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Optimal Multi-Phase Transition Paths

Toward a Global Green Economy

by

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1 Introduction

Economic growth thus far has been closely linked to the bulk conversion of energy stored in carbon based fuels (wood, coal, oil, natural gas) into useful work.[1] Burning such fuels gives rise to CO2-emissions. These emissions, together with other greenhouse gasses (GHG's) like methane, are now thought to be responsible for a considerable warming of the earth's atmosphere in the present and for years to come. That is bad news on a number of accounts: sea levels will rise, tropical diseases will become more wide-spread, storms will be more violent, patterns of rainfall will change (affecting agriculture), fresh water supply shortages will become a problem due to global glacier retreat, and so on. Most of these changes represent significant costs to society.[2] There is much to be said then for avoiding these consequences of global warming, by reducing emissions or by adapting to the new situation.

Adaptation will have to be part of the response to global warming, if only because much of the warming induced by man-made CO2 emissions is still in the pipeline. With the current level of (cumulative) emissions, it is doubtful whether global warming can be kept below an additional 2 degrees Celsius compared to pre-industrial times. The latter is considered to be a threshold that should not be crossed for fear of runaway global warming and corresponding runaway damages and the end of life as we know it.

Unfortunately, the mere acknowledgement of the existence of a threshold is not enough to ensure that CO2 and other GHG emissions will be sufficiently reduced. The BRICS countries have shown astonishing growth figures during the last two decades, and they feel entitled to welfare levels comparable to those in the West. It goes without saying that the West also feels entitled to such welfare levels. As a consequence, the provision of energy will have to keep pace with the continuing desire to grow. Unfortunately, many of the new power production plants currently built in China are coal fired power plants, which is the worst possible option from a CO2 emissions point of view.[3] At the same time, the tsunami disaster in Fukushima has forced governments to reconsider nuclear energy as a relatively carbon extensive power source. Only recently, the German government has announced to close down a number of nuclear power plants, and it is feared that this will entail a switch towards coal-fired power production again. However, the problem with coal fired power plants is that once they are there, they will be active for at least three decades or more, because these types of investments are irreversible in practice. In short, there are strong political pressures that tend to lead to a larger consumption of carbon-based energy which causes a persistent threat of runaway climate change.[4]

But even though we seem to be almost unavoidably heading towards a problematic climate future, it would be interesting to know **first** what economic theory has to say about combining the desire to grow with the 'need' to avoid disastrous climate change and what to do to end-up in a sustainable growth situation instead, and **secondly** what the role of both existing and new technologies

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[1] I.e. work in the physics sense of the word.

[2] See, for example, the Stern Review (Stern, 2006) for an extensive overview of the causes and consequences of climate change. See also (IPCC, 2007).

[3] The energy content of coal is 32.5 MJ/Kg while that of natural gas is 53.6 MJ/Kg, implying that the amount of energy per unit of CO2 is about 65 per cent larger for natural gas than for coal. (cf. Wikipedia, Energy Density).

[4] Obviously, such a threat may be mitigated by fundamental changes in lifestyles, at least for the well-to-do part of the global population. But it is hard to imagine that this will be politically acceptable, and so we will have to rely on 'technology' as a 'savior of last resort'.

and their embodiment in irreversible investment may be in such a transition process towards sustainable growth, and **third** whether such a process would be technically and/or politically feasible at all, within the limits set by the existence of an irreversible climate change threshold (further ICCT for short). If not, we may have to prepare ourselves for an interesting endgame.[5] If so, we should at least think again and try to take better informed decisions.

Investment (either in knowledge or in hardware) is the *'conditio sine qua non'* for a successful transition towards a sustainable future. The transition towards sustainability will therefore be determined by finding the right balance between two important aspects of investment: on the one hand we have to face the fact that the irreversibility of investment implies a certain degree of inertia to change, while on the other hand investment is literally the carrier of technological progress and so 'enables' (productivity-) change. This "double-role" of investment underlines the importance of the timing of investment decisions: it is unwise to invest too early because one runs the risk of missing out on potential productivity improvements still to come, and neither should one invest too late because of the rising opportunity cost of continuing to use old technologies instead of new, superior, ones. This setting naturally gives rise to such questions as how long to continue using and investing in present carbon based technologies, how much and how long to spend efforts on improving 'new' carbon free technology, and when to stop improving such carbon free technologies and start implementing and using them, this all in the face of having to stop cumulative emissions just in time and just below the ICCT.

To find answers to these questions, we formulate an optimum control model that borrows heavily from the AK-model from the endogenous growth literature (cf. Rebelo (1991) in particular), but that also expands upon this AK-setting in a number of ways. First, we allow for different technologies that can be used either next to each other or sequentially. Secondly, a technology is characterized not only by its capital productivity, but also by CO2-emissions per unit of capacity used. Obviously, a carbon free technology will not only be characterized by a zero emissions coefficient, but also by a lower capital productivity.[6] Third, we allow for the deactivation of existing technologies, i.e. the 'scrapping' of existing technologies, as in the vintage literature. We show how the time of deactivation of existing capacity depends on technological parameters but also on emission characteristics, in combination with the shadow price of emissions. The latter suggests that the position of the ICCT directly influences such replacement decisions through its impact on the shadow price of emissions. A tightening of the ICCT (for example if one would find evidence for runaway global warming occurring at less than 2 degrees warming) would tend to drive up the shadow price of emissions, and would lead to an earlier deactivation of carbon based technologies. Fourth, we allow for endogenous R&D based technical change. This requires a specification of the R&D function that is different from the ones found in Romer (1990), or Aghion and Howitt (1991), for example, because technical change in our model setting is not meant to compensate for the loss in capital productivity resulting from capital accumulation under neo-

[5] See, for example, Dyer (2010) and Lynas (2007) on this subject. Lynas provides a journalistic view on the consequences for the world of up to 6 degrees additional warming for each degree of extra warming, while Dyer provides scenarios regarding the struggle for resources necessary to fulfil basic needs, like water and arable (and habitable) land, as these are likely to arise with on-going global warming.

[6] The latter is a necessary assumption to make the model consistent with the observation that the present state of the economy is characterized by the intensive use of carbon based energy rather than carbon free energy. If the capital productivity of the latter technology would exceed that of the former, then the carbon free technology would be superior to the carbon based technology on all accounts, which is not really the case.

classical conditions, as in Lucas (1988) and Romer (1990), for instance. Rather, in an AK-setting, capital productivity is constant by assumption, and so technical change specified in the usual way would produce a continuously accelerating growth rate rather than just a higher, but still constant, rate of growth. We will come back to this in more detail later on. Fifth, we explicitly focus on the timing of the switches between investment in the one technology and in the other, and on the timing of the deactivation of old technologies, since the deactivation and activation of technologies that differ with regard to their emission-coefficients have a direct impact on the macro-emission rate and therefore on the time left until the ICCT will be hit.

The model that we are going to present extends the relatively small literature that is concerned with the optimal timing of switches between production technologies. Tahvonen and Salo (2001), Valente (2009) and Schumacher (2011) are three examples. Tahvonen and Salo (2001) focus on the timing of the switch between resource extraction technologies, based on differences in extraction costs, while Valente (2009) focuses on the switch between two macro-production technologies in a setting without any irreversible investment in production capacity. Moreover, Valente (2009) determines just one optimal switching moment in a standard dynamic optimization setting involving a CIES utility function, whereas the very requirement of the accumulation of physical production capacity BEFORE production using that capacity can actually take place, implies that one should consider at least two optimal switching moments, even if one has to switch between just two technologies.[7] Schumacher (2011) focuses on the timing of a switch towards a renewable resource regime induced by the increasing probability of climate disasters under a non-renewable production regime. However, he employs a production structure in which renewables and non-renewables form a 'complex' of perfectly substitutable inputs that together with 'generic' capital produce output. Schumacher therefore ignores the embodiment of technologies in technology specific capital goods and so downplays the need for a sufficient amount of time to build-up the carbon free capacity required to satisfy future consumption and investment needs. A recent paper is that by Boucekkine et al. (2012) who provide the theoretical backgrounds for formulating an optimal control resource extraction problem with irreversible ecological regimes.

The present paper summarizes part of the work conducted by the authors on the construction (and further extension) of a multi-phase transition model that incorporates the notion of embodiment of technical change on the one hand, the irreversibility of investment decisions, and the fact that the 'smooth' transition toward a carbon-free future will need to be prepared by means of the accumulation and subsequent run down of carbon-based production capacity, simply because capital, whether carbon-based or carbon free, is a produced means of production. The focus of the present paper is therefore on the selective build-up and deactivation of different types of capital stocks, the time this takes, and the implications this has for the development over time of welfare specified in terms of the flow of consumption.

The setup of the paper is as follows. In section 2, we present a description of the optimum control model. We provide four different versions. The simplest version, called the Basic Model (BAM for short), contains just two technologies, a carbon-based one and a carbon free technology, and we look at the

[7] There must be at least three phases: a pure carbon phase, a mixed carbon and carbon-free phase and a pure carbon free phase. The simple reason is that capital is a produced means of production, and the very first units of carbon free capacity must be produced using carbon based capacity if we start out with a pure carbon phase first.

timing of the various activities that need to get us from the business as usual situation which is carbon intensive to a carbon free sustainable future. In the second version we add endogenous R&D driven capital augmenting technical change to BAM. This version is called BAM+R&D. It can be shown that BAM+R&D and BAM have both three phases. However, the way in which decisions in each phase are linked together will differ in both models. The final version is called BAM+R&D+UCL and it extends the BAM+R&D model by allowing for extra weather related damages. We implement this by introducing another cumulative emissions threshold, above which technical decay occurs at a faster rate, due to increased weather related damages. In section 3, we present the outcomes generated for the three versions of the multiphase transition model. In section 4 we provide a summary and some concluding remarks.

2 The Model

2.1 Introduction

We use the simplest possible endogenous growth setting in which we have two broad (linear) technologies. One of them is an established technology (called the A-technology) with a relatively high productivity of capital that uses carbon based energy and that produces CO2 emissions in the process. As stated, these emissions add to the stock of GHG's and so affect the probability of the world getting into a situation of runaway global warming and catastrophe in the end. The alternative technology (further called the B-technology) does not generate CO2 emissions, but has a relatively low capital productivity that needs to be further developed (through R&D) and scaled up (through investment in physical capital) to a level in which it can contribute significantly to the consumption needs of the population at large.

The simplest possible growth setting that allows a relatively straightforward coverage of the features directly above is that of an 'AK+BK'-model, where AK reflects the old carbon based technology, and BK the new carbon free one. We set-up a sequential Hamiltonian system in which we allow for different phases in the transition from carbon based to carbon free production. In the first phase, called the business as usual phase (BAU for short), only the AK technology is active, but the basic innovations that together define the new BK technology have already been done. During the BAU phase the new technology is not active because it is outperformed by the old technology, in the sense that using the new technology would be relatively costly in terms of instantaneous consumption possibilities foregone. However, as cumulative emissions grow, continuing to use the A technology becomes ever less profitable relative to using the carbon free B technology.

A technology being active implies two things. First, the basic features of such a technology must be known, while secondly these features are embodied in new capital goods. A technology can be de-activated by not using the capital goods embodying that technology anymore. Because capital goods are technology specific, this implies that a new technology can take over from an old one only by actively investing in the capital goods that implement the new technology and by switching production from the old to the new technology.

In such a setting characterized by linear production technologies and linear cost functions, it can be shown that it is not optimal to invest in both technologies at the same time, if these technologies differ with respect to their net productivity (i.e. net of depreciation). The reason is that a unit of investment for both technologies would represent the same marginal cost in terms of consumption forgone, and so the technology with the highest net marginal product, would generate the highest net marginal welfare gain. Hence, if investment in the A technology is taking place, then investment in the B-technology will be zero and the other way around. We will also allow for the possibility that there is a phase in which no investment in A takes place, even though the existing capital stock is used to produce output while investment in and production using the B-technology is happening at the same time. This second phase will therefore be called the joint production phase (JPR for short). In the final phase, only investment in and production with the B-technology occurs, while the A-technology capital stock has been deactivated at the beginning of that phase. This is the carbon free phase (CFR for short).

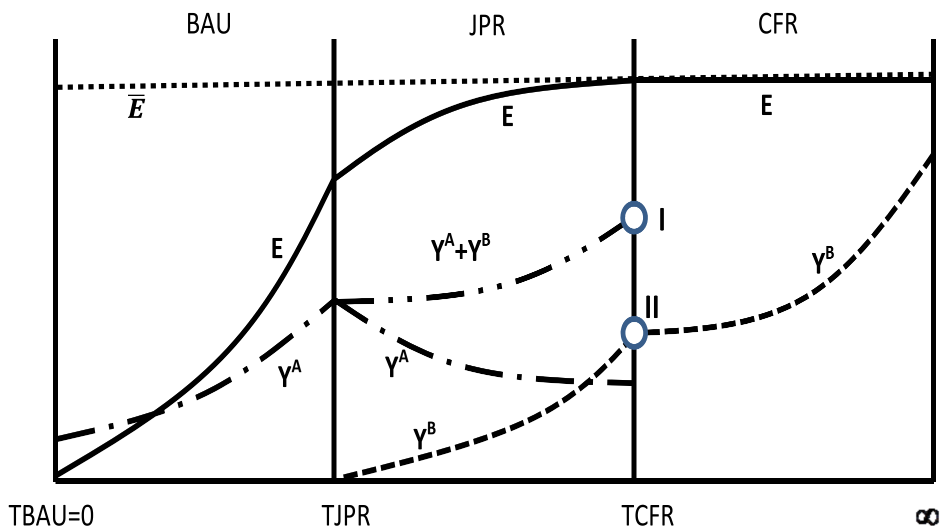


Figure 1. Transition Phases

The effects of the production and investment activities during the separate phases distinguished in the model can be summarized as in Figure 1. TBAU, TJPR and TCFR mark the moments in time at which the BAU, JPR and CFR phases begin. In the BAU phase, the cumulative emissions (labelled E) are increasing exponentially as the stock of carbon based capital is growing. In the JPR phase investment in technology A stops, and output using technology A (i.e. Y^A) is at its maximum level but starts to decrease over time, because of technical decay. The stock of technology B capital is built up from scratch starting with the arrival of the JPR phase. Production using technology B (i.e. Y^B) is at full capacity and exponentially increasing during the JPR phase. Cumulative emissions are still increasing, but at a decreasing rate as the stock of carbon based capital is run down. During the JPR phase, total output is still growing, but at a slower rate than the growth of Y^B. Phase CFR starts when cumulative emissions E hit the cumulative emissions threshold $\bar{E}$. During the final phase, only investment in and production with technology B is possible. Because the threshold has been reached, carbon based capital needs to be discarded. This leads to a drop in output, since production using the A technology ceases, even though there is still a positive amount of carbon based production capacity available. Hence, total output jumps from point I to point II and starts growing from the latter point again during the CFR phase.

It should be noted that the various phases in the model are qualitatively different. The first phase, i.e. the BAU phase, uses a high growth technology which unfortunately quickly raises the stock of cumulative CO2 emissions that is bounded from above by the ICCT. Before cumulative emissions will hit this threshold, investment in carbon free technology must have taken place during phase JPR to bring carbon free capacity up to a level where the switch from carbon-based to carbon free technology would not force changes in consumption levels that are too disruptive. The latter is implied by our assumption that consumers dislike consumption shocks (they want to smooth changes in consumption patterns over time to some extent, given the fact that they have a positive coefficient of relative risk aversion (i.e. θ)). From TCFR onwards the world is "green", and will have to grow at a relatively low but CO2 emission free rate.

In addition to the Basic Model, we implement two other versions. In the second version of the model, the productivity of the new technology can be improved through (endogenously determined) R&D before investment in the new technology takes place. The final version combines the endogenous R&D features with the existence of an emissions threshold that, once past, pushes the world into a high weather related damages regime/phase. The latter regime-shift is modelled by means of a one-time jump in the rates of technical decay. This regime change has an impact on the shadow price of emissions, and so feeds back to investment decisions prior to the high-damages switch point.

Using these models, we'll try to find out how R&D in combination with the embodiment of technology in investment in the face of the existence of (a) climate threshold(s), will determine the timing and the length of the various phases distinguished in our model.

2.2 The Basic Model[8]

The endogenous growth framework of BAM borrows heavily from the AK-model by Rebelo (1991). However, contrary to the original AK-setting, we distinguish between two types of capital: carbon based, or carbon based, capital further denoted by K_A and carbon free, or green, capital further denoted by K_B. The capital stocks in this model are subjected to exponential decay at rates δ_x for $x \in \{A, B\}$. Because of the linearity of the production functions in an AK-setting, and since one unit of capital takes one unit of consumption foregone for the two technologies distinguished, it follows that there will always by investment in just one type of capital at the time. Hence, gross investment in a particular technology is either equal to zero, or it is equal to total savings. Welfare in this setup comes from consumption only, and we use the CIES intertemporal welfare function to describe the total flow of welfare over time. The activities during the different phases are summarized in Table 1.

[8] BAM for short.

Activities	BAU Phase	JPR Phase	CFR Phase
Investment	$I_A > 0$	$I_B > 0$	$I_B > 0$
Production	$Y_A = A.K_A$	$Y_A = A.K_A$ $Y_B = B.K_B$	$Y_A = 0$ $Y_B = B.K_B$
Capital Accumulation	$\dot{K}_A = Y_A - \delta_A.K_A - C$	$\dot{K}_A = -\delta_A.K_A$ $\dot{K}_B = Y_A + Y_B - \delta_B.K_B - C$	$\dot{K}_B = Y_B - \delta_B.K_B - C$
CO2 Emissions	$\dot{E} = \varepsilon_A.K_A$	$\dot{E} = \varepsilon_A.K_A$	$\dot{E} = 0$

Table 1. BAM activities

In Table 1, I_x refers to the amount of investment in capital of type x, where $x \in \{A, B\}$ refers to 'carbon based' capital and carbon-free capital, respectively. Similarly, Y_x refers to the flow of output using K_x, where K_x is the stock of capital of type *x*. We also assume that Y_x is proportional to *Kx* with a constant productivity of capital as a factor of proportion. Typically, we use A and B to denote the capital productivities of technologies A and B, implying that Y_A=A.K_A, etcetera. Finally, the instantaneous flow of CO2 emissions is proportional to the capital stock in use with a constant factor of proportion ε_x for $x \in \{A, B\}$. Obviously, $\varepsilon_B = 0$. Note that when production on some type of carbon based capital ceases, this very fact initiates another phase, since this introduces a difference between the composition of activities between phases J and F. So Y_A=0 implicitly defines the arrival time of phase F, and the scrapping of carbon based capital at t=TF.

2.3 BAM: the intertemporal optimization setting

It should be noted that the fact that the final phase of BAM is a pure AK-setting, allows us to obtain the optimum consumption path for the CFR phase directly, given the initial value for K_B^{TF}.[9] As a consequence, we can immediately derive the optimum time path for welfare during the CFR phase for a 'given' initial value K_B^{TF}. The welfare generated during the CFR phase depends therefore on the terminal value of K_B at the end of the JPR phase. It follows that the distribution of a state variable over the entire path is optimal when the marginal costs of having to deliver an extra unit of the state variable in its role as a terminal value at some time t* is exactly matched by the marginal benefits that this extra unit of the state variable generates as the initial value for the optimum continuation from t*. Since these marginal benefits and marginal costs are captured by the co-state variables (see Leonard and Van Long (1992), ch 4), the latter need to be continuous along an optimum path: states and co-states don't jump.

An optimum path can be thought of as a combination of an optimum first step and an optimum continuation (as in dynamic programming problems), which allows us to interpret our multiphase transition model as a finite horizon optimum control problem with a free endpoint and a scrap value function, as described in Leonard and Van Long (1992, ch 7, further called LVL7). This is the situation that is of direct relevance in our case, since we do not know on beforehand when the next phase will start. However, on an optimum path, postponing or extending a particular phase by an infinitesimal amount of time, shouldn't change the valuation of the entire path. LVL7 show that the derivative of the value

[9] See, Barro and Sala-i-Martin (2004), chapter 4 in particular.

function (in our case the present value of total welfare) with respect to the terminal date (of a phase) matches the value of the Hamiltonian at that date. This makes sense, as the Hamiltonian at some moment in time measures the contribution to the value function of the optimal use of all resources available at that moment in time. But in a sequence of phases, lengthening the one phase by a unit of time implies shortening the next phase by the same unit of time. So we should keep on postponing the arrival of the next phase as long as the Hamiltonian of the earlier phase exceeds that of the later phase. The optimum switching moment between any two phases is therefore implicitly defined by the requirement that the Hamiltonians of two adjoining phases must be the same when evaluated at the moment of the phase-change. Again, this makes perfect economic sense, as the value of the Hamiltonian at the end of the current phase can be seen as the benefits of expanding the current phase by a marginal unit of time, while the Hamiltonian at the beginning of the next phase can be seen as the opportunity cost of that expansion. In practice, the equality of the Hamiltonians evaluated under the conditions relevant in either of the phases just before and just after a phase change, will result in a condition that needs to be met by a set of states and co-states evaluated at the moment of the phase-change.

The differences in the nature of the activities at the moment of a phase change can be used to implicitly describe the conditions that should be met at the moment of a phase change. For example, at t=TCFR it must be the case that carbon based capital is deactivated. For t>=TCFR is must therefore be the case that the shadow price of carbon based capital is zero, since the cumulative emission threshold has been reached and carbon based capital can therefore not be used anymore and has become worthless therefore.

2.4 BAM: formal structure

The overall welfare function consists of a summation of integral welfare derived from the flow of consumption during the three phases distinguished in the BAM:

$$W_0 = \int_0^{T_J} \frac{e^{-\rho \cdot t} \cdot (C_t)^{1-\theta}}{(1-\theta)} dt + \int_{T_J}^{T_F} \frac{e^{-\rho \cdot t} \cdot (C_t)^{1-\theta}}{(1-\theta)} dt + \int_{T_F}^{\infty} \frac{e^{-\rho \cdot t} \cdot (C_t)^{1-\theta}}{(1-\theta)} dt \qquad (1)$$

In equation (1) W_0 measures the present value of total welfare at time t=0, at which, by assumption, phase U begins. In equation (1), ρ is the rate of discount, while $1/\theta$ is the (constant) intertemporal elasticity of substitution. C_t is consumption at time t, while T_J and T_F are the moments in time at which phases J and F begin. Given the exposition on intertemporal optimization above, the time paths that would maximize (1) can be obtained by solving the time paths for the Hamiltonian problems that can be defined for the individual phases, while linking those time paths together by means of the requirements of optimum phase lengths (implying the equality of the Hamiltonians for phases U and J at t=TJ and for phases J and F at t=TF). Effectively this comes down to maximizing (1) with regard to the flows of consumption during each phase and with regard to the phase-lengths themselves, constrained by the technologies that are relevant in each phase, by the stocks inherited from previous phases, and by the thresholds that are relevant during the various phases. We will now solve the Hamiltonian problems for each individual phase.

Phase U

Using the superscripts U, J and F to denote the phase to which a particular variable pertains, while dropping the time subscript for ease of notation, the present value Hamiltonian H^U is given by:

$$H^U = \frac{e^{-\rho \cdot t} \cdot (C^U)^{1-\theta}}{(1-\theta)} + \lambda_{K_A}^U \cdot \left((A-\delta_A) \cdot K_A^U - C^U\right) + \lambda_E^U \cdot \varepsilon_A \cdot K_A^U \tag{2}$$

In equation (2), C is the only control variable, while K_A and E are the state variables and $\lambda_{K_A}^U$and λ_E^U are the corresponding co-states. As first order conditions we have:

$$\frac{\partial H^U}{\partial C^U} = e^{-\rho \cdot t} \cdot (C^U)^{-\theta} - \lambda_{K_A}^U = 0 \Rightarrow C^U = \left\{e^{\rho \cdot t} \cdot \lambda_{K_A}^U\right\}^{-1/\theta} \tag{3}$$

$$\frac{\partial H^U}{\partial K_A^U} = \lambda_{K_A}^U \cdot (A-\delta_A) + \lambda_E^U \cdot \varepsilon_A = -\dot{\lambda}_{K_A}^U \tag{4}$$

$$\frac{\partial H^U}{\partial E^U} = 0 = -\dot{\lambda}_E^U \tag{5}$$

$$\frac{\partial H^U}{\partial \lambda_{K_A}^U} = \dot{K}_A^U = \left((A-\delta_A) \cdot K_A^U - \left\{e^{\rho \cdot t} \cdot \lambda_{K_A}^U\right\}^{-1/\theta}\right) \tag{6}$$

$$\frac{\partial H^U}{\partial \lambda_E^U} = \dot{E}^U = \varepsilon_A \cdot K_A^U \tag{7}$$

Equation (6) is obtained by means of substitution of equation (3) into the macro-economic budget constraint which states that output is used for consumption and (gross) investment purposes. Equations (4)-(7) constitute a simultaneous system of differential equations that can be solved forward in time, given a set of initial values for the various state and co-state variables. This will give rise to terminal values of those same state and co-state variables at the terminal date of phase U, i.e. at t= TJ, the value of which is unknown so far.

Phase J

Phase J differs from phase U since investment in the A technology has stopped and that in the B-technology begins. However, the carbon based capital stock K_A is still used for production purposes. The present value Hamiltonian for phase J, i.e. H^J , is now given by:

$$H^J = \frac{e^{-\rho \cdot t} \cdot (C^J)^{1-\theta}}{(1-\theta)} + \lambda_{K_A}^J \cdot \left(-\delta_A \cdot K_A^J\right) + \lambda_{K_B}^J \cdot \left((B-\delta_B) \cdot K_B^J + A \cdot K_A^J - C^J\right) + \lambda_E^J \cdot \varepsilon_A \cdot K_A^J \tag{8}$$

As in the U phase, we have just one control, i.e. C^J, but three states K_A, K_B and E and corresponding co-states $\lambda_{K_A}^J$, $\lambda_{K_B}^J$and λ_E^J. As first order conditions we have:

$$\frac{\partial H^J}{\partial C^J} = e^{-\rho \cdot t} \cdot (C^J)^{-\theta} - \lambda_{K_B}^J = 0 \Rightarrow C^J = \left\{e^{\rho \cdot t} \cdot \lambda_{K_B}^J\right\}^{-1/\theta} \tag{9}$$

$$\frac{\partial H^J}{\partial K_A^J} = \lambda_{K_A}^J \cdot -\delta_A + \lambda_{K_B}^J \cdot A + \lambda_E^J \cdot \varepsilon_A = -\dot{\lambda}_{K_A}^J \tag{10}$$

$$\frac{\partial H^J}{\partial K_B^J} = \lambda_{K_B}^J \cdot (B-\delta_B) = -\dot{\lambda}_{K_B}^J \tag{11}$$

$$\frac{\partial H^J}{\partial E^J} = 0 = -\dot{\lambda}_E^J \tag{12}$$

$$\frac{\partial H^J}{\partial \lambda_{K_A}^J} = \dot{K_A^J} = -\delta_A \cdot K_A^J \tag{13}$$

$$\frac{\partial H^J}{\partial \lambda_{K_B}^J} = \dot{K_B^J} = (B - \delta_B) \cdot K_B^J + A \cdot K_A^J - \left\{e^{\rho \cdot t} \cdot \lambda_{K_B}^J\right\}^{-1/\theta} \tag{14}$$

$$\frac{\partial H^J}{\partial \lambda_E^J} = \dot{E^J} = \varepsilon_A \cdot K_A^J \tag{15}$$

Equation (14) is again obtained by means of substitution of optimum consumption levels (as given by equation (9)) into the macro-economic budget constraint. As before, equations (10)-(15) constitute a simultaneous system of differential equations that can be solved forward in time, given initial values for the various state and co-state variables. Note that the initial values in phase J for those state and co-state variables that both systems have in common, are the same as the terminal values for those variables at the end of phase U because of the continuity of state- and co-state variables along an optimum path. For a given value of TF, this system of differential equations allows the forward calculation of terminal values for the states and co-states at time t=TF, which will then function as the initial values for the states and co-states during the carbon free phase.

Phase F

Phase F differs from phase J in that the carbon based capital stock is discarded, and consequently the flow of CO_2 emissions drops to zero. From t=TF production is totally green. The present value Hamiltonian for phase F, i.e. H^F , is now given by:

$$H^F = \frac{e^{-\rho \cdot t} \cdot (C^F)^{1-\theta}}{(1-\theta)} + \lambda_{K_B}^F \cdot \left((B - \delta_B) \cdot K_B^F - C^F\right) + \lambda_E^F \cdot 0 \tag{16}$$

As in the U and J phase, we have just one control, i.e. C^F, but two states K_B and E and corresponding co-state variables $\lambda_{K_B}^F$ and λ_E^F. As first order conditions we have:

$$\frac{\partial H^F}{\partial C^F} = e^{-\rho \cdot t} \cdot (C^F)^{-\theta} - \lambda_{K_B}^F = 0 \Rightarrow C^F = \left\{e^{\rho \cdot t} \cdot \lambda_{K_B}^F\right\}^{-1/\theta} \tag{17}$$

$$\frac{\partial H^F}{\partial K_B^F} = \lambda_{K_B}^F \cdot (B - \delta_B) = -\dot{\lambda}_{K_B}^F \tag{18}$$

$$\frac{\partial H^F}{\partial E^F} = 0 = -\dot{\lambda}_E^J \tag{19}$$

$$\frac{\partial H^F}{\partial \lambda_{K_B}^F} = \dot{K_B^F} = (B - \delta_B) \cdot K_B^F - \left\{e^{\rho \cdot t} \cdot \lambda_{K_B}^F\right\}^{-1/\theta} \tag{20}$$

$$\frac{\partial H^F}{\partial \lambda_E^F} = \dot{E^F} = 0 \tag{21}$$

As before, equation (20) is obtained by substituting equation (17) into the macro-economic budget constraint. Again, equations (18)-(21) constitute a simultaneous system of differential equations that can be solved forward in time, given initial values for the various state and co-state variables inherited from phase J. However, in this case the terminal values for the states and co-states are implicitly described by the standard transversality conditions in an AK setting that require the present value of the carbon free capital stock to approach zero at the terminal date, i.e. in this case at time infinity. For cumulative emissions, the terminal value had already been reached at t=TJ, when cumulative emissions hit the threshold.

Transversality conditions

For phase F the standard transversality condition (further called TVC for short) applies regarding the value of carbon free capital at time infinity:

$$\lim_{t\to\infty} \lambda^F_{K_{B,t}} \cdot K^F_{B,t} = 0 \tag{22}$$

where we have now added time-subscripts. Note that (18) can be integrated directly to obtain the time path for $\lambda^F_{K_{B,t}}$ which can then be substituted into (20) to obtain the time path for $K^F_{B,t}$. We get:

$$\lambda^F_{K_{B,t}} = \lambda^F_{K_{B,TF}} \cdot e^{-(B-\delta_B)\cdot(t-TF)} \tag{23}$$

$$K^F_{B,t} = \frac{e^{-\frac{t\rho-(t-TF)(B-\delta\mathrm{B})}{\theta}}\theta\cdot\left(\lambda^F_{K_{B,TF}}\right)^{-1/\theta}}{\rho+(\theta-1)(B-\delta\mathrm{B})} + e^{(t-\mathrm{TF})(B-\delta\mathrm{B})}\left(K^F_{B,TF} - \frac{e^{-\frac{\mathrm{TF}\rho}{\theta}}\theta\cdot\left(\lambda^F_{K_{B,TF}}\right)^{-1/\theta}}{\rho+(\theta-1)(B-\delta\mathrm{B})}\right) \tag{24}$$

Equations (23) and (24) can be substituted into TVC (22), and we find that in order for TVC (22) to hold we should have that:

$$\frac{\rho+(\theta-1)(B-\delta_B)}{\theta} > 0 \tag{25}$$

$$K^F_{B,TF} = \frac{e^{-\frac{\mathrm{TF}\rho}{\theta}}\theta\cdot\left(\lambda^F_{K_{B,TF}}\right)^{-1/\theta}}{\rho+(\theta-1)(B-\delta\mathrm{B})} \tag{26}$$

When substituting (26) into (24), we find that:

$$K^F_{B,t} = K^F_{B,TF} \cdot e^{\frac{(B-\delta\mathrm{B}-\rho)(t-TF)}{\theta}} \tag{27}$$

It follows from (27) that if the structural parameters are such that (25) is met and if we pick consumption at time T_F (hence $\lambda^F_{K_{B,TF}}$ (see equation (17)) such that (26) is met, then the carbon free capital stock will grow at the steady state growth rate $(B - \delta\mathrm{B} - \rho)/\theta$ from time t=TF.
Apart from the TVC above, we require that:

$$\lambda^J_{K_{A,TF}} = 0 \tag{28}$$

Equation (28) states that the shadow price of carbon based capital at the end of the J phase (so at the moment it is discarded) should be zero, since having an additional unit of capital that will not produce anything is useless and therefore worthless.

Finally, there are two TVCs that pertain to the optimum length of phase U and phase J, and that require the equality of the Hamiltonians of the various phases at different points in time. For the optimum length of phase U (given by the value of TJ, since TU=0 by assumption), we must have that $H^U_{TJ} = H^J_{TJ}$, while the optimum arrival data of the F phase is determined by the requirement that $H^J_{TF} = H^F_{TF}$. Using the definitions of the Hamiltonians in (2), (8) and (16), as well as the FOCS regarding consumption in (3), (9) and (17) together with the continuity constraints on states and co-states that feature in both adjoining phases, we obtain implicit descriptions of the arrival dates of phases J and F, given by:

$$\lambda^J_{K_{A,TJ}} = \lambda^J_{K_{B,TJ}} \tag{29}$$

$$A \cdot \lambda^J_{K_{B,TF}} = -\varepsilon_A . \lambda^J_{E,TF} \tag{30}$$

Equation (29) states that investment in the carbon based technology should stop at the moment that the shadow price of the A technology is equal to (and then drops below) the shadow price of the B technology. Since the marginal cost of obtaining a unit of capital is the same in both cases (i.e. one unit of consumption foregone), equation (29) is consistent with the maximization of the (present value) welfare surplus associated with investment. Equation (30) states that production using carbon based capital should stop the moment that the benefits from continuing to use a unit of capital (the LHS of equation (30), as one unit of capital produces A units of output, and each unit of output is worth $\lambda^J_{K_{B,TF}}$ in present value welfare terms at t=TF) matches the cost of doing that (as given by the RHS of equation (30), since one unit of capital produces ε_A units of CO2 emissions, each at a cost of $-\lambda^J_{E,TF}$ [10]). Equation (30) is therefore consistent with the zero quasi-rent condition as an implicit description of the optimum moment to scrap existing capacity known from the vintage literature (cf Johansen (1995), Solow et al. (1966) and Boucekkine (2011), for example) that states that a vintage, once installed, should be discarded as soon as it's quasi-rents (consisting in this case of the present value welfare value of output less total variable (emission-) costs, also in present value welfare terms) become negative.

2.5 BAM: Sequential numerical solution of the systems of differential equations

The three systems of differential equations, further called SU, SJ and SF, can in principle be solved, as the initial and terminal values we have available for the state variables, and the TVCs that provide either some fixed points for the time paths of the co-states (cf. equation (28)), or link the co-states to a state-variable which time path has been fixed through a given initial value (cf. equation (26)), or that links different co-states at some point in time (cf. equations (29) and (30)) provide exactly enough information to obtain a fixed point for all time paths concerned. To see this, it should be noted that we need to obtain fixed points for the time paths for three different state variables (KA,KB and E), and for their corresponding co-state variables, as well as the optimum values of the phase lengths of phases U

[10] Note that the shadow price of emissions itself is negative, since an additional unit of cumulative emissions would reduce potential welfare rather than increase it.

and J. Hence, for BAM we need 8 pieces of information. We have initial values available for $K_{A,0}^{U} = \bar{K}_{A,0}^{U}$, $K_{B,TJ}^{J} = 0$, and $E_{0}^{U} = \bar{E}_{0}^{U}$. In addition, we have a terminal value for cumulative emissions $E_{TF}^{F} = \bar{E}_{TF}^{F}$. The transversality conditions given by equations (26),(28),(29) and (30) then provide the rest of the necessary information.

A numerical solution of SU can easily be obtained, conditional on some a priori values of $\lambda_{K_{A,0}}^{U}$, λ_{E}^{U} and TJ, given the initial values of $K_{A,0}^{U}$ and E_{0}^{U}. The solution of SU then provides terminal values for $\lambda_{K_{A,TJ}}^{U}$ and $\lambda_{E,TJ}^{U}$ that, on account of the continuity of states and co-states give rise to initial values for SJ, since we must have that $\lambda_{K_{A,TJ}}^{U} = \lambda_{K_{A,TJ}}^{J}$, $\lambda_{E,TJ}^{U} = \lambda_{E,TJ}^{J}$, $K_{A,TJ}^{J} = K_{A,TJ}^{U}$, $K_{B,TJ}^{J} = 0$, $E_{TJ}^{J} = E_{TJ}^{U}$. We only need an initial value for $\lambda_{K_{B,TJ}}^{J}$ as well as an a priori value for TF to be able to calculate SJ forward in time. That initial value is provided by the TVCs listed in equation (29). Given the a priori values for $\lambda_{K_{A,0}}^{U}$, λ_{E}^{U} , TJ and finally TF, we are able to calculate the terminal values for $\lambda_{K_{B,TF}}^{J}$, $\lambda_{E,TF}^{J}$, $K_{B,TF}^{J}$ and E_{TF}^{J} , which, again using the requirement of the continuity of states and co-states, imply that $K_{B,TF}^{F} = K_{B,TF}^{J}$, $E_{TF}^{F} = E_{TF}^{J}$, $\lambda_{K_{B,TF}}^{F} = \lambda_{K_{B,TF}}^{J}$ and $\lambda_{E,TF}^{F} = \lambda_{E,TF}^{J}$. The time paths thus obtained for all states and co-states, in combination with our guesses of the various phase lengths, can now be used to evaluate the differences between the RHS and the LHS of the terminal constraint $E_{TF}^{TF} = \bar{E}_{TF}^{TF}$ and those of the TVCs not used so far, i.e. of equations (26), (28) and (30). Obviously, if these constraints would be met by our a priori values of $\lambda_{K_{A,0}}^{U}$, λ_{E}^{U} , TJ and TF, then we have found the solutions for all the time paths that would maximize welfare. Being a guess, these differences will generally not be equal to zero, but we can use a search method (so far we have used a steepest descent algorithm) to find the initial values that would meet all the TVCs simultaneously.[11] It should be noted that a similar approach can be followed for versions of the model with more than 3 phases, as we will describe further below.

2.6 BAM+R&D: changes relative to BAM

A first modification of BAM involves the introduction of R&D driven endogenous technical change with respect to the carbon free technology. We have made the assumption that it takes the form

[11] In fact, given the simplicity of our system, it is possible to obtain analytical solutions for all time paths, even though these are in part quite intricate non-linear expressions involving hyper geometric functions. These integral equations link the various initial and terminal conditions as well as the values of TJ and TF together through a simultaneous system of non-linear equations. Using that system, we were able to find the fixed points for all time paths by numerically solving the non-linear system for the fixed points. The time paths for each of the state and co-state variables could then be obtained by substituting the numerical values thus found for the fixed points into the analytical solutions of all time paths involving these fixed points. This procedure proved to work, but is rather tedious and time consuming and it's feasibility depended crucially on the simplicity of the model. Minor deviations from the AK-set-up proved to make using the analytical method infeasible. This provided a strong incentive to use the sequential numerical solution method outlined in the main text. Both methods do generate the same results, as they should, but the sequential numerical method is much more efficient in terms of both obtaining the system of differential equations to be solved, and finding the solution. Nonetheless, more work is needed to improve upon the rather crude steepest descent search routine that we are employing at the moment. The latter converges rather slowly, if at all, for the more intricate versions of the model extensions we have built up to date and which are not reported here. Nonetheless, the numerical model does allow us to expand the model in ways that would make it impossible to solve using the analytical solution approach.

of increasing the value of B before the moment the technology is actually implemented, thus taking into account the notion of the embodiment of technical change and the irreversibility of investment decisions.

Since we are using an AK-setting, however, the standard specification for endogenous technical change as used in the endogenous R&D literature is not readily applicable. For example, a Lucas (1988) or Romer (1990) or Aghion and Howitt (1992) type of R&D function, would involve a specification such as:

$$\dot{B} = \varsigma \cdot R \cdot B \tag{31}$$

where R represents R&D resources measured as consumption foregone. However, in an AK setting, capital productivity is constant by construction, so that a continuous increase of the effective availability of another factor (say knowledge, or human capital) is not necessary to keep the marginal product of capital from falling as the economy is growing and capital is accumulating. If we would use (31) in a simple AK-setting, then, disregarding problems of embodiment for the sake of simplicity, growth would be explosive rather than steady. This is because for a positive value of R, capital productivity would tend to grow, thus raising both the incentive to engage in R&D and the possibility to allocate the resources, since the growth rate of output would be accelerating in the process. Hence, we have opted for a specification of the R&D process that allows for a decreasing marginal product of R&D and for a growth rate of B that asymptotically approaches zero for a constant positive allocation of R&D resources, so that the incentive to engage in R&D will diminish over time. We have:

$$\dot{B} = \varsigma \cdot R^{\beta} \cdot (\overline{B} - B) \tag{32}$$

where $\overline{B}$ is the asymptotic value for capital productivity and where $\varsigma > 0$ and $0 < \beta < 1$ are constants. Starting out with a basic idea that provides a value $0 < B_0 < \overline{B}$, the marginal product of R&D is falling as R increases and as B increases, rather than rising with B.[12] The notion underlying the specification of (32) is that the R&D process gives rise to discoveries of more efficient ways of doing things from a finite set of unknown possibilities, as in a Platonic world, with one of these possibilities simply being the best (and providing the value of $\overline{B}$). Equation (31) provides a principally different view, in that it allows for an R&D process that actively creates new and better ways of doing things in an unbounded fashion, instead of discovering the limited number of options that nature has been hiding so far.

Introducing (32) into the Basic Model gives rise to a number of modifications. First, the Hamiltonian has to be extended by adding the value of increases in B due to R&D. Secondly, the macro-economic budget constraint that describes the accumulation of capital has to be adjusted for the fact that one unit of R&D resources R requires one unit of output (i.e. consumption or investment foregone). Instead of just the one control variable C, we now have an additional control variable R, and an additional state variable, i.e. B (the carbon free productivity parameter of BAM). B has become a state

[12] Note that (32) is jointly concave in R and B (as opposed to (31)), which is a necessary condition for the welfare maximisation problem to have a solution.

variable since it can change as long as R&D is taking place, but must remain constant after R&D has ceased. Leonard and Van Long (1992, ch.7) describe how the latter situation can be handled. Their approach is to regard the Hamiltonian as a function of B, i.e. H(B), while substituting $\dot{B} = 0$ as the dynamic constraint for B for the phases without R&D (in which B doesn't change, therefore). In all phases we have that the equation of motion for the co-state variable associated with B is given by $\lambda_B = -\partial H / \partial B$. The reason for explicitly specifying the dynamic constraints on the co-state for B during the phases where B itself is not changing (because there is no R&D anymore) is that the terminal value of B when the R&D process stops, will influence the generation of welfare in the following phases without R&D. Thus, through the continuity of co-states and states, information about the future welfare effects of having a high value of B in the joint production phase and in the carbon free phase, can influence allocation decisions during the phase with positive R&D that would directly involve the trade-off between R&D and other uses of output, like consumption and tangible capital investment.

Note that It can be shown that once a B_0 is available, then it pays not to wait improving the B technology by engaging in R&D (see Appendix A). Hence, without loss of generality, we may assume that the basic idea underlying the carbon free technology is available from t=TBAU=0. In that case, R&D should start at t=TBAU, and it should stop at t=TJPR, i.e. the moment that investment in the carbon free technology commences. Consequently, the BAM+R&D model has the same number of phases as BAM. For the BAU phase, assuming that R&D is done from the very beginning, three equations need to be added, i.e.[13] $\lambda_B = -\partial H / \partial B$, $\dot{B} = \varsigma \cdot R^{\beta} \cdot (\overline{B} - B)$ and $\partial H / \partial R = 0$, while the capital accumulation constraint must be modified, giving $\dot{K}^A = (A - \delta^A).K^A - C - R$. In addition to this, the revised Hamiltonian for the BAU phase is now given by: $H = e^{-\rho.t} \cdot C^{1-\theta} / (1-\theta) + \lambda_A \cdot \{(A - \delta_A) \cdot K_A - C - R\} + \lambda_B \cdot \dot{B}$. Since there is now an additional state variable, there is also an additional TVC. As usual, we require that at infinity the present (utility) value of B should approach zero, i.e. $\lim_{t \to \infty} \lambda_{B,t} \cdot B_t = 0$. However, $B_t = B_{TJPR}\ \forall t \geq TJPR$, since R&D has ceased at t=TJPR, turning B effectively into a constant for $t \to \infty$, and so the additional TVC is reduced to the requirement that $\lim_{t \to \infty} \lambda_{B,t} = 0$. Using (25), it can be shown that the latter TVC implies:

$$\lambda_{B,TF}^F = K_{B,TF}^F \cdot \lambda_{K_{B,TF}}^F \cdot \frac{\theta}{(B_{TF}^F - \delta_B) \cdot (\theta - 1) + \rho} \tag{33}$$

It is hard to interpret (33). This goes a fortiori for the revised version of the transversality condition that now determines the optimum length of the BAU phase. That TVC is now different from the one we had before, since the R&D process is active during the BAU phase and inactive during subsequent phases. Consequently, the Hamiltonians of the BAU and JPR phase evaluated at t=TJPR, will now involve terms coming from the R&D function as well as the corresponding co-state evaluated at t=TJPR, resulting in a complicated expression linking the various states and co-states together at t=TJPR. Because it cannot

[13] Note that for ease of notation we have dropped the time subscript and the phase superscript except where the presence of the time subscript is needed.

readily be interpreted, the expression is not given here. Note that (33) in combination with the initial value for B, i.e. B_0, provide enough information to solve this revised system of differential equations.

2.6 BAM+ R&D+UCL

The final version concerns a combination of BAM and R&D that includes the introduction of a weather related damage threshold. We do this as follows. Suppose that the damage threshold lies at some value of cumulative CO2 emissions that is reached within the BAU phase. When the threshold is reached, the jump in the decay parameters occurs. At this moment in time, the BAU phase is effectively split into a low damage sub phase and a high damage sub phase. All the phases coming after the high damage BAU phase are also high damage phases from then on. Again, this is a situation that is described in Leonard and Van Long, where the regime shift from low damages to high damages initiated by a state variable hitting a particular threshold, implies a jump in the corresponding co-state variable. (cf. LVL, ch10). The only difference between the low damage sub phase of the BAU phase and the high damage sub phase is of course a change in the values of the technical decay parameters. Since the rate at which CO2 emissions are accumulated depends on production decisions, the moment in time at which the damage threshold will be hit, depends on these very production decisions too. Hence, the arrival time of the high weather related damage sub phases is subject to choice and therefore to optimal decision-making. Consequently, we can optimally choose the moment at which the high damage sub phase will arrive. We can do this again by requiring that the Hamiltonians evaluated at the moment of arrival of the high damage sub phase will be the same at either side of its arrival time. This implies that this model will have four different phases instead of three. It follows that we need to determine an extra phase length in addition to the size of the jump in the co-state for cumulative emissions at the time of arrival of the first high damage sub-phase. In addition to the given initial and terminal values of the BAM+R&D model as well as the corresponding transversality conditions, we have an additional terminal value in the form of the location of the damage threshold itself, in combination with the requirement of the equality of the Hamiltonians at both sides of the damage phase change. It turns out that the transversality condition for the arrival of the high damage sub phase during the BAU phase is given by:[14]

$$\lambda_{KA,TUH}^{U} \cdot \left(\delta_A^H - \delta_A^U\right) = \epsilon_{\mathrm{A}} \cdot \left(\lambda_{E,TUH}^{U,H} - \lambda_{E,TUH}^{U,L}\right) \tag{34}$$

Equation (39) should hold exactly at the arrival time of the high damage sub-phase, i.e. at t=TUH.[15] In equation (39), the superscripts H and L refer to low and high damage sub phases, while the superscripts you refers to the business as usual phase. The RHS of equation (34) contains the jump in the shadow price of cumulative emissions. It measures the change in the welfare costs associated with emissions per unit of capital before and after the jump. The LHS of equation (34) measures the welfare cost associated with the extra decay per unit of capital. Equation (34) implies the equality between the welfare cost of

[14] Obviously, the arrival time of the first high damage sub-phase could also be within the joint production phase. But ate this stage we are mainly interested in reporting on the principles involved, which would be the same in whichever phase the damages threshold would be situated.

[15] Note that TUH also denotes the end of the low damage sub-phase of the BAU phase. In that case, TUH comes one instant before the value of TUH that represents the beginning of the high damage sub-phase of the BAU phase.

using a unit of capital before and after the arrival of the high damage sub-phase. We conclude that the jump in the co-state must be such that the welfare cost of using a unit of capital remains unchanged. Since the depreciation costs are higher after the jump, the emission costs must be lower, implying a drop (in absolute terms) in the shadow price of emissions. The latter is consistent with the observation that a faster rate of decrease of the carbon based capital stock will lead to a reduction in the rate at which CO2 emissions are accumulated, other things remaining the same.

3 Outcomes

3.1 Model calibration

In order to show how the various models work, the parameters of the model need to calibrated or fixed a priori. To this end we have made use of the Nordhaus RICE 2010 data[16], as well as some direct assumptions, since the Nordhaus model and ours have a different nature. Nordhaus essentially uses a neoclassical growth setting with just one phase, while we use an AK-setting with multiple phases. This implies that simply copying the Nordhaus numbers into our model is not possible. In order, nonetheless, to reproduce growth rates and saving rates that have about the right size, we have made the assumption that the capital output ratio in the BAU phase is equal to 4, which is much higher than in the Nordhaus model. Then again, our (implicit) notion of capital is much broader, since it may be thought to also include human capital. We have also made the assumption that depreciation costs as a fraction of output is 15 per cent. This number is relatively high too, but not unrealistic given the much broader capital concept.

Using Nordhaus' data on TFP growth as well as output per capita and population growth, we arrive at an implied growth rate of output (and of capital in an AK-setting) equal to 0.03436. Using equation (27) to derive the steady state growth rate in an AK-setting, we find that if we make the assumption that output and (carbon based) capital stock grow a steady state rate equal to 0.03436, we must have that:

$$\hat{Y} = \hat{K} = \frac{A-\delta_A-\rho}{\theta} = 0.03436 = \frac{0.25*(1-\frac{\delta_A}{A})-\rho}{\theta} = \frac{0.25*(1-0.15)-\rho}{\theta} \qquad (35)$$

In equation (35), we have used the assumption that depreciation as a fraction of output is 15 per cent, implying that $\frac{\delta \cdot K}{A.K} = \frac{\delta}{A} = 0.15$. The latter implies that $\delta = 0.15 \cdot A = 0.15 \cdot 0.25 = 0.0375$. This value for the depreciation rate is much lower than the one used by Nordhaus, who uses a 10 per cent depreciation rate. Equation (35) implies combinations of θ and ρ given by:

$$\theta = 6.185 - 29.104 * \rho \qquad (36)$$

[16]See http://nordhaus.econ.yale.edu/RICEmodels.htm for further details. In order to obtain observations for 2010 from the ones listed for 2005 and 2015 in the RICE data, we use geometrical interpolation.

Plugging in the value $\rho = 0.015$ that Nordhaus also uses, we find that $\theta = 5.748$. This implies a much lower intertemporal elasticity of substitution than is used by Nordhaus, who uses a value of θ equal to 1.5. In both cases, however, $\theta > 1$, implying that felicity is negative, but rising in consumption levels. Marginal utility would still be positive and falling, however. Since welfare is the integral over (the present value) of felicity, welfare would be negative as well. We therefore added an additive term to felicity at all moments in time equal to the negative of felicity at time zero. This is equivalent to an upward shift of the felicity function to the positive quadrant. Since θ is relatively large, felicity will be relatively small, and so will be the present value of welfare. This implies that the numerical values of the co-states will be small as well, since they represent the change in welfare due to a 1 unit change in the corresponding states. For example, since Y_0=68.95 trillion dollars of 2005[17], our assumptions imply that K_0=275.8, but also that C0=(1-s).Y0=49.16. Hence, felicity at t=0 is equal to $F_0 = \frac{C_0^{1-\theta}}{1-\theta} = -1.958 * 10^{-9}$. Therefore we have added a multiplicative scaling factor to the felicity function as well equal to 10^9, resulting in values for states and co-states that are not many orders of magnitude apart.

Even though our implied intertemporal elasticity of substitution is rather low, in combination with the other parameter values, acceptable values for both the growth rate, and the saving rate can be generated. The implied value of the saving rate (s) is given by:

$$s = \frac{(\dot{K} + \delta \cdot K)}{Y} = \frac{\widehat{K}}{A} + \frac{\delta}{A} = 0.287 \qquad (37)$$

With respect to cumulative CO2 emissions and the location of the climate tipping point in terms of the cumulative emissions generated by our model, we have used the following procedure. From the Nordhaus data, we can infer that the ratio between the concentration of CO2 in the atmosphere measured in ppm and the concentration of carbon in the atmosphere measured in GTCs is given by:

$$ppm = 0.4695 * GTC \quad => \quad \Delta ppm = 0.4695 * \Delta GTC \qquad (38)$$

where Δ refers to the first difference operator. The present concentration of CO2 in the atmosphere amounts to 390 ppm.[18] The preindustrial concentration of CO2 is 280 ppm. The climate tipping point is generally associated with a temperature rise of 2° Kelvin above preindustrial levels. The equation describing the relation between temperature rises and CO2 concentrations in ppm relative to some base level is given by:[19]

$$ppm = ppm_0 \cdot e^{\Delta T/4.28} \qquad (47)$$

In equation (39), ΔT represents the temperature rise relative to a baseline temperature at a baseline concentration of CO2 given by ppm_0. It follows that the tipping point at 2° Kelvin relative to preindustrial levels is given by $280 \cdot e^{2/4.28} = 446.8$. For a 3° Kelvin temperature rise, the corresponding

[17] t=0 refers to 2010. The data for 2010 are obtained by means of geometrical interpolation between the Nordhaus data for 2005 and 2015.

[18] See the website on CO2 measurements on Mauna Loa, Hawaii http://www.esrl.noaa.gov/gmd/ccgg/trends/.

[19] See for more details http://en.wikipedia.org/wiki/Radiative_forcing.

concentration of CO2 would be 564.4 ppm. The current rate at which the CO2 concentration is rising equals about 2 ppm/year.[20] Assuming that this rate of increase will be growing at the rate of growth of output, we find that the room to emit provided by the 2° Kelvin tipping point is equal to 446.8-390=56.8, implying that there can be a further for 56.8/2=28.4 '2010 size batches' of gross emissions until the 2° Kelvin tipping point will be reached. According to Nordhaus, the gross emission rate in 2010 is about 10.63 GTC/year. Therefore, 28.4 batches of emissions represent 28.4.10.63=301.9 GTC of cumulative emissions in total. A fraction (1-q) of these emissions will be absorbed by the environment, while the remainder (q) will end up in the atmosphere. Using equation (37), the net emissions associated with a rise in 56.8 ppm is given by:

$$\Delta ppm = 0.4695 * \Delta GTC * q => q = \frac{56.8}{0.4695*301.9} = 0.4 \qquad (48)$$

It follows that if we rescale current cumulative emissions to zero, the net emissions until the 2° Kelvin tipping point would be reached is given by $0.4 * 301.9 \approx 120$ GTC. Similarly, the extra net emissions for 2.5° and 3° Kelvin would be about 240 GTC and 370 GTC, respectively.

For the value of output at the world level in 2010 we obtain the value 68.95 in trillions of dollars of 2005, again from the Nordhaus data. This implies an initial capital stock that is 4 times as high. In that case the flow of net emissions per unit of capital is given by:

$$\epsilon_A = 0.4 * \frac{10.63}{4*68.95} = 0.0154 \qquad (49)$$

Additional a priori parameter values and adjustments

For the carbon free technology we have made the assumption that depreciation is equal to that of the carbon based technology. In addition we have that in the Basic Model $B = 0.12, B_0 = 0.12, \bar{B} = 0.2$. For the R&D function we have made the assumptions that $\beta = 0.5, \zeta = 0.1$.

Finally, we have adjusted the climate tipping point in order to have a business as usual phase longer than just 5 or 10 years for the 2° Kelvin temperature rise. In fact, we have used a value of 325 GTC net which is consistent with a temperature rise of about 2.75° Kelvin. In order for the extra weather related damages to occur within the business as usual phase, we have introduced the extra damages threshold at a value of net cumulative emissions equal to 87 GTC (net).

3.2 BAM results

Preliminary parameter sensitivity analyses performed using the models show model reactions that are familiar from growth theory. Changes in the rate of discount or in the intertemporal elasticity of substitution all have the expected impact. This goes for the productivity parameters too. When emission thresholds get tighter, the shadow price of emissions rises (in absolute terms). When productivity parameters increase, so do the corresponding co-states of the associated state variables.

[20] See the website on CO2 measurements on Mauna Loa, Hawaii http://www.esrl.noaa.gov/gmd/ccgg/trends/.

Interestingly, the linking of various sequential phases introduces anticipatory behaviour that introduces transitional dynamics that are missing in an ordinary single-phase AK endogenous growth setting. The results for the parameter set and the initial values presented in the previous paragraph are listed in Figures 3.2.1 and 3.2.2.

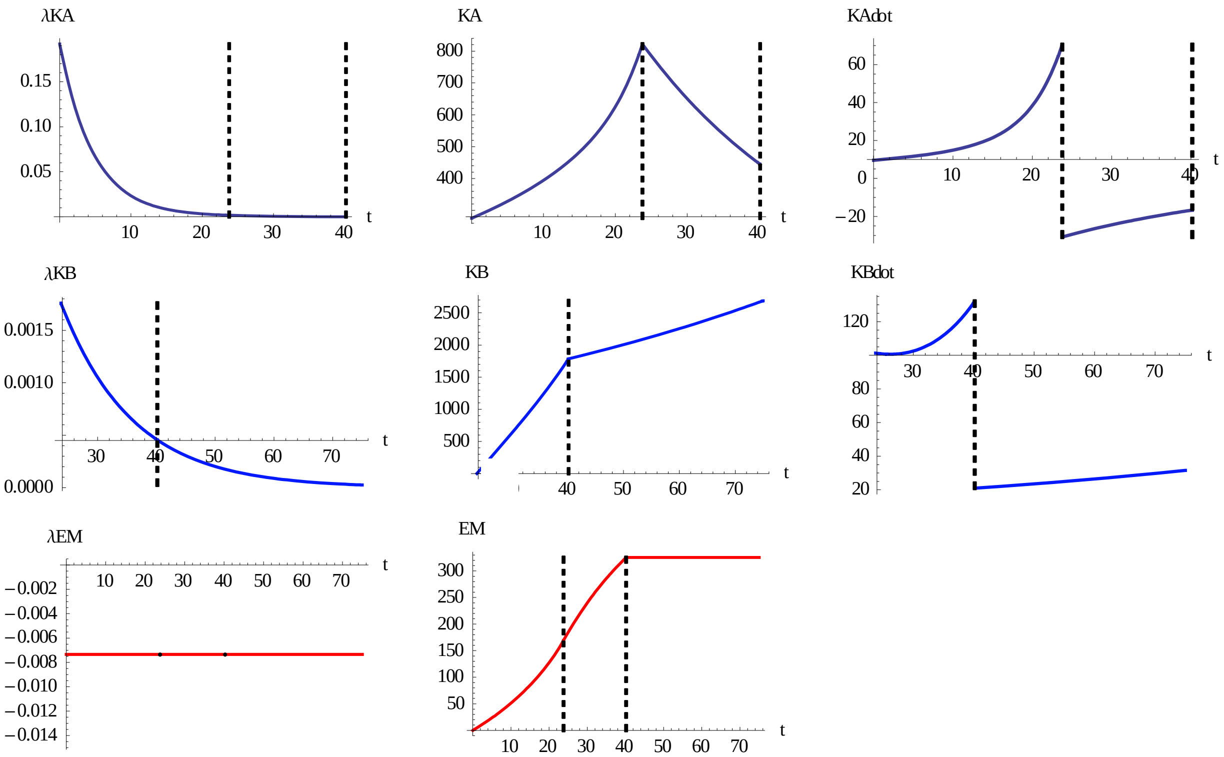


Figure 3.2.1 BAM baseline results for K_A, K_B and E

The first row of plots in Figure 3.2.1 show the outcomes with respect to carbon based capital KA. The vertical dotted lines mark the arrival times of the joint production phase and the carbon free phase. They are situated at TJ=23.78 and TF=40.17, implying that the business as usual phase takes slightly less than 24 years, while the joint production phase is slightly longer than 16 years. The first plot shows that the shadow price of carbon based capital steadily drops during both phases. The second plot shows the build-up of carbon based capacity during the business as usual phase and the subsequent run down of that capacity during the joint production phase. These events are mirrored in the third plot that shows the instantaneous rate of change over time of the carbon based capital stock. We see that net investment in carbon based capital accelerates towards the end of the business as usual phase, while it becomes negative during the joint production phase, but less negative towards the end as the absolute amount of capital lost due to technical decay is falling with the size of the capital stock still remaining.

The second row of plots shows the corresponding events for carbon free capital. Since the accumulation of carbon free capital begins at the start of the joint production phase, there is now just one dotted vertical that marks the arrival of the carbon free phase. It should be noted that net investment in carbon free capacity is rapidly increasing during the joint production phase in anticipation of the drop in capacity that will occur at the time of arrival of the carbon free phase, as carbon based capital will then be discarded. During the carbon free phase, net investment in the carbon free capital stock is much lower than during the joint production phase.

The third row of plots shows what happens to cumulative CO2 emissions. They rise exponentially during the business as usual phase. They keep on rising but at a rate that is slowing down during the joint production phase, and remain at the threshold level during the carbon free phase. The corresponding shadow price of cumulative emissions is negative (and constant), as expected.

In Figure 3.2.2 , we show the corresponding outcomes for the time paths of output (Y), consumption (C), welfare (W), felicity (F) and the instantaneous growth rate of output (gY) as well as the propensity to consume (PCONS). A striking feature of the Figure is the drop in output at the start of the carbon free phase. No such drop can be observed in the level of consumption, however. All of the drop in output comes at the expense of gross investment in carbon free capacity, as can be seen in the previous Figure, and is apparent from the changes in the propensity to consume.

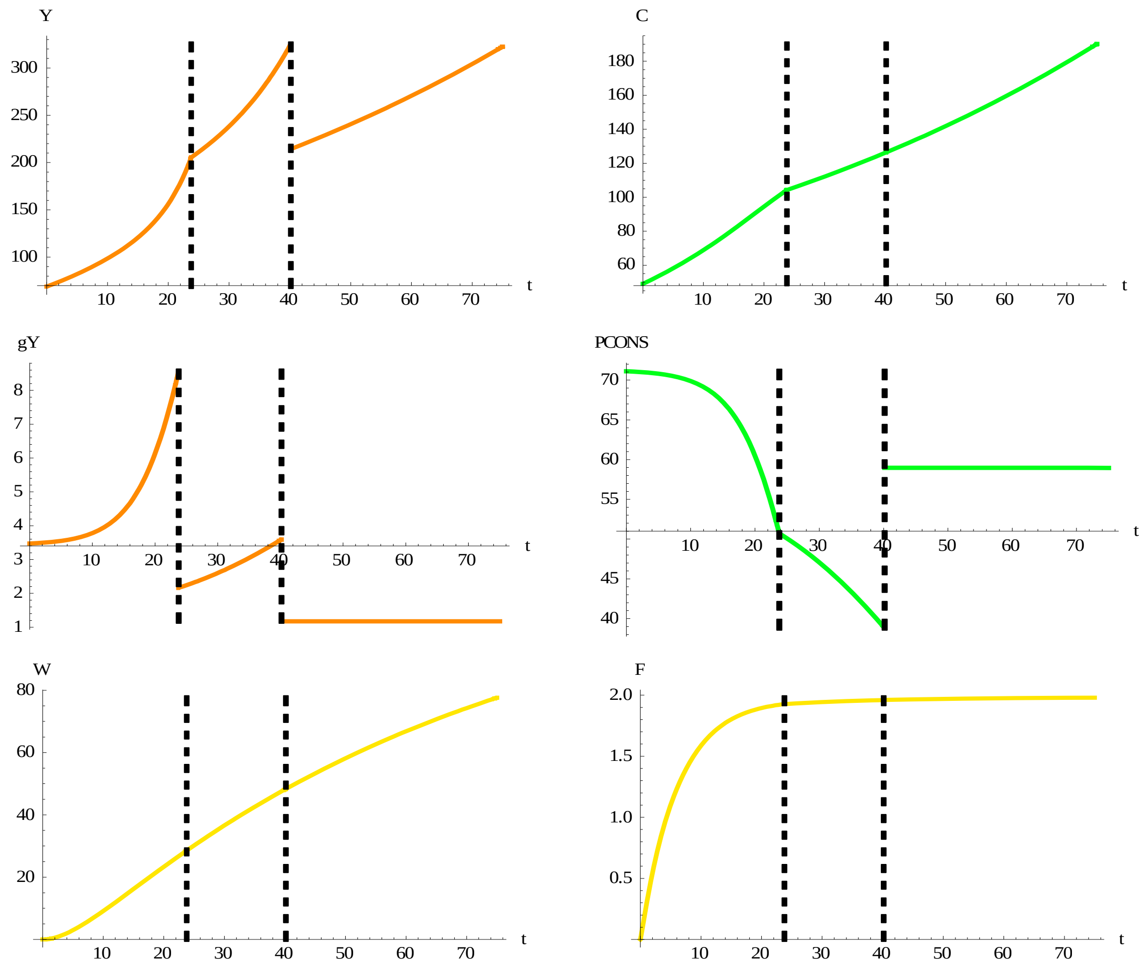


Figure 3.2.2 BAM baseline results for Y,C,F and W

The growth rate of output shows some anticipatory reactions to the changes that the arrival of a new phase will bring. For example, during the joint production phase, the average productivity of capital must fall, as the amount of relatively productive carbon based capacity decreases, and the amount of relatively unproductive carbon free capacity increases. This hold a fortiori for the arrival of the carbon free phase: when the remaining carbon based capacity is discarded, capital productivity suddenly drops to the level associated with carbon free capacity. In order to mitigate the effects on the consumption path of the corresponding drop in output, the build-up of carbon free capacity during the joint production phase should be speeded up towards the end of the production phase. A similar pattern can be observed for the build-up of carbon based capacity during the business as usual phase, as the build-up of carbon based capacity also allows a relatively high rate of investment in carbon free capacity during the next phase. Because capital is a produced means of production, a fast build-up of carbon free capacity, requires, certainly in the beginning of the build-up phase, a large carbon based capital stock.

Note finally, that the rescaling of Felicity has resulted in both positive felicity and positive welfare.

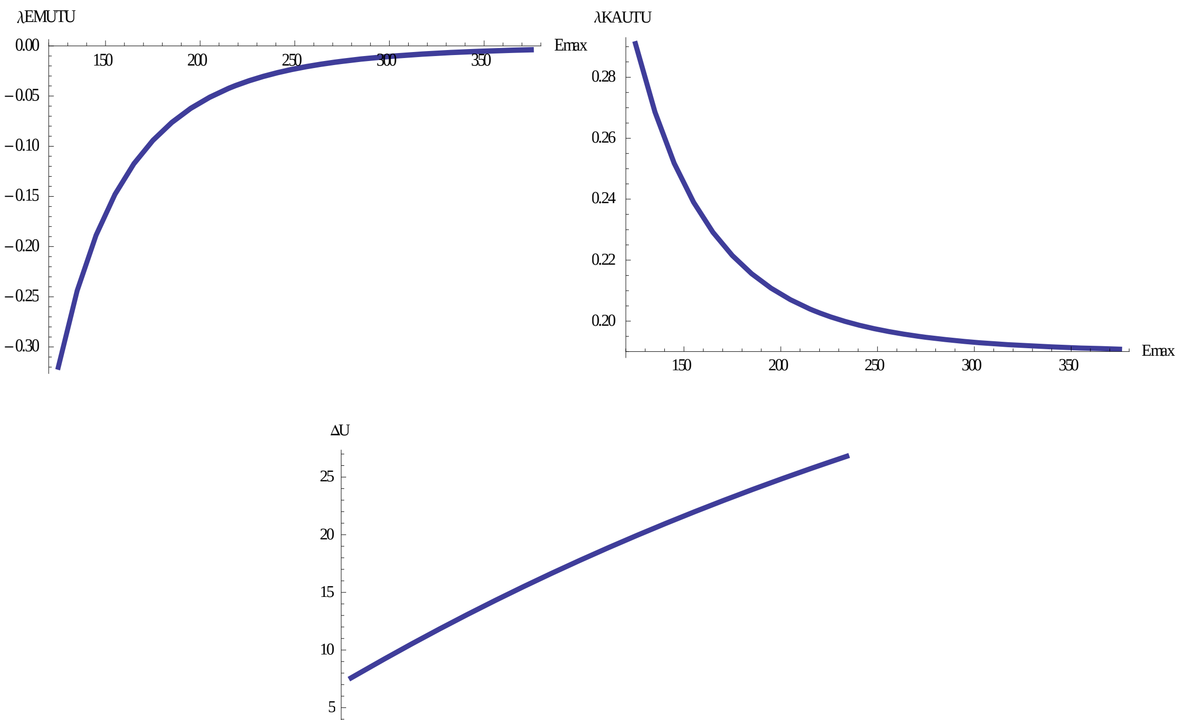


Figure 3.2.3 Sensitivity results BAM

In Figure 3.2.3 we report the consequences for the shadow prices of cumulative emissions and of carbon based capital of variations in the climate change threshold. . In Figure 3.2.3 , we present the initial values for the co-states of cumulative emissions ($\lambda EMUTU$), of carbon based capital

($\lambda KAUTU$) of the length of the BAU phase (ΔU). We also report the effects for the length of the business as usual phase. The length of the joint production phase is not affected, since it's length can be shown to depend on technological parameters only. In this sensitivity experiment we have varied the threshold value over the range 125-375 which is consistent with a temperature rise range of 2-3 degrees above preindustrial levels.

As stated earlier, a tighter thresholds induce a rise in the shadow price of emissions, in this case a more negative value of that shadow price. Maybe somewhat unexpectedly, the shadow price of carbon based capital rises as well as the threshold becomes tighter. This is a consequence of the fact that the business of usual phase shrinks to just a few years, and it becomes extremely important to have enough carbon based capital at the end of the business as usual phase in order to start building up carbon free capacity at a reasonable pace from then on. One could say that the value attributed to the carbon based capital stock is for an important part derived from the value of the carbon free stock that it is able to produce.

As one can see from the third plot in Figure 3.2.3 which represents the length of the BAU phase, the impact of a tightening of the emission constraint is considerable. In fact, for the 2° Kelvin rise in temperature, the length of the business as usual phase would be less than 10 years.

The outcomes of the sensitivity results for the other variables of the model will be presented in a different way. Each value within the range of threshold values generates its own set of time paths. We will put all of these paths together in one plot, but they will be collared differently. Low values within the emission threshold range will be associated with the low-frequency colours in the rainbow spectrum (the red part of the spectrum), while high values within the emission threshold range will be associated with the high-frequency (i.e. the violet) part of the spectrum. Intermediate threshold values will have a corresponding colour of the rainbow spectrum.

Since the carbon free end-phase has a constant steady state growth rate, we can limit yourselves to showing just the first part of the corresponding time paths, in our case until 75 years from now.

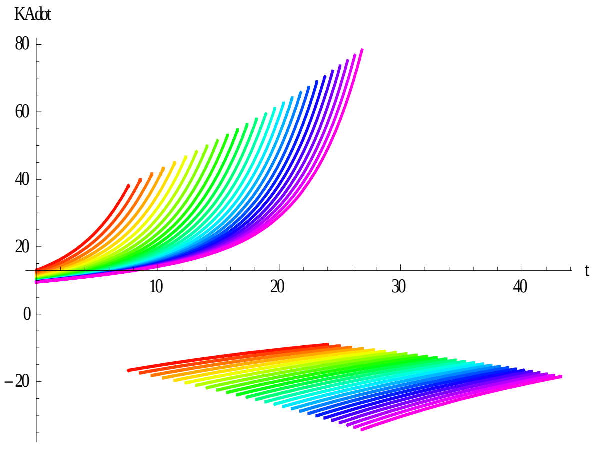


Figure 3.2.4.A $\dot{K}_A$: Emax=125-375

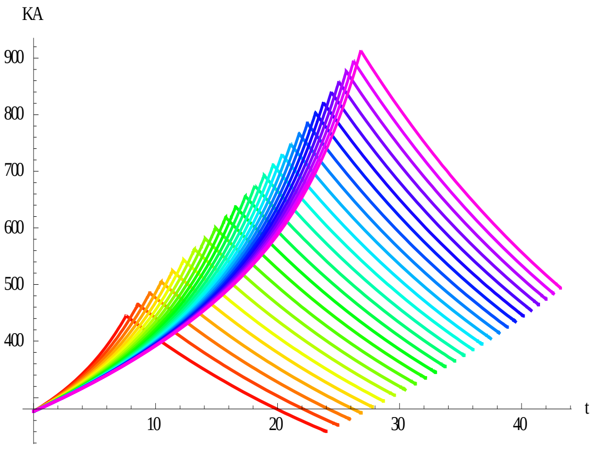


Figure 3.2.4.B K_A : Emax=125-375

Figures 3.2.4.A and 3.2.4.B show the time paths for the carbon based capital stock and for corresponding net investment. As stated above, the red lines correspond with a tight emission constraint and the violet lines are associated with the loosest emission constraint. We can observe that as the threshold becomes tighter the arrival of the carbon free phase is speeded up. Figure 3.2.4.A shows two blocks of lines. The leftmost block is associated with the business as usual phase. The rightmost block is associated with the joint production phase. Note that the endpoint of a particular time path in the leftmost block coincides in time, but not necessarily in value, with the initial point of that same time path in the rightmost block. In figure 3.2.4.B these points do not only share the same time coordinate, but also the same value of the capital stock. This is because states and co-states do not jump, whereas the time derivatives can. We see that looser emission constraints tend to lengthen the business as usual phase. For a given length of the joint production phase, this implies that the arrival time of the carbon free phase will be postponed by the same amount as the business as usual phase is lengthened.

There is a striking difference between the patterns for net investment of the carbon based capital stock, and of the carbon free capital stock. As the emission constraint is loosened, the whole of the net investment curve of carbon free capital is shifted upwards over the entire joint production period. In the case of carbon based capital, however, a loosening of the emission constraint implies both a lengthening of the business as usual phase and a downward shift of the net investment time path. At the end of the business as usual phase, however, the downward shift is more than compensated by the rise in net investment taking place over a longer stretch of time. The counterpart of this sequence of events is shown in figure 3.2.4.G which shows the time paths of consumption. Here we see that the longer business as usual phase calls for higher levels of consumption in the beginning, and correspondingly lower levels of net investment in carbon free capacity.

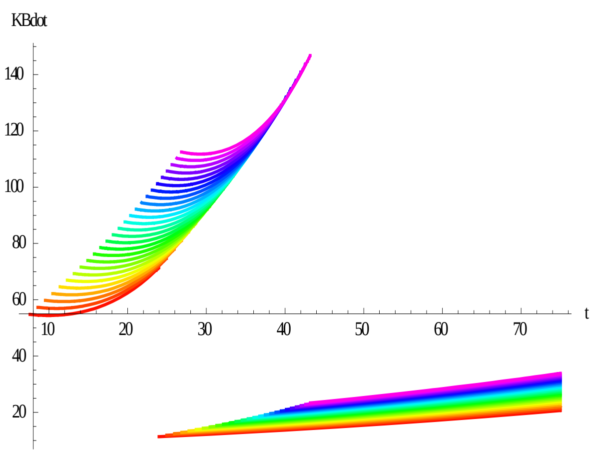


Figure 3.2.4.C $\dot{K}_B$: Emax=125-375

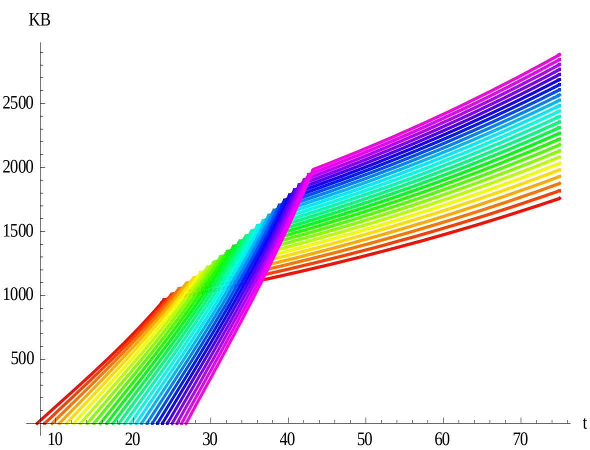


Figure 3.2.4.D K_B : Emax=125-375

The implications for the level of the carbon free capital stock are shown in Figure 3.2.4.D. With a tighter emission constraint the joint production phase comes earlier, while the capital stock which reaches a lower level at t=75 and hence will also be lower at $t = \infty$. Again, this is a consequence of the fact that capital is a produced means of production: if the carbon based capital stock is limited in size by the existence of binding cumulative CO2 emission constraints, then the implication is that the carbon free capital stocks will be limited in size as well.

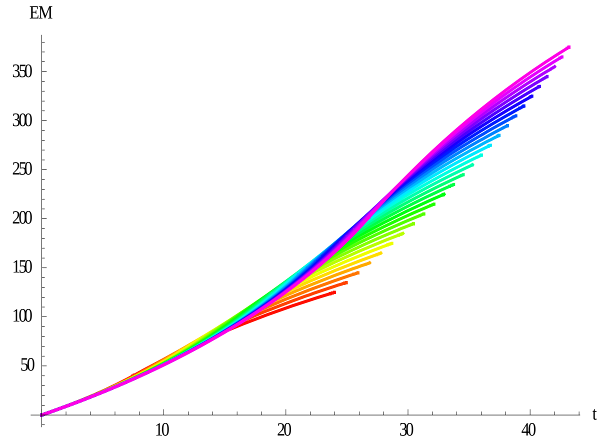
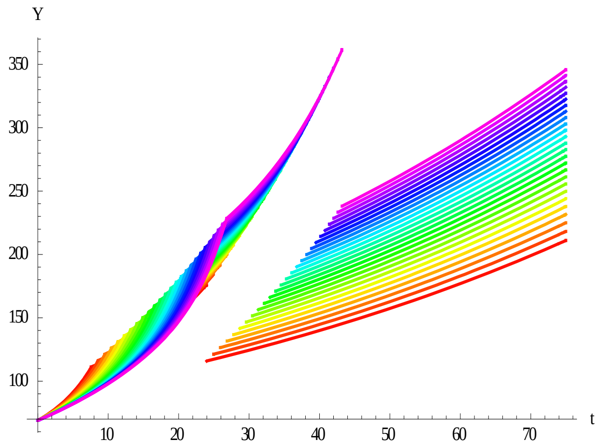


Figure 3.2.4.E Cumulative emissions E : Emax=125-375

Obviously, limiting emissions shows up directly in the time paths of cumulative emissions, both in terms of the value of the endpoints, but also in terms of the associated time coordinate, as can be seen from Figure 3.2.4.E. Events for output are more interesting, since output in the most constrained case reaches higher levels at the end of the BAU phase, but then loses out to the less constrained cases, as shown in Figure 3.2.F below.



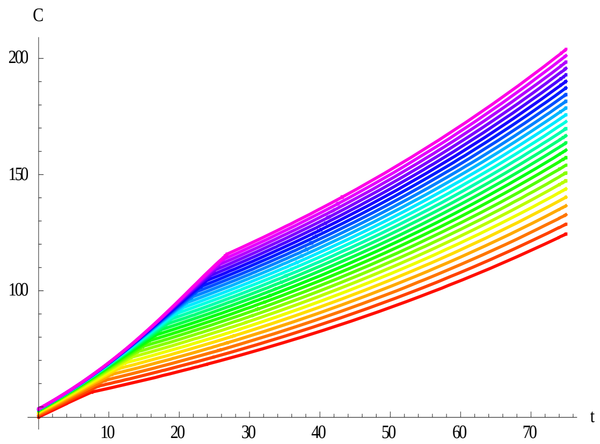


Figure 3.2.4.F Output : Emax=125-375

Figure 3.2.4.G Consumption : Emax=125-375

Output in the tighter constrained cases being taken over by output in the less constrained cases, is not reflected in the time paths for consumption, as shown in Figure 3.2.4.G. Here consumption in the least constrained case outperforms every other consumption time path: there is no taking over.

The overtaking mentioned earlier is reflected by the pattern of growth rates depicted in Figure 3.2.4.H. The values of the growth rate of output at the end of the business as usual phase are the same for all threshold values within the range. However, the periods of time during which these growth rates affect the level of output are very different, so much so that the positive effect on the level of output (see Figure 3.2.4.F) of an extension of the business as usual phase as the emission constraint becomes less tight, outweighs the negative effect of the initial drop in the growth rate of output as emission constraints become looser. The fact that the model converges to the same steady-state, but at different moments in time, is reflected by the horizontal line in the rightmost part of Figure 3.2.4.H

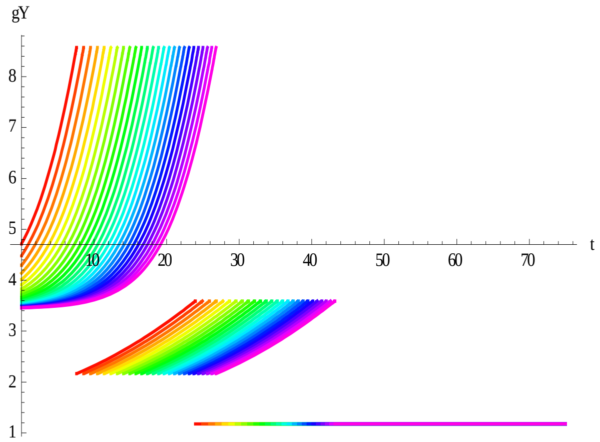


Figure 3.2.4.H Growth rate $\dot{Y}/Y$: Emax=125-375

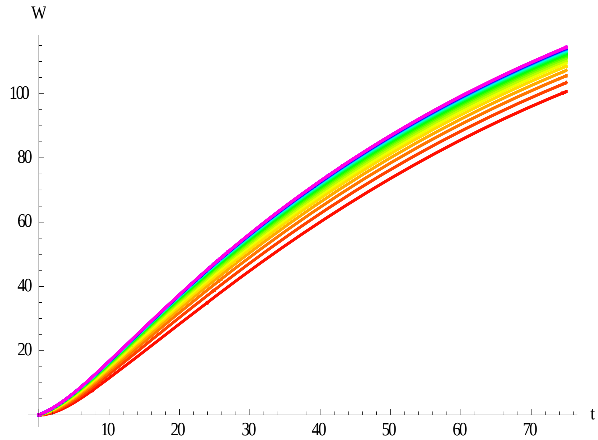


Figure 3.2.4.I Welfare W : Emax=125-375

The effect on welfare of looser emission constraints is positive as expected and as one can see from Figure 3.2.4.I. In addition to this, the time paths are getting closer together in the vertical direction, as emission constraints become looser, implying that the welfare effects associated with the trade-offs between consumption and investment during the business as usual phase and the joint production phase become more important as the business as usual phase decreases in length for a tightening of the emission constraint.

3.3 BAM+R&D results

The base line results for BAM+R&D are presented in Figure 3.3.1. The length of the BAU phase and the JPR phase are now 27.36 and 13.45 years. Hence, the arrival date of the carbon free phase has been ever so slightly postponed relative to BAM (TF=40.81 for BAM+R&D and TF=40.17 for BAM), but the main difference is in the relative lengths of the BAU and JPR phases. With endogenous R&D, the BAU phase is lengthened from a value of 23.8 to 27.4 years, whereas the length of the JPR phase is reduced from 16.4 in BAM to 13.4 in BAM+R&D. This is an illustration of the fact that accumulation of physical carbon free capital with low productivity for longer periods of time (as in BAM) is a substitute for the accumulation of high productivity carbon free capital for shorter periods of time (as in BAM+R&D) PLUS the accumulation of productivity change through R&D prior to the JPR and CFR phases.

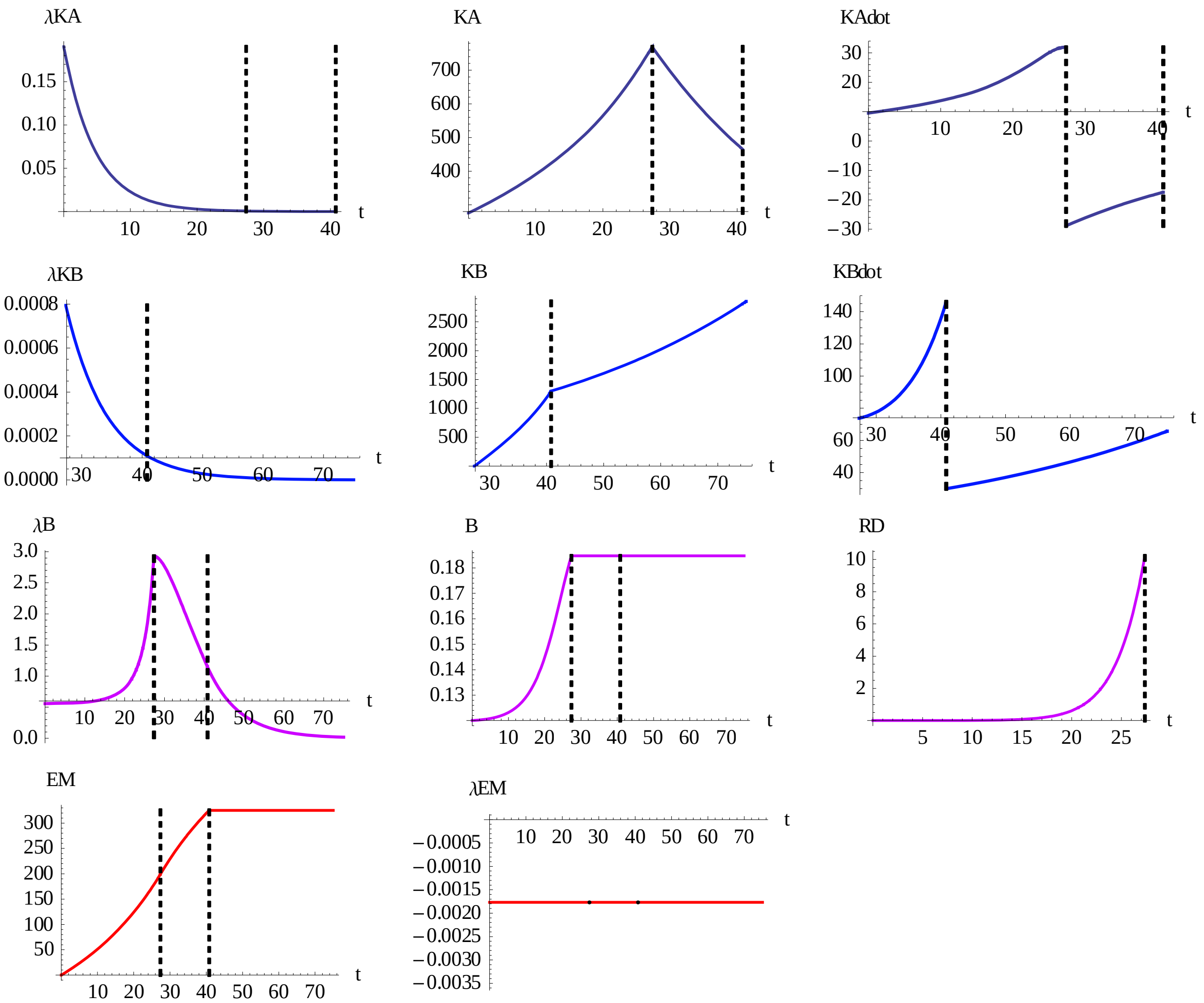


Figure 3.3.1 BAM+R&D baseline results for K_A, K_B and E

The big difference with respect to Figure 3.2.1 is the presence of the third row of plots that is now associated with the capital productivity B of the carbon free technology. The shadow price of capital productivity rises during the business as usual phase, and then falls. The reason why the shadow price rises at first is that during the business as usual phase the capacity to produce carbon free capital that will embody the new value of the capital productivity is rising. This represents an increase of the value of doing R&D. But since the productivity of doing R&D is high for low values of the productivity of capital, the corresponding impact on R&D levels is positive but limited, as long as B is relatively low. As B approaches its asymptotic value, however, the level of R&D activity increases exponentially, but at a relatively late stage in the business as usual phase. At the end of that phase, R&D activity ceases and B remains at a constant level (below its asymptotic value) while the shadow price of B will be falling from then on.

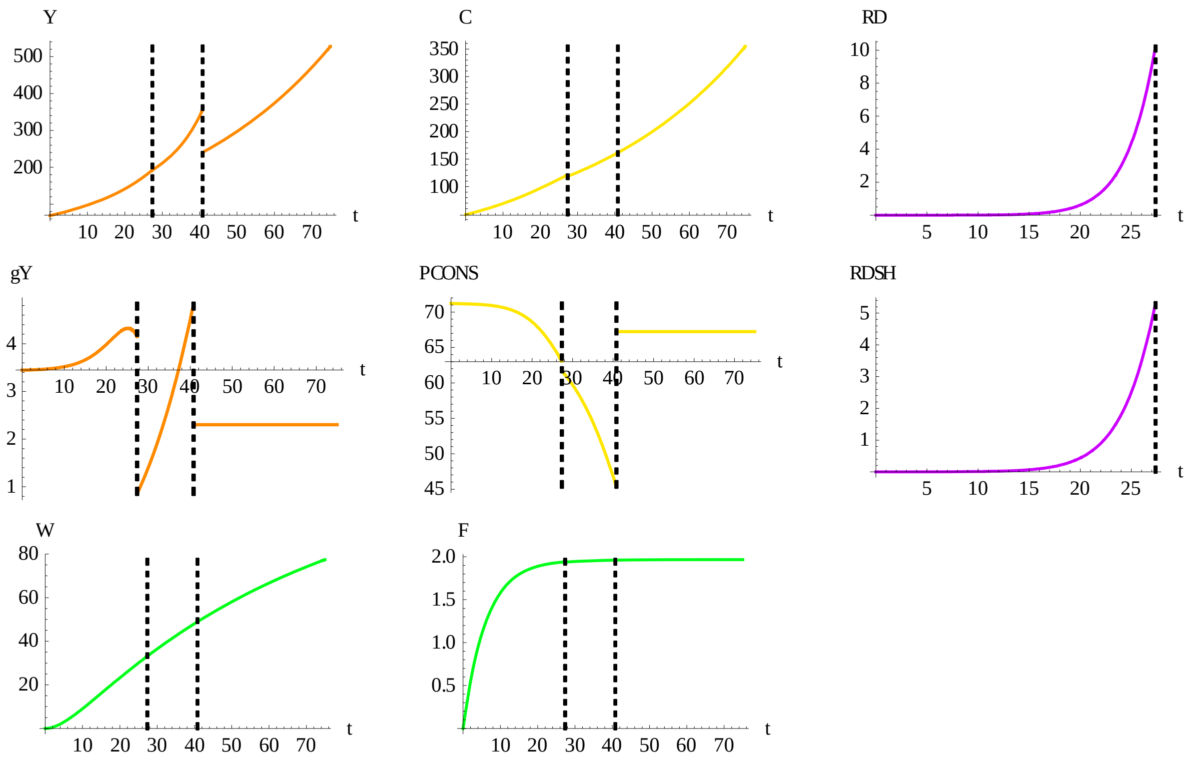


Figure 3.3.2 BAM+R&D baseline results for Y,C,R&D,F and W

In Figure 3.3.2, we show absolute R&D expenditures (RD) as well as the percentage share of R&D expenditures in total output. It should be noted that the exponential rise in R&D expenditures at the end of the BAU phase is reflected by the slowdown of the growth rate of Y at the end of the BAU phase. This is because net investment in the carbon based capital stock is slowing down as well to accommodate the increase in R&D expenditures. Note that during the joint production phase, the rate of net investment in carbon free capacity is significantly higher under BAM+R&D than in the BAM case because R&D in the previous phase has raised the return to investment in carbon free capacity relative to BAM.

For BAM+R&D we have run the same sensitivity experiment regarding the location of the cumulative emission threshold as for the BAM model. In Figure 3.3.3 , we present the initial values for the co-states of cumulative emissions ($\lambda EMUTU$), of carbon based capital ($\lambda KAUTU$)and of the productivity of carbon free capital($\lambda BUTU$), as well as the length of the BAU phase (ΔU) and the JPR phase (ΔJ). The plots for the initial values of the co-states of cumulative CO2 emissions, and of carbon based capital are very similar to the ones we had obtained for BAM. The plots for the initial value of the shadow price of carbon free capital productivity shows that tighter emission constraints tend to raise the value of doing R&D, as expected. We also find that the length of the business as usual phase is increased by a couple of years across the board. One of the reasons is that the R&D process itself being specified as a concave function in R&D expenditures, introduces a tendency to spread R&D expenditures over time. Hence, the longer the BAU phase, the higher the benefits that can be had from spreading R&D expenditures. A striking difference with respect to BAM is that the length of the joint production phase is no longer independent of the value of the emission constraint. In the discussion of the BAM results it was mentioned that the length of the JPR phase depends on technology parameters only. The latter were constant under BAM. Endogenous R&D activity now causes B to change over time.

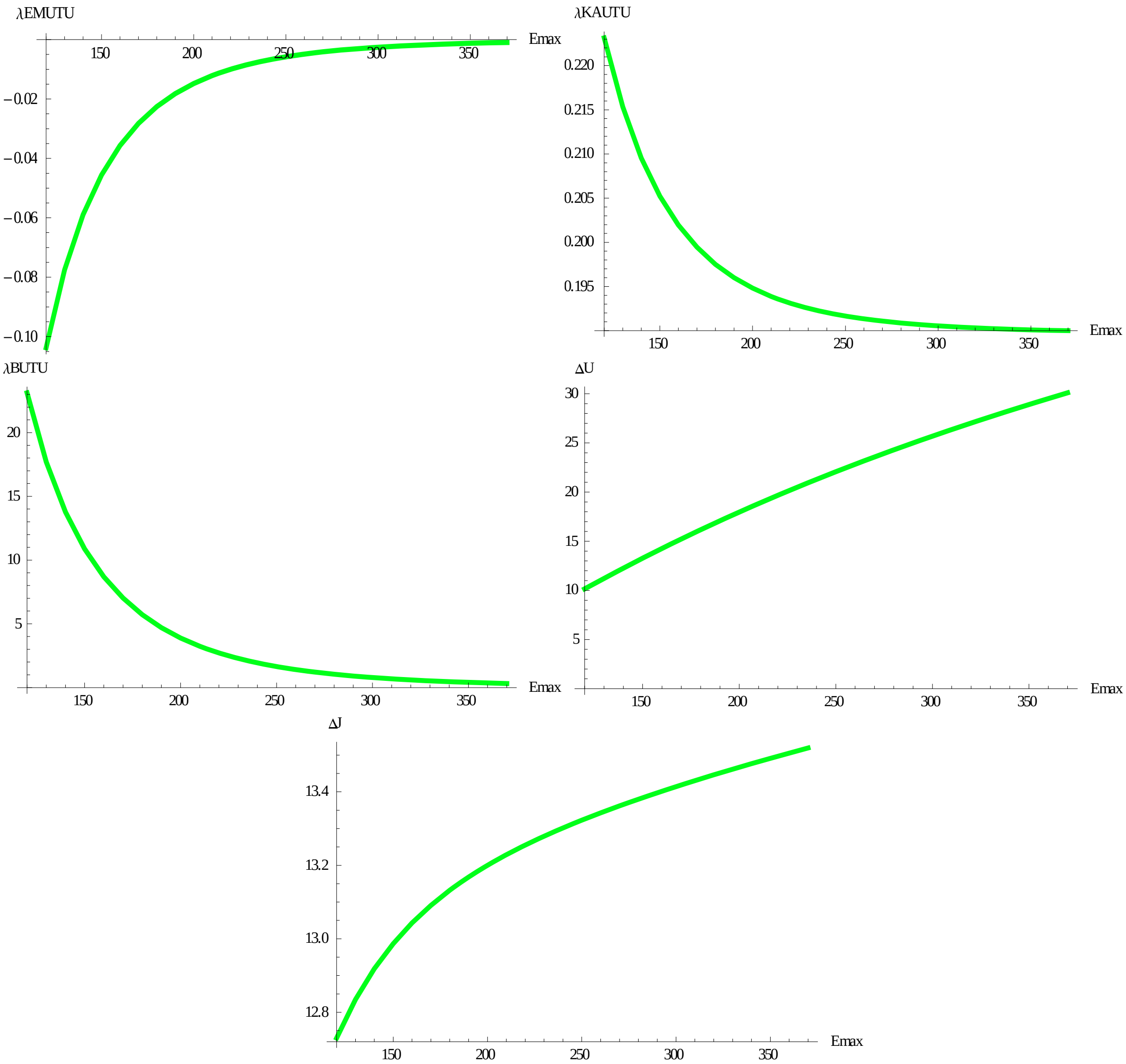


Figure 3.3.3 Sensitivity results BAM+R&D

In Figure 3.3.4 we show what happens to the development over time of carbon free capital productivity when emission constraints get less tight. We see first that the business as usual phase becomes considerably longer. We also see that the terminal value of carbon free capital productivity rises, implying a permanently higher growth rate during the carbon free phase.

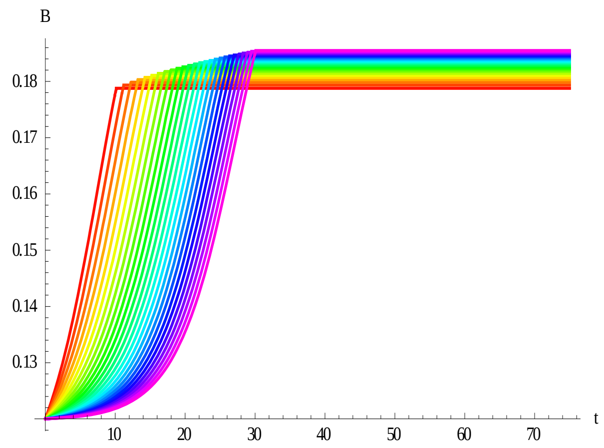
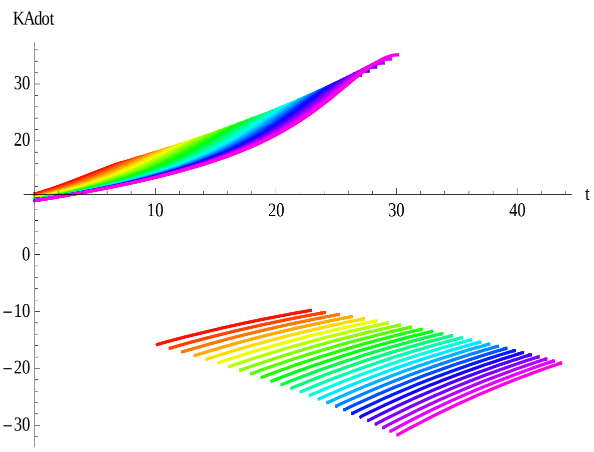


Figure 3.3.4 Time paths of capital productivity B

It should be noted from a comparison between Figures 3.3.3.A and 3.3.5.A that net investment in carbon based capital during the business as usual phase is stretched out over time as the emission constraint gets less tight, while, moreover, the level of net investment in carbon based capacity is lower under BAM+R&D than under BAM. Nonetheless, the terminal values of the carbon based capital stock at the end of the business as usual phase are of comparable magnitude, but the time paths of the carbon based capital stocks during the business as usual phase, are much closer together under BAM+R&D than under BAM. This is immediately apparent from the fact that the first block of lines in Figure 3.3.5.A is much denser than the first block of lines in Figure 3.3.3.A, implying a more even development over time of the carbon based capital stock under BAM+R&D than under BAM.



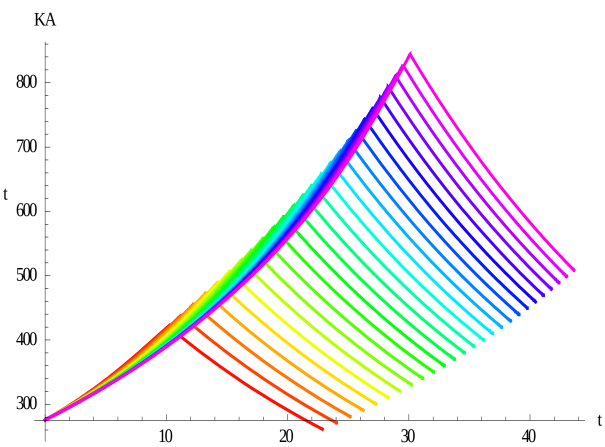


Figure 3.3.5.A $\dot{K}_A$: Emax=125-375

Figure 3.3.5.B K_A : Emax=125-375

For the carbon free capital stock, however, the rate of net investment under BAM+R&D rises more rapidly over time than under BAM, while, moreover, the periods during which the build-up of carbon free capacity is realized, is somewhat shorter. In all cases, however, the terminal value of the carbon free capital stock in the JPR phase under BAM+R&D exceeds that of the corresponding terminal value under BAM.

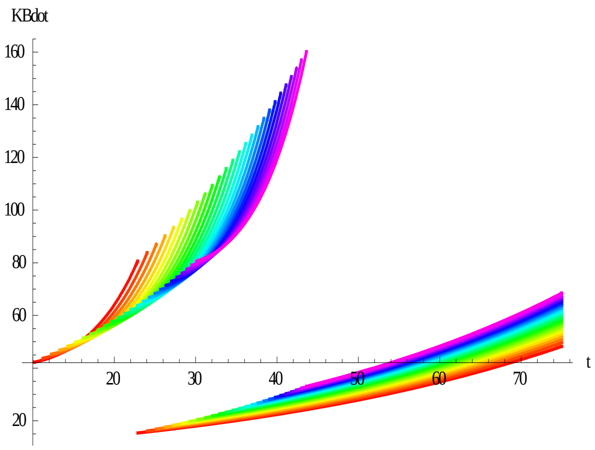


Figure 3.3.5.C capital $\dot{K}_B$: Emax=125-375

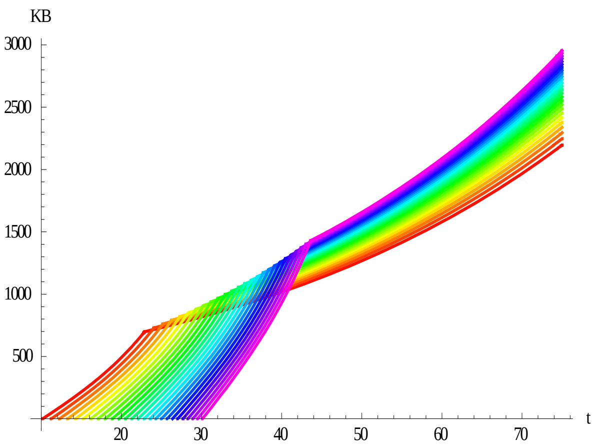


Figure 3.3.5.D K_B : Emax=125-375

The effect is of course that the terminal values of the carbon free capital stocks at t=75 under BAM+R&D exceed the ones under BAM. In addition to this, we see that the spread in terminal values for the carbon free capital stock at t=75 is much smaller under BAM+R&D than under BAM.

With respect to the time path of emissions, we see the shock absorbing nature of R&D activity at work during the business as usual phase, since the time paths of cumulative emissions virtually coincide for a large part of the BAU phase.

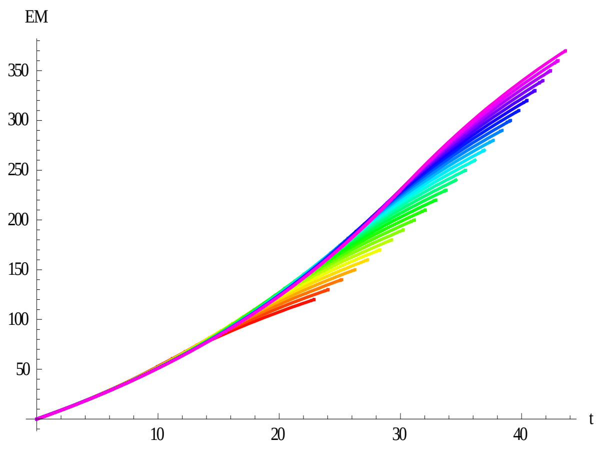


Figure 3.3.5.E Cumulative emissions E : Emax=125-375

The more even development over time under BAM+R&D than under BAM is also reflected in the plots regarding output. The kinky growth patterns observed under BAM are far less outspoken under

BAM+R&D (cf. Figures 3.3.5.& and 3.3.3.F as well as 3.3.5.H and 3.3.3.H). The same holds for consumption (cf. Figures 3.3.5.G and 3.3.3.G).

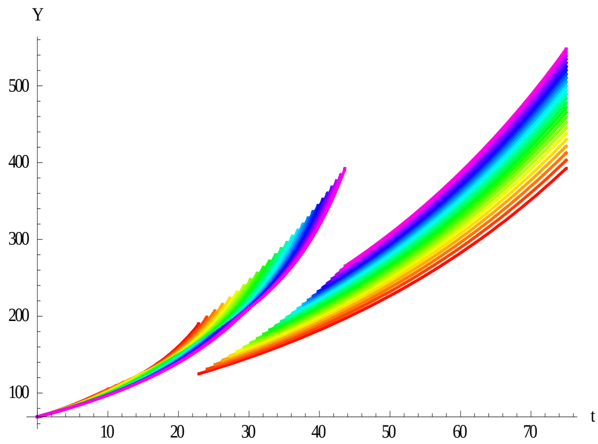


Figure 3.3.5.F Output : Emax=125-375

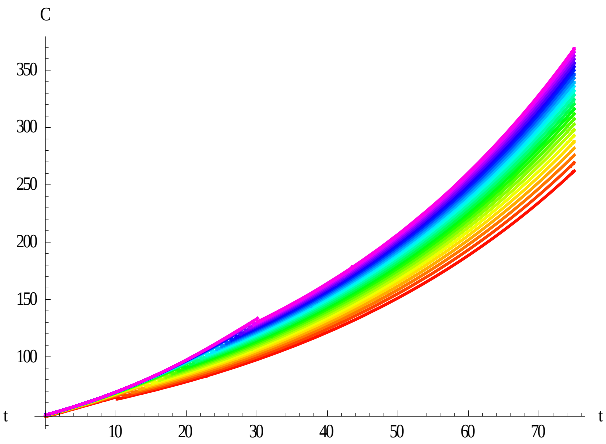


Figure 3.3.5.G Consumption : Emax=125-375

One difference between BAM+R&D and BAM is very noticeable by comparing Figures 3.3.5.H and 3.3.3.H). First of all, during the BAU phase under BAM+R&D the average growth rate of output is lower than the average growth rate under BAM. Secondly, under BAM+R&D, the average growth rate slows down at the end of the business as usual phase. During the joint production phase, however, the range of variation of the growth rate of output is much larger under BAM+R&D than under BAM, while, moreover, the steady state growth rate under BAM+R&D is higher than under BAM and slightly rising as emission constraints become less tight. All of this leads to a much more even development over time of consumption, welfare and felicity, as can be seen by comparing Figures 3.3.5.G and 3.2.5.G as well as Figures 3.3.5.I and 3.2.5.I. These Figures highlight the fact that having the possibility to change productivity through R&D helps to fight the negative effects of tightening emission constraints.

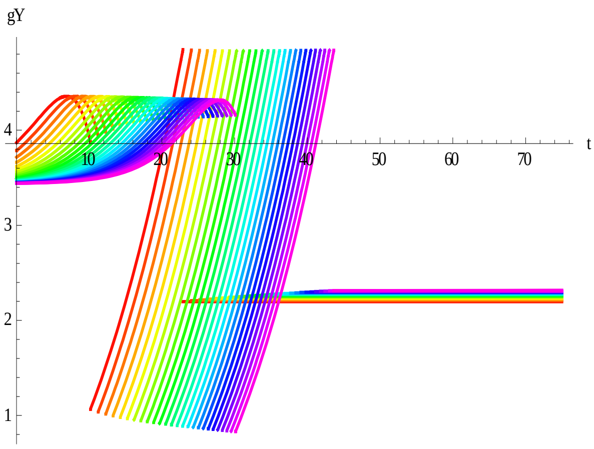


Figure 3.3.5.H Growth rate $\dot{Y}/Y$: Emax=125-375

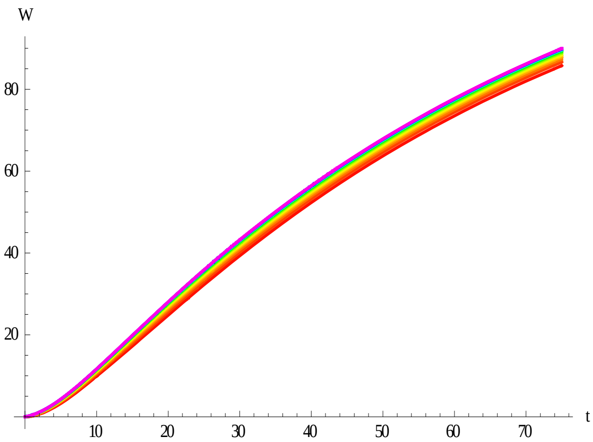


Figure 3.3.5.I Welfare W : Emax=125-375

3.4 BAM+R&D+UCL results

The results for this experiment have been obtained as follows. We have first made the assumption that the high damage depreciation parameters are exactly the same as the low damage parameters. In that case the model is reduced to the BAM+R&D model. Then we have introduced a value for the damage threshold that was well within the BAU phase of the BAM+R&D model. For a threshold situated at 87 GTC net cumulative emissions from current levels, the damage threshold was situated at about 15 years from now.

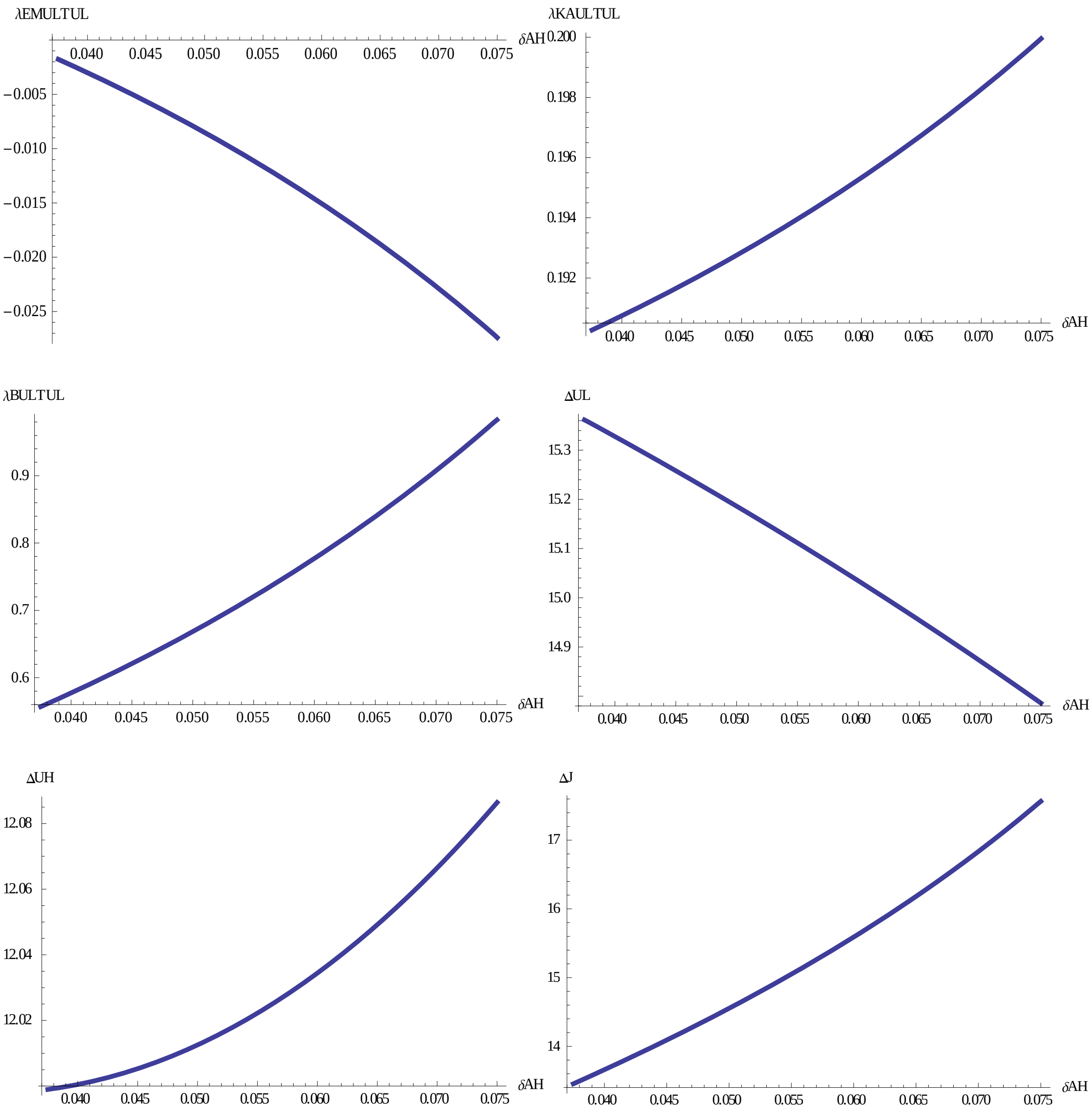


Figure 3.4.1 Sensitivity results BAM+R&D+UCL

Then we performed a sensitivity analysis in which we varied the depreciation parameter for carbon based capacity over the range 0.0375-0.075, i.e. twice the initial value. Because this model is so similar to BAM+R&D, we do not report the outcomes of the climate change threshold here. Rather, we focus on the experiment in which we account for extra damages for carbon based capital. The damage threshold effectively splits the BAU phase into a low damage first sub-phase and then a high damage sub-phase. The joint production phase and the carbon free phase will obviously also be high damage phases. The length of the various (sub-) phases for a value of $\delta_A^H = \delta_A^L = 0.0375$ are 15.36 for the low damage sub phase of BAU (i.e. ΔUL), 12.0 for the high damage sub-phase of the BAU phase (i.e. ΔUH), and 12.83 for the JPR phase (i.e. ΔJ). The sensitivity of the (sub-) phase lengths and the initial values for the co-states of carbon based capital $(\lambda EMULTUL)$ and carbon free capital productivity $(\lambda BULTUL)$ is shown in Figure 3.4.1 .

It is clear from this Figure that a rise in the decay parameter in the high damage phase will bring about a rise in the initial shadow price of emissions. This is also reflected in Figure 3.4.2 , showing the shadow price of cumulative CO2 emissions.

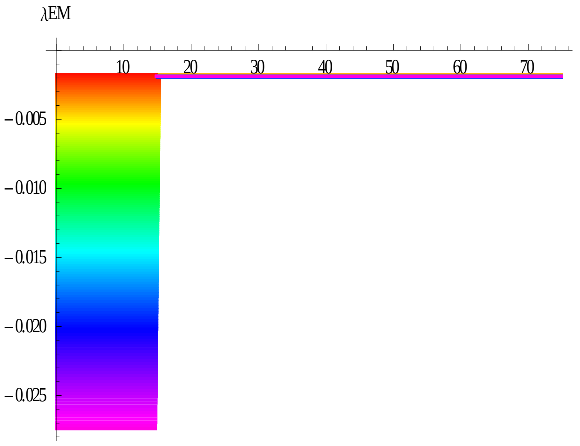


Figure 3.4.2 The shadow price of CO2 emissions

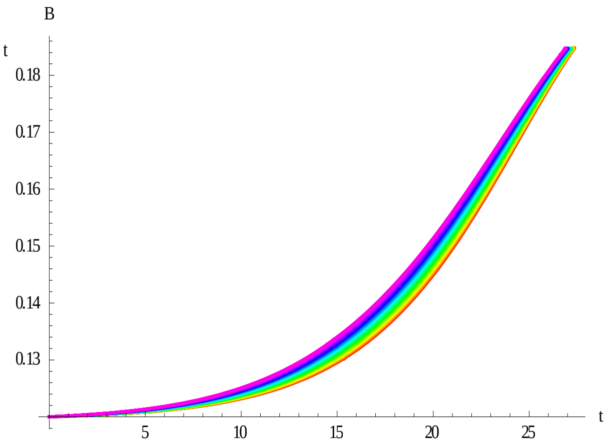


Figure 3.4.3 Carbon free capital productivity B

From Figure 3.4.2 it is clear that the jump in the shadow price of cumulative CO2 emissions is reflected by a rise (in absolute terms) of the shadow price pertaining to the low damage sub-phase. From the high damage BAU sub phase onwards, the shadow price remains unaffected by rises in the rate of decay, suggesting an absolute rise of the shadow price as compared to the BAM+R&D baseline results. The reason why this absolute rise happens is that the net rate of return on carbon based capital in the low damage sub phase has risen relative to the high damage sub phase. Consequently, there is more demand for carbon based capital during the low damage sub-phase, and therefore also more derived demand for emission space. The relative rise in the rate of return to carbon based capital in the low damage phase is reflected by the rise in its initial shadow price, as the rate of technical decay increases. Note that higher damages also make doing R&D more profitable. This is because due to higher damages a given volume of carbon based capacity will now produce less output, leading to a slower rate of physical carbon free capital accumulation. The negative long-term carbon free capacity effects this has, can be mitigated to some extent by making the lower volume of carbon free capacity more productive.

This is exactly what can be observed from Figure 3.4.3. We see that higher weather related damages tend to rotate the time path of carbon free capital productivity upwards, while, moreover, the arrival of the joint production phase is brought forward in time, albeit only slightly.

A higher rate of decay also makes the business as usual phase slightly shorter, especially because the low damage sub-phase decreases in length. The latter is caused by the fact that some of the investment in carbon based capacity during the high damage sub phase is brought forward in time, as can be seen in figure 3.4.4.A. In that figure, we see that the violet time paths are on top of the collection of low damage sub phase time paths, whereas during the high damage sub phase of the BAU phase, they are at the bottom of the collection. Note that the red time path doesn't show a break at all, because it reflects the baseline of the BAM+R&D model in which there was no difference between the low damage and the high damage sub phase of the BAU phase. A higher rate of carbon based capital accumulation in the low damage sub phase ultimately implies a higher volume of the capital stock at the end of the low damage sub phase. This would lead to a higher volume of capital to be discarded at a later date, unless the accumulation process itself stops earlier than before. Note that during the joint production phase, carbon based capital depreciates a lot faster than in the previous experiments.

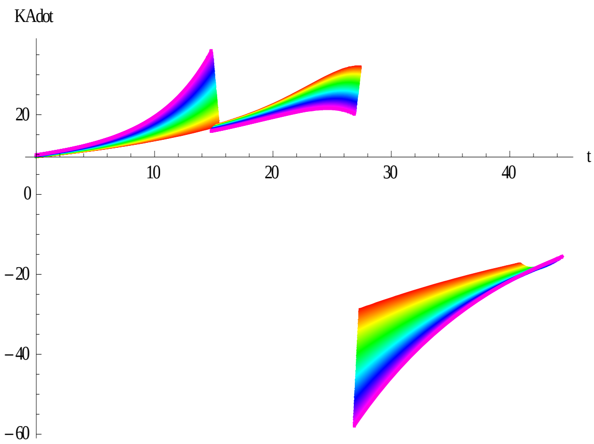


Figure 3.4.4.A $\dot{K}_A$: $\delta_A^H = 0.0375 - 0.075$

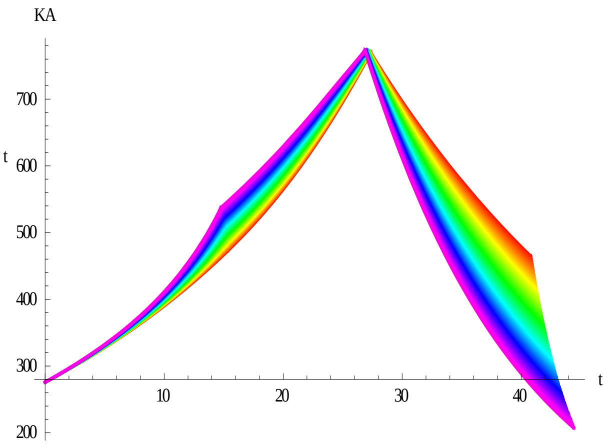


Figure 3.4.4.B K_A : $\delta_A^H = 0.0375 - 0.075$

The forward shift in time of carbon based net investment shows up as a distinct kink in the high decay parameter time paths. For a low decay parameters values, the BAM+R&D+UCL results are very close to the BAM+R&D baseline results that are identical to the reddest time path in figure 3.4.4.B.

The results for net investment in carbon free capacity show very different behaviour for low and for high damages. When damages are low, the rate of capital accumulation is rising exponentially over time. When carbon-based capital damages are high, the rate of net investment in carbon free capacity is initially higher than in the low damage phase, but at the end of the joint production phase the rate of net investment is even slightly lower than at the beginning. The reason is that due to increased damages of carbon based capital, the opportunity cost of investment in carbon free capacity have risen, since the volume of consumption is lower than on the low damage time paths. See also figure 3.4.4.H , that shows the time paths for consumption.

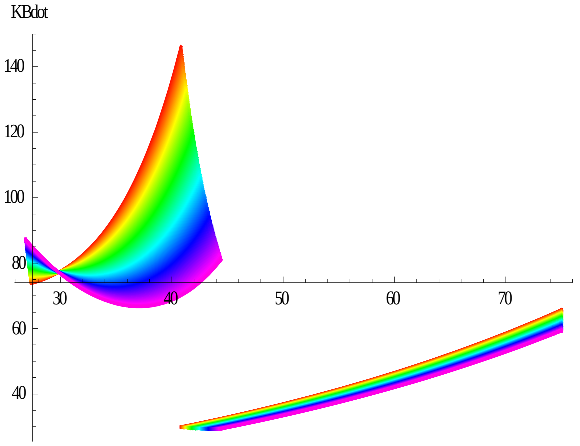


Figure 3.4.4.C $\dot{K_B}$: $\delta_A^H = 0.0375 - 0.075$

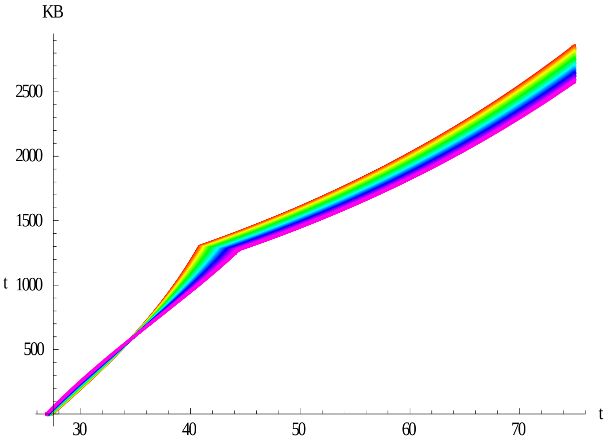


Figure 3.4.4.D K_D : $\delta_A^H = 0.0375 - 0.075$

The pattern in net investment observed in Figure 3.4.4.C is reflected in Figure 3.4.4.D, that shows the slightly higher carbon free capital stock in the beginning of the joint production phase for the high damage time paths, but after a while those high damage time paths show consistently lower levels of carbon free capital than the low damage time paths. Moreover, under high damages the arrival of the carbon free phase is postponed somewhat.

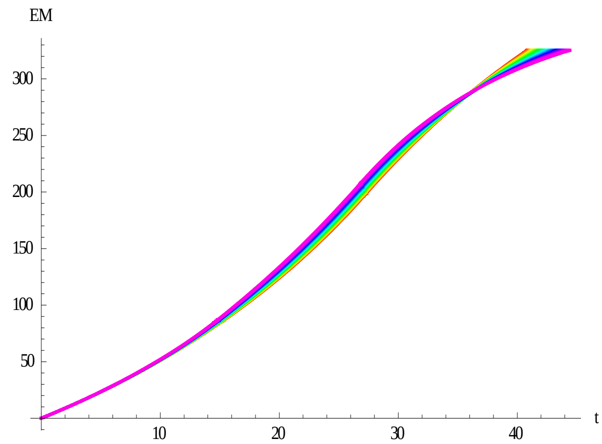
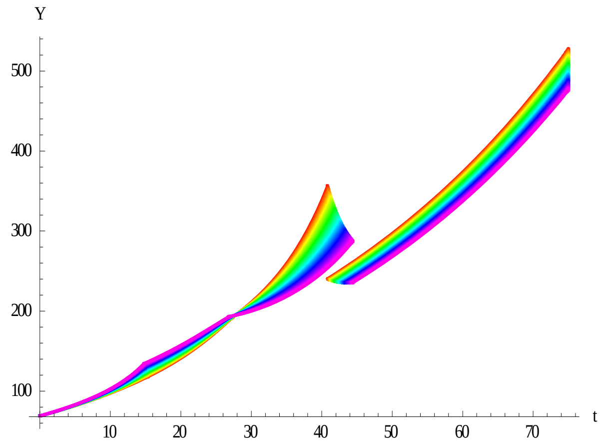


Figure 3.4.4.E Cumulative emissions E : $\delta_A^H = 0.0375 - 0.075$

In Figure 3.4.4.E , one can see that the extra net investment in the beginning of the business as usual phase under high weather-related damage conditions rotates the cumulative emission time path upwards for the largest part of the phases before the carbon free phase. The rate at which the flow of emissions decreases on the high damage time paths is however, so large that the upward shift in the beginning of the curve is more than compensated at the end of the curve, leading to an intersection of the high damage cumulative emissions curve and the low damage cumulative emissions curve a few years before the beginning of the carbon free phase.



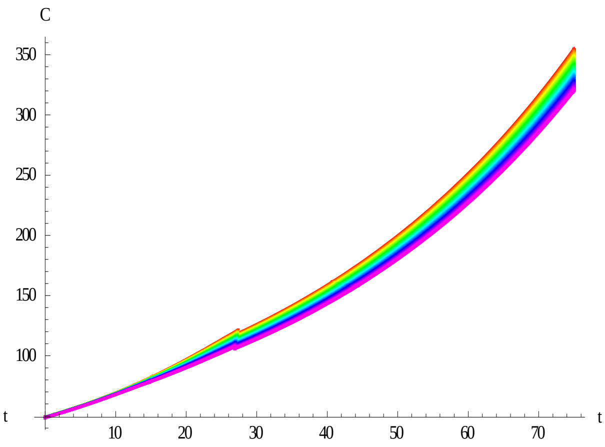
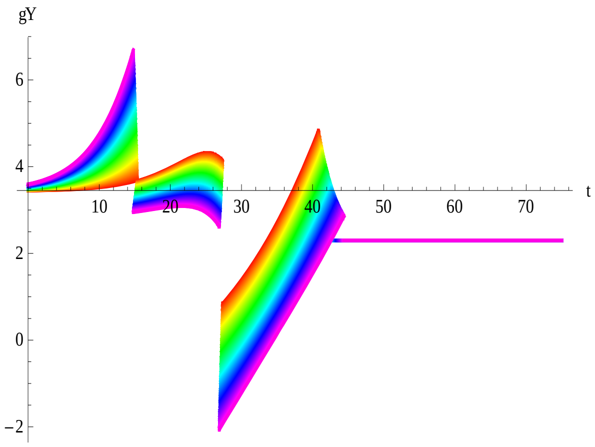


Figure 3.4.4.F. Output : $\delta_A^H = 0.0375 - 0.075$ Figure 3.4.4.G. Consumption : $\delta_A^H = 0.0375 - 0.075$

The time paths for output are provided in Figure 3.4.4.F. There is a major disruption when the carbon free phase begins, but this doesn't really affect the development over time of the corresponding consumption paths, as we have seen before, and now also in Figure 3.4.4.G. The only slight hiccup in consumption occurs at the end of the high damage sub-phase of the business as usual phase.



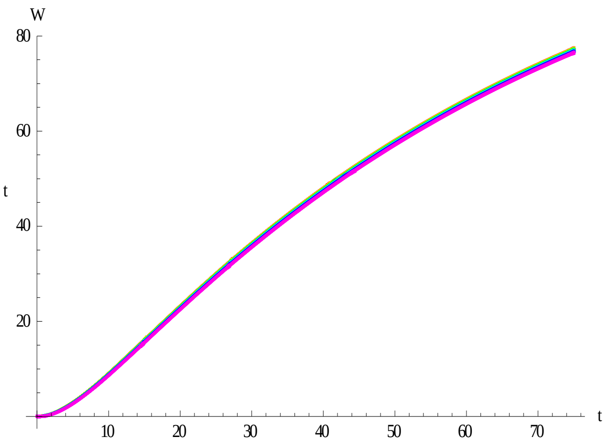


Figure 3.4.4.H Growth rate $\dot{Y}/Y$ Figure 3.4.4.I Welfare W : $\delta_A^H = 0.0375 - 0.075$

Figure 3.4.4.H shows the time paths of the growth rate of output. For the high decay parameter paths the drop in the growth rate of output at the beginning of the joint production phase is much higher than for the low decay parameter time paths. Nevertheless, welfare is hardly affected, as can be seen from Figure 3.4.4.I.

4 Summary and Conclusion

In this paper we have presented a multiphase transition model that provides the optimum investment rates necessary to make the transition from a carbon-based economy to a carbon free economy. The transition must be made just-in-time because of the existence of a cumulative CO2 emissions threshold that, when passed, throws the world into an irreversible runaway global warming regime. The irreversibility arises out of the existence of positive feedback-loops from global temperature rises to methane releases from melting permafrost and from deep sea hydrates. Methane is a much more potent GHG than CO2, and because so much of it is still stored in the permafrost and in the deep sea one would prefer (presumably *ex ante*, but certainly *ex post*) to avoid its spontaneous and unstoppable release at all costs. In order to model this, we have introduced a cumulative CO2 emissions threshold in our model, above which the irreversible climate change is assumed to be set in motion.

A central issue in our model is the fact that the transition towards a carbon free production system will require a switch in the deployment of production technologies, that will involve the build-up of carbon free capacity and the simultaneous rundown of carbon-based capacity, simply because the one type of capacity cannot be changed into the other type of capacity at little or no cost: a spade is indeed the spade. Therefore we have opted for a model setting in which we allow for two technologies that both can produce output, while one of these technologies produces CO2 emissions in the process. The latter technology is relatively productive: it has a high capital productivity, while the other technology has a low capital productivity but zero CO2 emissions. We use the setting of the AK endogenous growth model by Rebelo (1991), but extend it to two technologies. We also introduce three separate phases of production, each of them characterized by different technologies being active. It can be shown that, due to the linear production technologies employed, there will always be investment in just one of the technologies, even though output can be produced using both technologies simultaneously, as long as there is still room to emit. The first of the three phases is called the 'Business As Usual' or BAU phase, In which carbon-based capacity is still being built up, and the output produced is completely carbon-based. At some point in time the next phase arrives, in which investment in carbon-based capital ceases and the build-up and concurrent use of carbon free capacity begins. During that phase, production using carbon-based capital continues but the production level falls over time as the capital stock is worn down due to technical decay. The second phase is called the joint production phase (JPR phase for short), as both technologies are used to produce output. At the beginning of the third phase, called the 'Carbon Free' phase (CFR phase for short) the remaining carbon-based capacity is scrapped and production is from then on completely carbon free: the green future has arrived (just-in-time).

We extend the three-phase setting described above in two ways. First we introduce endogenous R&D driven technical change that improves the productivity of the carbon free technology up to the moment of the installation of the first unit that embodies the new technology. The reason to do this is that the embodiment of technology in fixed capital implies that the build-up of significant stocks of carbon free capital will take time which may turn out to be in short supply on the one hand, while on the other hand it will also draw upon the existing carbon-based capacity at the expense of consumption, which is the sole source of welfare in the standard AK-model and in our model too. To keep matters simple, we make the assumption that R&D driven technical change stops the moment the new technology starts to be implemented. The fact that the build-up of carbon free capacity to a level that is sufficient to mitigate the aggregate productivity drop associated with the scrapping of remaining carbon-based capacity at the beginning of the carbon free phase takes time, is one of the main reasons of the model's focus on the optimum timing of the changes between the different phases we have distinguished in the model. The R&D function itself, being incorporated in an AK-setting with an ultimately constant marginal product of capital, generates productivity levels that are bounded from above (asymptotic technical change), to reflect the 'fishing-out' effect known from Jones (1995). The latter effect seems to be particularly relevant with respect to the further development of specific technologies, as opposed to technical change occurring at the macro-level in the form of an expansion of a broad set of such specific technologies. The second extension pertains to the introduction of a weather related damage threshold, where we make the assumption that in this version of the model extra damages will occur (modelled as a jump in the rate of technical decay of the capital stocks), once a particular level of cumulative CO2 emissions has been surpassed. This damage threshold lies below the irreversible climate change threshold by assumption, since it could otherwise never become a truly [21]binding constraint.

The timing of the phase changes present in the various model versions are governed by transversality conditions following from the optimality condition that the Hamiltonians when evaluated at the moment of the phase change under the conditions pertaining to the phases just before and just after the phase change, are identical. In the Basic Model (further called BAM), i.e. the model without R&D and without extra weather related damages, the condition defining the optimum length of the BAU phase turns out to be the requirement that the shadow prices of carbon-based and carbon free capacity should be the same at the moment investment in the latter technology takes over from investment in the former. This makes perfect economic sense, as the opportunity cost per unit of investment in terms of consumption foregone is the same in both cases. For the timing of the change from the JPR phase to the CFR phase, the optimality condition results in a transversality condition that states that the moment that the CFR phase should begin is implicitly defined by the requirement that the benefits of continuing to use carbon-based capacity (the utility value of its output) is outweighed by the cost of doing so (the utility value of the corresponding emissions), which closely resembles the 'negative quasi-rent' scrapping condition known from the vintage literature with fixed factor proportions ex post (i.e. putty- clay and clay-clay models).[22] For the other model versions, more complicated transversality conditions arise out of the same general optimality conditions pertaining to the equality of Hamiltonians at the moment of phase change.

BAM contains two other transversality conditions, i.e. the standard one associated with the pure AK-setting of the CFR phase[23] and the one implied by the fact that from the start of the CFR phase this shadow price of carbon-based capital should be zero, as carbon-based capacity is worthless from then on. Using the requirement of the continuity of state and co-state variables at the moment of a phase change, the transversality conditions in combination with the given initial and terminal values for the state variables in the model allow us to use a (steepest descent) search method that, for a priori guesses

[21] The runaway global warming catastrophe may become more catastrophic still, but that by itself seems of little practical importance.

[22] Cf. Johansen (1959) and Solow et al. (1966).

[23] in the endogenous R&D version of the model, still another transversality condition is that as time goes to infinity, the present value of the welfare value of carbon free capital productivity should approach zero.

of the still missing initial values of a subset of co-states and of the phase lengths of the BAU and the JPR phases, solves the systems of differential equations resulting from the first order conditions of the Hamiltonian problems for each of the individual phases. Given this solution that is contingent on the a priori guesses, we can evaluate to which extent the different transversality conditions and the boundary condition for cumulative emissions are met. If one or more of these transversality conditions or the boundary condition are not met, the initial guesses need to be adjusted; if all the conditions are met then the optimum path has been found.

We then calibrated the parameters and initial values of the model, based to a large extent on Nordhaus' 2010 RICE model dataset and making some a priori assumptions regarding the capital productivity parameters and the parameters of the R&D function in particular.

Using this setup, we have run a number of simulations to investigate the sensitivity of the model for changes in the structural parameters featuring in the utility function and in the production functions, which all showed the expected outcomes known from the standard endogenous growth models. We then turned to a sensitivity analysis that involved the irreversible climate change threshold and the weather related damages threshold. In the simulation where we systematically reduced the climate change threshold in order to see what a more stringent application of the precautionary principle would mean, we find that in BAM it will be optimal to accumulate carbon-based capital at a faster pace than with a less tight cumulative emissions threshold, so that the arrival of the CFR phase is speeded up. To facilitate the latter, a quick build-up of the carbon-based capital stock is required to be able to switch relatively early to investment in carbon free capacity and to enable considerable rates of both investment and consumption once investment in carbon-based production and later on production using carbon-based capacity has ceased. Tightening the cumulative emissions constraint raises (in absolute terms) this shadow price of CO2 emissions, but also that of carbon free capital. But somewhat unexpectedly perhaps, it also raises the shadow price of carbon-based capital, simply because *the value attributed to the carbon-based capital stock is for an important part derived from the value of the carbon free stock that it is able to produce.*

When we allow for endogenous R&D reactions in this setting, we observe that the length of the BAU phase increases relative to the BAM case. The latter allows R&D activity to be more evenly spread over time, which, due to the concavity of carbon free capital productivity in R&D efforts raises the overall effectiveness of a given R&D budget. There is also more time to accumulate carbon-based capital, so that at the end of the BAU phase there can be a considerable terminal value of carbon-based capital: the extension of the BAU phase allows the economy to eat its carbon-based cake and still have a considerable amount of it left at the beginning of the JPR phase in the form of carbon-based capital. In contrast to the BAM case, the JPR phase is now shortened as the cumulative emissions threshold becomes tighter. The overall effect is that the CFR phase comes slightly earlier than in BAM, while the dispersion in the welfare effects is less than under BAM: *the possibility to change productivity through R&D helps to fight the negative welfare effects of tightening emission constraints.*

When we add extra weather related damages to this framework such that these occur within the BAU phase, we effectively introduce an extra phase so that this version of the model now contains four phases: a low damage BAU phase, a high damage BAU phase, and high damage JPR and CFR phases. The weather related damages introduce an additional incentive to engage in R&D. The reason is that the weather related damages pertain to physical capital, while the productivity of the surviving capital units, but more importantly also the future units, will remain untouched, so increasing the productivity of carbon free capital is a natural reaction to maintain a reasonable net return to future carbon free investment. In addition to the extra R&D activity, we also observe extra investment in carbon-based capacity during the low damage BAU phase. The reason is that the expectation of extra weather related damages induces a wedge in the rate of return on carbon-based investment during the low and high damage sub phases of the BAU phase. An increase in the extra damages rate also leads to a more even distribution of carbon free investment over time, in order to mitigate the negative effects on consumption of increased weather related damages. With low damage rates, investment in carbon free capacity during the JPR phase is strongly increasing in anticipation of having to cushion the drop in output associated with the scrapping of the remaining carbon-based capital stock at the beginning of the CFR phase. With higher damages, the drop will necessarily be less, *ceteris paribus*.

So what do we learn from all this? The embodiment of technical change in physical units of capital underlines the practical importance of the notion of capital as a produced means of production. It therefore stresses the need for a productive carbon-based capital production system in order to be able to produce the right amount and quality of the carbon free production units on which future welfare will exclusively come to depend. The results also underline that accounting for the embodiment of technical change seems to imply a worryingly short BAU phase. Furthermore, R&D is an important means to cushion the negative welfare effects of tighter emission thresholds and increasing weather related damages. The reason is that physical capital investment in carbon-based capacity is at the same time both a substitute for and a complement of R&D investment. It is a substitute from a pure production point of view, while it is a complement because of the embodied nature of technical change that needs physical investment to turn potential productivity improvements into real ones. Because the return to R&D and that to physical investment in carbon free capacity depend positively on each other, output itself and therefore consumption and investment possibilities are positively affected by having the possibility to engage in R&D. Hence, R&D efforts provide an additional means to both reduce the dispersion in welfare outcomes and to maintain or even increase future carbon free output levels and growth rates in the face of increasingly volatile weather events and corresponding damages and a rising probability of runaway global warming.

Appendix A. The timing of R&D

Consider the availability of $B_0>0$ at t=TBAU. Consider also the possibility to wait a while, say till T*, and then start the R&D process, still using only the A-technology for production. Then at T* the BAU phase is effectively split into two sub-phases, say U0 and U1, where U0 comes before U1. Assume, moreover, that the split at T* would be optimal. In that case the only difference between these sub-phases is the R&D process that needs to be fuelled by allocating part of output to the R&D process and that produces positively valued increases of the productivity parameter B from a given initial value B_0. So the Hamiltonian at T* for sub-phase U0 would be given by:

$$H_{T*}^{U0} = e^{-\rho.t} \cdot C_{T*}{}^{1-\theta} / (1-\theta) + \lambda_{T*}^{A} \cdot \{(A - \delta^{A}) \cdot K_{T*}^{A} - C_{T*}\} \qquad \text{(A.1)}$$

The first order condition for consumption can be solved for C, giving rise to $C_{T*} = f(\lambda_{T*}^{A}, T^*)$. Hence, the Hamiltonian can be rewritten as:

$$H_{T*}^{U0} = g(f(\lambda_{T*}^{A}, T^*)) + \lambda_{T*}^{A} \cdot \{(A - \delta^{A}) \cdot K_{T*}^{A} - f(\lambda_{T*}^{A}, T^*)\} \qquad \text{(A.2)}$$

For U1, we have:

$$H_{T*}^{U1} = g(f(\lambda_{T*}^{A}, T^*)) + \lambda_{T*}^{A} \cdot \{(A - \delta^{A}) \cdot K_{T*}^{A} - f(\lambda_{T*}^{A}, T^*) - R\} + \lambda_{T*}^{R} \cdot \varsigma \cdot R_{T*}^{\beta} \cdot (\overline{B} - B) \qquad \text{(A.3)}$$

The first order condition for an optimal allocation of R&D resources implies that:

$$\lambda_{T*}^{A} R_{T*}^{\beta} = \beta \cdot \lambda_{T*}^{R} \cdot \varsigma \cdot R_{T*}^{\beta} \cdot (\overline{B} - B) \qquad \text{(A.4)}$$

Hence, we must have that:

$$H_{T*}^{U1} = H_{T*}^{U0} + (1-\beta) \cdot \lambda_{T*}^{R} \cdot \varsigma \cdot R_{T*}^{\beta} \cdot (\overline{B} - B) > H_{T*}^{U0} \qquad \text{(A.5)}$$

for a positive level of R&D activity from T*, as we have assumed. It follows that the difference $H_{T*}^{U0} - H_{T*}^{U1} < 0$, and so T* should be reduced[24], in this case till the beginning of U0, i.e. till T*=TBAU, since the difference between both Hamiltonians would not vanish. This implies that the R&D process should begin at the moment that the B-technology is discovered. In our case we make the assumption that $B_0>0$ from t=TBAU=0.

[24] By reducing T* by 1 unit of time, i.e. let the U1 phase begin one unit of time earlier, we lose the Hamiltonian at the end of U0 and gain the Hamiltonian at the beginning of U1, which in this case would lead to a net gain.

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