welfare losses. It is shown that the expectation of extreme weather-related damages in such a setting may induce a welfare maximizing central planner to step-up carbon-based production rather than reducing such activity, for similar reasons as the green paradox arises. A further paradoxical result is that increases in the productivity of carbon-free technologies relative to carbon-based technologies do not necessarily imply an earlier phasing in of those carbon-free technologies, but rather allow for a more gradual (and more prolonged) transition towards a carbon-free future. Finally, it is shown that having a productive carbon-based production system facilitates the transition towards a carbon-free system, i.e. investment in carbon intensive technologies may be an effective instrument in securing a carbon-free future.

1 Introduction

The purpose of this article is to illustrate how the expectation of increased extreme weather-related damages may affect the transition from a carbon-based economy to a carbon-free economy. Such a transition must be made at least just-in-time in order for the world not to end up in a run-away global warming regime (see IPCC (2007), for example). Before ending up in a carbon-free world, however, society will have to cope with global warming induced increasing damages due to a rising frequency and intensity of extreme weather events, like hurricanes, floods etcetera. These extreme weather-related damages show an upward trend since the 1980's, with total reported losses due to natural disasters of \$3.8 trillion over the period 1980-2012, and with extreme weather-related damages of about \$ 2.6 trillion for that same period (cf. World Bank (2013), pp. 5), while current world GDP for 2013 is about \$75.6 trillion dollars (cf. http://data.worldbank.org/region/wld). So, current yearly damages are of the order of 0.1% of current GDP, but they are increasing rapidly and potentially uncontrollably if global temperatures continue to rise.

In climate assessment models like the DICE model by Nordhaus (2008), damages affect the productivity of the resources available for production. The damage functions used to cover the impact of climate change on productivity provide a link between temperature rises, themselves linked to changes in cumulative GHG emissions, and productivity losses. Such damage functions are continuous, and in intertemporally complete welfare maximisation problems as the ones that form the basis of the DICE model, that makes it hard, if not impossible, to disentangle the impact of expected damages and of actual current damages on current decisions. As the green paradox shows, expectations of future events may be extremely important in defining the effectiveness of announced carbon taxation measures (Sinn (2008)), and one could wonder whether the green paradoxical results associated with the announcement of a tax-driven reduction in future returns on the depletion of carbon-based fuel resources might not occur when forward looking people expect a reduction in the return on future investment because of a CO2-driven increase in the frequency and intensity of extreme weather events. Would those people be willing to postpone the arrival of such extreme weather-related events, as one might perhaps be inclined to think, or would these people do the unexpected, i.e. increase the CO2 emission rate by burning more fuels? And if the latter is the case, would this really be a 'bad' thing, or do our a priori notions about what is good or bad in a climate change setting depend more on gut feeling than on rational considerations?

Not surprisingly perhaps, we will make the case here that rationally acting decision makers anticipating extra damages will tend to speed up the arrival of a production regime with increased damages, rather than postponing that arrival by 'going easy' on the CO2 emissions (or economic activity in general). By using a setting in which a central planner maximises overall welfare under a global emission threshold and a temperature threshold below which we have no extreme weather-related damages and above which we do have extreme weather-related damages, we are able to show how the anticipation of additional extreme weather-related damages would influence the decisions taken by that central planner.

By using a temperature threshold value as a switch for the occurrence of extra damages, we diverge from the standard use of damage functions in climate assessment models such as DICE (cf. Nordhaus (2008)). Those damage functions link increases in global temperatures to total factor productivity. The problem with damages from such extreme weather events is that the connection between the occurrence and intensity of the events and the corresponding damage doesn't have to be one-to-one. Take flooding for example. In the Netherlands, the low lying regions of the country are protected using dykes that have been built to withstand high-water levels that will occur just once in a thousand years or so. They will break only if the event is extreme enough, implying a discontinuity (jump) in the corresponding damage function. Something similar goes for agricultural production. There is usually some temperature range within which crops will thrive, while outside this range these same crops will dwindle. At the level of individual events then, the use of a switch driven by some temperature threshold doesn't seem to be too farfetched, although we should admit that at the aggregate level, a clear-cut threshold would be obtained only when all switches would be flicked at the same temperature rise relative to pre-industrial levels. Be that as it may, for the purpose of identifying the anticipatory effects of the expectation of increased future damages caused by actions undertaken now, the switch at the aggregate level comes in very handy, because whatever would happen during the no-extra-damage initial phase of the welfare maximizing growth path after a change in the expected size of extra damages occurring in the high damage future, must be purely anticipatory.

Note that this setting implies that we can distinguish between two types of futures: one where damages are still non-existent followed by one where damages do occur because of extreme weather events. By making decisions regarding the emission of CO2 in the no-damage future, one implicitly decides about the arrival date of the future with extra damages. This is a situation that cannot be covered by intertemporal optimisation models such as DICE because in DICE the future is qualitatively homogeneous, whereas we distinguish between different types of futures, which we integrate in a multistage optimum control setting.

The remainder of the article is organised as follows. In section 2 we sketch the overall model setting and in section 3 we discuss the way in which the model is related to the existing literature. Section 4 contains the technical description of the model, while section 5 provides information regarding the model's calibration. Section 6 shows the outcomes of the base-run as well as those of a number of parameter sensitivity experiments, including the one regarding the relative size of the extreme weather-related damages. Section 7 concludes.

2. Overall setting

The multi-stage optimum control transition model provides the optimum investment rates necessary to make the transition from a carbon-based economy to a carbon-free economy. The transition must be made just-in-time because of the existence of a cumulative CO2 emissions threshold that, when passed, throws the world into an irreversible runaway global warming regime, and before that happens possibly into a regime with extreme weather-related damages. With the current level of (cumulative) emissions, it is doubtful whether global warming can be kept below an additional 2^0 K compared to pre-industrial times.[1] The irreversibility of climate change arises out of the existence of positive feedback-loops from global temperature rises to methane releases from melting permafrost and from deep sea hydrates. Methane is a much more potent greenhouse gas (further denoted as GHG) than CO2, and because so much of it is still stored in the permafrost and in the deep sea, one would prefer (presumably *ex ante*, but certainly *ex post*) to avoid its spontaneous and unstoppable release at all costs. In order to model this, a cumulative CO2 emissions threshold has been introduced in the model, above which the irreversible climate change is assumed to be set in motion (cf. IPCC (2007), IPCC (2014)).

A central issue in the model is the fact that the transition towards a carbon-free production system will require a switch in the deployment of production technologies, which will involve the build-up of carbon-free capacity through investment and the simultaneous rundown of carbon-based capacity, simply because the one type of capacity cannot be changed into the other type of capacity at little or no cost: a spade is indeed a spade. At the same time, investment is the '*conditio sine qua non'* for a successful transition towards a sustainable future. This transition will therefore require finding the right balance between on the one hand a certain degree of systemic inertia to change arising out of the irreversibility of investment, while on the other hand investment is literally the carrier of technological progress and so 'enables' environmental quality improvements at the macro/global level. This "double-role" of investment underlines the importance of the timing of investment decisions; it is unwise to invest too early because one runs the risk of missing out on potential productivity improvements still to come, nor should one invest too late because of the rising opportunity cost of continuing to use old technologies instead of new, superior, ones. This setting naturally gives rise to such questions as how long to continue using and investing in present relatively productive carbon-based technologies, and when to start building up less productive but more climate-friendly production capacity, this all in the face of having to stop cumulative emissions just in time and just below the climate threshold/tipping point.

To find answers to these questions, we employ an optimum control model using the perspective of a central planner and resembling a dynamic requirements planning setting which borrows heavily from the AK-model known from the endogenous growth literature (cf. Rebelo (1991) in particular), but that also extends this AK-setting in a number of ways. First, we allow for different technologies that can be used either next to each other or sequentially. Secondly, a technology is characterised not only by its capital productivity, but also by CO2-emissions per unit of capacity used. Obviously, a carbon-free technology will not only be characterised by a zero emissions coefficient, but also by a lower capital productivity.[2] Thus, technology is embodied in physical capital goods. Third, we allow for the deactivation or 'scrapping' of existing technologies, as in the vintage literature. We show how the time of deactivation of existing capacity depends on technological parameters but also on emission characteristics, in combination with the shadow price of emissions. The latter suggests that the position of the climate

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[1] 2^0 K warming above pre-industrial levels (or a CO2 concentration of 450 ppm) is now generally considered to be the threshold for runaway global warming and corresponding runaway damages (Oppenheimer and Petsonk (2005)).

[2] The latter is a necessary assumption to make the model consistent with the observation that the present state of the economy is characterised by the intensive use of carbon-based energy rather than carbon-free energy. If the capital productivity of the latter technology would exceed that of the former, then the carbon-free technology would be superior to the carbon-based technology in all economically relevant respects, which is not the case in practice.

tipping point (further called CTP) directly influences such replacement decisions through its impact on the shadow price of emissions. A tightening of the CTP (for example if one would find evidence for runaway global warming occurring at less than 2^0 K warming) would tend to drive up the shadow price of emissions, and would lead to an earlier deactivation of carbon-based technologies. Fourth, we explicitly focus on the timing of the switches between investment in the one technology and in the other, and on the timing of the deactivation of old technologies, since the deactivation and activation of technologies that differ w.r.t. their emission rates have a direct impact on the macro-emission rate and therefore on the time left until the CTP will have been reached.

In its simplest configuration, our multi-stage optimum control set-up consists of just three stages: an initial "business as usual" stage which is completely carbon-based and in which the carbon-based capital stock is further expanded, a "joint production" stage in which the carbon-free capital stock is built-up and used next to the already existing carbon-based capacity which is decreasing through technical decay, and a final "carbon-free" stage starting with the scrapping of the last units of carbon-based capacity at the moment cumulative emissions would reach the CTP. By changing the aggregate emission rate, controlled by differential rates of (dis-) investment in carbon-based and carbon-free capacity, one is able to regulate the arrival time of the carbon-free future. We extend this basic framework by allowing for additional decay of the capital stocks due to extreme weather-related damages, which arrival is triggered by cumulative emissions surpassing an additional emission threshold that lies below the CTP (but that is still above present cumulative emission levels, by assumption). Instead of using a damage function that is continuous in cumulative emissions (or, equivalently, in the change in global temperatures relative to pre-industrial levels (cf. Nordhaus (2010)), we use a cumulative emissions threshold as a switch that induces a jump in the rates of technical decay of the capital stocks. In this way, we are able to show how extra damages that occur in the future may give rise to purely anticipatory adjustments in the present.[3] These adjustments may seem somewhat counterintuitive at first sight, but they follow directly from the logic of the model and from the observation that capital is a produced means of production; in order to build a lot of carbon-free capital (so as to mitigate the drop in welfare associated with the transition to a carbon-free economy), one needs productive carbon-based capital (cf. the notion of round-about production in von Böhm-Bahwerk (1891)).

3. Relevant literature

The environmental multi-stage optimum control framework that we have designed allows explicitly for heterogeneity in production technologies and irreversibilities in investment. These technological features of the macro-economic production process define the degree to which it will be possible to mitigate the welfare drop that is necessarily associated with the carbon transition. In order to *optimally* mitigate such a drop, it needs to be decided when to switch between the various production technologies that differ in efficiency and in their corresponding environmental externalities.

By focusing on these real-life and relatively down-to-earth features of production technologies, our model extends but also crucially differs from the relatively small literature that is concerned with the

[3] These adjustments are purely anticipatory, because present cumulative emission levels do not affect current output, as it would be the case when using an ordinary damage function a la Nordhaus (2008).

optimal timing of switches between production technologies or fuel types, such as Valente (2011), Tahvonen and Withagen (1996), Boucekkine et al. (2011, 2012), Schumacher (2011) and Saglam (2011). Saglam (2011) differs from the rest since his model lacks an environmental dimension, but does feature a relatively detailed vintage dimension. He uses an AK-setting where the effective volume of the capital stock is corrected for productivity differences in investment flows (associated with exogenous embodied technical change, thus introducing continuous time vintages) and where the capital stock in efficiency units is further corrected for technology induced scrapping. He defines a rate of capital depreciation that depends positively on the level of embodied technical change of the new vintage falling asymptotically to zero from its highest level at the moment of scrapping old capacity. This scrapping cost feature provides an economic incentive to limit the number of scrapping events (which in turn implicitly define separate stages in between these now *discrete* scrapping events). Even though Saglam (2011) does not have any environmental features, he does show how scrapping costs may give rise to the occurrence of stages. However, the ad hoc nature of the induced scrapping correction which leads to discrete scrapping events (hence a finite number of stages) as well as the finite horizon of the model are relatively weak points of the analysis. Valente (2011), by contrast, focuses on the switch between two macro-production technologies in a setting without any irreversible investment in production capacity, but with endogenous technical change, so disregarding embodiment issues altogether. Moreover, he identifies only a single optimal switching moment in a standard dynamic optimisation setting, whereas the very requirement of the accumulation of physical production capacity *before* production using that capacity can actually take place, implies that one should consider at least two optimal switching moments, even if one has to switch between just two technologies.[4] Schumacher (2011) focusses on the timing of a switch towards a renewable resource regime induced by the increasing probability of climate disasters under a non-renewable production regime. However, he employs a production structure in which renewables and non-renewables form a 'complex' of perfectly substitutable inputs that together with 'generic' capital produce output. Schumacher therefore ignores the embodiment of technology in physical capital goods and therefore downplays the need for a sufficient amount of time to build up the carbon-free capacity required to satisfy future consumption and investment needs. A recent paper by Boucekkine et al. (2012) provides the theoretical background for formulating an optimal control resource extraction problem with irreversible ecological regimes as in Boucekkine et al. (2011), where the focus is on the interaction between ecological thresholds and technological regimes. Here too, capital is generic, i.e. not technology-specific. A relatively early paper that makes use of a two-stage optimal control setting is that by Tahvonen and Withagen (1996). Their stages are defined to cover the situation of a U-shaped pollution decay function, where the one stage covers the situation in which the left part of the decay function applies and the other stage covers the situation where the right part of the decay function applies. In this paper too, capital stocks, hence the notion of the embodiment of different technologies in physical capital, are missing, just as in Valente (2011). In all of the papers mentioned above (except Saglam (2011)), the problem of sinking irrecoverable investment costs does not arise, either because there is no physical capital at all, as in Valente (2011) and in Tahvonen and Withagen (1996), or because

[4] There must be at least three stages (hence two switching moments): a pure carbon stage, a mixed carbon and carbon-free stage and a pure carbon-free stage. The simple reason is that the notions of embodiment and irreversibility in combination with the fact that capital is a produced means of production, imply that the very first units of carbon-free capacity must be produced using carbon-based capacity if we start out with a pure carbon stage.

capital is generic, as in Schumacher (2011) and Boucekkine (2011,2012), thus limiting the number of state variables in the multi-stage optimum control problem, but also, and more importantly, limiting the practical relevance of their theoretical settings for studying the transition towards a sustainable energy future.

There are good practical reasons for limiting the number of state variables (stocks) in an optimum control problem, since each state comes together with a co-state, and the interaction between the various states and co-states gives rise to systems of differential equations that are most of the time hard to solve analytically, if that would be possible at all. Even the AK-setting that we have chosen cannot be solved analytically, although it is probably the simplest setting one could imagine. That is why we have had to use the 'multiple-moving-target-shooting-method' as described in more detail in van Zon and David (2012). [5] This method enables us to numerically obtain the optimum stage lengths in combination with the optimum time paths for all the states and co-states during each of the individual stages. As far as we are aware of, the literature on numerically solving optimum control problems does not cover the type of problem that we have: numerical methods are either concerned with finding optimum stage lengths in settings where total transition time needs to be optimised (cf Maurer et al. (2005)), or they are defined for situations in which stage-lengths are determined a priori and where events during individual stages are then optimised given these stage-lengths (cf. standard shooting methods, as described in Judd (1998), for example).

4 The Model

4.1 Introduction

We use the simplest possible endogenous growth setting in which we have two broad (linear) technologies. One of them is an established technology (called the A-technology) with a relatively high productivity of capital that uses carbon-based energy and that produces CO2 emissions in the process. As stated, these emissions add to the stock of GHG's and so raise global temperatures as well as the probability of the world getting into a situation of runaway global warming. The alternative technology (further called the B-technology) does not generate CO2 emissions, but has a relatively low capital productivity. In such a setting with linear production technologies and linear cost functions, it can't be optimal to invest in both technologies at the same time, if these technologies differ with respect to their productivity net of depreciation. The reason is that a unit of investment for both technologies would represent the same marginal cost in terms of consumption forgone, and so the technology with the highest net marginal product, would generate the highest net marginal welfare gain. Hence, there will be investment in the A-technology or in the B-technology, but not in both technologies at the same time. We start out with a stage (called BAU for "business as usual") in which investment in just the A-technology takes place, but which is followed by a stage in which there will be only investment in the B-

[5] Multiple targets, since we need to "guess" or "hit" initial values for all co-states as well as all the switching moments. Moving targets, because the switching moments themselves need to be "guessed" as well. As we will see, the transversality conditions that apply to our model, define equality constraints that would need to be met at the switching moments between stages. The constraints can be seen as the targets to be hit, and the switching moments as the distances between the shooter and the targets. For further details, see van Zon and David (2012).

technology, while the existing A-capital stock is still being used to produce output next to the B-capital stock and where the A-capital stock is decaying in the process (this stage is called the “joint production stage” or JPR, for short). Finally, the remaining A-capital stock is scrapped when cumulative emissions reach the CTP, and from then on the carbon-free stage (called CFR) begins.

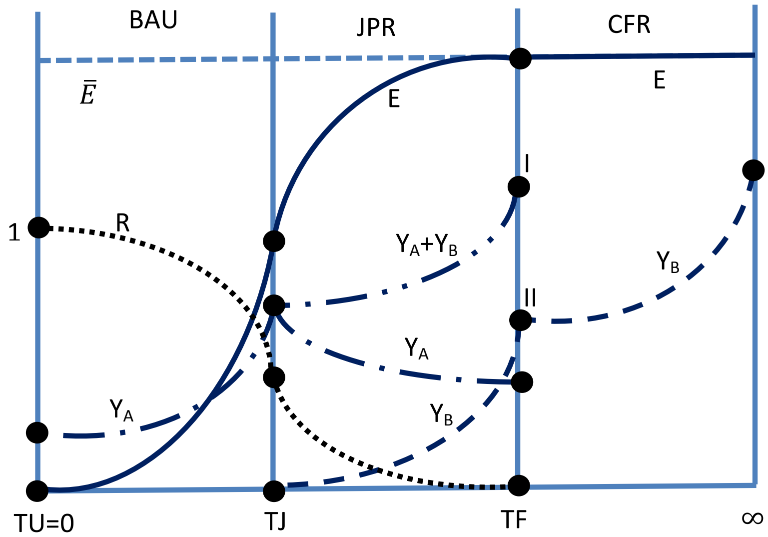

Figure 1. Transition Stages

The effects of the production and investment activities during the separate stages distinguished in the model can be summarised as in Figure 1. TU, TJ and TF mark the moments in time at which the BAU, JPR and CFR stages begin. In the BAU stage, cumulative emissions (labelled E) are increasing exponentially as the stock of carbon-based capital is growing. In the JPR stage investment in technology A stops, and output using technology A (i.e. Y^A) is at its maximum level but starts to decrease over time, because of technical decay. The stock of B-capital is built up from scratch from the very beginning of the JPR stage. Production using technology B (i.e. Y^B) is at full capacity and exponentially increasing during the JPR stage. Cumulative emissions are still increasing, but at a decreasing rate as the stock of carbon-based capital is decaying. During the JPR stage, total output is still growing, but at a slower rate than the growth of Y^B. Stage CFR starts when cumulative emissions E reach the threshold $\bar{E}$. During the final stage, only investment in and production with technology B is possible. When $\bar{E}$ has been reached, carbon-based capital is discarded. This leads to a drop in output, since production using the A technology ceases, even though there is still a positive amount of carbon-based production capacity available. Hence, total output jumps from point I to point II and starts growing from the latter point again during the CFR stage. Note that Figure 1 shows the vertical distance between the CTP labelled $\bar{E}$ and cumulative emissions labeled E. This represents the remaining capacity to emit, which is further called R. The basic model will be specified in terms of R and changes in R rather than in terms of cumulative emissions E.[6]

In an extended version of the basic model, we have introduced an additional threshold for cumulative emissions, which, when surpassed, would induce extra technical decay thus mimicking the arrival of extreme weather-related damages in addition to 'normal' decay. We have chosen this threshold such that its arrival will happen during the BAU stage, thus effectively splitting the BAU stage into a low-damage sub-stage and a high-damage sub-stage. The JPR and CFR stages will therefore become high-damage stages too.

4.2 The basic framework

We distinguish between two types of capital: carbon-based capital further denoted by K_A and carbon-free capital further denoted by K_B. The capital stocks in this model are exponentially decaying at rates δ_A and δ_B. Gross investment in either K_A or K_B (never in both) is equal to total savings. Welfare in this setup comes from consumption only, and is described using the CIES intertemporal welfare function.

Carbon-based capital generates a flow of emissions that is proportional to the amount of capital in use. This leads to a fall in the remaining capacity to emit. Setting *TU=0*, the remaining capacity to emit *R* as a percentage of the initial capacity to emit is given by $R = (\bar{E} - E)/(\bar{E} - E_0)$. Consequently, the flow of emissions (i.e. $\dot{E} = \varepsilon_A \cdot K_A$) will give rise to a corresponding change in *R*, as shown in Table 1 which provides an overview of the various activities during the individual stages:

Activities	BAU Stage	JPR Stage	CFR Stage
Gross Investment	$I_A > 0$	$I_B > 0$	$I_B > 0$
Production	$Y_A = A.K_A$	$Y_A = A.K_A$ $Y_B = B.K_B$	$Y_A = 0$ $Y_B = B.K_B$
Capital Accumulation	$\dot{K_A} = Y_A - \delta_A.K_A - C$	$\dot{K_A} = -\delta_A.K_A$ $\dot{K_B} = Y_A + Y_B - \delta_B.K_B - C$	$\dot{K_B} = Y_B - \delta_B.K_B - C$
CO2 Emissions	$\dot{R} = -\widetilde{\varepsilon_A}.K_A$[7]	$\dot{R} = -\widetilde{\varepsilon_A}.K_A$	$\dot{R} = 0$

Table 1. Activities

In Table 1, I_A and I_B refer to gross investment in carbon-based and carbon-free capital, while Y_A and Y_B are the levels of output produced using the corresponding stocks of capital K_A and K_B. *A* and *B* are the corresponding capital productivities.

The intertemporal optimisation setting

It should be noted that the fact that the final stage of the model is a pure AK-setting, allows us to obtain the optimum consumption path, hence welfare, for the CFR stage directly, given the value of

[6] More precisely, R will measures the current remaining capacity to emit as a fraction of the initial remaining capacity to emit, i.e. $R = (\bar{E} - E)/(\bar{E} - E_0)$. Hence in Figure 1, R_0=1.

[7] $\widetilde{\varepsilon_A}$ is defined as $\widetilde{\varepsilon_A} = \varepsilon_A/(\bar{E} - E_0)$ and is constant.

$K_{B,TF}$.[8] The welfare generated during the CFR stage depends therefore on the terminal value of K_B at the end of the JPR stage. It follows that the distribution of a state variable over the entire path is optimal when the marginal costs of having to deliver an extra unit of the state variable in its role as a terminal value at some time t* is exactly matched by the marginal benefits that this extra unit of the state variable generates as the initial value for the optimum continuation from t*. Since these marginal benefits and marginal costs are captured by the co-state variables (see Leonard and Van Long (1992), Ch 4), the latter need to be continuous along an optimum path: co-states don't jump, except in the case of pure state-constraints becoming binding (cf. Leonard and Van Long (1992, Ch. 10)).

An optimum path can be thought of as a combination of an optimum first step and an optimum continuation (as in dynamic programming problems), which allows us to interpret our multi-stage transition model as a finite horizon optimum control problem with a free endpoint and a scrap value function, as described in Leonard and Van Long (1992, Ch 7, further called LVL7), where the present value of welfare of the CFR stage provides the scrap value function. On an optimum path, postponing or extending a particular stage by an infinitesimal amount of time, shouldn't change the valuation of the entire path. LVL7 show that the derivative of the value function (in our case the present value of total welfare) with respect to the terminal date (of a stage) matches the value of the Hamiltonian at that date.[9] But in a sequence of stages, lengthening the one stage by a unit of time implies shortening the next stage by the same unit of time. So the arrival of the next stage should be postponed as long as the Hamiltonian of the earlier stage exceeds that of the later stage. The optimum switching moment between any two stages is therefore implicitly defined by the requirement that the Hamiltonians of two adjoining stages must be the same when evaluated at the moment of the stage-change. In practice, the equality of the Hamiltonians evaluated under the conditions relevant in either of the stages just before and just after a stage change, will result in a constraint that needs to be met by a set of states and co-states evaluated at the moment of the stage-change.

Formal structure

The overall welfare function consists of a summation of integral welfare derived from the flow of consumption during the three stages distinguished in the basic model:

$$W_0 = \int_0^{TJ} \frac{e^{-\rho \cdot t} \cdot (C_t)^{1-\theta}}{(1-\theta)} dt + \int_{TJ}^{TF} \frac{e^{-\rho \cdot t} \cdot (C_t)^{1-\theta}}{(1-\theta)} dt + \int_{TF}^{\infty} \frac{e^{-\rho \cdot t} \cdot (C_t)^{1-\theta}}{(1-\theta)} dt \tag{1}$$

In equation (1) W_0 measures the present value of total welfare at time t=TU=0, when, by assumption, stage BAU begins. In equation (1), ρ is the rate of discount, while $1/\theta$ is the (constant) intertemporal elasticity of substitution. C_t is consumption at time t. Given the exposition on intertemporal optimisation above, the time paths that would maximise (1) can be obtained by solving the time paths for the Hamiltonian problems that can be defined for the individual stages, while linking those time paths together by means of the requirements of optimum stage lengths (implying the equality of the Hamiltonians for stages BAU and JPR at t=TJ and for stages JPR and CFR at t=TF).

[8] See Barro and Sala-i-Martin (2004), chapter 4 in particular.

[9] This makes sense, as the Hamiltonian at some moment in time measures the contribution to the value function of the optimal use of all resources available at that moment in time.

Effectively this comes down to maximizing (1) w.r.t. the flows of consumption during each stage and w.r.t. the stage-lengths themselves, constrained by the technologies that are relevant in each stage, by the stocks inherited from previous stages, and by the thresholds that are relevant during the various stages. We will now solve the Hamiltonian problems for each individual stage.

The BAU stage

Using the superscripts U, J and F to denote the BAU, JPR and CFR stages to which a particular variable pertains while dropping the time subscript for ease of notation, the present value Hamiltonian H^U is given by:

$$H^U = \frac{e^{-\rho\cdot t}\cdot(C^U)^{1-\theta}}{(1-\theta)} + \lambda_{KA}^U \cdot \Big((A-\delta_A)\cdot K_A^U - C^U\Big) - \lambda_R^U \cdot \widetilde{\varepsilon_A} \cdot K_A^U \tag{2}$$

In equation (2), C is the only control variable, while K_A and R are the state variables and $\lambda_{KA}^U$and λ_R^U are the corresponding co-states. As first order conditions we have:

$$\frac{\partial H^U}{\partial C^U} = e^{-\rho\cdot t}\cdot (C^U)^{-\theta} - \lambda_{K_A}^U = 0 \Rightarrow C^U = \left\{e^{\rho\cdot t}\cdot \lambda_{K_A}^U\right\}^{-1/\theta} \tag{3}$$

$$\frac{\partial H^U}{\partial K_A^U} = \lambda_{KA}^U \cdot (A-\delta_A) - \lambda_R^U \cdot \widetilde{\varepsilon_A} = -\dot{\lambda}_{KA}^U \tag{4}$$

$$\frac{\partial H^U}{\partial R^U} = 0 = -\dot{\lambda}_R^U \tag{5}$$

$$\frac{\partial H^U}{\partial \lambda_{K_A}^U} = \dot{K_A^U} = \Big((A-\delta_A)\cdot K_A^U - \{e^{\rho\cdot t}\cdot \lambda_{KA}^U\}^{-1/\theta}\Big) \tag{6}$$

$$\frac{\partial H^U}{\partial \lambda_E^U} = \dot{R}^U = -\widetilde{\varepsilon_A}\cdot K_A^U \tag{7}$$

Equation (6) is obtained by means of substitution of equation (3) into the macro-economic budget constraint which states that output is used for consumption and (gross) investment purposes. Equation (5) implies that the shadow price of the remaining capacity to emit should remain constant over time. Equations (4)-(7) constitute a simultaneous system of differential equations that can be solved forward in time, given a set of initial values for the various state and co-state variables. This will give rise to terminal values of those same state and co-state variables at the terminal date of stage BAU, i.e. at t= TJ, the value of which is unknown so far.

The JPR stage

This stage differs from the BAU stage since investment in A-capital has stopped and that in B-capital begins. However, the carbon-based capital stock K_A is still used for production purposes, but it is falling over time due to technical decay. The present value Hamiltonian for the JPR stage, i.e. H^J , is now given by:

$$H^J = \frac{e^{-\rho\cdot t}\cdot(C^J)^{1-\theta}}{(1-\theta)} + \lambda_{KA}^J \cdot \left(-\delta_A\cdot K_A^J\right) + \lambda_{KB}^J \cdot \Big((B-\delta_B)\cdot K_B^J + A\cdot K_A^J - C^J\Big) - \lambda_R^J \cdot \widetilde{\varepsilon_A}\cdot K_A^J \tag{8}$$

As in the BAU stage, we have just one control, i.e. C^J, but three states K_A, K_B and R and corresponding co-states λ_{KA}^J , $\lambda_{KB}^J$and λ_R^J. Completely analogous to the previous stage, we have:

$$\frac{\partial H^J}{\partial C^J} = e^{-\rho\cdot t}\cdot (C^J)^{-\theta} - \lambda_{KB}^J = 0 \Rightarrow C^J = \left\{e^{\rho\cdot t}\cdot \lambda_{KB}^J\right\}^{-1/\theta} \tag{9}$$

$$\frac{\partial H^J}{\partial K_A^J} = \lambda_{KA}^J\cdot -\delta_A + \lambda_{KB}^J \cdot A - \lambda_R^J\cdot \tilde{\varepsilon}_A = -\dot{\lambda}_{KA}^J \tag{10}$$

$$\frac{\partial H^J}{\partial K_B^J} = \lambda_{KB}^J\cdot (B-\delta_B) = -\dot{\lambda}_{KB}^J \tag{11}$$

$$\frac{\partial H^J}{\partial R^J} = 0 = -\dot{\lambda}_R^J \tag{12}$$

$$\frac{\partial H^J}{\partial \lambda_{K_A}^J} = \dot{K}_A^J = -\delta_A\cdot K_A^J \tag{13}$$

$$\frac{\partial H^J}{\partial \lambda_{K_B}^J} = \dot{K}_B^J = (B-\delta_B)\cdot K_B^J + A\cdot K_A^J - \left\{e^{\rho\cdot t}\cdot \lambda_{KB}^J\right\}^{-1/\theta} \tag{14}$$

$$\frac{\partial H^J}{\partial \lambda_E^J} = \dot{R}^J = -\widetilde{\varepsilon_A}\cdot K_A^J \tag{15}$$

Note that the initial values in stage JPR for those state and co-state variables that the systems for stages BAU and JPR have in common, are the same as the terminal values for those variables at the end of stage BAU because of the continuity of state- and co-state variables along an optimum path. For a given value of TF, this system of differential equations allows the forward calculation of terminal values for the states and co-states associated with stage JPR at time t=TF, which will then function as the initial values for the states and co-states during the CFR stage.

The CFR stage

Stage CFR differs from stage JPR in that the carbon-based capital stock is discarded, and consequently the flow of CO_2 emissions will drop to zero from the beginning of the stage. The present value Hamiltonian for stage F, i.e. H^F , is therefore given by:

$$H^F = \frac{e^{-\rho\cdot t}\cdot (C^F)^{1-\theta}}{(1-\theta)} + \lambda_{KB}^F\cdot \Big((B-\delta_B)\cdot K_B^F - C^F\Big) + \lambda_R^F\cdot 0 \tag{16}$$

As in the BAU and JPR stage, we have just one control, i.e. C^F, but two states K_B and R and corresponding co-state variables λ_{KB}^F and λ_R^F. As first order conditions we have:

$$\frac{\partial H^F}{\partial C^F} = e^{-\rho\cdot t}\cdot (C^F)^{-\theta} - \lambda_{KB}^F = 0 \Rightarrow C^F = \{e^{\rho\cdot t}\cdot \lambda_{KB}^F\}^{-1/\theta} \tag{17}$$

$$\frac{\partial H^F}{\partial K_B^F} = \lambda_{KB}^F\cdot (B-\delta_B) = -\dot{\lambda}_{KB}^F \tag{18}$$

$$\frac{\partial H^F}{\partial R^F} = 0 = -\dot{\lambda}_R^J \tag{19}$$

$$\frac{\partial H^F}{\partial \lambda_{K_B}^F} = \dot{K}_B^F = (B-\delta_B)\cdot K_B^F - \{e^{\rho\cdot t}\cdot \lambda_{KB}^F\}^{-1/\theta} \tag{20}$$

$$\frac{\partial H^F}{\partial \lambda_E^F} = \dot{R}^F = 0 \tag{21}$$

As before, equations (18)-(21) constitute a simultaneous system of differential equations that can be solved forward in time, given initial values for the various state and co-state variables inherited from stage JPR. However, in this case the terminal values for the states and co-states are implicitly described by the standard transversality conditions in an AK setting which require the present value of the carbon-free capital stock to approach zero at the terminal date, i.e. in this case at time infinity. For the remaining capacity to emit *R*, the terminal value of zero had already been reached at t=TF, when cumulative emissions *E* hit the threshold $\bar{E}$, or, equivalently, when *R=0*.

Allowing for extreme weather-related damages

It is now straightforward to include a switching moment in between a low-damage BAU sub-stage and a high-damage BAU sub-stage, initiated when R would hit its damage threshold value $\bar{\bar{R}}$ or, equivalently, when cumulative emissions E would hit their extra damages concentration $\bar{\bar{E}} = \left(1 - \bar{\bar{R}}\right) \cdot \bar{E} + \bar{\bar{R}} \cdot E_0$. The description of the events during the low-damage BAU sub-stage will be identical to equations (2)-(7), while the high-damage BAU sub-stage is a copy of equations (2)-(7) but with δ_A being replaced by $\delta_A + \Delta\delta$, where $\Delta\delta \geq 0$ represents the jump in the rates of decay δ_A and δ_B due to extreme weather-related damages. The JPR and CFR stages also become high-damage stages, and so every occurrence of δ_A and δ_B in equations (8)-(21) would need to be replaced by $\delta_A + \Delta\delta$ and $\delta_B + \Delta\delta$, respectively. Note that the objective function (1) also needs to be extended to cover the additional sub-stage arising from splitting the BAU stage.

Transversality conditions for the basic model

For stage CFR the standard transversality condition (further called TVC for short) applies regarding the value of carbon-free capital at time infinity:

$$\lim_{t\to\infty} \lambda_{KB,t}^F \cdot K_{B,t}^F = 0 \tag{22}$$

where we have now added time-subscripts. Note that (18) can be integrated directly to obtain the time path for $\lambda_{KB,t}^F$ which can then be substituted into (20) to obtain the time path for $K_{B,t}^F$. Thus we get:

$$\lambda_{KB,t}^F = \lambda_{KB,TF}^F \cdot e^{-(B-\delta_B)\cdot(t-TF)} \tag{23}$$

$$K_{B,t}^F = \frac{e^{-\frac{t\rho-(t-TF)(B-\delta\mathrm{B})}{\theta}}\theta\cdot\left(\lambda_{KB,TF}^F\right)^{-1/\theta}}{\rho+(\theta-1)(B-\delta\mathrm{B})} + e^{(t-\mathrm{TF})(B-\delta\mathrm{B})}\left(K_{B,TF}^F - \frac{e^{-\frac{\mathrm{TF}\rho}{\theta}}\theta\cdot\left(\lambda_{KB,TF}^F\right)^{-1/\theta}}{\rho+(\theta-1)(B-\delta\mathrm{B})}\right) \tag{24}$$

Equations (23) and (24) can be substituted into TVC (22), and we find that in order for TVC (22) to hold we should have that:

$$\frac{\rho+(\theta-1)(B-\delta_B)}{\theta} > 0 \tag{25}$$

$$K_{B,TF}^F = \frac{e^{-\frac{\mathrm{TF}\rho}{\theta}}\theta\cdot\left(\lambda_{KB,TF}^F\right)^{-1/\theta}}{\rho+(\theta-1)(B-\delta\mathrm{B})} \tag{26}$$

Substituting (26) into (24), we find that:

$$K_{B,t}^F = K_{B,TF}^F \cdot e^{\frac{(B-\delta\mathrm{B}-\rho)(t-TF)}{\theta}} \tag{27}$$

It follows from (27) that if the structural parameters are such that (25) is met and if we pick consumption at time TF (hence $\lambda_{KB,TF}^F$ (see equation (17)) such that (26) is met, then the carbon-free capital stock will grow at the steady state growth rate $(B-\delta\mathrm{B}-\rho)/\theta$ from time t=TF.

Apart from the TVC as given by (26), we require that:

$$\lambda_{KA,TF}^J = 0 \tag{28}$$

Equation (28) states that the shadow price of carbon-based capital at the end of the JPR stage (so at the moment that carbon-based capital is discarded) should be zero, since having an additional unit of capital that will not produce anything is useless and therefore worthless.

Finally, there are two TVCs that pertain to the optimum length of stages BAU and JPR, and that require the equality of the Hamiltonians of the various stages at the moment of a switch between two consecutive stages. For the optimum length of stage BAU (given by the value of TJ, since TU=0 by assumption and the length of the BAU stage is equal to TJ-TU=TJ), we must have that $H_{TJ}^U = H_{TJ}^J$, while the optimum arrival time of the CFR stage is determined by the requirement that $H_{TF}^J = H_{TF}^F$.[10] Using the definitions of the Hamiltonians in (2), (8) and (16), as well as the FOCS regarding consumption in (3), (9) and (17) together with the continuity constraints on states and co-states that feature in the relevant adjoining stages, we can obtain implicit descriptions of the arrival times of stages JPR and CFR, given by:

$$\lambda_{KA,TJ}^J = \lambda_{KB,TJ}^J \tag{29}$$

$$A \cdot \lambda_{KB,TF}^J = \tilde{\varepsilon}_A \cdot \lambda_{R,TF}^J \tag{30}$$

Equation (29) implies that *investment* in the carbon-based technology should stop at the moment that the shadow price of A-capital is equal to (and then drops below) the shadow price of B-capital. Since the marginal cost of obtaining a unit of capital is the same in both cases (i.e. one unit of consumption foregone), equation (29) is consistent with the maximisation of the (present value) welfare surplus associated with investment. Equation (30) implies that *production* using carbon-based capital should cease from the moment that the benefits from continuing to use a unit of capital (the LHS of equation (30), as one unit of capital produces A units of output, and each unit of output is worth $\lambda_{KB,TF}^J$ in present value welfare terms at t=TF) matches the cost of doing that (as given by the RHS of equation (30), since one unit of capital uses $\widetilde{\varepsilon_A}$ units of the remaining capacity to emit at a cost of $\lambda_{R,TF}^J$ per unit). Equation (30) is therefore consistent with the zero quasi-rent condition as an implicit description of the optimum

[10] It should be noted that the time-index TJ in the BAU stage represents the final moment of the BAU stage, while it also signals the first instant of the JPR stage. So, both TJ's should be regarded as being infinitesimally close to each other and therefore numerically indistinguishable from each other in practice.

moment to scrap existing capacity known from the vintage literature (cf. Johansen (1995), Solow et al. (1966) and Boucekkine (2011), for example) which states that a vintage, once installed, should be discarded as soon as it's quasi-rents (consisting in this case of the present value welfare value of output less variable (emission-) costs, also in present value welfare terms) become negative.

A transversality condition regarding extreme weather-related damages

When allowing for extreme weather-related damages, an additional transversality condition arises for the optimum arrival time of the high-damage BAU sub-stage (called TUH in this case), which involves the equality of the Hamiltonians of the low-damage and the high-damage sub-stages at t=TUH. In this case, however, the equality of the Hamiltonians in combination with the terminal constraint $R_{TUH} = \bar{\bar{R}}$ translates directly into a jump of the co-state of the remaining capacity to emit at t=TUH and the optimal value of TUH. Thus we get:

$$\left(\lambda_{R,TUH}^{UL} - \lambda_{R,TUH}^{UH}\right) \cdot \tilde{\varepsilon}_A = \Delta\delta \cdot \lambda_{KA,TUH}^{UH} = (\delta_A + \Delta\delta) \cdot \lambda_{KA,TUH}^{UH} - \delta_A \cdot \lambda_{KA,TUH}^{UL} \qquad (31)$$

$$\lambda_{R,TUH}^{UH} = \lambda_{R,TUH}^{UL} - \Delta\delta \cdot \lambda_{KA,TUH}^{UH} / \tilde{\varepsilon}_A \qquad (32)$$

Note that the LHS of (31) measures the decrease in the emission cost per unit of capital when switching from the low-damage regime to the high-damage regime, while the RHS of (31) measures the increase in the cost of decay per unit of capital associated with this switch.[11] So, equation (31) states that at the arrival time of the high-damage regime, the cost of using carbon-based capital in both regimes should be the same, i.e. an increase in the welfare cost of extra decay should be matched by a drop in the cost of using the remaining capacity to emit. (31) then implies (32), which shows that the arrival of the high damage sub-stage induces a drop in the shadow price of the remaining capacity to emit. The latter is in line with the fact that higher decay rates (so positive values of $\Delta\delta$) will reduce the net productivity of capital, hence the (welfare-) value of an extra unit of *R* which would allow the use of such capital for a longer period of time.

Solution of the basic model

The three systems of differential equations can in principle be solved, as the initial and terminal values which we have available for the state variables and the TVCs (which provide either some fixed points for the time-paths of the co-states (cf. equation (28)), or which link the co-states to a state-variable which time path has been 'fixed' through a given initial value (cf. equation (27)), or which link different co-states at some point in time (cf. equations (28) and (29))) provide exactly enough information to provide 'fixed points' for all time paths concerned using the 'multiple moving targets shooting method' mentioned before.

[11] Note that we use the continuity of the co-state for K_A here, i.e. $\lambda_{KA,TUH}^{UH} = \lambda_{KA,TUH}^{UL}$, where the moments in time when the co-states are evaluated (i.e. t=TUH) are at different sides of the (time-) boundary of the high-damage and low-damage sub-stages (denoted by the superscripts UH and UL, respectively). Also note that $\lambda_{KA,TUH}^{UL}$=$\lambda_{KA,TUH}^{UH}$ is the shadow price of a unit of capital in welfare terms, so $\Delta\delta \cdot \lambda_{KA,TUH}^{UH}$ measures the loss in welfare per unit of capital due to extra decay.

5. Model calibration

In order to show how the model works, its parameters need to calibrated or fixed a priori. To this end, we have made use of the Nordhaus DICE 2010 data (Nordhaus (2010)). However, Nordhaus essentially uses a single-stage neoclassical growth setting, while we use an AK-setting with multiple stages. This implies that simply copying the Nordhaus numbers into our model is not possible. In order, nonetheless, to produce growth rates and saving rates that have about the right size, we have made a number of assumptions regarding the size of the long-term growth rate of GDP (2.5%), the gross savings rate (30%), the share of depreciation charges in GDP (10%) as well as about the subjective rate of discount (5%), that allow us to obtain reasonable values for some of the behavioural and technical parameters of the model under the assumption that the world can be characterised up to now as an AK-setting with just the A-technology in place. For, in the latter case we would have that:

$$\hat{Y} = \hat{K} = \frac{A-\delta_A-\rho}{\theta} = 0.025 = \frac{A-\delta_A-0.05}{\theta} \tag{33}$$

where a hat over a variable denotes the instantaneous proportional growth rate of that variable. Furthermore, in an AK-setting, the steady state savings rate would be given by:

$$s = \frac{(\dot{K}+\delta_A \cdot K)}{Y} = \frac{\hat{K}}{A} + \frac{\delta_A}{A} = 0.3 \tag{34}$$

while the share of depreciation charges in GDP is given by:

$$\frac{\delta_A \cdot K}{A.K} = \frac{\delta_A}{A} = 0.1 \tag{35}$$

Equations (33)-(35) allow us to obtain the implied values for A (0.125), δ_A (0.0125) and θ (2.5).

With respect to cumulative CO2 emissions and the location of the climate tipping point in terms of the cumulative emissions generated by our model, we have again used the Nordhaus (2010) data to obtain the change in cumulative CO2 emissions (measured in ppm) relative to the amount of capital. To do so, we have divided output at the world level in 2010 by the output-capital ratio A to obtain the implied volume of capital that, in an AK-setting, would be able to produce the observed amount of output. Then assuming that 45% of CO2 emissions remain in the atmosphere while the rest is absorbed by terrestrial and ocean sinks (Canadell et al. 2010)[12], the Nordhaus data give rise to $\epsilon_A = 0.00406$ and $K_{A,0}$=553.

Finally, we have combined our concept of (relative) remaining capacity to emit with the value of the global temperature rise above pre-industrial levels, ΔT, which is linked to present cumulative emissions *E* as given by:[13]

[12] Note that the airborne fraction (i.e. the ratio of the atmospheric CO2 increase in a given year over that year's total emissions) has been around 0.45 in the previous decade, but that Canadell et al. (2010) point to the possibility that it can be increasing in the future due to degradation of the capacity of natural sinks sequestering CO2. Dealing with these uncertainties is beyond the scope of this article.

[13] http://en.wikipedia.org/wiki/Radiative_forcing.

$$\Delta T = 0.8 \cdot 5.35 \cdot Log\left(\frac{E}{E_{pi}}\right) \Rightarrow \Delta T = -24.1169 + 4.28 \cdot Log(\bar{E} \cdot (1-R) + E_0 \cdot R) \tag{36}$$

where E_{pi}=280 ppm represents the pre-industrial CO2 concentration, and E_0=380 ppm, i.e. the recently observed concentration of CO2.[14]

When we set $\bar{E} = 450$, there is precious little time left for the build-up of carbon-free capacity under the parameterisation above: the calculated length of the BAU stage is just a few years.[15] This may or may not be true, but for our illustrative purposes this leaves too little numerical room to experiment with the sensitivity of the outcomes to alternative parameter values. Henceforth, we have made the assumption that the tipping point is at 600 ppm, which amounts to a rise of global temperatures of about 3.2^0 K above pre-industrial levels.

Additional a priori parameter values and adjustments

For the carbon-free technology we have made the assumption that depreciation as well as the sensitivity to damages are equal to that of the carbon-based technology. In addition, we assume that $B = 0.08$. This is a reasonable assumption given the lower productivity of renewable energy technologies relative to thermal capacity at the current scale – as an example from the energy sector.

Base-run parameter set

The parameters used for the simulations below, are summarised in Table 2.

Parameter	Value	Parameter	Value	Parameter	Value
A	0.125	ε_A	0.00406	$\bar{E}$	600
B	0.08	ρ	0.05	E_0	380
$\delta_A = \delta_B$	0.0125	θ	2.5	$K_{A,0}$	553

Table 2. Base-run parameters

6. Model Outcomes

The base-run

Preliminary parameter sensitivity analyses performed using the model show reactions that are familiar from growth theory. Changes in the rate of discount or in the intertemporal elasticity of substitution all have the expected impact. This goes for the productivity parameters too. When emission thresholds get tighter, the shadow price of emissions rises. When productivity parameters increase, so do the corresponding co-states of the associated state variables. Because these particular results show nothing unexpected, we do not report them in any further detail here. What is of interest, though, is that the linking of various sequential stages introduces anticipatory behaviour that generates transitional

[14] See the website on CO2 measurement on Mauna Loa, Hawaii http://www.esrl.noaa.gov/gmd/ccgg/trends/.
[15] This will be shown in section 5.

dynamics which are missing in an ordinary single-stage AK endogenous growth setting. The results for the base-run parameter set in Table 2 are listed in Figures 5.1 and 5.2.

The first row of plots in Figure 5.1 shows the outcomes with respect to carbon-based capital K_A. The vertical dotted lines mark the arrival times of the JPR stage and the CFR stage. They are situated at TJ=31.02 and TF=60.73. The first plot shows that the shadow price of carbon-based capital steadily drops during both stages, until it reaches a zero-level at t=TF, as required by equation (28). The second plot shows the build-up of carbon-based capacity during the BAU stage and the subsequent run down of that capacity during the JPR stage. These events are mirrored in the third plot that shows the instantaneous rate of change over time of the carbon-based capital stock. We see that net investment in carbon-based capital accelerates towards the end of the BAU stage, while it becomes negative during the JPR stage because of technical decay.

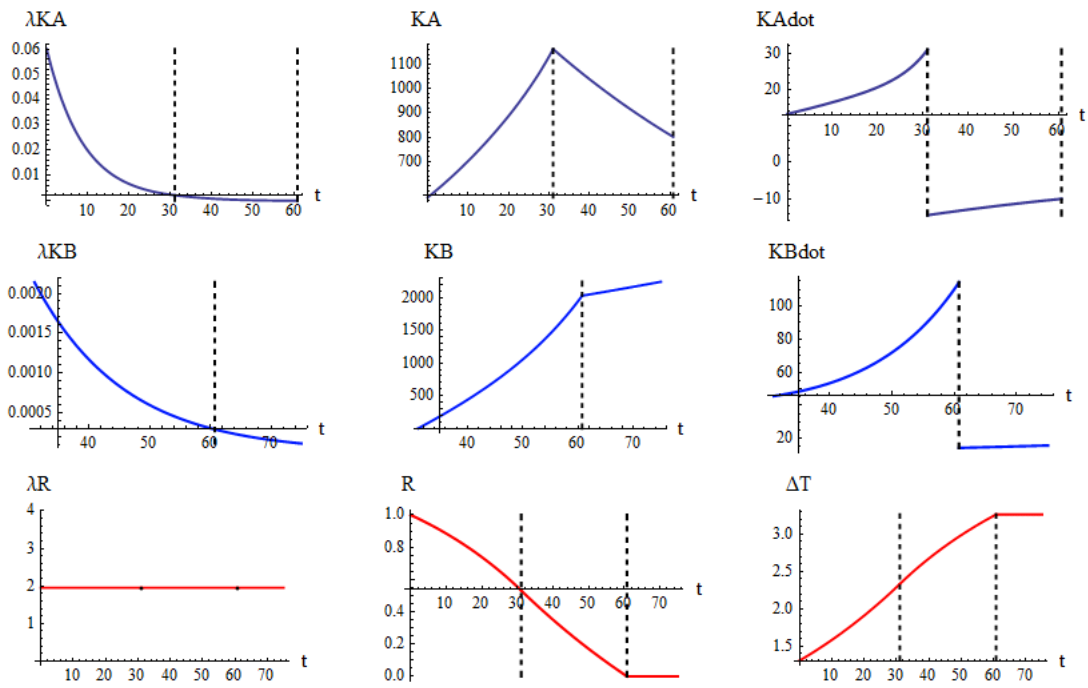

Figure 5.1 Baseline results for K_A, K_B and R

The second row of plots shows the corresponding events for carbon-free capital. Since the accumulation of carbon-free capital begins at the start of the JPR stage, there is now just one dotted vertical that marks the arrival of the CFR stage at t=TF. It should be noted that net investment in carbon-free capacity is rapidly increasing during the JPR stage in anticipation of the drop in capacity that will occur at the time of arrival of the CFR stage, as carbon-based capital will then have to be discarded. During the CFR stage, net investment in the carbon-free capital stock is much lower than during the JPR stage.

The third row of plots shows what happens to the remaining capacity to emit R. The latter falls gradually during the BAU and JPR stages, hitting zero at t=TF. The corresponding shadow price of the remaining capacity to emit remains constant, since a fall in *R* does not have a continuous direct negative effect on this economy as would be the case with an ordinary damage function.[16] Note, moreover, that the terminal value of the increase in temperatures above pre-industrial levels, i.e. ΔT, is about 3.2 K^0 for $\bar{E} = 600$ ppm.

In Figure 5.2 , we show the corresponding outcomes for the time paths of output (Y), consumption (C), and the instantaneous growth rate of output (gY) as well as the propensity to consume (PCONS), both in percentage terms. A striking feature of the Figure is the drop in output at the start of the carbon-free stage because of the scrapping of carbon-based capital. No such drop can be observed in the level of consumption, however. All of the drop in output goes at the expense of gross investment in carbon-free capacity, as can be seen in Figure 5.1.

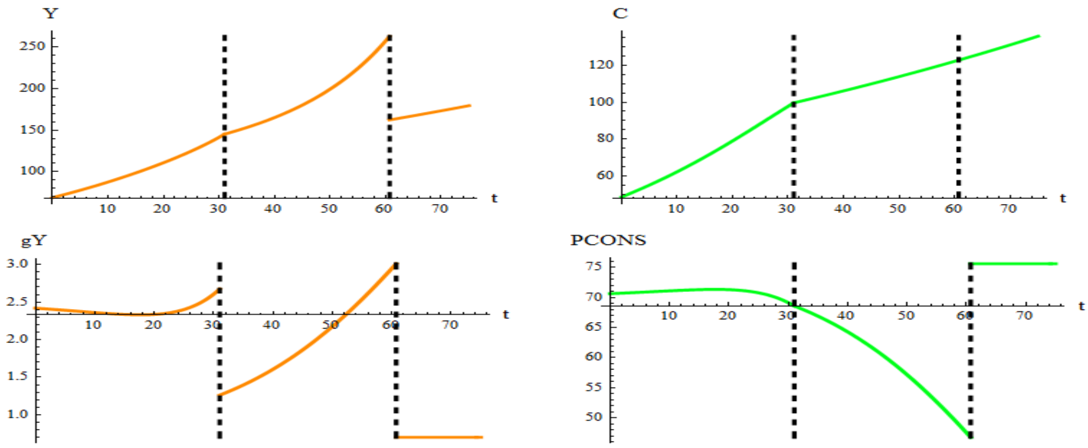

Figure 5.2 Baseline results for Y, gY, C and PCONS

The growth rate of output shows some anticipatory reactions to the changes that the arrival of a new stage will bring. For example, during the JPR stage, the average productivity of capital must fall, as the amount of relatively productive carbon-based capacity decreases, and the amount of relatively unproductive carbon-free capacity increases. This hold *a fortiori* for the arrival of the CFR stage when the remaining carbon-based capacity is discarded and aggregate capital productivity suddenly drops to the level associated with carbon-free capacity. In order to mitigate the effects on the consumption path of the corresponding drop in output, the build-up of carbon-free capacity during the JPR stage is speeded up towards the end of that stage. A similar pattern can be observed for the build-up of carbon-based capacity during the BAU stage, as the build-up of carbon-based capacity also allows a relatively high rate of investment in carbon-free capacity during the next stage. *One could say then, somewhat provokingly perhaps, that a fast build-up of carbon-free capacity, may require a large carbon-based capital stock simply because capital is a produced means of production.*

[16] With an ordinary damage function, the positive shadow price of R would come from the damages avoided by having a higher *R* (or less cumulative emissions E) , just like the positive (but constant) value of the shadow price of *R* in our model comes from the fact that the arrival of the low-productivity JPR and CFR stages can be postponed to some extent by having higher R/lower E (so in ordinary damage-function terms, 'relative productivity damages' can (temporarily) be 'avoided' by postponing their arrival).

Sensitivity analysis: variations in $\bar{E}$

In Figure 5.3 we report the consequences for the initial values of the shadow prices of the remaining capacity to emit (i.e. $\lambda RUTU$) and of carbon-based capital (i.e. $\lambda KAUTU$) of variations in the climate change threshold. The Figure also shows the length of the BAU stage (called ΔU). The length of the JPR stage remains unaffected by variations in $\bar{E}$, since it's length can be shown to depend on technological parameters only, which are left unchanged in this experiment. Instead, we have varied $\bar{E}$ over the range of 450-600 ppm, which is consistent with temperature rises of 2-3.2 degrees above pre-industrial levels.

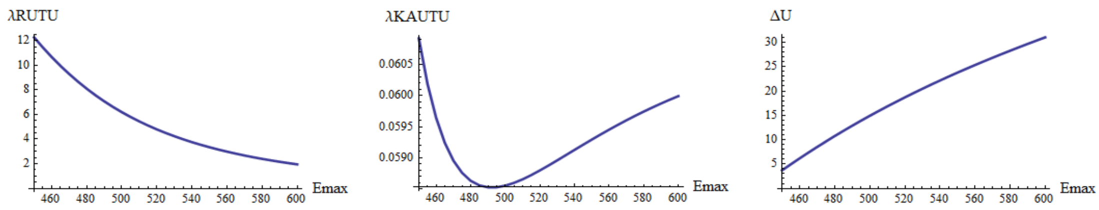

Figure 5.3 Sensitivity results: $\bar{E} = 450 - 600$

As stated earlier, a tighter threshold induces a rise in the shadow price of emissions. Maybe somewhat unexpectedly, the plot of the initial value of the shadow price of carbon-based capital is U-shaped. The shadow price first falls as $\bar{E}$ rises from 450 ppm to about 490 ppm and then rises again for further increases in $\bar{E}$. This U-shape comes from the facts that the BAU stage shrinks to just a few years as $\bar{E}$ is reduced to 450 ppm. Such short horizons make the initial endowment of capital increasingly more important which is reflected by the rising shadow price as the BAU stage length drops as $\bar{E}$ gets ever closer to the 450 ppm mark. For values of $\bar{E}$ from about 490 ppm $\lambda KAUTU$ rises again, because the lengthening of the BAU stage allows carbon-based capital to be longer in business (since the length of the JPR phase doesn't vary with $\bar{E}$). So the part of the U-shape with a positive slope is due to the implied increase in economic lifetime of carbon-based capital as $\bar{E}$ increases from about 490 ppm to 600 ppm.[17]

The outcomes of the sensitivity results for the other variables of the model will be presented in a different way. Each value within the range of threshold values generates its own set of time paths for the variables of the model. We will put all these paths together in one plot, but the individual paths will be coloured differently. Low values within the emission threshold range will be associated with the low-frequency colours in the rainbow spectrum (the red part of the spectrum), while high values within the emission threshold range will be associated with the high-frequency (i.e. the violet) part of the spectrum. Intermediate threshold values will have a corresponding colour of the rainbow spectrum. Note that since the carbon-free end-stage is pure AK with a constant steady state growth rate, we can limit ourselves to showing just the first part of the corresponding time paths, in our case until 75 years from now.

Figure 5.4 shows the changes in both capital stocks. The plot for KAdot (the rate of change over time of K_A), shows two patches of colour. The top-patch is associated with the BAU stage, and the bottom patch with the JPR stage. The top-patch in the plot for KBdot pertains to the JPR stage, however, while

[17] Given that the length of the JPR stage depends on technological parameters only, a change in the length of the BAU stage implies a corresponding change in the economic lifetime of carbon-based capital, *ceteris paribus*.

the bottom-patch is associated with the (truncated) CFR stage. It should be noted that the moment in time at which a path during the BAU stage ends is one instant before the moment in time at which the path of the same colour continues during the JPR stage. So we can see that for $\bar{E} = 450$ (i.e. the reddest line in the plot) the JPR stage begins indeed after about 3 years, while for $\bar{E} = 600$ (i.e. the violet lines in the plot) the JPR stage begins at about 31 years.

There is a striking difference between the patterns for net investment of the carbon-based capital stock, and of the carbon-free capital stock. As the emission constraint is loosened, the whole of the net investment curve of carbon-free capital is shifted upwards over the entire joint production period. In the case of carbon-based capital, however, a loosening of the emission constraint implies both a lengthening of the BAU stage and a downward shift of the net investment time path. At the end of the BAU stage, however, the downward shift of net investment in K_A is more than compensated by the rise in net investment taking place over a longer stretch of time. The counterpart of this sequence of events is shown in figure 5.6 below which shows the time paths of consumption. Here we see that the longer BAU stage calls for higher levels of consumption in the beginning, and correspondingly lower levels of net investment in carbon-free capacity.

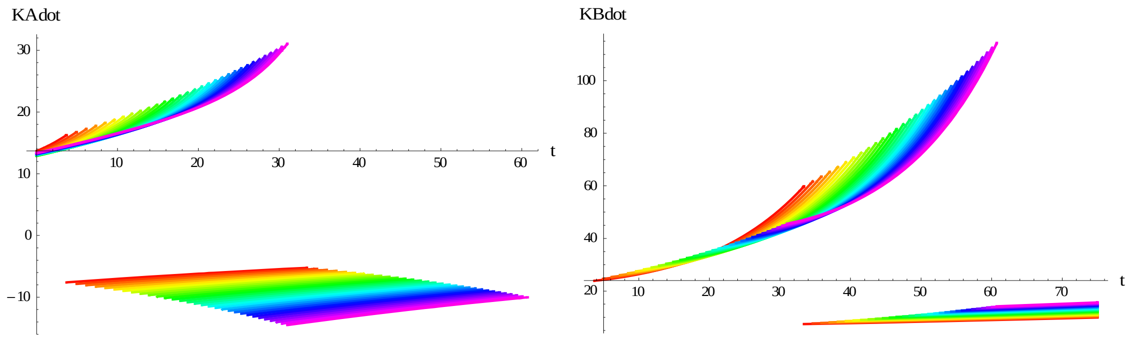

Figure 5.4 Net investment in K_A and K_B: $\bar{E} = 450 - 600$

The implications of the variations in net investment for the level of the carbon-free capital stock are shown in the LHS-panel of Figure 5.5. With a tighter emission constraint the JPR stage comes earlier, while the capital stock will reach a lower level at t=75 and hence will also be lower at $t = \infty$. Again, this is a consequence of the fact that capital is a produced means of production: if the carbon-based capital stock is limited in size because of the existence of binding cumulative CO2 emission constraints, then the carbon-free capital stocks will necessarily be limited in size as well, *ceteris paribus*.

The RHS-panel of Figure 5.5 shows how the growth rate of output reacts to variations in $\bar{E}$. The values of the growth rate of output at the end of the BAU stage are practically the same for all threshold values of $\bar{E}$ within the range. However, the periods of time during which these growth rates affect the level of output are very different, so much so that the positive effect on the level of output (see Figure 5.6) of an extension of the BAU stage as the emission constraint becomes less tight, outweighs the negative effect of the initial drop in the growth rate of output as emission constraints become looser. The fact that the model converges to the same steady-state growth rate during the CFR stage, but at different moments in time, is reflected by the presence of a multi-coloured horizontal line in the RHS-panel of Figure 5.5.

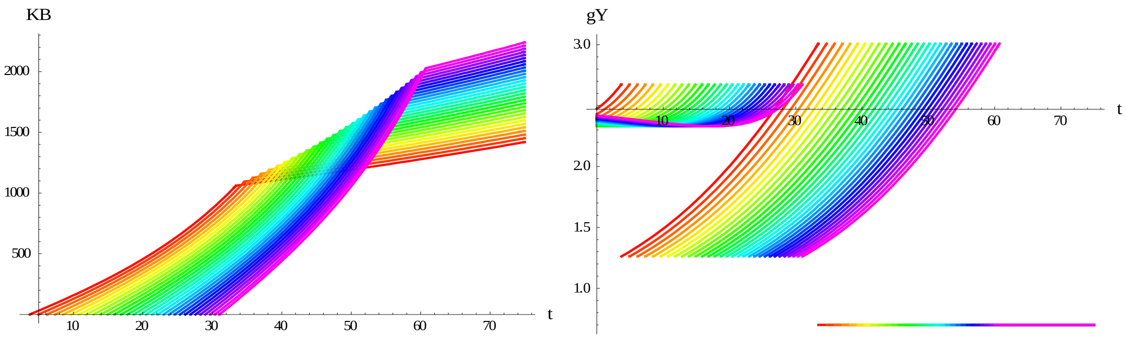

Figure 5.5 The carbon-free capital stock K_B and output growth gY: $\bar{E} = 450 - 600$

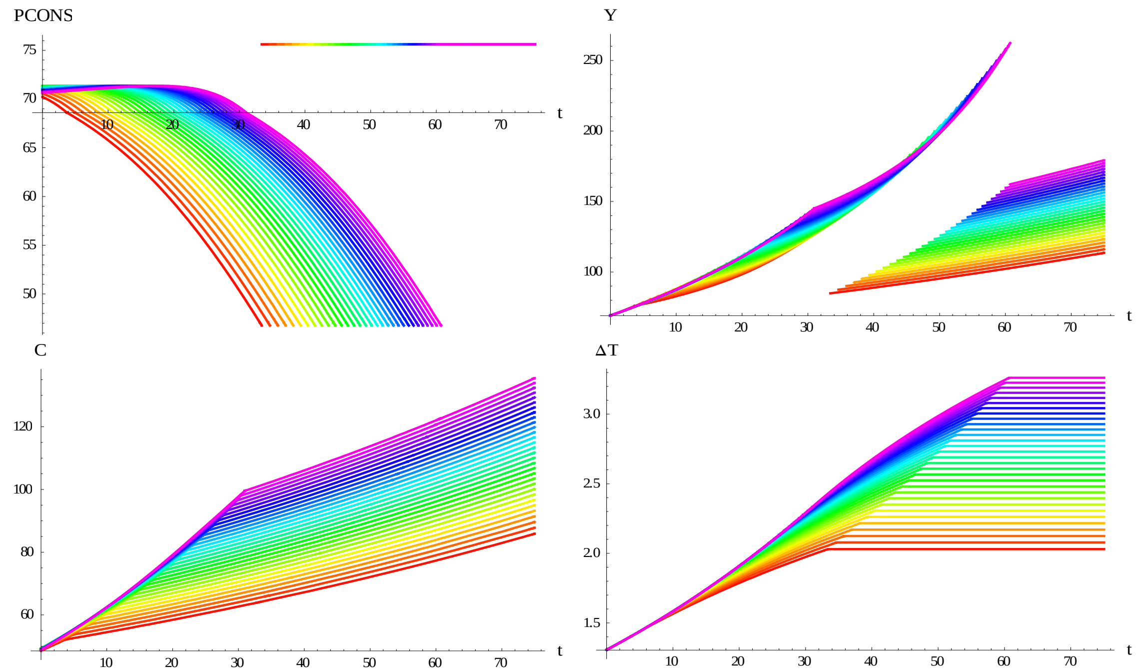

Figure 5.6 PCONS,Y,C and ΔT: $\bar{E} = 450 - 600$

A striking feature of the top-LHS-panel of Figure 5.6 is that the propensity to consume (PCONS) converges to the same value at the end of the JPR stage and then jumps up in all cases to the same level at t=TF again. However, the corresponding levels of output (Y) are very different, lower for the red curves and higher for the violet curves, and so are consumption expenditures therefore. It should be noted that output itself drops during the switch from stage JPR to CFR, as can be seen from the top-RHS panel in Figure 5.6. At the same time, the propensity to consume jumps up, so that consumption (C) itself remains the same at the moment of the switch (cf. bottom-LHS-panel) The drop in output is seemingly inconsistent with the growth rates of output being positive during both the JPR and the CFR stage, but it is caused by the one-time scrapping of carbon-based capital. Temperature rises relative to pre-industrial levels (ΔT) remain limited to 2^0 K for $\bar{E} = 450$ and increase to about 3.2^0 K for $\bar{E} = 600$.

Sensitivity analysis: variations in carbon-free productivity B

We now show some of the results for an experiment in which we make the capital productivity of the carbon-free technology change over the range from 0.08 to 0.12, while still remaining less productive than the carbon-based technology.

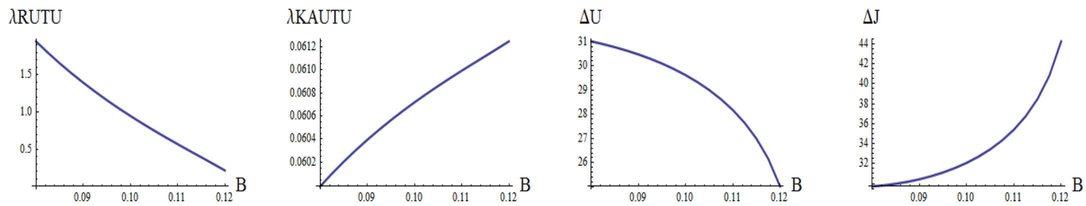

Figure 5.7 Sensitivity results: $B = 0.08 - 0.12$

Figure 5.7 shows the initial values of the co-states of *R* and K_A as well as the lengths of the BAU and JPR stages, going from left to right. As the productivity of carbon-free capital goes up, the shadow price of carbon-based capital rises too, as it is (in part at least) used to produce the now more productive carbon-free capital. The more productive carbon-free capital reduces the need for capacity to emit, and so the shadow price of R falls in the process. The length of the BAU stage also falls (from about 31 to 25 years), while that of the JPR stage increases from about 30 to 43 years, so *more productive carbon-free capital tends to prolong the use of carbon-based capital*, simply because the need to burn a lot of carbon quickly in order to be able to rapidly produce carbon-free capacity is reduced as the produced carbon-free capacity becomes more and more like carbon-based capacity from a productivity perspective: who needs carbon-based capital when carbon-based and carbon-free capital goods are (almost) equally productive? The rise in carbon-free capital productivity opens up the possibility to spread the use of carbon-based capacity more evenly over time, and so reduce the cost of scrapping the last units of carbon-based capacity at the end of the JPR stage. This is apparent from the fact that the arrival date of the CFR stage is actually *postponed* by about 7 years.

The reduction in the volume of carbon-based capital scrapped at the beginning of the CFR stage can be seen from the LHS-panel in Figure 5.8. For low values of B, the accumulation of K_A takes place at roughly the same rate as for high values of B, but over a longer period. The consequence is that the maximum value of K_A for low values of B exceeds the maximum values of K_A for high values of B. Hence, for a given remaining capacity to emit, more carbon-based capacity will need to be scrapped for low values of B than for high values of B. This scrapping represents a cost to the economy, since capital is consumption foregone. Having more productive carbon-free capacity enables a reduction of such waste, as can be seen from the fact that the terminal values of K_A at the end of the JPR stage are lower for the high B paths than for the low B paths. The consequence for the stock of carbon-free capital is that its accumulation will start at an earlier date, as can be seen from the RHS-panel of Figure 5.8, allowing for higher terminal values of K_B at the end of the JPR stage in general, but certainly for higher levels (and higher growth rates) of the steady state growth path starting at t=TF. Note that the K_B-path associated with the highest B is temporarily overtaken by other paths with lower values of B. However, the terminal values of K_B which are associated with the kinks in the K_B-paths at t=TF, are still unambiguously rising with B.

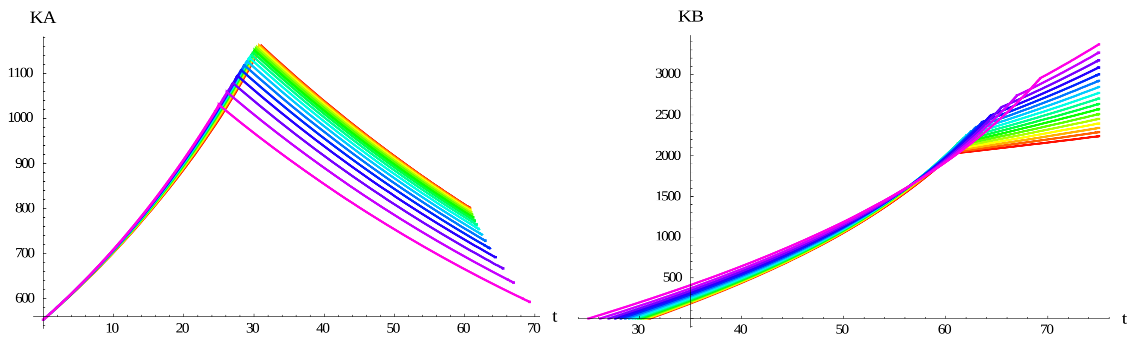

Figure 5.8. KA and KB: B = 0.08-0.12

The overtaking is more visible when looking at the growth rates of output, shown in the LHS-panel of Figure 5.9. For higher values of B, growth is generally higher during the BAU stage (except at the very end), but then the drop in growth performance for the high B cases is far less pronounced than for the low B cases when the JPR stage begins. Nonetheless, the growth rate of the highest B path is overtaken in absolute terms by some of the paths with lower (but still relatively high) values of B.

As can be seen from the RHS-panel of Figure 5.9, the drop in the level of output (due to the scrapping of still remaining carbon-based capacity) is highest for the low B paths and smallest for the high B paths. Nothing of the overtaking in the previous Figure is visible in Figure 5.10 for consumption in absolute terms: higher B's imply higher consumption levels particularly from the JPR phase onwards, while the outcomes for the propensity to consume for the highest values of B are ever so slightly below those for lower values of B during the BAU phase. This is due to the fact that if B is low, then the opportunity cost of consuming now rather than later are low as well. This is corroborated by the fact that the initial value of the co-state for carbon-based capital is relatively low for low values of B and is rising for increasing values of B.

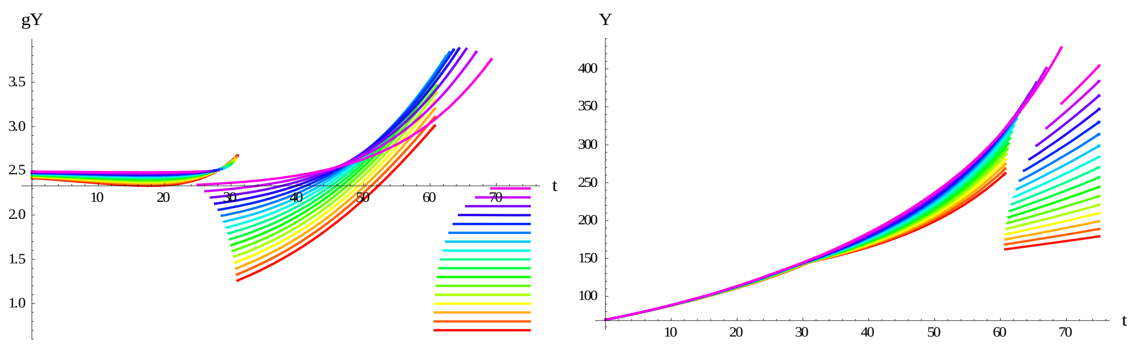

Figure 5.9. gY and Y: B = 0.08-0.12

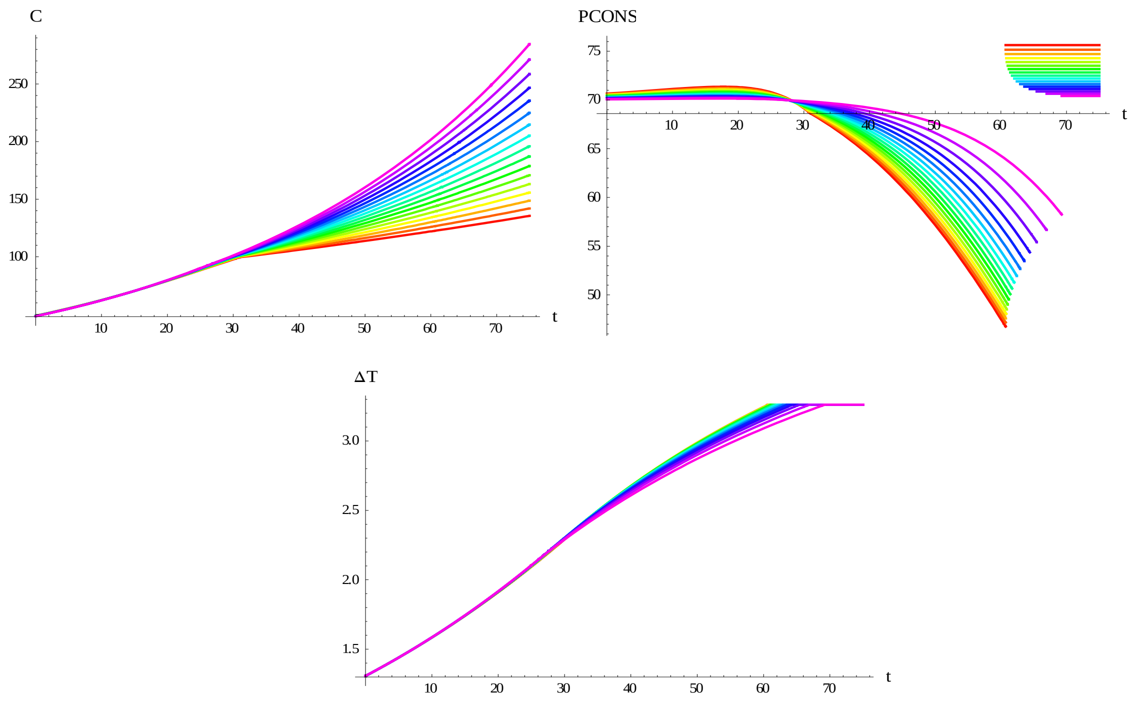

Figure 5.10. C, PCONS and ΔT: B = 0.08-0.12

Temperature-wise, a rise in B simply postpones the arrival date of the CFR phase by about 8 years, but Figure 5.8 shows that the arrival date of the JPR stage is brought forward in time when B rises, as can be seen from the shift in the top of the K*A*-curves toward the vertical axis.

Extreme weather-related damages

In Figure 5.11 we show the results for a shock in the rates of depreciation by 1 percentage point, raising both δ_A and δ_B from 0.0125 to 0.0225 at a value of ΔT of 2^0 K. This introduces a split in the BAU stage at t=22.4 years, which is also the length of the low-damage sub-stage, while the length of the high-damage sub-stage of the BAU stage is 9.7 years, and that of the high-damage JPR stage becomes 32.4 years. The arrival date of the CFR stage becomes TF=64.5. So, relative to the base-run, the arrival dates of both the JPR and the CFR stages will be postponed by about 1 and 3.5 years, respectively.

As one can see from Figure 5.11, K_A is growing during both the low-damage and the high-damage sub-stages of the BAU stage, but its rate of growth falls somewhat with the arrival of extreme weather-related damages. The most striking result is the considerable drop in the shadow price of the remaining capacity to emit as damages increase. This drop more than halves the initial value of the shadow-price, even though the rate of decay is less than doubled. An alternative interpretation of what happens to the shadow-price is that its value during the low damage sub stage rises relative to that of the high-damage sub-stage. This interpretation is in line with the plot depicting the jump in the shadow price of R in Figure 5.12, where the range of variation of the shadow price during the low damage sub-stage is much larger than that during the high damage sub-stage. Moreover, the latter interpretation is in line with the observation that extra damages make owning carbon-based capital during the high-damage-regime less valuable than owning such capital during the low-damage-regime. Consequently, one would expect that in a sensitivity experiment with extra damages growing from 0 to 1.5% of both capital stocks, investment in carbon-based capital would be shifting towards the low-damage present, which is confirmed in Figure 5.14 further below.

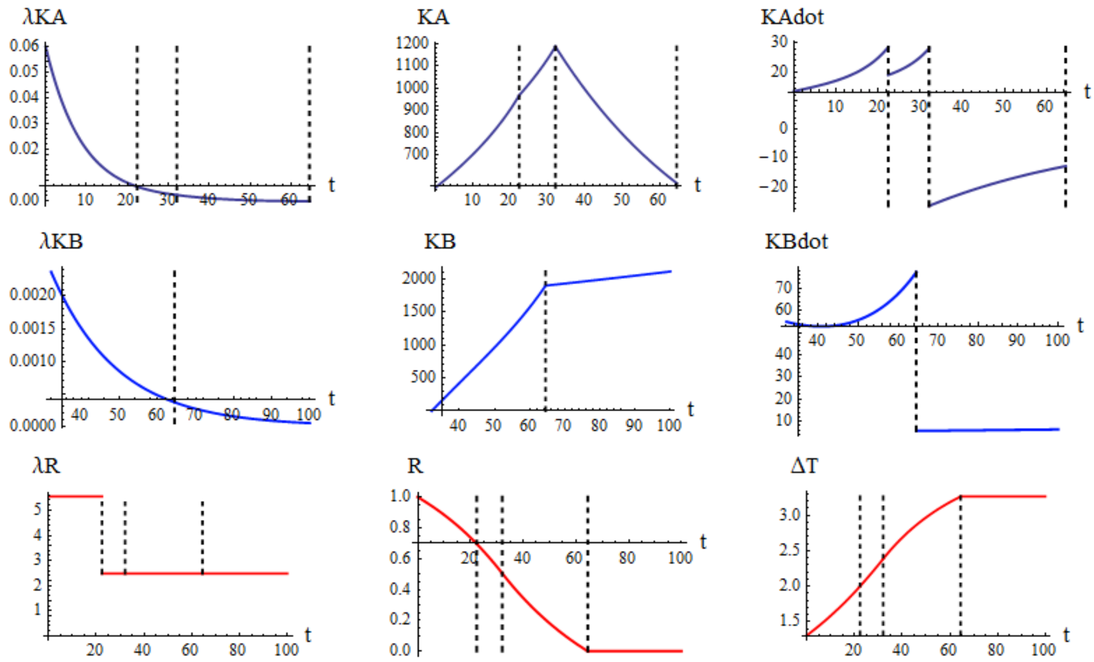

Figure 5.11 Extreme weather-related damages results

Figure 5.12 shows the sensitivity results for the arrival times of the various (sub-) stages and for the initial values of the co-states for *R* and K_A. From inspection of this Figure we can conclude that the expectation of future damages increases the value of the remaining capacity to emit, but for the low-damage BAU sub-stage more than for the high-damage BAU sub-stage. This is due in part to investment in carbon-based capital being brought forward in time: *the expectation of future damages makes it more worthwhile to use (new) capacity now rather than later*.

For another part it is caused by the fact that since net output per unit of consumption foregone is negatively affected by the extra damages, there is an incentive to use equipment that will, on average, be less productive than without the extra damages, over longer periods of time, but this would require additional capacity to emit, hence the rise in the shadow price of *R*. Nonetheless, the shadow price of carbon-based capital is negatively affected, because the net rate of return of both carbon-based capital and of carbon-free capital are negatively affected too by the extra damages, while carbon-free capacity must in part be built up using carbon-based capital. The incentive to stretch the economic life time of carbon-based capital to compensate in part for the fall in its net productivity is apparent from the fact that the low-damage BAU sub-stage length (called ΔUL here) is slightly falling, whereas that of the high damage BAU sub-stage (ΔUH) is rising more than ΔUL is falling, thus increasing the length of the whole of the BAU-stage. The JPR stage is also extended by several years, *thus in fact postponing the arrival of the carbon-free stage by keeping the carbon-based capital stock longer in business.* This result is paradoxical[18], because a natural reaction to receiving the message of increased future damages would seem to be to cut back on doing the things that would cause these damages. Instead, the message of this model is to *'get it over and done with'* by increasing investment in carbon-based capacity during the low damage sub-stage and then extending the duration of the use of the culprit technology over its 'normal' duration. The counterintuitive result we find arises directly out of the intertemporal asymmetries in the rates of return on carbon-based capital caused by these damages. Apparently, the subtleties of intertemporal optimisation are not readily captured by pure intuition.

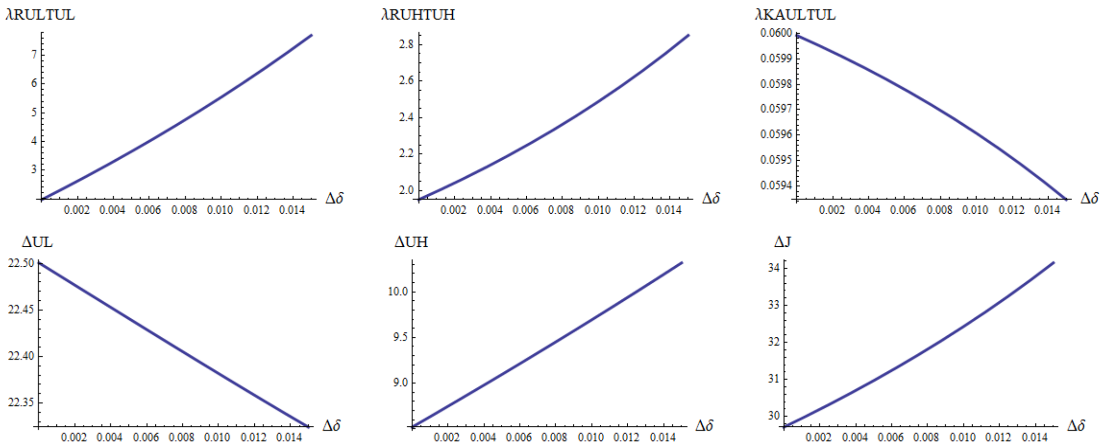

Figure 5.12. Sensitivity results: $\Delta\delta = 0 - 0.015$

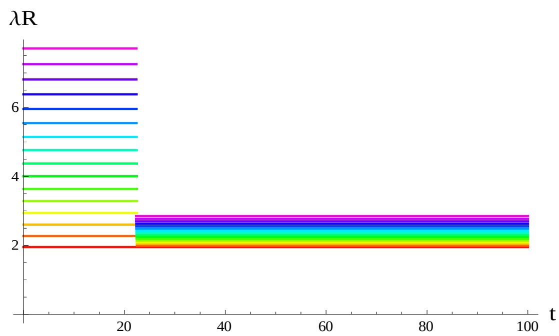

Figure 5.13. The shadow price of R: $\Delta\delta = 0 - 0.015$

Figure 15.13 shows the range of variation for the shadow prices of the remaining capacity to emit for both the low-damage-regime and the high-damage-regime. We see that the shadow price during the low-damage regime has a tendency to rise with the level of the extra damages. This also happens during the high-damages regime, but the rise is far less outspoken. Note that the bottom time-path for the shadow-price doesn't exhibit a jump, because the extra damages are zero by construction in this case. This implies that it is easy to infer from Figure 5.14 that indeed investment in carbon-based capital is

[18] The 'green paradox' covers the case where a future tax on the extraction of carbon-based fuels speeds up extraction in the present, for similar reasons as we have here (cf. Sinn (2008)).

brought forward in time, because all the paths for rising extra damages lie above the zero-extra damage path in the low-damage BAU sub-stage (i.e. the red line), and below the zero-extra damage path during the high damage BAU sub-stage.

For net investment in carbon-free capacity, we see that increasing damages lead to a much more uniform distribution of net-investment over time, involving higher net investments at the beginning of the JPR stage and lower net investments at the end in the high extra damage cases compared to the no-extra damage case. This more even distribution of carbon-free investment over time, mitigates the negative effects on consumption of increased damages. With low or no extra damages, investment in carbon-free capacity during the JPR stage is strongly increasing in anticipation of having to cushion the drop in output associated with the scrapping of the remaining carbon-based capital stock at the beginning of the CFR stage. With higher damages, the drop will necessarily be less, *ceteris paribus*, and investment itself can be more evenly distributed over time, also allowing consumption to be more evenly distributed over time.

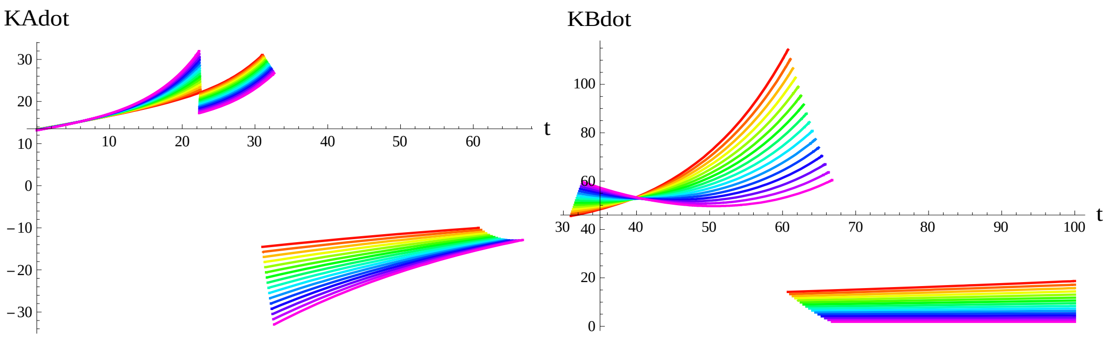

Figure 5.14. Net investment in K_A and K_B: $\Delta\delta = 0 - 0.015$

As can be seen from Figure 5.15 this leads to a somewhat accelerated build-up of the carbon-based capital stock during the low damage BAU sub-stage, a deceleration of that build-up during the high-damage BAU sub-stage, both having the effect of ending up with slightly higher terminal values of the carbon-based capital stock at the end of the slightly extended BAU stage, and a corresponding slight postponement of the arrival of the carbon-free stage. The time-paths for K_A show that the fact that extra damages creates an incentive to bring investment in carbon-based capital forward in time generates a higher initial value of that stock at the beginning of the JPR stage, in anticipation too of the higher rate of decay compared to the no-extra damages case. The spread in the terminal value of K_A at the arrival date of the CFR stage is far larger than the spread in the terminal values of K_B at the same date, which is just as well, because this terminal value of K_B at t=TF is the only capacity available for production an instant later, since carbon-based capacity will have to be discarded because of the 600 ppm (or 3.2^0 K) threshold.

Figure 5.16 shows the growth rate of output and its corresponding level. During the BAU-stage, output growth rates would first be positively then negatively affected by the expectation of extra damages. The net effect on output levels during the BAU-stage is limited, however. Growth rates during the other high-damage stages would all be negatively affected, and significantly so. However, the level drop in output at the arrival date of the CFR stage is less outspoken for the high-damages cases than for the low damage cases, since the range of variation of the terminal values for Y at the end of the JPR stage is at least twice as large as the range of variation of the initial values for Y at the beginning of the CFR stage. This level jump in a particular path is associated with the discarding of carbon-based capital. Nonetheless, having lower extra damages is still the preferred option, since this leaves higher production possibilities at the start of the CFR stage.

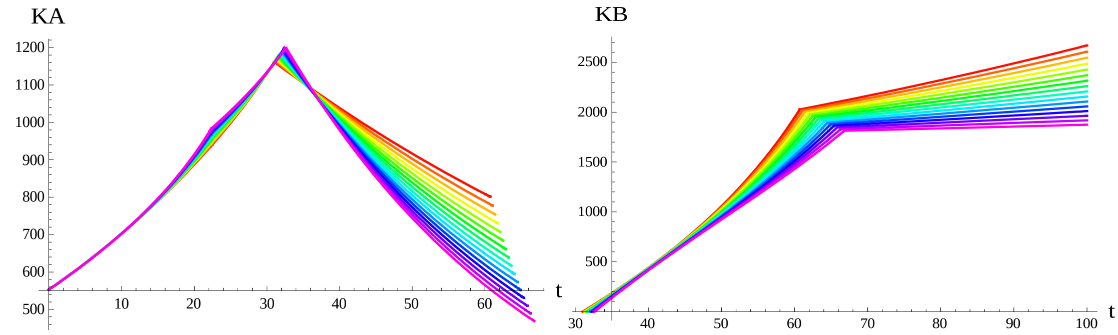

Figure 5.15. The capital stocks K_A and K_B: $\Delta\delta = 0 - 0.015$

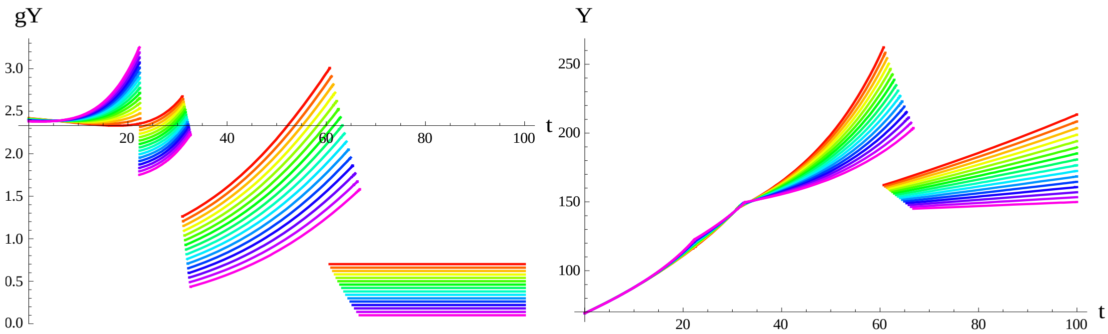

Figure 5.16. Output growth (gY) and output levels (Y): $\Delta\delta = 0 - 0.015$

7. Conclusion

The model we have presented in this article is a strongly simplified, but intertemporally complete, transition model that emphasises the intertemporal aspects of a just-in-time transition from a carbon-based economy to a carbon-free economy, while taking account of the fact that capital is a produced means of production and that investment is irreversible. The latter implies that in order to make a successful just-in-time transition, some carbon-based capacity will need to be scrapped just before cumulative emissions would reach the climate tipping point that is present in our model.

In a number of sensitivity experiments, we show that tightening the cumulative emissions constraint raises the shadow price of CO2 emissions, and also that of carbon-free capital. But somewhat unexpectedly perhaps, *it also raises the shadow price of carbon-based capital, simply because the value attributed to the carbon-based capital stock is in part derived from the value of the carbon-free stock that it is able to produce.* We also find that in this case it is optimal to accumulate carbon-based capital at a faster pace than with a less tight cumulative emissions threshold, so that the arrival of the CFR stage is speeded up. To facilitate the latter, *a quick build-up of the carbon-based capital stock is required to be able to switch relatively early to investment in carbon-free capacity* and to enable considerable rates of both investment and consumption once investment in carbon-based capacity and later on production using carbon-based capacity has ceased.

When we increase the productivity of carbon-free capital, we find that this positively affects the shadow price of carbon-based capital, *underlining the importance of carbon-based capacity for the creation of a carbon-free future*. We also find, again somewhat paradoxically, that *a rise of the productivity of carbon-free capacity tends to prolong the use of carbon-based capital*, simply because the need to burn a lot of CO2 quickly in order to enable a fast build-up of carbon-free capacity is less pressing when carbon-free capital goods become more like carbon-based capital goods from a productivity point of view.

When we add extreme weather-related damages to this framework we observe that the rate of investment in carbon-based capacity is increased rather than being reduced, because the expectation of extreme weather-related damages introduces a wedge between the rates of return on carbon-based investment during the low and high damage sub-stages of the BAU stage. This makes it worthwhile to use (new) carbon-based capacity now rather than later. An increase in the extra damages rate also leads to a more even distribution of carbon-free investment over time, in order to mitigate the negative effects on consumption of increased weather-related damages. With low damage rates, investment in carbon-free capacity during the JPR stage is strongly increasing in anticipation of having to cushion the drop in output associated with the scrapping of the remaining carbon-based capital stock at the beginning of the CFR stage. With higher damages, the drop will necessarily be less, *ceteris paribus,* and so will be the need to mitigate the drop.

So what do we learn from all this? The embodiment of technology in physical units of capital underlines the practical importance of the physical capital stock as a produced means of production. This in turn stresses the need for a productive carbon-based production system in order to be able to produce the right amount and quality of the carbon-free production units on which future welfare will exclusively come to depend. The results also show that accounting for the embodiment of technology in physical capital goods in combination with the irreversibility of investment seems to imply a worryingly short BAU stage. In addition to this, the optimum response to an expected increase in extreme weather-related damages shows that investment decisions that seem to be intuitively ‘right’ (i.e. cutting down on investment in the technologies that ‘produce’ these extra damages), are not necessarily sensible from an intertemporal welfare optimisation point of view.

References

- Barro, R., & Sala-i-Martin, X. , 2004, *Economic Growth* (2nd ed.): MIT Press.
- Boucekkine, R., De la Croix, D., & Licandro, O., 2011, Vintage Capital Growth Theory: Three Breakthroughs. Unpublished Working Paper. Barcelona GSE.
- Boucekkine, R., A. Pommeret and F. Prieur, 2011, "Technological vs ecological switch and the environmental Kuznets curve", Document de Travail no. 2011-41, GREQUAM.
- Boucekkine, R., A. Pommeret and F. Prieur, 2012, *Optimal Regime Switching and Threshold Effects: Theory and Application to a Resource Extraction Problem under Irreversibility*, Document de Recherche nr 2012-14, Unité de Formation et de Recherche d'Economie, Montpellier.
- Canadell, J.G.; C. Le Quere, M.R. Raupach, C.B. Field, E.T. Buitenhuis, P. Ciais, T.J. Conway, N.P. Gillett, R. A. Houghton, and G. Marland, 2007, Contributions to accelerating atmospheric CO2 growth from economic activity, carbon intensity, and efficiency of natural sinks. *PNAS* **104** (47): 18866–18870.
- IPCC, 2007, *Climate change 2007: the physical science basis: contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge; New York: Cambridge University Press.
- IPCC, 2014: *Climate Change 2014: Synthesis Report*. *Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change* [Core Writing Team, R.K. Pachauri and L.A. Meyer (eds.)]. IPCC, Geneva, Switzerland.
- Johansen, L., 1959, Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis. *Econometrica, 27*(2), 157-176.
- Judd, K., 1998, *Numerical Methods in Economics*, MIT Press.
- Leonard, D., & Van Long, N., 1992, *Optimal control theory and static optimization in economics*. Cambridge: Cambridge Univ. Press.
- Maurer, H., C. Büskens, J.H.R. Kim and C.Y. Kaya, 2005, "Optimization methods for the verification of second order sufficient conditions for bang-bang controls", *Optimal Control Applications and Methods*, 26, pp. 129-156.
- Nordhaus W.D., 2010, Economic aspects of global warming in a post-Copenhagen environment. *Proceedings of the National Academy of Sciences* 107(26):11721-11726.
- Nordhaus W.D., 2008, *A Question of Balance: Weighing the Options on Global Warming Policies*, Yale University Press.
- Oppenheimer M, Petsonk A., 2005, Article 2 of the UNFCCC: historical origins, recent interpretations. *Climatic Change* 2005;73:195–226.
- Rebelo, S., 1991, Long-Run Policy Analysis and Long-Run Growth, Journal of Political Economy, 99, pp. 500-521.
- Saglam, C., 2011, Optimal pattern of technology adoptions under embodiement: A multi-stage optimal control approach, *Optimal Control Applications and Methods*, 32, pp. 574-586.
- Schumacher, I., 2011, WHEN SHOULD WE STOP EXTRACTING NONRENEWABLE RESOURCES? *Macroeconomic Dynamics, 15*(4), 495-512.

- Sinn, H.W., 2008, Public Policies against global warming: a supply side approach, *International Tax Policy Public Finance*, 15, pp. 360-394.
- Solow, R. M., Tobin, J., von Weizsäcker, C. C., & Yaari, M., 1966, Neoclassical Growth with Fixed Factor Proportions. *The Review of Economic Studies, 33*(2), 79-115.
- Tahvonen, O. and S. Salo, 2001, "Economic growth and transition between renewable and nonrenewable energy resources", European Economic Review, 45, 1379–1398.
- Valente, S.,2011, Endogenous Growth, Backstop Technology Adoption, and Optimal Jumps. *Macroeconomic Dynamics, 15*(3), 293-325.
- Van Zon, Adriaan and Paul David, 2012, Optimal Multi-Stage Transition Paths toward a Global Green Economy, UNU-MERIT Working Paper 2012-079.
- Von Böhm-Bawerk, Eugen, 1891, *The Positive Theory of Capital.* London: Macmillan. Translated by William Smart. Available online at: http://www.econlib.org/library/BohmBawerk/bbPTC.html.
- Worldbank, 2013, Building Resilience: Integrating Climate and Disaster Risk into Development, the World Bank.

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