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Michael Verba

Maastricht Economic and social Research institute on Innovation and Technology (UNU-MERIT)
email: info@merit.unu.edu | website: http://www.merit.unu.edu

Maastricht Graduate School of Governance (MGSoG)
email: info-governance@maastrichtuniversity.nl | website: http://mgsog.merit.unu.edu

Keizer Karelplein 19, 6211 TC Maastricht, The Netherlands
Tel: (31) (43) 388 4400, Fax: (31) (43) 388 4499

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Growth and Innovation in the Presence of Knowledge and R&D Accumulation Dynamics

By Michael Verba*

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This article develops a model of growth and innovation in which accumulation dynamics of knowledge and R&D are explicitly considered. The model is based on a more general knowledge production process than commonly used in Endogenous Growth Theory and R&D productivity literatures, reconciling as special cases of a broader framework disparate analytical approaches. The model of knowledge dynamics highlights the role of human capital, physical capital, and accumulation in the creation of innovations and establishes the theoretical possibility of long-run idea-driven growth without the razor-edge assumption of Romer (1990) and in the absence of growth in R&D employment stipulated by Jones (1995). This analysis also predicts the structure of estimation biases that can result from omission of relevant factors and failure to take into account the accumulation dynamics of knowledge and R&D. Empirical estimation supports these predictions. Findings provide recommendations for future empirical studies aiming to explain innovation.

JEL: O30, O31, O32, O40
Keywords: Growth theory; innovation; R&D; productivity; knowledge production function; accumulation

I. Introduction

Scholars from different academic disciplines, and working with different methodologies, argue that accumulation of technological knowledge is a key driver of economic growth. The claim of centrality of knowledge accumulation to economic growth extends to many discussion streams in economics, particularly subdisciplines concerned with production, management and innovation. This argument can also be found in historical accounts of the economic development of nations, which observe how mastery of new technologies had accompanied spurts of industrialisation (Gerschenkron, 1962). Technological knowledge also occupies

* United Nations University Maastricht Economic and Social Research Institute on Innovation and Technology (UNU-MERIT), Boschstraat 24, 6211AX, Maastricht, Limburg, The Netherlands, Email: mverba@alumni.princeton.edu.

a central role in growth theory, where it acts as a key input, alongside capital and labour, in models of aggregate production (Solow, 1956, 1957; Romer, 1990). Finally, this perspective persists in policy discussion, both in advanced and developing economies, where promotion of innovation is seen as a pressing concern. The premise of this paper is that if knowledge is an important component in modern economic growth, knowledge dynamics processes deserve attention and their own formal model.

In this paper we present a model of knowledge dynamics designed to capture the process of knowledge creation and accumulation. We begin with an exercise in reconceptualisation. The intellectual history of oft-repeated terms, like "R&D capital," "human capital," and "knowledge" is revisited in a summary way, and definitional lines are drawn. In the study, we make a distinction between research effort and the resulting knowledge flows. Competing conceptions of knowledge generation are reconciled, before the foundational stones of a model of knowledge dynamics are set in place. The modelling exercise starts by extending the standard knowledge production function encountered in models of economic growth, particularly the literature from Endogenous Growth Theory (EGT). It proceeds by incorporating features of "R&D capital" accumulation encountered in the literature on R&D and productivity.

The building block of the model describing the process of knowledge generation includes two stocks: a knowledge stock consisting of the sum total of disembodied technologically relevant ideas, and an R&D stock, representing accumulated embodied research effort. R&D stock contains human and physical capital components, allowing for a role of accumulated research effort in the creation of knowledge. Maintaining separate stocks allows us to capture the separate but interconnected flow and accumulation dynamics of knowledge and R&D.

The analytical exercise has implications for balanced growth and the measurement of impact of R&D on the flows of innovation. The model suggests the possibility of idea-driven growth without the razor-edge assumption of Romer (1990) and in the absence of growth in R&D employment stipulated by Jones (1995). These results also add a dimension to the paradox first noted by Jones (1995), that in recent decades, an increase in the size of the R&D sector in developed economies has not led to a commensurate rise in technical change. Finally, our model of knowledge dynamics reveals the structure of mismeasurement and estimation biases that can result from exclusion of accumulable components, such as physical and human capital, from measures of research effort, and from the failure to take into account accumulation dynamics. Section XII consists of an empirical exercise lending support to our conception of knowledge dynamics.

II. Reflections on Models of Knowledge Production

The *production function* is one of the core analytical tools in economics. By relating output to its factor inputs it describes the production process and links to other notions from production theory, such as efficiency and productivity. Production functions used in contemporary economics trace their lineage to the 1928 work of Charles Cobb and Paul Douglas (Berndt and Christensen, 1973), although the idea of expressing production as a mathematical function that relates output to a set of inputs is older, dating at least as far back as 1894, when philosopher Philip Wicksteed published his essay on distribution, and to even earlier work of Johann von Thünen (Mishra, 2007; Wicksteed, 1894).

The *knowledge production function* (KPF) framework represents an important methodological approach to the study of innovation and technical change, the alternatives to which are qualitative and historical studies (Griliches, 1979). While in a generic production function an index of outputs is related to measures of factor inputs, in a knowledge production function an index of innovation is related to factors determining innovative activity. The index of innovation is either incorporated into a broader production function framework which includes output, or is used standalone. The former is the standard approach of theoretical studies, while the latter is more frequently the case in empirical research that is concerned primarily with explaining patterns of innovative activity.

The knowledge production function approach has been applied to assess the impact of R&D on output and total factor productivity (Griliches, 1988; Verspagen, 1995; Abdih and Joutz, 2006), to estimate the rate of return to R&D (Bernstein, 1989; Jones and Williams, 1998), to understand factors determining the intensity of innovative activity across industries and at various spatial scales (Porter and Stern, 2000; Mohnen, Mairesse and Dagenais, 2006), and to measure knowledge spillovers (Jaffe, 1986; Griliches, 1992; Coe and Helpman, 1995; Audretsch and Feldman, 1996).[1]

Knowledge production in the R&D productivity literature

Knowledge production functions come standard with the literature on productivity. An early discussion in this mould can be found in Griliches (1979). The departure point for studies in the relationship between R&D and productivity is the aggregate production function:

$$Y = F(A, K, L) \tag{1}$$

in which a measure of output Y is related to inputs, where K and L represent capital and labour, respectively, and A stands for the level of technological knowledge. The literature posits a relationship between the level of technological knowledge and investments in knowledge production in the form of research and development, and sets before itself the task of estimating the impact of R&D

[1] A recent survey of work on the R&D-productivity nexus is Mohnen and Hall (2013). For an overview of studies estimating the rate of return to R&D see Hall, Mairesse and Mohnen (2010). Surveys of literature on spillovers can be found in (Branstetter, 1998) and Cincera and de la Potterie (2001); a more recent survey on this topic is Belderbos and Mohnen (2013). The study by Eberhardt, Helmers and Strauss (2013) provides a critique of the methods used in the R&D productivity literature; similar to the present study, it tries to build a bridge between disparate literatures.

activities on output.

The role of the knowledge production function in this literature is to describe the relationship between knowledge and R&D investment. The KPF is of the form:

$$\dot{A} = R + v \tag{2}$$

which we may term, for referential convenience, the "Griliches knowledge production function." In this equation, $\dot{A}$ is the output of new knowledge and R is input into knowledge discovery effort by way of R&D expenditure; v represents other unobserved influences on knowledge production (Griliches, 1990). [2]

Knowledge stock in the Griliches framework is a cumulation of current and prior additions to knowledge resulting from the stream of R&D expenditures. In the absence of knowledge depreciation, knowledge can be described simply as the sum of the current and past R&D investments:[3]

$$\mathbf{A}_t = \sum_{i=-\infty}^{t} \dot{A}_i = \sum_{i=-\infty}^{t} R_i. \tag{3}$$

However, in deriving knowledge stock from the knowledge production function, knowledge depreciation must be taken into account. Knowledge depreciation is a phenomenon analogous to depreciation in capital theory (Benhabib and Rustichini, 1991; Hall, 2007). Because it is considered that over time technical knowledge loses its relevance, prior R&D expenditures contribute less to the current knowledge stock than current expenditures. In the notation provided by Griliches (1979), the stock of technologically relevant knowledge is expressed as a function of the R&D expenditure stream using the following equation:[4]

$$\mathbf{A}_t = G(W(B)R, v). \tag{4}$$

In Eq. (4), $\mathbf{A}$ is the stock of technological knowledge and $W(B)R$ is an index of current and lagged R&D expenditures. As in Eq. (2), here too the variable v represents all other factors influencing the stock of knowledge, so that Eq. (4) expresses knowledge stock as a function of the sum of the current-period R&D and depreciated R&D from prior periods, plus the residual factors v. If we set aside residual factors, the function $G(W(B)R)$ can be re-expressed as

[2] Griliches (1990) uses K and u to represent knowledge, and other influences, respectively. The original notation has been changed to maintain consistency with the nomenclature used throughout this article.

[3] In the following discussion we denote stock variables using boldface font.

[4] Eq. (4) is sometimes called a "knowledge production function" or "knowledge function" (Esposti and Pierani, 2003), although, strictly speaking, it is a function describing knowledge accumulation.

$$\mathbf{A}_t = R_t + (1-\gamma)\mathbf{A}_{t-1}, \tag{5}$$

which will be recognised as the Perpetual Inventory Method (PIM) for calculating stocks. In the PIM R&D stock equation, current stocks are calculated as the sum of current-period investments (R_t) and the stocks left over from the previous period adjusted for depreciation ($(1-\gamma)\mathbf{A}_{t-1}$). The depreciation rate is given by the parameter γ. Supplemental discussion of the relationship between the Griliches KPF and the PIM approach can be found in the Appendix.

Although the Griliches framework keeps the door open for inclusion of factors besides R&D spending, these residual factors have not played much of a role in empirical construction of R&D stocks. One review of studies on R&D and productivity found that "[a]lmost all... have used a simple perpetual inventory or declining balance methodology with a single depreciation rate to construct the knowledge capital produced by R&D investments" (Hall, Mairesse and Mohnen, 2010, p. 15).

It is worthy of notice that while the knowledge production function in Eq. (2) is at the heart of the Griliches framework, it is not very salient. The aim of this literature is to study the effect of knowledge, created by R&D, on productivity. Because in the productivity literature Eq. (2) serves simply as a transition point on the way to calculation of R&D stock, given by the knowledge accumulation equation in Eq. (5), it is easy to miss. For the purposes of modelling innovation, however, the Griliches equation offers a theory of knowledge dynamics that is of paramount interest.

Knowledge Production and the Theory of Endogenous Growth

The knowledge production function features most prominently in growth theory. Although the knowledge production sector is only one element of a complete endogenous growth model, it is of focal importance, since the growth rate of knowledge determines the growth rates of all other variables in the system. The standard knowledge production function encountered in Endogenous Growth Theory, which we term the Romer-Jones knowledge production function, is of the form: [5]

[5] Above is the parametrized KPF adapted from Romer (1990) by Jones (1995). Variations exist, based on slightly different interpretations of the the labour variable, restrictions on parameters λ and ϕ and utilisation of a different nomenclature for variables. In Romer (1990), the exact notation used was $\dot{A}=\delta H_A A$, with knowledge represented by A and H_A denoting the amount of human capital. Jones uses the form $\dot{A} = \delta L_A^{\lambda} A^{\phi}$; where knowledge stock is represented by $\mathbf{A}$, L_A is labour employed in R&D, and δ is the arrival rate of innovations. While Jones (1995) takes the "number of scientists and engineers" as a measure of R&D labour, in Romer (1990) the same measure is used as a stand-in for human capital. In empirical estimation the difference in notation has no practical consequence. For consistency we have kept to the Jones (1995) notation throughout the study.

$$\dot{A} = \delta L_A^{\lambda} \mathbf{A}^{\phi}, \tag{6}$$

where $\dot{A}$ is knowledge flow, $\mathbf{A}$ is knowledge stock, L_A is labour employed in the R&D sector, λ is a parameter measuring the return of knowledge from R&D labour, ϕ is the intertemporal spillover parameter and δ is the productivity of knowledge discovery. This rendition of the knowledge creation process includes knowledge stock ($\mathbf{A}$) on the right-hand side to account for the possibility that knowledge output depends on the stock of already accumulated knowledge.

Even a cursory glance at Eqs. (2) and (6) reveals that the Griliches and Romer-Jones KPFs present quite different theories of knowledge formation. In the literature on returns to R&D, knowledge production is synonymous with research effort (Hall, Mairesse and Mohnen, 2010). In endogenous growth theory too, new technologically relevant ideas involve research effort, but the arrival rate of innovations is also conditioned by the stock of previously accumulated knowledge (Romer, 1990; Aghion and Howitt, 1992; Jones, 1995). Furthermore, the measure of research effort in the two models is different. The Romer-Jones knowledge production function proxies research effort using the quantity of labour employed in the R&D sector, while the Griliches framework measures research effort with R&D expenditure, which is a broader measure incorporating the labour—and physical capital—components of the knowledge discovery effort. Finally, the two models of knowledge production differ in their approach to accumulation. In the Griliches knowledge accumulation equation research effort accumulates, contributing to the stock of knowledge. In the Romer-Jones model, knowledge accumulates, but research effort does not. The Romer-Jones KPF includes two factors: the existing body of knowledge ($\mathbf{A}$) and the number of scientists and engineers in the R&D sector (L_A). The former is a stock but the latter is a flow variable.

It might appear at first glance that the count of scientists and engineers can be considered a stock variable. In a literal sense it is, but not from the point of view of the production system, where the input into production is not the stock of labour itself, but labour's effort in production. From the standpoint of the production system, labour's contribution to production is the services it renders to the production process. The work of labour in the R&D sector produces ideas in the current period, but it also enhances the production of ideas in subsequent periods, by, for example, adding to human and physical capital that remain for a certain time as assets in the production of ideas. To illustrate this point, we can consider the case of an industrial research lab that involves n scientists full-time on the development of a clinical vaccine in year 1, and the same number of scientists in year 2—the year when the vaccine is successfully developed. To count as labour input to knowledge production only the number of personnel working on the vaccine in year 2 would be to ignore half of total effort.

For our purposes, whether L_A is a stock or a flow depends on whether it includes labour services rendered in prior periods. If only current-period labour count is included in production, as in the case of Jones (1995), and in many empirical studies, the labour variable is a measure of the flow of current research services from labour.

Differences between the Griliches and Romer-Jones conceptions of knowledge production can have profound implications for modelling and empirical estimation. If existing knowledge stock serves as a factor in knowledge production, then its omission from the Griliches KPF will result in omitted variable bias and skew the resulting elasticity estimates, a point raised previously by Jones and Williams (1998). Omission of accumulated research effort from the KPF of Endogenous Growth Theory can be expected to lead to biases of its own. Exclusion of a cumulable component of effort from knowledge production is another potentially important drawback.

What, then, is a better way to model knowledge production? What factors should be included in a knowledge production function? In the next section we consider inputs into knowledge production and their inter-relationships. This exercise leads to three observations which serve as a scaffolding on which we build a more general knowledge production function, in Section IV, of which both Griliches and Romer-Jones functions are special cases.

III. R&D Capital, Human Capital and Knowledge

In economic theory knowledge has more than one alias. The variable "**A**" in Eqs. (4) and (6), representing the stock of knowledge, also goes under the names "technology" (Benhabib and Spiegel, 2005; Los and Verspagen, 2000) and "total factor productivity" (Caselli and Coleman, 2006). Changes, or new additions to the body of knowledge, the variable "$\dot{A}$" in (2) and (6), alternate between the labels "technical change" (Griliches, 1988), "technological change" (Verspagen, 1995), "new knowledge" (Abdih and Joutz, 2006), "invention" (Griliches, 1979) and "innovation". Finally, the units of measure into which "the stock of general knowledge" (Branstetter, 1998) can be divided have been discussed in terms of "ideas" (Porter and Stern, 2000), "blueprints" (Grossman and Helpman, 1991), "patents" (Sequeira, 2012), "designs" (Romer, 1990; Branstetter, 1998), "inventions" (Jones, 1995), and, once more, "innovations." The word "innovation" has two senses. It can mean "a novelty," or "an act or process of creating or introducing something new." Both meanings are in currency in the literature, with the former definition used as another term for a unit of measure of knowledge (as, for example, in Porter and Stern (2000), and the later as a description of the process of creation of new knowledge (e.g.: Freire-Seren (2001)). It is important to discern the underlying concept that hides behind the fog of nomens. This is the task of this Section.

Both knowledge and R&D are frequently invoked concepts in economics. Yet, in existing literature, the definitions of knowledge and R&D and their correspondence are not always made clear, nor are these features consistent across studies. Existing literature has treated research effort and knowledge in one of two ways, either by equating research effort with knowledge or by describing a process by which research effort is turned into knowledge. In the first, "effort as knowledge" perspective, the two concepts are either treated as synonymous, or a measure of one is used as a close proxy for the other. A mark of the "effort as knowledge" literature is that terms pertaining to knowledge and R&D are used interchangeably. Griliches (1992), Esposti and Pierani (2003), and Hall, Mairesse and Mohnen (2010) are just a handful of many excellent studies in which the variable $\boldsymbol{A}$ in the accumulation equation, Eq. (4), or its equivalent, is referred to alternatively as "knowledge capital" or "R&D capital", "knowledge stock" or "R&D stock", because such identification follows naturally from the utilised theoretical framework.

The "effort as knowledge" perspective has been dominant in studies of productivity. This is a perspective hardwired into the Griliches knowledge production function—which sets a sign of equality between $\dot{A}$ and R. In the accompanying PIM accumulation equation, Eq. (5), R&D turns into knowledge seamlessly. One becomes the other, with adjustment only for depreciation. Aside from the reduction owing to depreciation, the model implies that R&D and knowledge are consubstantiate. In this view, knowledge is nothing more than accumulated R&D expenditure.

In "effort to knowledge" discourse, R&D and knowledge can be recognised as distinct concepts. The relationship between them is described as a process by which knowledge arises from research effort. In this framework, the flow of new technological ideas ($\dot{A}$) is driven by the allocation of resources to research. The EGT literature has adopted this perspective, describing a process by which knowledge rises from reseach effort, which is represented by the research labour or human capital component of R&D and measured by the number of scientists and engineers. Because the concepts "knowledge" and "R&D" are so central to the study of innovation, we pause to reflect on them.

Observation 1: Knowledge is Distinct from R&D

In discussing the attributes of knowledge some authors stress two features: nonrivalness and partial excludability. The nonrival nature of knowledge allows multiple agents to use it at the same time. Knowledge is nonrival because it is "disembodied" (Benhabib and Spiegel, 2005), that is, "independent of any physical object" (Romer, 1990). It is partially excludable because even though it can be used by multiple agents, there might be a mechanism through which it might be possible to restrict some agents from using it, as is the case when monopoly on its use is provided through patents or copyrights, or its availability is restricted by trade secrets. Further, the stock of knowledge has no obvious natural bound; in principle it can grow without limit.

In contrast to the disembodied nature of knowledge, R&D is embodied. Factor inputs involved in research are readily measurable and conveniently expressed in terms of expenditure. The Frascati Manual defines R&D expenditure as consisting of several categories. These include capital expenditures on land and buildings, instruments and equipment and computer software, various labour expenses, and "non-capital purchases of materials, supplies and equipment to support R&D performed" (OECD, 2002, p. 109). Most of these factors are rival and excludable. This is particularly the case of research-related real estate, such as land and buildings that host labouratories, but also of research assets linked to labour. A scientist is a rival asset who cannot be put to work in multiple locations at the same time. Computer software is an exception on this list, being nonrival but excludable.

While a measure of knowledge output based on R&D inputs is convenient and can be appropriate for some purpuses, clearly, ideas and R&D are different. There are two key differences between knowledge and R&D: one has to do with the extent of embodiment and the second, with the emplacement within the innovation process.

Knowledge is disembodied. It resides in replicable patterns—arrangement of human brain neurons, books, media, data and patents. Research and development expenditure, on the other hand, purchases scientific instruments, raises labouratories and pays the salaries of scientists and engineers. Knowledge is the sum total of useful ideas. R&D is the expenditure made with the aim of discovering new useful ideas, and the assets and activities associated with this expenditure.

Knowledge's situation in the innovation process is also distinct from R&D. R&D is an input into knowledge creation, while knowledge is an output from R&D effort. The relationship between knowledge and R&D can be conceptualised by locating the place of each in the production process. In any given period of time, society has a fixed amount of aggregate output which it can spend on different activities. A fraction of society's output is allocated to R&D. R&D, or "R&D expenditure" to be more precise, are resources devoted to the discovery of new knowledge. Additions to the stock of knowledge are the propitious results of the search process.

Finally, the amount of accumulated knowledge is one of the inputs defining the productive capacity of society. Consequently, knowledge produced as a result of research and development contributes to economic productivity and increases total output, some of which can be allocated to R&D. The 3-step process from research, to knowledge, to output is visualised as a cycle in Figure 1.

Observation 2: Knowledge is Distinct from Human Capital

Our next observation pertains to the relationship between knowledge and the concept of "human capital" as introduced into economic discourse by Mincer (1958), developed by Schultz (1961, 1964), formalised by Becker (1962) and incorporated into EGT by Ziesemer (1991). The primary significance of this investigation, for our purposes, is to determine whether knowledge is distinct from human capital, or is subsumed by it.

There is no agreement in the literature on where, exactly, the boundary between

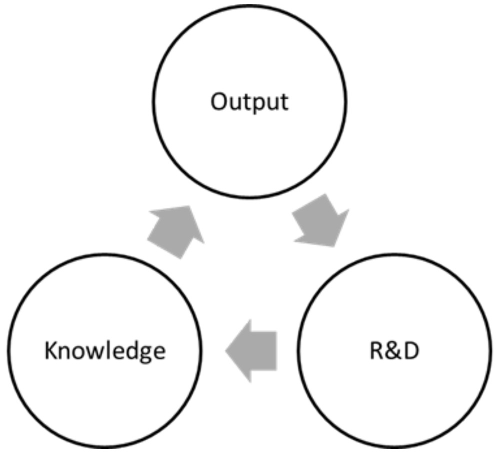

FIGURE 1. INNOVATION CYCLE

knowledge and human capital should lie. Some confusion is to be expected because in many instances the distinction is not immediately obvious. For example, is the ability to read "human capital" or "knowledge"? If the distinction between knowledge and human capital is blurry, it is also in part because neither term has a consistently-adhered-to definition in the economics literature.

For Mincer, "human capital" is a factor influencing earning capacity that is a result of in-school and on the job training. If part of training involves transmission of knowledge, then knowledge is a sub-category of human capital. Becker views human capital as an "intangible resource" that includes schooling, information and knowledge. In Foray (2004), it is knowledge that is the broader concept subsuming aspects of human capital. Foray (2004) defines knowledge as "expertise," a definition that captures a swath of the territory belonging to "human capital," as proposed by Mincer.

In Romer (1990), Ziesemer (1991, 1995), and growth theory generally, human capital and knowledge are formalised as distinct quantities. Romer (1990) argues for a sharp distinction between knowledge and human capital based on the property of embodiment. He argues that the former is disembodied and nonrivalrous, while the notion of human capital lacks either attribute. Human capital is not disembodied since it is linked to human beings. It is also rivalrous, because a human possessing a certain skill cannot exercise that skill in multiple places concurrently. Additionally, unlike replication of knowledge, duplication of human skills is not relatively costless: xeroxing a document comes at negligible cost, but "[t]raining the second person to add is as costly as training the first" (Romer, 1990, p. S75).

Ziesemer (1995) introduces a model with three kinds of knowledge: H, A, and B. H is human capital; A is private, firm-specific knowledge; and B is public knowledge resulting from basic research. The distinction between knowledge types is made based on the source of provision, as well as their innate characteristics. He defines human capital as "knowledge people have personally as a result of schooling" (Ziesemer, 1995, p. 4). Public knowledge is provided by the government, firm-specific knowledge by firms, and human capital by households. Firm-specific knowledge is approbriable to the firm, whereas basic research is a non-appropriable public factor. A is an externality bounded by the organisational limits of the firm, whereas B is a social externality. Human capital is distinguished from the other two types of knowledge by perishability, since knowledge embodied in humans is limited by the human life-span, whereas public and firm-specific knowledge can persist.

For reasons of conceptual and formal clarity, we will adopt the following perspective in the present study. For us, pure knowledge possesses the properties of disembodiment and nonrivalry. Economic factors that lack either attribute are viewed as aspects of human capital or as other categories of input. However, it should be noted that our analytical exercise does not depend on one rigid set of definitions of knowledge and human capital. While the boundary between knowledge and human capital may be drawn differently, its exact sinuation is not crucial for conceptual consistency or for our formal model. As long as there is agreement that a line between these two concepts can be drawn *somewhere*, we can proceed with a model in which knowledge and human capital are two distinct variables. The definition of human capital can be left open, based on attributes linked to humans, but exclusive of pure knowledge.

Observation 3: "R&D Capital" Includes R&D labour

Finally, we take care to avoid a misconception that might arise out of the notion of "R&D capital", alternatively referred to in the literature as "knowledge capital", "R&D stock" and "knowledge stock". In the productivity literature it has been widely recognised that R&D expenditures "act as capital", that is, R&D expenditures should be viewed as investments that continue to have an effect post-expenditure and should not be "assumed to be instantaneously depreciated" (Terleckyj, 1980, p. 57). That is why when estimating the elasticity of output with respect to R&D and the rate of return on R&D investment, a measure of "R&D capital" is derived, which consists of cumulated R&D expenditures depreciated at some rate γ.

It is important to observe, however, that "R&D expenditures are composed of labour, capital, and material costs" (Hall, Mairesse and Mohnen, 2010, p. 13). Strangely, while in productivity literature the preferred measure of knowledge discovery effort—termed "R&D capital"—also includes labour, it is precisely the physical capital component of R&D that is typically excluded from the measure of research effort adopted in Endogenous Growth Theory. Therefore, the term "R&D capital" can be misleading. For this reason we prefer the more neutral term "R&D stock", understood to contain a physical capital component and a human component.

IV. An R&D-Based Knowledge Production Function

In rethinking the two common forms of the knowledge production function, we aim to address the shortcomings of existing frameworks for modelling knowledge creation. The Griliches KPF does not view knowledge as distinct from R&D. It is, however, amenable to modelling an accumulation dynamic. The Romer-Jones functional form has two weaknesses. First, it does not include the full spectrum of effort devoted to knowledge discovery. Embedded in Eq. (6) is the assumption that discovery effort comes only from research labour, excluding physical capital used in R&D.[6] The second shortcoming of the Romer-Jones KPF is that it does not consider accumulation in effort applied to idea creation. Although the Romer-Jones model of the knowledge sector incorporates accumulated knowledge stocks as a factor in knowledge production, it includes only current-period effort devoted to the discovery of new knowledge, as measured by research labour (L_A).

From literature on R&D we know that knowledge discovery is subject to lags (Hall, 2007). Consequently, current-period discovery effort is important, but so is effort made in prior time periods. Because the Romer-Jones KPF sets up research effort as a flow, it is unable to take into account its accumulation. In short, Griliches KPF is a step in a model of knowledge accumulation; the Romer-Jones KPF is mostly about production. But in studying knowledge, innovation, and growth, we are interested in both. A more general model should capture the production and accumulation of both knowledge and research effort.

Below we follow up the discussion of KPF functional forms and the relationship between KPF factor inputs with an alternative knowledge production function that addresses the weaknesses of earlier approaches. What are the positive recommendations for the construction of this KPF? Observation 1 militates against the tautology between knowledge and R&D of Eq. (2). From Observation 2 we conclude that an R&D human capital component should be included as a factor in the KPF, distinct from the knowledge stock factor. Observation 3 argues for inclusion of an R&D physical capital variable. Finally, some factor variables should enter the KPF as stocks, in order to account for lagged effects.

In a generic knowledge production function new knowledge is an output related to a list of inputs. We can conceive of a knowledge production process in which new knowledge results from research effort, modulated by the stock of already existing knowledge. Knowledge generation can be modelled with a compact R&D-based knowledge production function:

$$\dot{A} = F(\mathbf{R}, \mathbf{A}), \tag{7}$$

[6] In Romer (1990) human capital is included as a factor representing research effort; in Jones (1995) knowledge discovery effort comes from labour employed in R&D.

where $\mathbf{R}$ represents accumulated R&D effort, or R&D stock and $\mathbf{A}$ is existing knowledge stock. $\mathbf{R}$ is a composite input consisting of human effort and physical capital components:

$$\mathbf{R} = G(\mathbf{E}_A, \mathbf{K}_A), \tag{8}$$

where $\mathbf{E}_A$ denotes the effort of labour employed in the R&D sector and $\mathbf{K}_A$ is the stock of physical capital applied to research. $\mathbf{E}_A$ is also a composite input, combining labour and human capital. Human effort applied to research is measured in efficiency units of labour:

$$\mathbf{E}_A = J(L_A, \mathbf{H}_A), \tag{9}$$

with L_A as the number of workers engaged in the discovery of new knowledge and $\mathbf{H}_A$ representing average units of human capital per researcher. The accumulation of human effort in research ($\mathbf{E}_A$) occurs out of the augmentation of the stock of human capital of researchers ($\mathbf{H}_A$) through their current-period research activities (L_A).[7] In addition, labour contributes to the creation and accumulation of physical capital ($\mathbf{K}_A$).

Collating Eqs. (7), (8) and (9) results in an extended R&D-based knowledge production function:

$$\dot{A} = F(L_A, \mathbf{H}_A, \mathbf{K}_A, \mathbf{A}). \tag{10}$$

In the above model of the idea-generating process, the flow of ideas is a result of the number of researchers employed in research, the human and physical capital employed in R&D, and the stock of previously generated knowledge. Assuming a Cobb-Douglas functional form, we can express the extended R&D-based KPF as:

$$\dot{A} = \delta L_A^{\bar{\lambda}} \mathbf{H}_A^{\bar{\chi}} \mathbf{K}_A^{\bar{\kappa}} \mathbf{A}^{\phi}. \tag{11}$$

In Eq. (11) variables L_A, $\mathbf{H}_A$, $\mathbf{K}_A$, and $\mathbf{A}$ are as defined above, while parameters $\bar{\lambda}$, $\bar{\chi}$, and $\bar{\kappa}$ measure the elasticity of knowledge with respect to labour, human capital and physical capital, respectively. The intertemporal knowledge spillover parameter ϕ measures the contribution of extant knowledge stock to the production of new knowledge. Finally, δ is the R&D productivity parameter.

The Cobb-Douglas analogue of the compact R&D-based knowledge production function from Eq. (7) can be recovered by realising that R&D stock $\mathbf{R}$ is a com-

[7]There exist several alternative models of production of human capital (for examples, see Rebelo (1991), Ziesemer (1991) and Ziesemer (1995)). We avoid committing ourselves to a specific human capital production process since it is not essential for understanding knowledge dynamics that are the focus of this study. One essential feature of our knowledge production function is to provide a way for past activities of researchers to influence current production of ideas. If human capital is excluded from the knowledge production function, accumulation of physical capital applied in research could provide for a similar intertemporal dynamic.

posite input consisting of labour, human capital, and physical capital components:

$$\mathbf{R} = L_A^{\lambda}\mathbf{H}_A^{\chi}\mathbf{K}_A^{\kappa}, \tag{12}$$

where the superscripts λ, χ, and κ represent the share of the corresponding factor in total R&D stock and $\lambda + \chi + \kappa = 1$. Under the assumption of perfect competition, factor shares will equal elasticities: $\lambda = \bar{\lambda}$, $\chi = \bar{\chi}$, and $\kappa = \bar{\kappa}$.[8]

Eq. (11) can then be rewritten, in summary form, as:

$$\dot{A} = \delta\mathbf{R}^{\zeta}\mathbf{A}^{\phi}, \tag{13}$$

without loss of generality for the model of the knowledge creation process. Parameter ζ in Eq. (13) measures the elasticity of knowledge with respect to the composite of research effort. In the following pages we will work with the compact R&D-based KPF of Eq. (13) whenever parsimony is desired, and with the extended version in Eq. (11) when we want greater detail on the role of the several input factors.

It must be noted that although the R&D-based knowledge production function here adopted is not the standard functional form in growth theory, there are precedents in the literature that either provide a KPF that is like it or at the very least come close to suggesting a similar functional form. However, although a knowledge production of the type in Eq. (11) shimmers in casual discussion of earlier theoretic work, it is incomplete, not fully developed, or trimmed back out of technical considerations, theoretical qualifications, or for the sake of exposition or modelling convenience. In Shell (1966) the same inputs used in production of output are also used in the production of technical knowledge, leading to an implicit function: $\dot{A} = F(L, K, A)$. In one variation of the Phelps (1966) model, technological output is set as a function of employment in the research sector, total employment, capital, and lagged existing technology level.[9] In neither model is human capital included as a knowledge-producing factor since the concept of human capital, was not fully developed at the time.

Ziesemer (1991) models the flow of new ideas as a function of existing knowledge stock and the ratio of human capital to labour, excluding physical capital. A KPF formulation similar to Eq. (13) is found in Jones and Williams (1998) with the difference that in the above study the authors measure research effort with the flow of R&D.[10]

The functional form in Eq. (11) is mentioned as a possibility in Rivera-Batiz and Romer (1991*b*, p. 975, Eq. (2)) but is set aside in favour of the standard KPF where "neither labour nor durables are used in research" (Rivera-Batiz and

[8] The assumptions of Cobb-Douglas knowledge production technology under perfect competition implies constant returns to scale. However, a parametrized version of Eq. (13) similar to the parametrized KPF in Jones (1995) can be imagined, which would relax this restriction.

[9] See Phelps (1966, p. 143, Eq. 35).

[10] The Jones and Williams (1998) KPF can be expressed as $\dot{A} = \delta R^{\zeta}\mathbf{A}^{\phi}$ using our boldface notation for differentiating between stock and flow variables.

Romer, 1991*b*, p. 979). Rivera-Batiz and Romer (1991*a*) work with two alternative models of the knowledge sector. The "knowledge-driven" specification—what we have termed the Romer-Jones knowledge production function in the present study—is a model of the knowledge sector in which new ideas are a function of human capital and already existing knowledge stock. In the alternative "lab-equipment" specification: "human capital, unskilled labour, and capital goods (such as personal computers or oscilloscopes) are productive in research. But in contrast to the previous specification, knowledge per se has no productive value" (Rivera-Batiz and Romer, 1991*a*, p. 536). From the standpoint of our model, both specifications are incomplete. The knowledge-driven specification excludes labour and physical capital, while the lab-equipment specification does not include knowledge stock.

Eq. (11) can be viewed as a synthesis of the two Rivera-Batiz and Romer (1991*a*) specifications. By bringing knowledge stock into the lab-equipment model, it opens us to the possibility that the knowledge-creation process, while relying on the full range of embodied inputs, is also conditioned by the stock of knowledge created by prior generations.

The R&D-based KPF makes three contributions to the analysis of knowledge dynamics. The first advantage of the R&D-based KPF is completeness. This formulation diverges from the standard EGT approach that treates the number of scientists and engineers is a proxy for research effort, where it has been argued that in the R&D sector labour inputs are most of what matters for knowledge creation because of the assumed dominant role of labour in R&D. Yet, creation of new knowledge requires research labs as well as research lads. In fact, R&D is more capital-intensive than the productive sector (Porter and Stern, 2000). Cross-country data on R&D presented in Table A1 show that the non-labour share in R&D is not negligible and therefore should not be discarded uncritically. The advantage of Eq. (11) from the point of view of theory is that it brings into view a wider spectrum of inputs involved in knowledge production.

Explicit recognition that research effort is a composite input brings advantages from the perspective of empirical research. In empirical estimation, the recognition that knowledge output depends on an input other than research labour can further more complete empirical models of knowledge and R&D dynamics and provide more accurate estimates of the relationship between knowledge inputs and outputs. Recognition that research effort consists of labour, human capital and physical capital components can be expected to have the practical effect of correcting for an omitted variable bias in empirical estimates.

The second contribution is synthesis. The theoretical usefulness of our R&D-based knowledge production function is that it lets us pinpoint the differences between the models of knowledge dynamics originating in Endogenous Growth Theory and research on R&D and productivity. We note that the Romer-Jones KPF is a special case of Eq. (11), under restrictions $\bar{\chi} = 0$ and $\bar{\kappa} = 0$. Likewise, the Griliches equation is a different special case of Eq. (11) with the restrictions

$\delta = 1$, $\phi = 0$, and 100% depreciation for lags of accumulable factors $\mathbf{H}_A$ and $\mathbf{K}_A$. Alternatively, the Griliches KPF can easily be recognised as a special case of Eq. (13) under the restrictions $\delta = 1$, $\phi = 0$, $\zeta = 1$ and the composite R&D input factor treated as a flow variable. Seeing Romer-Jones and the Griliches knowledge production functions as part of a broader framework enables us to lay a bridge between disparate literatures. This, in turn, might help separated literature streams interact.

The third benefit of the R&D-based KPF is that it takes us a step closer to our ultimate goal of modelling the accumulation dynamics of knowledge and R&D by serving as a key building block in the full model of knowledge dynamics. The R&D-based knowledge production function forms the core of our model of knowledge dynamics. But, by itself, the knowledge generation process is insufficient to explain the full range of knowledge dynamics since it neither takes into account the accumulation of the stocks that serve as factors of knowledge production nor accumulation of knowledge itself. What remains to be done is to embed the knowledge production equation within a framework of accumulation. That is the task of the next two sections.

V. Building Blocks of a Knowledge Dynamics Model

In developing a model of the knowledge sector we embed the knowledge generation function in a broader framework that includes accumulation processes. This section presents, in general terms, the elements of this framework. Our knowledge dynamics model consists of four components. The first, is a rule by which investments are allocated to the R&D sector. By this rule a stream of flows into the R&D sector is generated. The second module is a process of R&D accumulation, that takes into account depreciation, or obsolescence, of aged R&D stocks. A knowledge production process represented by a KPF is the third component. The fourth module is a model of knowledge accumulation, which works similarly to the R&D accumulation process. In this and following sections, wherever expositional simplicity is desired, we work with the more compact form of the R&D-based KPF (Eq. (7) or (13)), featuring composite factor $\mathbf{R}$ that represents the stock of accumulated R&D effort.

R&D investment

Knowledge creation begins with provision of resources for research. In each time period some economic resources are allocated towards research and development. This incremental addition to R&D is described by an ***R&D investment equation*** the general form of which is:

$$R_I = G(V(...)), \tag{14}$$

where R_I represents the economic resources devoted to R&D, and G is a function of the vector of variables V that determine R_I. The allocation of resources for

R&D can be made on the basis of a fixed proportion of total resources (i.e. a set percentage of GDP) or follow from some other allocation rule. One can imagine a number of societal R&D investment rules. In principle, we can treat the R&D increment as constant, as a variable growing at a constant rate, or as a variable governed by a more complex functional form. In an another alternative, R_I can be derived from a profit-maximisation rule. Current-period R&D investment could also be formulated to depend on prior-period R&D, or on past or current macroeconomic conditions, or simply grow at a constant rate, starting from a base value R_{I0}.

R&D stock accumulation

Next, we turn to consider the accumulation dynamics of R&D stock. R&D accumulation consists of two processes: investment and depreciation. The stock of R&D increases as a result of R&D investment. At the same time, the R&D stock is subject to depreciation. The law of motion for R&D is described by the ***R&D stock accumulation equation***:

$$\dot{R} = R_I - \gamma_R \times \mathbf{R}, \tag{15}$$

where $\dot{R}$ represents the net change in R&D stock, R_I is the incremental addition to R&D described by Eq. (14), $\boldsymbol{R}$ is extant R&D stock, and γ_R is the R&D depreciation rate.

Knowledge production

Current-period incremental increase in knowledge is described by a ***knowledge production function*** F_A, the general form of which is:

$$A_I = F_A(O(...)), \tag{16}$$

where O is a vector of variables representing knowledge production factors. Competing forms of this function were considered in Sections II and IV. If knowledge production is given by the R&D-based KPF, Eq. (16) can be replaced by Eq. (13). The R&D stock variable will then be present in the knowledge production function, as well as in the R&D accumulation equation, a feature that allows closure of the model of the knowledge sector with respect to knowledge and R&D dynamics.

Knowledge stock accumulation

Much like R&D stock and physical capital, knowledge too has been theorised to exhibit accumulation dynamics, that is, being subject to creation and depreciation (Griliches, 1990). The notion of depreciation in the context of physical capital is based on the physical phenomena of wear and breakdown. For R&D stock, the meaning of depreciation is linked to the wear and tear of equipment used in research (depreciation of physical capital employed in research), as well as obsolescence of capabilities embodied in humans working on the creation of new ideas. In the context of knowledge accumulation, the concept of depreciation relates to obsolescence of ideas in their capacity to contribute to the creation of new ideas.[11]

Mathematically, the treatment of accumulation dynamics in knowledge stock is identical to that of R&D stock. Accumulated knowledge stock can be defined as the sum of all additions to knowledge, adjusted for depreciation. In each time period, the change in knowledge stock $\dot{A}$ is determined by the amount of knowledge currently produced (A_I) minus depreciated stock. The evolution of knowledge stock is described by a ***knowledge stock accumulation equation***:

$$\dot{A} = A_I - \gamma_A \times \mathbf{A} \tag{17}$$

that has a precedent in the work by Shell (1966). Combining knowledge production with knowledge accumulation we get the following general form for the law of motion of knowledge stock:

$$\dot{A} = F_A(O(...)) - \gamma_A \times \mathbf{A} \tag{18}$$

The knowledge stock in any period is the result of accretion of the above knowledge flows. Integrating $\int_{-\infty}^{m} \dot{A}(t)\, dt$ will give us knowledge stock at time m.

VI. A Model of Knowledge Dynamics

We build the model by giving concrete functional forms to the four building block equations previously specified in implicit form. The building block equations can then be integrated into a complete model of knowledge and R&D dynamics. A full model will show the state of knowledge and R&D stocks at any given point in time. It will also reveal the short-term and long-term growth rates of the two stocks and their sensitivity to parameters in the production and accumulation equations. Finally, with proportional growth rates in hand it will be possible to study the conditions under which a double-stock model with accumulation of knowledge and R&D would be consistent with balanced growth.

[11] Note that an obsolete idea can continue to be useful in the physical economy while losing its capacity to contribute to creation of new ideas. A bicycle, once invented, can continue to be manufactured and used while yielding its significance to more cutting edge technological innovations in transportation.

R&D STOCK AND GROWTH

Let us assume that in every time period, a certain amount of resources, $R_I(t)$, is allocated towards research and development. Let us further assume that this recurring R&D investment increment starts from a base value of R_{I0} in time period $t = 0$ and grows over time at a constant rate θ_R:

$$R_I(t) = R_{\mathrm{I0}} e^{t\theta_R}. \tag{19}$$

The above R&D investment equation defines the stream of R&D flows. Eq. (19) describes gross allocations to R&D, not adjusted for depreciation.

R&D stock itself evolves according to the previously discussed R&D stock accumulation equation:

$$\dot{R}(R_I(t), \mathbf{R}(t), t) = R_I(t) - \gamma_R \times \mathbf{R}(t), \tag{20}$$

which is net of depreciation.

We can obtain the formula for R&D stock in two steps. The first step is to substitute the R&D investment equation, Eq. (19), into the implicit R&D stock accumulation equation, Eq. (20), to get:

$$\dot{R}(\mathbf{R}(t), t) = R_{\mathrm{I0}} e^{t*\theta_R} - \gamma_R \times \mathbf{R}(t), \tag{21}$$

which yields the law of motion for $\mathbf{R}$. In step two, solving for $\mathbf{R}$ as a function of time and solving for the constant of integration produces an equation for the evolution of R&D stock:

$$\mathbf{R}(t) = \frac{R_{I0} e^{t\theta_R}}{\gamma_R + \theta_R} - \frac{R_{I0} e^{-t\gamma_R}}{\gamma_R + \theta_R} + R_0 e^{-t\gamma_R}, \tag{22}$$

where R_0 is the level of R&D stock at $t = 0$.

As $t \to \infty$, the last two terms of Eq. (22) approach zero so the evolution of R&D stock in the long run will be described by Eq. (23):

$$\mathbf{R}(t) = \frac{R_{I0} e^{t\theta_R}}{\gamma_R + \theta_R}. \tag{23}$$

Several observations follow from the above equation regarding the effect of model parameters on R&D stock. First, the greater the depreciation rate γ_R, the lower the R&D stock. R&D stock, however, is positively dependent on the size of the initial R&D increment (R_{I0}). As for the effect of θ_R on the stock of R&D, the picture is slightly more complicated because this parameter appears twice in the expression, once in the numerator and again in the denominator of the right-hand-side of Eq. (23). But we can say that for large values of t the exponent with the θ_R parameter predominates over the θ_R term in the denominator, which means that the long-run effect of a high growth rate for the R&D investment increment is positive, as we would expect from intuition.[12]

In the special case where $\theta_R = 0$, meaning the R&D effort increment is identical in each period, the equation for the long-run capital stock at the asymptotic limit simplifies further to:[13]

$$\mathbf{R}(t) = \frac{R_{\mathrm{I}0}}{\gamma_R}. \tag{24}$$

Knowledge stock and growth

Armed with essential information on the evolution of the stock of R&D, we now consider the production and accumulation of knowledge stock, in which the former plays a key part. The incremental addition to knowledge stock (A_I) is determined by the R&D-based knowledge production function:

$$A_I(\mathbf{R}(t), \mathbf{A}(t), t) = \delta(\mathbf{R}(t))^{\zeta}(\mathbf{A}(t))^{\phi}. \tag{25}$$

Substituting the KPF given by Eq. (25) into the knowledge stock accumulation equation we arrive at the following law of motion for knowledge stock:

$$\dot{A}(\mathbf{R}(t), \mathbf{A}(t), t) = \delta(\mathbf{R}(t))^{\zeta}(\mathbf{A}(t))^{\phi} - \gamma_A \mathbf{A}(t). \tag{26}$$

Solving the differential equation from Eq. (26) for $\mathbf{A}(t)$ will give us an equation describing the time-path of knowledge stock. To arrive at the equation for long-run knowledge stock, as t becomes arbitrarily large, we proceed in two steps. First, we substitute Eq. (23) giving the long-run research stock, into Eq. (26). The next step is to solve for $\mathbf{A}(t)$, and eliminate the term with the constant of integration, which approaches 0 in the limit. The solution of the differential equation, presented in Eq. (26), is a continuous-time equation describing the evolution of knowledge stock in the long run, as t approaches ∞:

[12] We can see from the derivative of $\mathbf{R}$ with respect to the growth exponent θ_R (Eq. (23)) that the positive term predominates for arbitrarily large values of t:

$$\frac{\partial \mathbf{R}(t)}{\partial \theta_R} = R_{\mathrm{I}0}\left(e^{t\theta_R}\right)\left(\frac{t}{\gamma_R + \theta_R} - \frac{1}{(\gamma_R + \theta_R)^2}\right).$$

[13] Note that $\theta_R = 0$ does not imply that the R&D increment is zero. Under assumption $\theta_R = 0$ the R&D investments are a constant stream equaling $R_{\mathrm{I}0}$ in each period.

$$\mathbf{A}(t) = \left(\frac{\delta \left(\frac{R_{I0} e^{t\theta_R}}{\gamma_R + \theta_R} \right)^{\zeta} (1-\phi)}{\zeta\theta_R + \gamma_A(1-\phi)} \right)^{\frac{1}{1-\phi}}. \tag{27}$$

Together with Eq. (23) describing long-run evolution of R&D stock, Eq. (27) is one of two key derivations of our model. If productive capacity of economies depends on their level of accumulated knowledge, a considerable weight rests on this one variable, $\mathbf{A}(t)$. We now know how $\mathbf{A}(t)$ evolves over time, the parameters determining its level, and their composition into the output variable. And if knowledge stock, in turn, is determined jointly by a small number of parameters, these parameters also acquire importance through their effect on the stock of knowledge. Section VII considers the relationship between knowledge stock and the parameters of the model.

VII. Knowledge Stock Comparative Statics

In our model, knowledge stock is determined jointly by time, initial R&D outlays R_{I0}, and the knowledge and R&D production and accumulation parameters: δ, ζ, ϕ, θ_R, γ_A, γ_R. Relationships between parameters of the model and knowledge stock can be measured by the elasticity of knowledge stock ($\mathbf{A}$) with respect to parameters in the model. The elasticity of Y with respect to X represents the percentage change in variable Y as a result of a percentage change in variable X. For measures of elasticity of Y with respect to X we adopt the notation σ_{YX}.

How does knowledge stock respond to changes in parameters and other determinants of knowledge accumulation? In the model presented here, R&D stock is the ultimate tangible factor involved in the creation of knowledge stock. The elasticity of $\mathbf{A}$ with respect to $\mathbf{R}$ represents the percentage increase in the technological sophistication of the economy in response to a percentage increase in R&D stock, and can be shown to be:

$$\sigma_{AR} = \frac{\zeta}{1-\phi}. \tag{28}$$

The interesting aspect of this result is that the sensitivity of knowledge stock to R&D stock is independent of the parameters of the model pertaining to *accumulation*: the rates of depreciation γ_R and γ_A, and the R&D increment growth rate, θ_R. The elasticity σ_{AR} depends solely on the two parameters of knowledge *production*: ζ and ϕ. The elasticity σ_{AR} is positively related to ζ and ϕ, provided that $\phi < 1$.

The relationship between knowledge stock and the rate of investment in R&D is captured by the following approximation to the elasticity of knowledge with respect to θ_R:

$$\sigma_{A\theta_R} \approx \frac{\zeta\theta_R}{1-\phi}t; \text{for large values of } t \tag{29}$$

The elasticity parameter $\sigma_{A\theta_R}$ depends positively on ζ, $\phi < 1$, and the R&D depreciation rate γ_R. The sensitivity of $\mathbf{A}$ to changes in θ_R varies with θ_R itself; the greater θ_R, the higher is the elasticity. Furthermore, the elasticity is time-dependent, increasing with the progression of time.

As can be expected, knowledge stock is negatively affected by R&D depreciation. The change in $\mathbf{A}$ in response to change in γ_R is given by the equation for $\sigma_{A\gamma_R}$:

$$\sigma_{A\gamma_R} = -\frac{\zeta\gamma_R}{(\gamma_R + \theta_R)(1-\phi)}. \tag{30}$$

Higher values for knowledge production parameters ζ and $\phi < 1$ increase the absolute value of the elasticity of knowledge with respect to the R&D depreciation rate. An increase in the R&D growth rate θ_R, on the other hand, reduces the absolute value of $\sigma_{A\gamma_R}$. Elasticity $\sigma_{A\gamma_R}$ tends to be negative under realistic assumptions for values of the other parameters in Eq. (30). For example, if γ_R, θ_R, and ζ are greater than zero and $0 \leq \phi < 1$, $\sigma_{A\gamma_R}$ is negative—meaning that an increase in the depreciation rate of R&D leads to a lower knowledge stock.

A similarly negative relationship holds between knowledge stock and the knowledge depreciation rate γ_A:

$$\sigma_{A\gamma_A} = -\frac{\gamma_A}{\zeta\theta_R + \gamma_A(1-\phi)}. \tag{31}$$

Under the assumptions regarding the values of θ_R, ζ, ϕ, discussed in the preceding paragraph, and assuming, furthermore, that γ_A is positive, $\sigma_{A\gamma_A}$ will be less than zero. The elasticity of knowledge stock with respect to the knowledge stock depreciation rate abates, in absolute value terms, at higher values of ζ and θ_R. More robust pace of allocation of new resources for research (reflected in higher θ_R) and greater productivity of R&D resources in the generation of new knowledge (observed as higher ζ) ameliorate the negative effects of knowledge depreciation on the technology level. Higher values of ϕ have an opposite effect, leading to greater elasticity of knowledge stock to knowledge depreciation.

In summary, we turn our attention to the following observations. First, there exists a tractable relationship between each parameter of the model of knowledge dynamics and the knowledge stock level $\mathbf{A}(t)$, as measured by the elasticity of knowledge stock with respect to the parameters. Second, the relationship between long-term knowledge stock and long-run R&D stock is exclusively a function of the parameters of knowledge production (ζ and ϕ), from which the asymptotic percentage change in knowledge level resulting from a percentage change in R&D stock can be easily calculated. Third, in most cases, the elasticity of knowledge with respect to a parameter is static, that is, time-independent. This means that a change in the parameter will result in the corresponding effect on knowledge level if given sufficient time to persist, irrespective of the specific point in time when the change occurred. Furthermore, the elasticity of long-run knowledge stock with respect to the parameter is constant in time. However, the elasticity of knowledge level with respect to the growth rate of the R&D investment increment is not static. The elasticity $\sigma_{A\theta_R}$ varies with time. The impact of a change in resources devoted to knowledge discovery increases over time, and is infinite as $t \to \infty$. Finally, Eq. (28) and (29) are interesting from a policy-planning perspective as they allow calculation of the impact of resources devoted to knowledge discovery on long-term technology level of society.

VIII. Growth in R&D and Knowledge

Having analyzed R&D and knowledge stock levels, we consider their growth in time. Taking the derivative of $\mathbf{R}$ in Eq. (22) with respect to time, gives us the equation for growth of R&D stocks:

$$\dot{R}(t) = \frac{\theta_R R_{I0} e^{t\theta_R}}{\gamma_R + \theta_R} + \frac{\gamma_R R_{I0} e^{-t\gamma_R}}{\gamma_R + \theta_R} - \gamma_R R_0 e^{-t\gamma_R}. \tag{32}$$

Asymptotically, as $t \to \infty$, the last two terms approach zero and the equation giving growth in $\mathbf{R}$ in the long run simplifies to the form in Eq. (33):

$$\dot{R}(t) = \frac{\theta_R R_{I0} e^{t\theta_R}}{\gamma_R + \theta_R}. \tag{33}$$

The long-run proportional growth rate for R&D stock $\mathbf{R}(t)$, is given by the ratio of the right-hand sides of Eq. (33) and (23), which reduces to the growth rate of the R&D increment θ_R:

$$\frac{\dot{R}(t)}{\mathbf{R}(t)} = \theta_R. \tag{34}$$

The result is surprising in its simplicity. The R&D stock ($\mathbf{R}$) and its growth rate ($\dot{R}(t)$) depend on a number of parameters, and the short-term dynamics of the proportional growth rate are specified by the ratio of Eqs. (32) and (22)—a considerably complicated expression. Despite this, the asymptotic proportional growth rate is a function only of one parameter—the growth rate of the string of

R&D investments. Even the R&D depreciation parameter γ_R does not appear in the expression for the proportional growth rate.[14]

We obtain the rate of growth of knowledge stock by taking a derivative of Eq. (27). In its simplest form, the rate of change in the stock of knowledge can be expressed as a function of current knowledge stock and time:

$$\dot{A}(\mathbf{A}(t), t) = \frac{\zeta\theta_R}{1-\phi}\mathbf{A}(t) \tag{35}$$

Dividing Eq. (35) by $\mathbf{A}(t)$ provides the asymptotic proportional growth rate:

$$\frac{\dot{A}(t)}{\mathbf{A}(t)} = \frac{\zeta\theta_R}{1-\phi}. \tag{36}$$

Knowledge stock grows at a rate proportional to the growth rate of the R&D investment increment θ_R. There is a positive relationship between the proportional growth rate of knowledge, the R&D growth parameter θ_R, the R&D stock elasticity of knowledge parameter ζ, as well as the intertemporal elasticity of knowledge ϕ. Because Eq. (36) is undefined at the point $\phi = 1$, we need to impose a technical restriction $\phi \neq 1$.

IX. Along the Balanced Growth Path

Now that we are in the possession of proportional growth equations, let us stop to consider growth along a balanced growth path. A balanced growth path is an idealised scenario when key macroeconomic variables grow at a constant rate. Let us assume that the economy is growing at such a constant rate θ_Y^*. If the economy allocates a fixed percentage of its total output to R&D, the growth rate of the R&D increment will equal the growth rate of output:

$$\theta_R^* = \theta_Y^*. \tag{37}$$

The fixed-proportion assumption can be justified on theoretical grounds as arising from the logic of balanced growth.[15]

In Eq. (34), above, we have shown that the rates of growth of R&D increment equals growth of the overall R&D stock. It follows that along the balanced growth path R&D stock will increase at the same rate as aggregate output:

[14] The net growth rate $\gamma_R + \theta_R$ is present in both the numerator $\dot{R}(t)$ and the denominator $\mathbf{R}(t)$, leading to its cancellation.

[15] It is also in line with business and policy practice, supported by the observation that firms gravitate towards a fixed-proportion heuristic in budgeting for R&D, which is adjusted infrequently (Nelson and Winter, 1982). Furthermore, at the level of national policy, governments often commit to spend a target proportion of GDP on research (European Commission, 2003).

$$\frac{\dot{R}(t)}{R(t)} = \theta_R^* = \theta_Y^*. \tag{38}$$

Previously, in Eq. (36), we found the proportional growth rate of knowledge. Along a balanced growth path, as output, R&D increment and R&D stock grow at the same rate θ_R^*, the proportional growth of the knowledge stock will equal:

$$\frac{\dot{A}(t)}{\mathbf{A}(t)} = \frac{\zeta\theta_R^*}{1-\phi}; \phi \neq 1. \tag{39}$$

If the model of the real economy is defined as in Romer (1990), along a balanced growth path the growth rate of knowledge will determine the growth rate in other variables, including output, so that:

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{R}(t)}{\mathbf{R}(t)} = \frac{\dot{A}(t)}{\mathbf{A}(t)}. \tag{40}$$

Eqs. (37) through (40) are reconciled under the following additional restriction:

$$\zeta + \phi = 1 \tag{41}$$

which imposes constant returns to scale for the two inputs of knowledge production.

In summary, our double-stock model of knowledge dynamics is consistent with a balanced growth path. Along a balanced growth path, the growth rate of the key variables of the model is given by the proportional growth equations for knowledge and R&D stocks derived in Section VIII. Balanced path growth also requires the assumption that the proportion of output allocated to R&D remains fixed. The proportional growth equations, together with the fixed-proportion assumption, constant returns to scale, and the relationship between the knowledge and the real sector as in Romer (1990) provide for a balanced growth path.

X. Cumulable Factors and Their Omission—Implications for Growth

What are the implications of our model of knowledge dynamics for growth theory? The question can be decomposed into two parts. First, are there implications of using as measures of research effort factors that are cumulable? Second, are there implications of incorporating additional factors in the knowledge production function? We consider these question by comparing the results from our model to the key predictions from Romer (1990) and Jones (1995).

The study by Jones (1995) works with a parametrized knowledge production function:

$$\dot{A} = \delta L_A^{\lambda} \mathbf{A}^{\phi}, \tag{42}$$

where L_A is labour employed in the research sector, $\mathbf{A}$ is knowledge stock, λ is a parameter measuring the return of knowledge from R&D labour and ϕ is the intertemporal spillover parameter. The functional form adopted by Romer is similar, but with parameters λ and ϕ set to 1. Variations on the knowledge production functions by Romer and Jones have been widely used in endogenous growth literature (Freire-Seren, 2001; Sequeira, 2012) and in empirical research on innovation (Porter and Stern, 2000).

Our model is based on a knowledge production function introduced in Eq. (13) and reproduced below for easy reference:

$$\dot{A} = \delta \mathbf{R}^{\zeta} \mathbf{A}^{\phi},$$

where $\mathbf{R}$ is R&D stock and ζ measures the elasticity of knowledge with respect to R&D. R&D stock represents the sum of lagged and appropriately depreciated research effort. It is a cumulable composite factor consisting of labour, human capital and physical capital components.

What, if any, are the implications for models of economic growth of using a knowledge production function where the factor representing research effort accumulates? Jones' knowledge production function implies a proportional growth rate of knowledge given by:

$$\frac{\dot{A}}{\mathbf{A}} = \frac{\lambda\left(\frac{\dot{L_A}}{L_A}\right)}{1-\phi}. \tag{43}$$

According to Eq. (43) the growth rate of the knowledge stock is driven by the growth rate of the R&D sector labour force $(\frac{\dot{L_A}}{L_A})$.

Our model of knowledge dynamics produces a proportional growth rate for knowledge stock that is similar in many respects. In Section VIII we showed that according to our model long-term growth in knowledge will be given by:

$$\frac{\dot{A}}{\mathbf{A}} = \frac{\lambda\left(\frac{\dot{R}}{\mathbf{R}}\right)}{1-\phi} \tag{44}$$

and that, in fact, because in the long run the rate of growth of R&D stock will approach the growth rate of the R&D increment θ_R, determining the flow of R&D investments ($\frac{\dot{R}}{\mathbf{R}} = \theta_R$, as per Eq. (34)), growth in knowledge stock will be driven by θ_R, as well. In other words, a knowledge production function with an input factor that accumulates leads to the same growth rate—in the long run—as a KPF where the input factor is treated as a flow. We find that the Romer-Jones and our R&D-based knowledge production function make similar predictions about the long-term rate of growth.

Yet, although accumulation makes no difference in the very long run, there is still a difference in our framework compared to Romer-Jones, the source of which is a difference in the constitution of the knowledge discovery input factor. R&D stock in our model is a composite factor consisting of labour, human capital, and physical capital components. Therefore, the growth rate $\frac{\dot{R}}{\mathbf{R}}$ will be a weighted average of the growth rates of its constituent factors. To see how this fact will affect the long-run growth rate of knowledge we derive from the extended R&D-based KPF in Eq. (11) the proportional growth rate of the knowledge stock:

$$\frac{\dot{A}}{\mathbf{A}} = \frac{\zeta\left(\lambda\frac{\dot{L}_A}{L_A} + \chi\frac{\dot{H}_A}{\mathbf{H}_A} + \kappa\frac{\dot{K}_A}{\mathbf{K}_A}\right)}{1-\phi}. \tag{45}$$

In our model, the engine of idea-based growth can be any singe one or a combination of the constituent factors of R&D: the size of the labour force devoted to research, human capital, or physical capital in R&D. Unlike the model in Jones (1995), in our model, zero growth in L_A does not automatically imply dissipation in the growth rate of ideas. In the case when $\frac{\dot{L}_A}{L_A} = 0$ idea-based growth can be fuelled by accumulation in human or physical capital. Since the R&D sector requires an educated labour force, and is fairly capital-intensive (Porter and Stern, 2000), an idea-driven growth model is in some sense incomplete without taking these factors into account. This point, however, is most relevant for empirically-focused studies, where reliance on current-period employment in R&D to measure overall research effort might lead to inaccurate estimates of the contribution of research effort to the rate of creation of new ideas.

Our framework, incorporating cumulable inputs in knowledge production, is different in another aspect. If we move away from analysis of long-run knowledge growth, we find that accumulation dynamics are important and can lead to substantial differences in estimates vis-à-vis a flow-based model. In our model, in the short run, defined by the period $t = 0$, the proportional growth rate of a stock-based input factor will be given by:

$$\frac{\dot{R}}{\mathbf{R}} = \frac{R_I}{\mathbf{R}} - \gamma_R. \tag{46}$$

Restating this expression in plain terms, we can say that the growth rate in R&D stock is the ratio of the R&D increment to pre-existing R&D stock, minus the depreciation rate of R&D.

While in the long run depreciation is unimportant, in the short run it surely is. The practical consequence of this finding is obvious, since empirical work on innovation is typically based on data series containing information on a relatively small number of time periods. A measure of growth of the factor used in knowledge production that does not treat that factor as a cumulable stock will be inflated upward. This, in turn, will impact the estimates of the relationship between inputs and outputs to knowledge production. If we had accurate estimates of the knowledge production technology parameters, ζ and ϕ, the upward bias in the measure of effective research effort would lead us to an inflated expectation for the proportional growth rate of knowledge.

Another observation our model makes evident is that in the short run, the effect of knowledge depreciation can also be important. Dividing Eq. (17)—the knowledge stock accumulation equation—by $\mathbf{A}$, gives us the proportional growth rate for knowledge:

$$\frac{\dot{A}}{\mathbf{A}} = \frac{A_I}{\mathbf{A}} - \gamma_A \tag{47}$$

The growth rate of the knowledge stock is the difference in the rate of creation of new knowledge ($\frac{A_I}{\mathbf{A}}$) and the rate of obsolescence of the extant knowledge stock (γ_A).

In constructing an index of innovation, the literature typically works with measures of gross knowledge stock, not adjusting for depreciation in the value of ideas. Together, exclusion of depreciation in accumulated research effort and depreciation of the stock of knowledge might contribute to explaining the paradox observed in Jones (1995): an economy characterised by high growth rates in scientists and engineers, but much lower growth in innovative performance.

XI. R&D Accumulation Dynamics and Econometric Estimation

In this section we look at the R&D-based knowledge production function and its Griliches and Romer-Jones alternatives through the lens of econometric estimation. If our model of knowledge dynamics is more complete, then the omission of knowledge stock from the Griliches KPF can be viewed as a simple omitted variable problem that can lead to biased and inconsistent estimates of the elasticity of innovation with respect to R&D. The use of current-term employment in place of R&D stocks is similarly problematic from the perspective of econometric estimation. This substitution can also be expected to lead to mis-estimation of the ζ parameter.

Suppose we try to estimate the knowledge accumulation equation in Eq. (26), which we reproduce below in a slightly modified form, having incorporated the depreciation term into the dependent variable:

$$\Big(\dot{A}(t) + \gamma_A \mathbf{A}(t)\Big) = \delta(\mathbf{R}(t))^{\zeta} * (\mathbf{A}(t))^{\phi}. \tag{48}$$

On the surface this looks similar to the Romer-Jones KPF in Eq. (6). The difference between the two equations is in the $\mathbf{R}$ variable and its content. In our R&D-based KPF, $\mathbf{R}$ consists of labour, human capital, and physical capital components of R&D, the last two of which are measured as stocks:

$$\begin{aligned}\mathbf{R}(t) &= L_A(t)^{\lambda}\mathbf{H}_A(t)^{\chi}\mathbf{K}_A(t)^{\kappa}\\ &= \left(\dot{L}(t)\right)^{\lambda}\left(\frac{\dot{H}(t)}{\gamma_H+\theta_H}\right)^{\chi}\left(\frac{\dot{K}(t)}{\gamma_K+\theta_K}\right)^{\kappa}.\end{aligned} \tag{49}$$

In the Romer-Jones KPF, besides knowledge stock, only current-period labour flows $\dot{L}$ are included as input in knowledge production. We can separate $\mathbf{R}(t)$ into included and excluded components:

$$\mathbf{R}(t) = \dot{L}(t)^{\lambda}\mathbf{X}(t). \tag{50}$$

The variable $\mathbf{X}$ is a composite of omitted factor components.

Econometric estimation of (48) will typically involve linearisation through logarithmic transformation. The log-transformed equation:

$$\begin{aligned}\ln\left(\dot{A}(t)+\gamma_A * A(t)\right) &= \ln(\delta)+\phi\ln(A(t))+\zeta\ln\left(\dot{L}(t)^{\lambda}\mathbf{X}(t)\right)+\epsilon\\ &= C+\phi\ln(A(t))+\tilde{\zeta}\ln(\dot{L}(t))+\zeta\ln\left(\mathbf{X}(t)\right)+\epsilon,\end{aligned} \tag{51}$$

can then be estimated. Here C is a constant and ϵ is the stochastic error term. The parameter measuring the contribution of labour to innovation $\tilde{\zeta}=\lambda\zeta$ consists of the R&D elasticity parameter ζ multiplied by the labour share λ. Since $\lambda < 1$, if R&D labour is the sole measure of R&D effort, $\tilde{\zeta}$ will underestimate the contribution of R&D to innovation—even if $\tilde{\zeta}$ is measured accurately. However, accurate measurement of $\tilde{\zeta}$ is itself unlikely.

Estimation of the regression in Eq. (51), with $\mathbf{X}(t)$ factors excluded, leads to an omitted variable scenario. With an omitted variable, the expected value of the parameter estimated might be biased. The estimator $\tilde{\zeta}^*$ from the mis-specified model will be given by:

$$\tilde{\zeta}^* = \hat{\tilde{\zeta}}+\hat{\zeta}\frac{\hat{\mathrm{Cov}}(\ln(\dot{L}),\ln(\mathbf{X}))}{\hat{\mathrm{Var}}(\ln(\dot{L}))}, \tag{52}$$

where $\hat{\tilde{\zeta}}$ is the estimator of $\tilde{\zeta}$ from the correctly specified model. The expectation of $\tilde{\zeta}^*$ is:

$$E[\tilde{\zeta}^*] = \tilde{\zeta} + \zeta \frac{\text{Cov}(\ln(\dot{L}), \ln(\mathbf{X}))}{\text{Var}(\ln(\dot{L}))}, \tag{53}$$

which can in turn be expressed as:

$$E[\tilde{\zeta}^*] = \zeta(\lambda + \rho), \tag{54}$$

where ρ is the ratio of the covariance between the log of labour and the log of the excluded component $\mathbf{X}$ over the variance of the log of labour.

The sign and magnitude of ρ will determine the direction and degree of distortion of the expected value of the estimator of $\tilde{\zeta}$. Of prime interest is whether exclusion of $\mathbf{X}$ is likely to result in an over or-underestimate of $\tilde{\zeta}$ vis-á-vis the true elasticity of innovation with respect to total R&D, given by ζ.

Recall that the parameters λ, χ, κ represent the share of each factor in the R&D composite and must therefore sum to 1. If $\rho = \chi + \kappa$, the estimate will equal to true ζ; with $\rho > \chi + \kappa$, $\tilde{\zeta}$ will overestimate ζ; and if $\rho < \chi + \kappa$ then $E[\zeta^*]$ will be an underestimate.

Exploiting the well-known properties of omitted variable bias, it can be shown that ρ is determined by the relationship between labour flows and the omitted variables, so that when the growth and depreciation parameters $\theta_H, \theta_K, \gamma_H, \gamma_K$ are constant:

$$\rho = \chi \frac{\text{Cov}(\ln \dot{L}, \ln \dot{H})}{\text{Var}(\ln \dot{L})} + \kappa \frac{\text{Cov}(\ln \dot{L}, \ln \dot{K})}{\text{Var}(\ln \dot{L})}. \tag{55}$$

Because variance is always positive, the sign of the covariance between labour flows and the two omitted factors will determine the direction of estimation bias.

Here we show that the condition $\rho > \chi + \kappa$, leading to an overestimate—although it cannot be excluded—is not very likely. The inequality $\rho > \chi + \kappa$ obtains only under the following condition:

$$\chi \frac{\text{Cov}(\ln \dot{L}, \ln \dot{H})}{\text{Var}(\ln \dot{L})} + \kappa \frac{\text{Cov}(\ln \dot{L}, \ln \dot{K})}{\text{Var}(\ln \dot{L})} > \chi + \kappa. \tag{56}$$

Under what scenarios will the condition in Eq. (56) hold? Consider the structure of estimation bias that emerges when labour is included but capital is excluded from the knowledge production function. If the covariance between labour flows and capital flows is negative, as would be the case if one input is used to substitute for the other, the condition in Eq. (56) is less likely to hold, meaning that $E[\zeta^*]$ is more likely to be an underestimate. If covariance has a positive sign, the magnitude of bias will depend on the ratio of the covariance term to $\text{Var}\left(\ln\left(\dot{L}\right)\right)$.

If $\mathrm{Cov}\left(\ln\left(\dot{L}\right), \ln\left(\dot{K}\right)\right) > \mathrm{Var}\left(\ln\left(\dot{L}\right)\right)$, as might be expected if R&D labour and physical capital are strongly complementary, then $E[\tilde{\zeta^*}]$ could be greater than true ζ. But this is not likely. Even if labour and capital are complements and move in the same direction on average, changes in physical capital allocations would have to be extremely sensitive to changes in R&D employment in order to meet the conditions for an over-estimate. It can be shown by application of analogous arguments that the consequences for the relationship between $E[\tilde{\zeta^*}]$ and ζ of exclusion of the human capital variable are similar.

Although the true direction and magnitude of bias can only be ascertained from the data, the chips are stacked against a finding of overestimation. Our reasoned expectation is that the ratios of covariance to variance in Eq. (55) will be positive but less than 1, for terms associated with human capital and physical capital, which will produce an underestimate of the true impact of R&D activities on innovation.

When it comes to estimation of the R&D elasticity parameter based on the Griliches specification, the expected value of the estimate will be:

$$E[\zeta^*] = \hat{\zeta} + \hat{\phi}\frac{\mathrm{Cov}(\ln(\mathbf{R}), \ln(\mathbf{A}))}{\mathrm{Var}(\ln(\mathbf{R}))}. \tag{57}$$

Here the direction of estimation bias is less ambiguous. We can conjecture that changes in R&D stock will be positively correlated with knowledge accumulation: the term $\mathrm{Cov}\left(\ln\left(\mathbf{R}\right), \ln\left(\mathbf{A}\right)\right)$ will be positive. We would also expect positive values for the knowledge production parameters $\hat{\zeta}$ and $\hat{\phi}$. The conclusion that follows is that $E[\zeta^*]$ is likely to overestimate the true elasticity of innovation with respect to R&D stock.

In summary, our analysis suggests that the the Griliches KPF will overestimate the elasticity of innovation with respect to R&D, while the Romer-Jones KPF will underestimate the relationship between innovation performance and the measure of research effort it utilises. The prediction of our analysis is that the elasticity of innovation with respect to a measure of research effort, based on the R&D-based KPF, will be bounded by the aforementioned two estimates.

XII. Empirical Estimation

In this section we test key predictions of our theory of knowledge dynamics by comparing how estimates of the R&D-based KPF fare compared to estimates of its two alternatives. We rely on a country panel for 40 countries spanning the years 1985-2010 obtained from the OECD (OECD, 2014). The dataset contains for each country information on the total Triadic patents granted to domestic inventors, annual R&D expenditures,[16] as well as country total number of scientists and

[16] R&D expenditures are in constant-price 2005 U.S. dollars and adjusted for purchasing power parity.

engineers.

Table 1—The International Knowledge Production Function—Pooled OLS

Dep. Var.:	*ln(Patents)*					
Reg.:	(1)	(2)	(3)	(4)	(5)	(6)
Estim.:	Pooled	Pooled	Pooled	Pooled	Pooled	Pooled
Spec.:	R&D	R-J	G	R&D	R-J	G
lnA	1.097***	1.095***				
	(0.04)	(0.05)				
lnR	-0.059		1.029***			
	(0.05)		(0.15)			
lnSE		0.019				
		(0.10)				
L3.lnA				0.800***	0.934***	
				(0.12)	(0.05)	
L.lnR				0.288**		1.018***
				(0.15)		(0.15)
L.lnSE					0.224*	
					(0.12)	
Constant	-2.056***	-3.141***	-7.119***	-3.936***	-4.441***	-6.947***
	(0.45)	(0.98)	(1.64)	(1.00)	(1.20)	(1.62)
R-sqr	0.707	0.578	0.389	0.594	0.504	0.387
N	905	522	905	785	471	865
F	580.3	272.4	48.3	211.3	247.9	48.3

Significance level: * 10 percent; ** 5 percent; *** 1 percent

R&D stocks were calculated on the basis of annual R&D expenditures using the Perpetual Inventory Method (PIM). Initial R&D stocks were estimated by dividing the base year R&D outlays by the sum of estimated R&D growth rate and depreciation. The R&D stock growth rate was calculated for each country separately using the first 10 years for which R&D expenditures were available. We also went with the assumption of a 5% depreciation in R&D stocks, which is standard in the productivity literature. In a similar fashion, using PIM, country knowledge stock was calculated from the annual flow of Triadic patents in conjunction with a 15% depreciation rate assumption for knowledge, and a country-specific estimate for the growth in knowledge stock in the base year, calculated from the first 10 years of the data.

These variables allow us to estimate the three different specifications of the knowledge production function and compare their results. In each empirical model we regress a patent-based measure of innovation on factors featured in the corresponding theoretical model of knowledge production. In Table 1 we report the results of the first 6 regressions. All regressions in this table represent pooled OLS estimates. The empirical models contained in columns (1), (2) and (3) correspond, respectively, to R&D-based, Romer-Jones, and Griliches KPFs.

The first three regressions are econometrically naive OLS estimates that do not control for endogeneity between the independent variable and factor inputs. Besides the log of knowledge stock, none of the other factors in the first three regressions produce statistically significant coefficients. The first three regressions are reported for comparison purposes, but the presence of endogeneity is highly likely, especially between knowledge stock and the dependent knowledge flow variable, so we should not place too much confidence in the estimated coefficients from models (1) through (3).

In regressions (4) through (6) of Table 1 we control for endogeneity by using lagged regressors. R&D stock and count of scientists and engineers are lagged one period; knowledge stock is lagged three years. In the R&D-based specification, the parameters on lagged knowledge stock and R&D stock are 0.8, and 0.288, both statistically significant at least at the 5 percent significance level. The regression indicates that an increase in knowledge stock of 100% would lead to an 80% increase in innovative output. The doubling of R&D stock would increase innovation by about 29%.

In the Romer-Jones specification, the coefficient on the variable measuring research input is lower, at 0.224, and with weaker statistical significance—at the 10% significance level. However, the loss of predictive power of the research variable is compensated by a greater magnitude of the knowledge stock parameter. Albeit the coefficient in the R&D-based KPF is slightly lower, the two models that include knowledge stock as a regressor yield qualitatively similar estimates for the inter-temporal knowledge spillover parameter. The Griliches function produces the highest estimate of the elasticity of innovation with respect to research effort. The value of the parameter is 1.018, highly statistically significant.

The results are generally in line with our predictions. Here we see that exclusion of a variable measuring knowledge stock (in the Griliches specification) leads to inflated measures of the impact of R&D inputs on innovation, as we suspected in Section XI. We also see that basing research effort purely on a measure of labour inputs (in the Romer-Jones model) underestimates the impact of research on innovation. If one measure of elasticity of innovation with respect to research input is an underestimate and the other an overestimate, then "truth is in the middle." The R&D-based estimate of elasticity of innovation with respect to research effort is sandwiched between the estimates from the other two specifications.

In Table 2 we exploit the panel nature of our data by estimating fixed-effects and random-effects regressions for our three KPF specifications. All three knowledge production models produce atypical results when estimated using fixed-effects. In the Romer-Jones model the coefficient on the scientists and engineers variable is negative; in the Griliches model the R&D stock parameter estimate is approximately 75% lower than in Table 1, column (6); and in the R&D-based model there is, again, a negative return to research effort.

The random-effects regressions in columns (4) through (6) provide much more sensible parameter estimates, but the Hausman test recommends fixed-effects for all three specifications. We are set before a dilemma in which the estimation technique recommended by the Hausman test leads to results that are not plausible and in contradiction of prior findings in the literature.

The source of this dilemma is likely in the features of the dataset. The main dif-

TABLE 2—THE INTERNATIONAL KNOWLEDGE PRODUCTION FUNCTION—WITH COUNTRY EFFECTS

Dep. Var.:	*ln(Patents)*					
Reg.:	(1)	(2)	(3)	(4)	(5)	(6)
Estim.:	FE	FE	FE	RE	RE	RE
Spec.:	R&D	R-J	G	R&D	R-J	G
L3.lnA	0.249**	0.877**		0.603***	0.957***	
	(0.10)	(0.35)		(0.07)	(0.08)	
L.lnR	-1.170***		0.155***	0.314***		0.327***
	(0.22)		(0.06)	(0.12)		(0.06)
L.lnSE		-3.358***			0.024	
		(0.55)			(0.16)	
Constant	15.334***	34.388***	2.362***	-3.104***	-2.477*	0.278
	(2.12)	(4.79)	(0.64)	(1.01)	(1.48)	(0.66)
R-sqr	0.709	0.630	0.696	0.593	0.501	0.387
N	785	471	865	785	471	865
F	14.1	21.5	6.9			
χ^2				255.5	221.1	33.5

Significance level: * 10 percent; ** 5 percent; *** 1 percent

ference in the underlying assumptions of fixed-effects and random-effects models is that random effects assumes that unobserved country-level effects are uncorrelated with the other regressors, while fixed effects allows for correlation. Furthermore, fixed-effects estimation relies exclusively on variation within countries over time, to the exclusion of between-country variation. Since variables in our panel tend to be relatively stable across time, and most of the variation is between countries, and not within countries across time, it is not at all surprising to see that estimations exploiting within-country variation are not precisely estimated. Random-effects estimates, on the other hand, provide greater precision in estimates but can show biased coefficient estimates if the country-effects are correlated with other regressors.

Dieleman and Templin (2014) has considered this bias-precision trade-off and provided evidence that in panel datasets with large differences between groups but little variation within groups the random-effects estimator will tend to be more appropriate than fixed-effects, and the Hausman test will often wrongly reject the random-effects model for these data. More specifically, the authors find that in relatively small samples of approximately several hundred observations or less, where within-group variation is 20% or less of total variation, random-effects estimates are to be preferred. These attributes roughly correspond to our country panel.[17] This guidance, and the general harmony between the random-effects models of Table 1 and pooled OLS models of Table 2 makes us place greater faith in the random-effects regressions.

In the R&D-based knowledge production function estimated using random-effects, presented in column (4) of Table 2, the coefficient estimate on knowledge stock is 0.603, slightly lower than in the OLS R&D-based model. This is likely

[17]Approximately 31% of variation in patent flow is explained by the within-country component. Within-country variance constitutes 9%, 28%, and 5% of variance in knowledge stock, R&D stock and number of scientists and engineers, respectively.

because in the OLS model the knowledge stock variable, being relatively constant over time, would capture part of the variation in the data owed to country effects. The elasticity of innovation with respect to R&D is highly significant and slightly greater in the random-effects model, at 0.314, but is qualitatively similar to the OLS estimate.

In Table 2, column (5), presenting the Romer-Jones variation of the KPF, the estimate for knowledge stock does not change appreciably from the analogous model estimated via OLS, in column (5) of Table 1. However, once we capture country-effects, the coefficient on scientists and engineers drops considerably and becomes statistically insignificant. For the Griliches specification, located in column (6) of the same table, the parameter estimate on R&D stock is two-thirds lower than when the model is estimated via OLS. Here we will notice that the Griliches parameter estimate on R&D stock is higher than the estimate from the R&D-based function, but the difference is not very large. The likely explanation for this result is that, in this empirical model, country-effects are capturing a host of country-specific attributes explaining differences in knowledge production and, with knowledge stocks being relatively constant over time, they are effectively controlled for via country random-effects. Thus, the Griliches specification, if it adequately controls for country-effects in a panel data context, can give us a good estimate of the elasticity of knowledge with respect to R&D stock. The set of random-effects regressions reproduces the predicted pattern whereby the ζ estimate from the R&D-based regression lies between the estimates from the other two models.

In reviewing overall model performance, we observe that the R&D-based KPF generally provides a better fit to the data than its Romer-Jones and Griliches counterparts. Whether using OLS, fixed-effects, or random-effects estimation, the R-squared is higher for the R&D-based model than for its alternatives. In the instance of the random-effects estimation approach, the R-squared is 0.593, 0.501 and 0.387, respectively, for the R&D-based, Romer-Jones and Griliches models.

In addition to the conclusions drawn from econometric models, we note that basic indicators of innovative activity support the the claim that R&D stock is a better predictor of innovative performance than measures of scientists and engineers. As an illustration, Figure 2 compares the level of U.S. R&D stock and researchers to the number of domestic patent grants at the USPTO. The indexes show that, short-term volatility in the patent indicator aside, patent grants have tracked the evolution of domestic R&D stock quite closely. On the other hand, the gap between research employment and patenting has widened over time. Between 1985 and 2010 patent grants to domestic applicants at the USPTO increased 173%. During the same period the U.S. R&D stock increased in similar proportion, to 167% of its level in the baseline year. On the other hand, the number of full-time-equivalent researchers in the U.S. economy increased only 98% over the same period. The gap between the research employment and patent trendlines reinforce the conclusion that R&D, measured as stocks, is the more

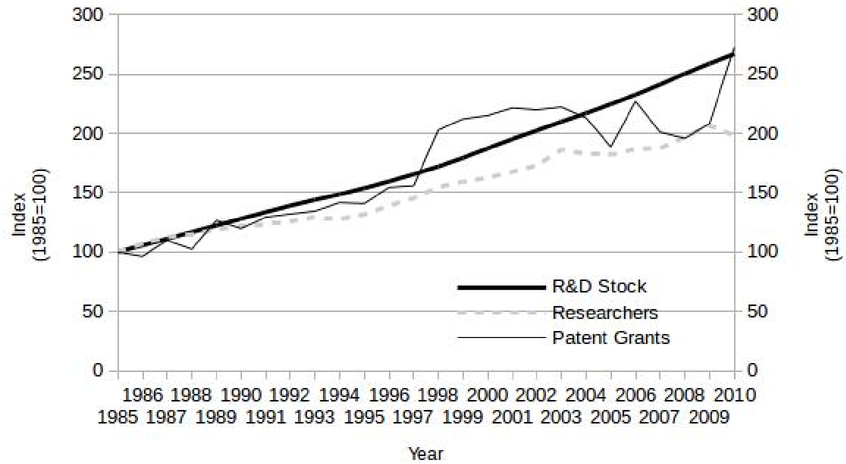

Figure 2. U.S. Knowledge output vs. R&D stock

Note: Figure presents indexes for U.S. R&D stock, total researchers (full-time-equivalent) in the labour force, and U.S.-origin patent grants at the USPTO. Base year 1985=100.
Source: R&D stock from author's calculations based on data from OECD; Total full-time-equivalent researchers from OECD; U.S.-origin patent grants from USPTO.

relevant measure of research effort.

The empirical results presented in this section provide a measure of validation for our knowledge dynamics framework. On the basis of our econometric estimations we conclude that the R&D-based KPF provides a better approximation of true knowledge dynamics than either Romer-Jones or Griliches knowledge production functions. Nevertheless, greater certainty in the results can come only from additional confirmatory research, with more, better and different data, and more sophisticated estimation techniques. These goals are beyond the scope of any single study, but are well within the reach of future research.

XIII. Discussion and Conclusion

This study set out to develop a model of innovation and knowledge dynamics. The path toward this objective led through re-conceptualization and synthesis of existing methods, concepts and frameworks in studies of innovation. The model presented includes two accumulable factors: a knowledge stock consisting of the sum total of technologically relevant ideas, and a separate R&D stock, representing the accumulated effort devoted to the discovery of new knowledge. Prior research on economics of innovation tended to spotlight only one of these two stocks and conflate knowledge and R&D. Our approach, on the other hand, insists on conceptual clarity with respect to R&D effort and knowledge. While R&D is an aggregate of embodied research factors in the production of innovation, knowledge is a disembodied outcome of that process, that in accumulated form also serves as an input to innovation. Maintaining the distinction between knowledge and R&D paved the way first, for a framework, and then for a formal model capturing the essential dynamics of the knowledge generation-accumulation process.

At the heart of our model of knowledge dynamics is an R&D-based knowledge production function that combines accumulated R&D and knowledge stocks in the production of new knowledge. The R&D-based knowledge production function is distinct from functional forms commonly appearing in research on rates of return to R&D—which have considered R&D stocks, but apart from knowledge accumulation. It is also different from the Romer-Jones knowledge production function from Endogenous Growth Theory, which includes only current period discovery effort from labour employed in the R&D sector.

The theoretical analysis of knowledge production draws a relationship between existing models used in the study of innovation, and proposes a synthesis that offers to reconcile alternative knowledge production functions as special cases of our functional form, while promising a better way to model knowledge dynamics. The R&D-based knowledge production function bridges the gap between the two alternative formulations of knowledge dynamics from growth theory and R&D productivity literatures. It is hoped that this feature serves as a land bridge between the two literatures and provides a footing for fruitful interaction between them.

In our model, the R&D-based function itself is embedded within a framework for accumulation of knowledge and R&D effort, completing the general model of knowledge and R&D dynamics and setting the stage for analysis of its implications for innovation and growth. Taking R&D and knowledge stocks into account brings into the field of vision aspects of growth and innovation that theory previously left out of sight. As one example, it brings into view the possibility of ideas-driven growth that relies neither on Romer's razor-edge restriction nor on Jones' requirement of positive growth in R&D employment. Finally, the model paves the way for modelling innovation and growth processes while avoiding unnecessary estimation biases.

The R&D-based knowledge production function offered a set of predictions that could be tested empirically. The empirical exercise produced estimates which were consistent with the original predictions. This result lends support to the R&D-based knowledge dynamics framework. Our estimations offer a number of preliminary observations about past empirical work, and lessons for work to come. First, prior studies adopting a flow-based measure of research effort likely underestimated the contribution of research to innovative performance. In the future, measures of effort incorporating non-labour contribution to R&D, and adjusting for accumulation should be pursued. The second lesson following from this study is that prior research investigating the marginal effect of R&D on measures of innovation may have produced imprecise, and probably inflated, estimates, although studies on panel data with controls for country-effects could have provided sufficiently accurate estimates. The message remains—results of application of research effort depend on context. The current state of knowledge plays a powerful mediating role between research effort and innovative outcomes. Future studies on the relationship between R&D and productivity, even while controlling for country-specific effects, should not ignore accumulated knowledge stock in their estimations. The third lesson from the empirical exercise is that the R&D-based knowledge production provides a closer fit to the data and therefore a better explanation of innovation dynamics than its alternatives. Future studies interested in explaining innovation can start with, and extend, the R&D-based specification.

One way to view this study is as an attempt to synthesise the micro-level "research lab" perspective on innovation, and the aggregate knowledge production framework from country-level growth models with endogenous technical change. Although the empirical exercise and much of the discussion are framed to address macro-level questions, and endogenous growth—one of the base theories considered—was developed to explain patterns in development at the country level, the general model of knowledge dynamics presented is just as applicable to micro contexts. The model of knowledge dynamics achieves a mechanical description of the evolution of the level, change, and growth rates of knowledge and R&D stocks. A model in which innovations produced by a unit of observation other than a country—say, a firm—result from combination of acquired knowledge stocks and accumulated research effort, can also make use of this framework. As long as other necessary assumptions of the model hold, these mechanics will be universal, ready to be applied at any scale.

While the model can claim a number of advantages, it also has certain limitations. In developing the model, we abstracted away from market structure and the decision rule for spending on research. Furthermore, we did not include a separate accumulation process for physical capital and human capital. Extensions for physical capital accumulation, and processes for creation and accumulation of human capital along the lines of Rebelo (1991) or Ziesemer (1995) could very well have been included. But these features were left out in order to keep the model general and focused on the process of knowledge dynamics.

Another potential limitations of the above model of knowledge dynamics, it might be argued, is its focus on R&D. In the model, the ultimate engine of technological progress is resources allocated to research and development. For this reason, the model can be subjected to the criticism of being not just "R&D-based" but "R&D-centric." What about economic resources engaged in activities that are not, in a strict accounting sense, R&D, but that nevertheless have innovation as a propitious by-product? What of innovation resulting from "learning by doing" activities such as investment in capital goods (Arrow, 1962)? And what about determinants of innovation that are not a category of primary inputs? The amount of knowledge created in a society at any given time can be conditioned by factors such as the suitability of the general political and economic environment for innovation, quality of institutions, and the intellectual property regime, among others. How do they enter into the proposed framework?

Firstly, our perspective on knowledge dynamics does share with the view of Karl Shell that R&D activity has a preeminent role in explaining the creation of technologically significant ideas and that "the rate of production of technical knowledge can be increased by increasing the allocation of economic resources explicitly devoted to inventive activity" (Shell, 1966, p. 62). This is particularly true of the most valuable ideas, such as those inscribed in Triadic patent filings. While "learning by doing" has an important role in the innovation system, its role is greater in the diffusion, adoption and adaptation of new ideas, and lesser in their creation.

Secondly, the model is certainly open to the possibility of additional input factors. Currently, the residual term δ of the R&D-based knowledge production function is a catch-all for other determinants of the innovation arrival rate. Additional determinants of innovation can be "extracted" from this catch-all term. Future empirical work can use the R&D-based KPF as a starting point and add additional explanatory variables for a fuller description of the dynamics of knowledge. For example, a decomposition of knowledge stock into its domestic and borrowed components, the latter capturing external knowledge spillovers, should be pursued. We leave this extension for future work. Yet, even as it stands currently, in a rather restricted form, the R&D-based knowledge production function at the heart of the model of knowledge dynamics proposed here, represents an improvement over alternative specifications and explains up to 60% of variation in the innovative performance of nations.

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Appendix

Griliches knowledge production and its relationship to the Perpetual Inventory Method

The stock of technologically relevant knowledge $\mathbf{A}$ is given by the following equation:

$$\mathbf{A} = G(W(B)R). \tag{A1}$$

Here, $W(B)$ represents a lag polynomial, in which B is the backward shift operator, so that:

$$W(B)R = w_0R_t + w_1R_{t-1} + w_2R_{t-2} + ... = \sum_{i=-\infty}^{t} w_{t-i}R_i. \tag{A2}$$

If the constant R&D depreciation rate is γ, the lag polynomial is a geometric series with common ratio $(1-\gamma)$. Knowledge stock at time t can then be expressed as:

$$\mathbf{A}_t = \sum_{i=-\infty}^{t} (1-\gamma)^{(t-i)}R_i. \tag{A3}$$

Note that Eq. (A3) is a modification of Eq. (3) that takes into account the depreciation of stocks over time.

The current-period investment increment is not adjusted for depreciation—the addition to the stock in period t equals R_t. Previous investments, however, are adjusted for depreciation. Extracting R_t from the right-hand side of Eq. (A3), the total stock at time t can be decomposed into the sum of R_t and the depreciated stock from the previous period $t-1$, leading to the perpetual inventory method (PIM) equation for calculating stocks:

$$\mathbf{A}_t = R_t + (1-\gamma)\mathbf{A}_{t-1}. \tag{A4}$$

Table A1—Share in R&D Expenditure

Country	Period	Capital[a]	labour[b]	Other[c]	Total
Argentina	1998-2011	0.09	0.71	0.20	1.00
Australia	1981-2008*	0.10	0.47	0.44	1.00
Austria	1981-2011*	0.10	0.51	0.39	1.00
Belgium	2000-2011	0.09	0.60	0.31	1.00
Chile	2007-2010	0.25	0.46	0.21	1.00
China	1998-2012*	0.19	0.24	0.56	1.00
Chinese Taipei	1998-2012	0.11	0.48	0.41	1.00
Czech Republic	1995-2012	0.14	0.38	0.48	1.00
Denmark	1981-2011*	0.09	0.56	0.35	1.00
Estonia	2005-2011	0.25	0.43	0.31	1.00
Finland	1981-2011*	0.06	0.52	0.42	1.00
France	2002-2011	0.10	0.58	0.32	1.00
Germany	1981-2011*	0.10	0.58	0.32	1.00
Greece	1995-2005*	0.15	0.59	0.26	1.00
Hungary	1992-2011*	0.13	0.43	0.39	1.00
Iceland	1981-2011*	0.10	0.58	0.32	1.00
Ireland	1981-1993	0.15	0.53	0.32	1.00
Israel	1993-2012	0.07	0.74	0.19	1.00
Italy	1981-2011*	0.12	0.56	0.32	1.00
Japan	1981-2011	0.13	0.43	0.45	1.00
Korea	1995-2011	0.14	0.38	0.47	1.00
Mexico	1993-2007*	0.19	0.56	0.24	1.00
Netherlands	1981-2011*	0.10	0.57	0.33	1.00
New Zealand	2005-2011*	0.10	0.52	0.38	1.00
Norway	1981-2011*	0.09	0.56	0.35	1.00
Poland	1994-2011	0.21	0.41	0.38	1.00
Portugal	1982-2011	0.18	0.58	0.24	1.00
Romania	1995-2011	0.12	0.49	0.39	1.00
Russian Federation	1994-2012	0.05	0.53	0.42	1.00
Singapore	1998-2012	0.20	0.42	0.38	1.00
Slovak Republic	1996-2012	0.12	0.44	0.44	1.00
Slovenia	1993-2011	0.11	0.55	0.34	1.00
South Africa	2001-2010*	0.12	0.45	0.43	1.00
Spain	1999-2011	0.17	0.56	0.27	1.00
Sweden	2007-2011*	0.04	0.40	0.32	1.00
Switzerland	1992-2008*	0.07	0.57	0.35	1.00
Turkey	2001-2011	0.17	0.47	0.36	1.00

Note: Table provides a country comparison of the cost structure of total intramural R&D for the time period indicated in the second column. Total intramural R&D includes R&D spending by government, business enterprises, higher education and private non-profit entities.
Asterisk (*) indicates that data was not available for some years during the period indicated.
[a] Consists of expenditure on equipment and buildings. [b] Expenditure on salaries. [c] Other current costs.
Source: OECD STAN Database

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