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# **Endogenous Economic Growth through Connectivity**

by Adriaan van Zon and Evans Mupela  
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## **Abstract**

In this paper we show the benefits of regional connectivity and specialization to growth. Starting with one region we show how welfare measured by utility per head increases as the number of connected regions increase. We assume a common connectivity infrastructure implemented by satellite, through which the 'Great Connector' (GC) is able to add new regions to the pool of connected regions by taking a tax from those already connected. We find that increasing production costs leads to faster transitions towards the steady state whereas increasing transportation and communication costs tends to lengthen the transition. The results point to reductions in transportation and communication costs in particular as a suitable vehicle to speed up growth. The results also show a strong positive effect of reductions in the cost of making new connections. This has a significant impact on both the steady state growth rate and on transitional growth, while significantly reducing the transition period.

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## 1. Introduction

Since the days of Adam Smith and David Ricardo, it is widely known that the specialization of production activities and the subsequent trading of the fruits of such specialization is able to generate higher benefits from resource use than would be possible in the case of pure self sufficiency. Smith provides the famous example of the pin factory, where the set of all workers specialized in different sub-tasks of pin-making taken together are more productive than the same set of workers if each individual worker would have to cover all sub-tasks by himself. With David Ricardo, the benefits from international specialization arise from the so-called comparative advantages of countries in particular parts of the tradable goods-spectrum. The sources of welfare increase are indeed concentration of productive activity on comparative advantage goods and the subsequent trading/exchanging of the products/services produced among countries. This requires a high degree of connectedness between trading partners that is assumed a priori but does not have to exist in actual fact.

Take rural areas in African developing countries, for example. Communities in such areas are often relatively disconnected from other communities, since both means of transportation like cars, trains, and complementary infrastructure (roads, bridges, railroads) and means of tele-communication are often lacking. This implies that such communities are forced to be self-reliant to a large extent, and, if such communities are relatively small, which they usually are, then it may be difficult for such communities to attain a level of welfare through the specialization of production activities that is attainable for larger communities.

In this paper, we want to look into this matter more closely, by formulating a very simple or even simplistic model of growth through specialization that relies on the provision of communication and transportation infrastructure for welfare growth to take off. The growth in welfare is due to communities becoming connected through communication channels and through transportation infrastructure, providing the possibility of trade in goods and services. Hence, in our model, the provision of information and transportation infrastructure is a *condition sine qua non* for growth to occur.

To find out how large the impact of the provision of such infrastructure is on economic growth, we formulate a model that is largely based on a stripped-down version of Krugman (1979), except that we use it as a template for the description of how a community works when left on its own. Then we add an additional top-layer to the Krugman template in which we allow different communities to communicate and trade with each other. Moreover, by allowing the 'local' community to go 'global' in this way, all communities that are connected to each other can concentrate on their comparative strengths, and trade. By explicitly introducing costs of trading, and costs of being connected, it is not self-evident that being connected pays off. Nonetheless, we try to find out how a benevolent central planner would have to play his role as the 'Great Connector' (further called GC) in such a way as to optimize the development over time of utility per head for all people 'touched' by the GC. We will show that under particular parameter constraints, the GC will want to connect more and more people, because it is in the interest of the people already connected to do so. We look into the steady state rate of the expansion of such connections, but also at the corresponding transitional dynamics.

We find that increasing production costs leads to faster transitions towards the steady state whereas increasing transportation and communication costs tends to lengthen the transition. The results point to reductions in transportation and communication costs in particular as a suitable vehicle to speed up growth.

The results also show a strong effect of reductions in the cost of making new connections. This has a significant impact on both the steady state growth rate and on transitional growth, while significantly reducing the transition period.

The organization of the paper is as follows. In section 2 we describe the model. Section 3 provides the outcomes of some sensitivity analyses, while section 4 is devoted to the policy implications of the model. Section 5 concludes.

## **2. The Model**

### **2.1 Krugman Preliminaries**

In this section we provide the elements that we will be using from the Krugman (1979) model. We will leave out the technology features of Krugman (1979), assuming that the set of

varieties of goods/services that could be imitated is so large and that people are so good in imitating that the rate of imitation is unbounded in principle, but for the existence of fixed costs in producing a particular variety. We also leave out the North-South asymmetry present in Krugman (1979), and so end up with a South-South type of model instead, where all ‘countries’ connected to each other would in principle be able to cover the same spectrum of goods as they should when they would be self-reliant. Finally, we drop the notion that countries are engaging in trade with each other, but rather adopt the view that it is organized communities of people that do the specialization and trading, and that there could well be many communities inside a country. To keep things as simple as possible, we have but one factor of production, i.e. labour, and as we want to expand the set-up with potentially infinitely many communities, we make use of the assumption that all communities are identical, except for the fact that they may be connected or not.

*The demand for goods and Community welfare*

As regards the demand for goods and services, we assume that if there are  $N$  different goods that could be produced by a community, then the utility that an individual belonging to that community could gain from spending a budget  $B$  on the consumption of these  $N$  varieties is given by (1). The utility function is a standard CES function with equal contribution of all varieties to utility, which, by a suitable choice of units of measurement then boils down to:

$$U = \left\{ \sum_{i=1}^N (x_i)^\rho \right\}^{1/\rho} \quad (1)$$

In equation (1), which is the well-known Dixit-Stiglitz-Spence utility function,  $\sigma = 1/(1 - \rho)$  is the elasticity of substitution between varieties. The inverse demand function for a particular variety will be given by:

$$p_i = \partial U / \partial x_i / \lambda = U^{1-\rho} \cdot x_i^{\rho-1} / \lambda \quad \forall i = 1..N \quad (2)$$

Multiplying (2) by  $x_i$  and subsequently summing over  $i$  gives the familiar result that  $\lambda = U / B$ , i.e. the Lagrange multiplier of the utility maximization problem of which (2) is the first order condition (further called FOC) equals utility per dollar spent on consumption in the consumption optimum.

If prices are identical for all varieties consumed, i.e.  $p_i = \bar{p} \forall i$ , then the level of consumption of each variety would be the same as well,  $x_i = \bar{x} \forall i$ , and so the budget would have to be distributed evenly over all varieties, implying that:

$$\bar{x} = (B / N) / \bar{p} \quad (3)$$

(3), when substituted in (1), implies that:

$$U = B / \bar{p} \cdot N^{1/\rho-1} \quad (4)$$

For  $\rho < 1$ , i.e. an elasticity of substitution greater than one, (4) shows the impact of Love of Variety on utility: the greater  $N$ , the greater total utility, *ceteris paribus*. Note that in equation (4),  $B / \bar{p}$  actually represents the level of utility for  $N=1$ .

Let  $\bar{L}$  be the size of the total community in terms of the number of persons. Then the utility for the community,  $UC$  would be given by:

$$UC = \bar{L} \cdot B / \bar{p} \cdot N^{1/\rho-1} \quad (5)$$

## 2.2 Connecting with another Community

Now assume that instead of all goods commanding the same consumer price, there are two groups of goods commanding different prices. The first group of goods will be thought to be produced within the community, while the second group of goods is obtained from a different source external to the community. These then are goods 'imported' into the

community. The internally produced goods will in part have to be exported to the external community to pay for the imports.

Let  $x$  now stand for domestically produced (and therefore exported) goods, and let  $m$  be the common level of imported goods. Furthermore let  $q$  be the corresponding price of  $m$ . In that case, the level of individual utility would be given by:

$$U = (N_x \cdot \bar{x}^\rho + N_m \cdot \bar{m}^\rho)^{1/\rho} \quad (6)$$

where  $N_x$  is the number of varieties that are domestically produced and exported, and  $N_m$  is the number of varieties of imported goods. The corresponding budget constraint is then given by:

$$B = N_x \cdot \bar{p} \cdot \bar{x} + N_m \cdot \bar{m} \cdot \bar{q} \quad (7)$$

Maximizing (6) subject to (7) by choosing the individual<sup>1</sup> levels of  $x_i \forall i | 1 \leq i \leq N_x$  and  $m_j \forall j | 1 \leq j \leq N_m$ , results in:

$$\bar{x} = (\bar{p} \cdot \lambda)^{-\sigma} \cdot U \quad (8.A)$$

$$\bar{m} = (\bar{q} \cdot \lambda)^{-\sigma} \cdot U \quad (8.B)$$

Furthermore, when substituting (8.A) and (8.B) into (6) we find that:

$$\lambda = (N_x \cdot \bar{p}^{1-\sigma} + N_m \cdot \bar{q}^{1-\sigma})^{-1/(1-\sigma)} \quad (9)$$

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<sup>1</sup> To obtain the individual levels, the summations over  $i$  and  $j$  should be substituted back into the utility function, and the partial derivatives w.r.t.  $x_i$  and  $m_j$  should be evaluated first, and only then the symmetry assumptions  $m_j = \bar{m}$ ,  $x_i = \bar{x}$ ,  $p_i = \bar{p}$ ,  $q_j = \bar{q}$  should be substituted in the FOCS. However, in case of equations (6) and (7), both maximization w.r.t. individual  $x$ 's and  $m$ 's or w.r.t. average  $x$ 's and  $m$ 's would generate the same results, as the FOC's would be the same due to the cancellation of the  $N_x$ 's and  $N_m$ 's at both sides of the FOC's.

In addition, multiplying (8.A) and (8.B) by  $N_x \cdot \bar{p}$  and  $N_m \cdot \bar{q}$ , respectively, and adding up the results would give us:

$$N_x \cdot \bar{p} \cdot \bar{x} + N_m \cdot \bar{q} \cdot \bar{m} = B = \lambda^{-\sigma} \cdot (N_x \cdot \bar{p}^{1-\sigma} + N_m \cdot \bar{q}^{1-\sigma}) \cdot U = \lambda^{-\sigma} \cdot \lambda^{\sigma-1} \cdot U \Rightarrow \lambda = U / B \quad (10)$$

where we have used (9) to get rid of the bracketed term in (10). Equation (10) can be used to substitute for  $U$  in equations (8.A) and (8.B) leaving the levels of consumption of domestic and imported goods as a function of the available budget and the corresponding consumer prices and the Lagrange multiplier only:

$$\bar{x} = \bar{p}^{-\sigma} \cdot \lambda^{1-\sigma} \cdot B \quad (11.A)$$

$$\bar{m} = \bar{q}^{-\sigma} \cdot \lambda^{1-\sigma} \cdot B \quad (11.B)$$

### 2.3 Goods and Services Supply with Two Connected Communities

The supply of each individual good can be modeled using the standard assumption that each individual supplier is of measure zero, i.e. his own actions do not noticeably affect the average cost of a 'util' (i.e. a unit of utility), hence  $1/\lambda$ , hence  $\lambda$ . So, from the perspective of an individual supplier, both the budget and the Lagrange multiplier are given in equations (11.A). For a profit maximizing supplier of domestic goods, the resulting profit function will therefore be given by:<sup>2</sup>

$$\pi = p \cdot x + q \cdot z - w \cdot (\alpha \cdot x + (\alpha + \beta) \cdot z + \bar{l}) \quad (12)$$

In equation (12),  $\pi$  represents the profit flow for the local producer of each variety. This producer sells his produce on the domestic market at price  $p$ , but also to the external market at price  $q$ . The corresponding volumes sold are  $x$  and  $z$ . These price/volume combinations have to be consistent with the respective demand equations, such as those given by (11.A). For the

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<sup>2</sup> From now, we drop the subscripts indicating a particular variety, since we assume that the production technologies are symmetric as well.

export volume  $z$ , the corresponding demand for imported goods by the external community would function as the relevant demand constraint i.e. equation (11.B) would be relevant, but then with the foreign budget, and foreign domestic prices and foreign import prices (i.e. the export price  $r$  in this case), replacing  $B$ ,  $p$  and  $q$ , respectively.

As regards the production technology, we have assumed that the production of a variety requires the input of labor at a wage rate  $w$  to perform three different functions. The production of each variety requires  $\bar{l}$  units of labor as fixed set-up costs. Moreover, the variable cost of producing a variety amounts to  $\alpha$  units of labor per unit of output. Finally, if a unit is shipped to an external community, then that requires  $\beta$  additional units of labor per unit of output to cover per unit communication and transportation resource requirements. Note that for reasons of simplicity we assume that  $\alpha$  and  $\beta$  are independent of the variety and the community.

Maximization of (12) conditional on the demand constraints being met, then results in the profit maximizing prices given by:

$$p = \alpha \cdot w / \rho \tag{13.A}$$

$$q = (\alpha + \beta) \cdot w / \rho \tag{13.B}$$

Note that, as usual, we need to assume that  $0 < \rho < 1$ , since otherwise profits would be negative.

## 2.4 Benefits from Additional Connections

Under the symmetry assumptions employed so far (i.e. same utility functions, same production technologies) and adding a further one by assuming that communities are of the same size, it must be the case that if we have  $W$  connected communities, then the outside world to which each individual communities is connected consists of  $W-1$  communities in turn. In addition to this, if the local community exports  $N_x$  varieties to the outside world, then, because of the symmetry assumptions made before, the outside world must be exporting  $(W-1) \cdot N_x$  varieties to the local community in turn. This begs the question what the value of  $N_x$  would be?

We can determine the value of  $N_x$  by using the assumption of free entry up to the point that profits per variety drop to zero. To this end, we can substitute equations (13) as well as our observation that  $N_m = (W - 1) \cdot N_x$  into (9) while taking into account that the foreign budget equals  $(W-1) \cdot B$ , while, moreover, the Lagrange multipliers in all communities must be the same, because of the symmetry assumptions made above. In that case, we find after some tedious algebra that:

$$\pi = (B \cdot (1 - \rho) - \bar{l} \cdot N_x \cdot w) / N_x = 0 \Rightarrow N_x = B \cdot (1 - \rho) / (w \cdot \bar{l}) \quad (14)$$

Note that in equation (14),  $B / (\bar{l} \cdot w)$  is the absolute maximum of the number of varieties that could be produced, because for this value of  $N_x$  total expenditures  $B$  are just enough to cover the total set-up cost, leaving no resources to actually produce a strictly positive level of the  $N_x$  varieties. Note moreover that if the elasticity of substitution between varieties would increase, i.e. if  $\rho$  would go up, then the number of varieties supplied to the market would go down. This is because in that case the profit margin would go down, *ceteris paribus*.<sup>3</sup> This would make it harder to recover the fixed set-up cost per variety.

Using (14), the total number of varieties ( $V$ ) consumed by  $W$  connected communities would be given by:

$$V = W \cdot B / (\sigma \cdot w \cdot \bar{l}) \quad (15)$$

It should now be noted that since profits are zero because of the free entry assumption, all income generated must be wage-income. Hence, the consumer budget in each community is given by  $B = w \cdot \bar{L}$ . Substituting this result into (15) then gives rise to:

$$V = W \cdot (\bar{L} / \bar{l}) / \sigma \quad (16)$$

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<sup>3</sup> As the reader can easily verify for himself using (13), the profit margin on local and external sales equals  $1 - \rho$  as a percentage of marginal cost.

Again,  $\bar{L}/\bar{l}$  is the absolute maximum of the number of varieties that each community would be willing to support. Hence,  $W$  times that quantity is the absolute maximum number of varieties that all communities could produce. Since  $\sigma > 1$ , the actual number of varieties produced by all communities taken together is strictly smaller than  $V$ .

We can now obtain utility per capita (further called  $UPC$ ) in each of the  $W$  connected communities by substituting the previous results into (6):

$$UPC = U/\bar{L} = \{\bar{L}/\bar{l}\}^{1/(\sigma-1)} \cdot (\sigma-1) \cdot \sigma^{\sigma/(1-\sigma)} \cdot (\alpha^{1-\sigma} + (W-1) \cdot (\alpha+\beta)^{1-\sigma})^{1/(\sigma-1)} \quad (17)$$

Since we must have that  $\sigma > 1$ , it follows directly that  $\partial UPC/\partial W > 0$ ,  $\partial UPC/\partial \bar{L} > 0$ ,  $\partial UPC/\partial \bar{l} < 0$ ,  $\partial UPC/\partial \alpha < 0$ ,  $\partial UPC/\partial \beta < 0$ , i.e. under these parameter values and symmetry assumptions, rational communities would have an interest in extending the number of connections with other communities. In addition, utility per capita would rise with the size of each individual community, while it would fall with the level of fixed set-up labor cost. Finally, a rise in transportation and communication costs would negatively affect utility per capita in all connected communities.

The analysis above still leaves the following questions unanswered:

- a) Having established that  $\partial UPC/\partial W > 0$ , does an optimum  $\dot{W}$  exist that maximizes utility for all connected communities?
- b) if an optimum  $\dot{W}$  exists, how would it depend on the parameters of the model?

Finding answers to these questions is the subject of the next section.

## 2.5 Optimum Network Expansion Rates

Let us now assume that it takes some labor resources to connect thus far disconnected communities, by building ground-stations in the newly connected communities as well as transportation infrastructure.<sup>4</sup> As before, we make the simplest assumption possible, i.e. that the

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<sup>4</sup> To keep things as simple as possible, we assume that there are only fixed set-up costs in doing this, so that the infrastructure is infinitely lived.

resources needed to make new connections are proportional to the number of newly connected communities. Thus we get:

$$\dot{W} = \delta \cdot L_w \quad (18)$$

In equation (18),  $L_w$  are the total labor resources used for expanding the number of connected communities. Let each connected community contribute a fraction  $\tau$  of its available labor force to this activity. Then we must have that  $L_w = \tau \cdot \bar{L} \cdot W$ , and consequently it follows from (18) that:

$$\hat{W} = \delta \cdot \bar{L} \cdot \tau \quad (19)$$

In equation (19),  $\hat{W}$  is the instantaneous growth rate of the number of communities that is connected at any time.

If a fraction  $\tau$  of total real resources is used for connecting communities, then the new real budget available for spending on goods and services within each community must be equal to  $(1 - \tau) \cdot B$ . This change in the real budget would not change optimum price setting behavior, but it would change both the supply of goods and services and the optimum number of varieties produced within each community. When we redo the analysis above, but with  $(1 - \tau) \cdot B$  replacing  $B$ , we find that the new number of varieties by community becomes a fraction  $1 - \tau$  of the old number of varieties:

$$N_x = (\bar{L} / \bar{l}) \cdot (1 - \tau) / \sigma \quad (20)$$

According to (20), the introduction of the costs of connecting communities therefore reduces the number of varieties supplied by each community. Utility per capita would fall on that account, but for the fact that the number of communities connected (i.e.  $W$ ) increases as well, and therefore also, potentially at least, the total number of varieties available to all

connected communities (cf. (17)). Substituting (20) and (13) into (6), while taking into account that  $N_m = (W - 1) \cdot N_x$ , we find that the new expression for utility per capita becomes:

$$UPC = \{\bar{L} / \bar{l} \cdot (1 - \tau)\}^{1/(\sigma-1)} \cdot (\sigma - 1) \cdot \sigma^{\sigma/(1-\sigma)} \cdot \left( \alpha^{1-\sigma} + (W - 1) \cdot (\alpha + \beta)^{1-\sigma} \right)^{1/(\sigma-1)} \quad (21)$$

It follows from (21) that  $\partial UPC / \partial \tau < 0$ .

We can now construct an optimum control problem in which the GC would want to maximize the present value of total utility in all connected communities, while using (21) in the objective function to be maximized. An alternative objective function would be the maximization of the utility per head of community initiating the integration process communities. As the communities are assumed to be symmetric, it should be the case that if for the initiating community it would be beneficial to expand the network of connected communities, then it would have to be beneficial for the newly connected communities as well. The corresponding Hamiltonian reads:

$$H = \exp(-\mu \cdot t) \cdot UPC^{1-\theta} \cdot \bar{L} \cdot W / (1 - \theta) + \psi \cdot \dot{W} \quad (22)$$

In equation (22),  $\mu$  is the rate of discount, while  $1/\theta$  is the intertemporal elasticity of substitution and  $\psi$  is the co-state variable associated with the state variable  $W$ .<sup>5</sup> The control variable of the system is  $\tau$ . The corresponding FOC's to this problem are implicitly given by the requirements  $\partial H / \partial \tau = 0$ ,  $\partial H / \partial W = -\dot{\psi}$ ,  $\partial H / \partial \psi = \dot{W}$  and the transversality condition that requires  $\lim_{t \rightarrow \infty} \psi(t) \cdot W(t) = 0$ . Doing the algebra, results in a set of non-linear differential equations, that, under certain conditions, converges to a steady state when  $W(t)$  approaches infinity. The system of differential equations is given by:

$$\dot{\psi} = \psi \cdot A \cdot \{W \cdot (1 - \tau) / ((W - 1) + B) + C \cdot \tau - 1\} \quad (23.A)$$

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<sup>5</sup> Note that we disregard the case where  $\theta = 1$ .

$$\dot{\psi} = \psi \cdot \left\{ \hat{W} \cdot W / \{(1+C) \cdot (W-1+B)\} + C \cdot \dot{\tau} / ((1-\tau) \cdot (1+C)) - \mu \right\} \quad (23.B)$$

$$\hat{W} = A \cdot \tau \quad (23.C)$$

with  $A = \bar{L} \cdot \delta$ ,  $B = ((\alpha + \beta) / \alpha)^{\sigma-1}$ ,  $C = (\sigma + \theta - 2) / (1 - \theta)$ .

It should be noted that for  $W$  approaching infinity, the ratio  $(W-1)/W$  will approach 1, in which case the system becomes more manageable. Using the requirement that in the steady state  $\tau$  must be constant (otherwise  $\hat{W}$  can't be constant, cf. equation (19)), it follows that in the steady state  $\dot{\tau} = 0$ , implying that the steady state solution is given by:

$$\bar{\tau} = \frac{(1+C) \cdot (A \cdot (2+C) - \mu)}{A \cdot C \cdot (2+C)} = \frac{(\sigma-1) \cdot (\bar{L} \cdot \delta \cdot (\sigma-\theta) - (1-\theta) \cdot \mu)}{\bar{L} \cdot \delta \cdot (\sigma-\theta) \cdot (\sigma+\theta-2)} \quad (24.A)$$

$$\bar{\hat{W}} = \frac{(1+C) \cdot (A \cdot (2+C) - \mu)}{C \cdot (2+C)} = \frac{(\sigma-1) \cdot (\bar{L} \cdot \delta \cdot (\sigma-\theta) - (1-\theta) \cdot \mu)}{(\sigma-\theta) \cdot (\sigma+\theta-2)} \quad (24.B)$$

$$\bar{\dot{\psi}} = \frac{A \cdot (2+C) - (1+C)^2 \cdot \mu}{C \cdot (2+C)} = - \frac{(\sigma-1)^2 \cdot \mu - \bar{L} \cdot \delta \cdot (\sigma-\theta) \cdot (1-\theta)}{(\sigma-\theta) \cdot (\sigma+\theta-2)} \quad (24.C)$$

where a bar over a variable denotes the steady state value of that variable.

#### *Parameter Constraints*

The transversality condition requires that  $\bar{\hat{W}} + \bar{\dot{\psi}} < 0$ , which gives rise to the following parameter constraint:

$$\frac{\bar{L} \cdot \delta \cdot (\sigma-\theta) - (\sigma-1) \cdot \mu}{\sigma+\theta-2} < 0 \quad (25.A)$$

In order to be able to have positive steady state growth in the number of connected communities, we require that  $\bar{\hat{W}} > 0$ , which implies that:

$$\frac{(\sigma-1) \cdot (\bar{L} \cdot \delta \cdot (\sigma-\theta) - (1-\theta) \cdot \mu)}{(\sigma-\theta) \cdot (\sigma+\theta-2)} > 0 \quad (25.B)$$

In order to get the ‘standard’ results that the growth rate depends negatively on the rate of discount  $\mu$ , we require that:

$$\frac{\partial \bar{W}}{\partial \mu} < 0 \Rightarrow \frac{(\sigma-1) \cdot (1-\theta)}{(\sigma-\theta) \cdot (\sigma+\theta-2)} > 0 \quad (25.C)$$

If, moreover, we want a rise in the productivity of connection resources to have a positive impact on the steady state growth rate, we should have:

$$\frac{\partial \bar{W}}{\partial \delta} > 0 \Rightarrow \frac{\bar{L} \cdot (\sigma-1)}{\sigma+\theta-2} > 0 \quad (25.D)$$

Since  $\sigma > 1$  by assumption and since  $\bar{L}$  must be strictly positive, it follows from (25.D) that we must have  $\sigma + \theta - 2 > 0$ . But then it follows from (25.C) that the ratio  $(1-\theta)/(\sigma-\theta) > 0$ , implying that either  $\theta < 1 < \sigma$  (further called Case I) or  $\theta > \sigma > 1$  (further called Case II).

The parameter constraints for the two cases are summarized in Table 1. It follows from the Table that Case II is the least restrictive case of the two in terms of choosing  $\mu$ . However, Case II also implies a lower intertemporal elasticity of substitution than Case I, and hence a lower willingness to (temporally) divert resources.

Constraint	Case I ( $\sigma > 1 > \theta > 0$ )	Case II ( $\theta > \sigma > 1$ )
Transversality	$\mu > \bar{L} \cdot \delta \cdot (\sigma - \theta) / (\sigma - 1) > 0$	$\mu > \bar{L} \cdot \delta \cdot (\sigma - \theta) / (\sigma - 1) < 0$
$\bar{W} > 0$	$\mu < \bar{L} \cdot \delta \cdot (\sigma - \theta) / (1 - \theta) > 0$	$\mu < \bar{L} \cdot \delta \cdot (\sigma - \theta) / (1 - \theta) > 0$
$\partial \bar{W} / \partial \delta > 0$	$\sigma + \theta - 2 > 0$	$\sigma + \theta - 2 > 0$
$\partial \bar{W} / \partial \mu < 0$	$(1 - \theta) / (\sigma - \theta) > 0$	$(1 - \theta) / (\sigma - \theta) > 0$
$\bar{\tau} < 1$	$\mu > \bar{L} \cdot \delta \cdot (\sigma - \theta) / (\sigma - 1) > 0$	$\mu > \bar{L} \cdot \delta \cdot (\sigma - \theta) / (\sigma - 1) < 0$

Table 1. Parameter Constraints

## 2.6 Transitional Dynamics

The system of differential equations given by (23) is non-linear in the variables  $W, \tau$  and  $\psi$ . By means of substituting out the differential equation for the co-state variable, we can reduce (23) to a two-dimensional system that is still non-linear, but that is saddle-path stable under certain parameter conditions (see above) and that features steady state growth. When we would introduce an auxiliary variable  $Z$  defined as  $Z = W / (W + B - 1)$  (cf. (23.B)), the quasi-state variable  $Z$  converges to 1, for  $W$  goes to infinity. In addition, the growth rate of  $Z$  is given by  $\hat{Z} = \hat{W} \cdot (B - 1) / (W + B - 1)$ . Substituting these relations into (23) and substituting out the growth rate of  $\psi$ , then leaves a two-dimensional system, with a constant steady state, given by:

$$\hat{Z} = A \cdot (1 - Z) \cdot \tau \quad (26.A)$$

$$\hat{\tau} = \frac{-(\mu \cdot (1 - C) + A \cdot (1 + C + Z) \cdot (1 + C \cdot (1 - \tau))) \cdot (1 - \tau)}{C \cdot \tau} \quad (26.B)$$

The locus of combinations of  $Z$  and  $\tau$  for which  $Z$  and  $\tau$  are not growing is obtained by setting  $\hat{Z} = 0$  in (26.A) and setting  $\hat{\tau} = 0$  in (26.B), giving:

$$Z = 1 \quad (27.A)$$

$$\tau = (1 + C) \left\{ \frac{\mu}{A \cdot ((1 + C) - C \cdot \tau)} - 1 \right\} \quad (27.B)$$

The  $\hat{Z} = 0$ -locus is given by (27.A) and the  $\hat{\tau} = 0$ -locus by (27.B). The steady state value of  $Z$  is obviously equal to 1, as it should be, while the steady state value of  $\tau$  is given by (27.B) after substituting (27.A), which gives the same results as (24.A), after substituting the definitions of  $A$ , and  $C$  in terms of the structural parameters of the system. Note that under the assumptions of Case II, the  $\hat{\tau} = 0$ -locus has a vertical asymptote at  $\tau' = (1 + C)/C < 1$  since  $C$  must then be negative, and  $(C + 1)/C = (\sigma - 1)/(\sigma + \theta - 2) = ((\sigma - 1)/(\sigma - 1 + \theta - 1)) < 1$ , as Case II implies  $\theta > \sigma > 1$ . Under the Case I assumptions the asymptote lies at a value of  $\tau > 1$ , and we need a further constraint on the parameters to ensure that the steady state value of  $\tau < 1$  (see also the last line of Table 1). Moreover, system (26.A) can be shown to be saddle-path stable in both cases. The difference between the two cases is therefore that Case I is more constrained than case II, while in Case II, the intertemporal elasticity of substitution between periods is smaller than the elasticity of substitution between goods within periods. Henceforth, we will focus on Case II.

The saddle-path stability of both cases follows readily from differentiating (26.B) with respect to  $Z$ , giving:

$$\frac{\partial \hat{\tau}}{\partial Z} = \frac{-A \cdot (1 + C \cdot (1 - \tau)) \cdot (1 - \tau)}{C \cdot \tau} \quad (28)$$

It follows that  $sign(\partial \hat{\tau} / \partial Z) = sign(-(1 + C)/C + \tau) < 0 \forall \tau | 0 < \tau < (C + 1)/C$ . Hence, if we would move vertically from a point on the  $\hat{\tau} = 0$ -locus for a given value of  $Z$ ,  $\hat{\tau}$  would become negative, while the opposite would be the case if we would move downward. Note that this would hold for any value of  $Z$ , since  $\partial \hat{\tau} / \partial Z$  is independent of  $Z$ . Note, moreover, that for combinations of  $Z$  and  $\tau$  above the  $\hat{Z} = 0$ -locus, the growth rate of  $Z$  becomes negative, while the opposite holds for points below the  $\hat{Z} = 0$ -locus. Consequently, the phase-diagram associated with (26) and (27) looks like Figure 1.

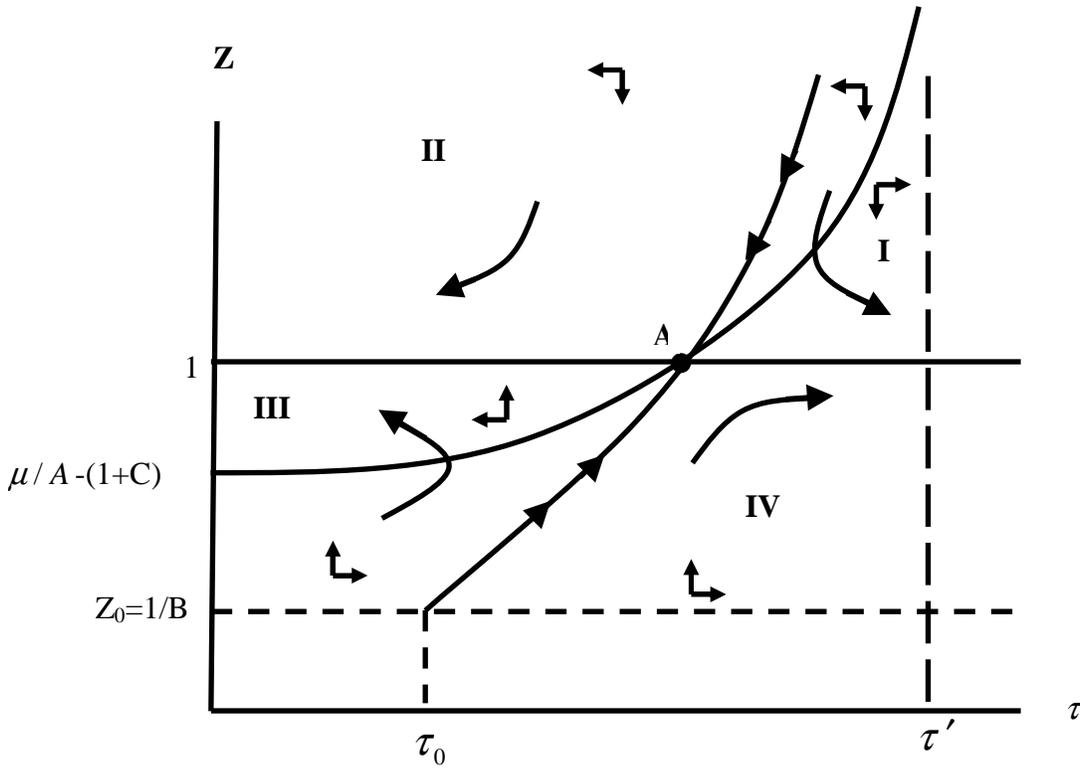


Figure 1. The Phase-Diagram

Point A in Figure 1 represents the steady state. The four areas labeled I-IV in Roman numerals show that under the Case II parameter restrictions, the model exhibits saddle-path stability. The horizontal solid line at  $Z=1$  is the  $\hat{Z} = 0$ -locus, while the solid convex curve is the  $\hat{\tau} = 0$ -locus. The saddle-path is the remaining solid curve. The horizontal dotted line is the value of  $Z$  at time zero. The  $\hat{\tau} = 0$ -locus has a point of intersection with the vertical in the positive quadrant. It depends on the actual parameter values whether it is above or below  $Z_0$ , but where exactly it is relative to  $Z_0$  doesn't make a qualitative difference. To keep the Figure as simple as possible, we have drawn it as it is.

It follows from the Figure that the optimum path for the allocation of communication resources and the corresponding path for the expansion of  $Z$  can relatively easily be obtained by means of the method of Backward Integration, i.e. obtain the time-path for  $Z$  (and  $\tau$ ) by making time run backward, starting at  $Z=1$  and decreasing time up to the moment when  $Z=Z_0$ . In that

way we can obtain the corresponding initial value of  $\tau_0$ , and consequently the entire time-path for  $\tau$  is thus easily obtained since the steady state value of  $\tau$  is known (cf. Equation (24.A)).

## 2.7 Solving for the Transition Path Using Backward Integration

Because of the non-linearity of (26), the solutions for the speed of connection and the corresponding resource allocation requirements can unfortunately not be obtained by analytical means. Therefore, in this section, we will use Mathematica to show how the adjustment path itself, but also the overall shape of the transition path changes with the parameters of the system. To do this, we use the method of Backward Integration (Brunner and Strulik (2002)). The method is suitable, since we know where the transition path ends (i.e. in the steady state), while we also ‘know’ where the quasi-state variable  $Z$  starts (i.e. at  $Z_0 = 1/B$ ). Hence by integrating (26) backwards in time, and ‘waiting’ till  $Z$  hits the  $Z_0$ -mark in Figure 1, we also obtain the corresponding initial value of  $\tau$ , i.e.  $\tau_0$  in Figure 1. The only problem is that when we would start integrating backward while being **exactly** in the steady state, we wouldn’t be able to get away from there, since the speed of adjustment in the steady state is exactly equal to zero. Consequently, we need to move slightly outside the steady state, while being on the transition path, and then start the integration process. In order to do that we can draw a tiny circle around the steady state (with radius  $\varepsilon$ , thus defining a (circular)  $\varepsilon$ -region around the steady state) and pick the point of intersection of that circle with the transition path as the initial values for  $\tau$  and  $Z$  in the backward integration process. This in turn requires that we linearize (26) around the steady state, and obtain the Eigen values of the linearized system, which then can be written as:<sup>6</sup>

$$\begin{pmatrix} \dot{Z} \\ \dot{\tau} \end{pmatrix} = \begin{pmatrix} \frac{-(1+C) \cdot (A \cdot (2+C) - \mu)}{C \cdot (2+C)} & 0 \\ \frac{(1+C) \cdot \mu \cdot (A \cdot (2+C) - (1+C) \cdot \mu)}{A \cdot C^2 \cdot (2+C)^2} & \frac{-A \cdot (2+C) + (1+C) \cdot \mu}{C} \end{pmatrix} \cdot \begin{pmatrix} Z - \bar{Z} \\ \tau - \bar{\tau} \end{pmatrix} \quad (29)$$

---

<sup>6</sup> Cf. Barro and Sala-i-Martin (1995), appendix 1.

with corresponding Eigen values  $\zeta_1 = \frac{-(1+C) \cdot (A \cdot (2+C) - \mu)}{C \cdot (2+C)}$  (implying that

$$\zeta_1 = \frac{(\sigma-1) \cdot ((\theta-1) \cdot \mu + \bar{L} \cdot \delta \cdot (\sigma-\theta))}{(\theta+\sigma-2) \cdot (\theta-\sigma)} \text{ and } \zeta_2 = \frac{-A \cdot (2+C) + (1+C) \cdot \mu}{C} \text{ (which implies that}$$

$$\zeta_2 = \frac{\bar{L} \cdot \delta \cdot (\theta-\sigma) + \mu \cdot (\sigma-1)}{\theta+\sigma-2} \text{). In order to have saddle-path stability, we need one of the}$$

Eigen values to be negative, and the other one to be positive. Since it must be the case that  $\theta + \sigma - 2 > 0$  (cf. (25.D)), this implies that under Case II,  $\zeta_2$  must be the positive Eigen value, and hence  $\zeta_1$  is the negative Eigen value provided that  $\mu < \bar{L} \cdot \delta \cdot (\sigma - \theta) / (1 - \theta)$ , which is a parameter constraint that should hold in both cases anyway (see section 2.5 above, in particular the constraint associated with  $\widehat{W} > 0$ ). Under Case I the transversality condition implies that  $\mu > \bar{L} \cdot \delta \cdot (\sigma - \theta) / (\sigma - 1)$ , which would make  $\zeta_2$  the positive Eigen value again, and in order for  $\zeta_1$  to be negative we would need that  $\mu < \bar{L} \cdot \delta \cdot (\sigma - \theta) / (1 - \theta)$ , which was the requirement associated with having  $\widehat{W} > 0$  in both cases. We conclude that the parameter restrictions outlined in section 2.5 imply the saddle path-stability of the optimization problem and that  $\zeta_1$  is the negative Eigenvalue, with corresponding Eigenvector  $v$ , where  $v$  is given by:

$$v = \begin{pmatrix} \frac{\bar{L} \cdot \delta \cdot (\bar{L} \cdot \delta \cdot (\theta-1) \cdot (\theta-\sigma) - \mu \cdot (\sigma-1)^2) \cdot (\theta-\sigma) \cdot (\sigma+\theta-2)}{(\theta-1)^2 \cdot \mu \cdot (\bar{L} \cdot \delta \cdot (\theta-\sigma) + \mu \cdot (\sigma-1)) \cdot (\sigma-1)} \\ 1 \end{pmatrix} \quad (30)$$

Consequently, the slope of the stable arm in the steady state is given by the top element of  $v$ , the numerical value of which we will further call  $s$ , for reasons of simplicity.

Now consider a circle with radius  $\varepsilon$  and center coordinates  $\{\bar{Z}, \bar{\tau}\}$  in the  $Z, \tau$ -plane. Also consider a straight line through that center with slope  $s$ . The points of intersection of this line with the circle can be found by solving the simultaneous system  $\{ (Z - \bar{Z})^2 + (\tau - \bar{\tau})^2 = \varepsilon^2, (Z - \bar{Z}) = s \cdot (\tau - \bar{\tau}) \}$  giving as the relevant solution for our case with an upward sloping stable arm (see Figure 1):

$$Z' = \bar{Z} - \varepsilon \cdot s / \sqrt{1 + s^2} \quad (31.A)$$

$$\tau' = \bar{\tau} - \varepsilon / \sqrt{1 + s^2} \quad (31.B)$$

The point  $\{Z', \tau'\}$  lies below and to the left of the steady state given by the point  $\{\bar{Z}, \bar{\tau}\}$ , since both  $\varepsilon, s > 0$  under the parameter constraints outlined above. By choosing increasingly smaller values of the radius  $\varepsilon$ , we could get infinitely close to the steady state. The time spent during the transition from that point towards the initial point (or vice versa, which is what we really want) would become correspondingly longer. In the next section, we will present the results of the sensitivity analysis performed using the Backward Integration Method.

### 3. Sensitivity Analysis

The results obtained using the Backward Integration Method for the transitional dynamics are associated with (variations on) the “base-run” parameter vector listed in Table 2 below. The values used for this vector are all consistent with the case II parameter constraints outlined in the previous section. They generate moderate but positive growth rates for the number of connected communities and transition paths that take several centuries before hitting the  $\varepsilon$ -region around the steady state.

Parameter	Value	Parameter	Value
$\alpha$	0.5	$\bar{L}$	1
$\beta$	0.1	$\theta$	2
$\sigma$	1.25	$\delta$	0.1
$\bar{l}$	1	$\mu$	0.05

Table 2. The Base-Run Parameter vector

The parameters can be divided into three different groups. The first group consists of the production cost parameters  $\alpha$  and  $\beta$ . As is clear from equations (23) and (24), these cost parameters do not influence the steady state itself, but affect the transitional dynamics only. The

other structural parameters  $\sigma, \bar{L}, \delta, \mu, \theta$  do have an impact on both the steady state and the corresponding transitional dynamics, whereas the fixed-cost parameter  $\bar{l}$  does not influence either the steady state or the transitional dynamics. It does have an impact on welfare, though, since higher fixed labour costs imply lower numbers of varieties and hence lower welfare, *ceteris paribus*.

Using the parameter-vector above, we have performed a sensitivity analysis for all the elements in the vector separately. Each element has been varied over the range  $c^*(1-x), c, c^*(1+x)$  where  $c$  is the central value taken from Table 2, and  $x = 0.5$  for the parameters  $\alpha$  and  $\beta$ , and  $x=0.1$  for all the other parameters. These relative shocks are all still compatible with the case II parameter constraints. For the parameters  $\alpha$  and  $\beta$ , the shocks are relatively high, because otherwise the effects on the transition path would hardly be visible. The corresponding results are depicted in the Figures further below. In these Figures, we first see the variables  $Z, W, \tau$  graphed against time, and then the implied graph of  $Z$  against  $\tau$ , as in the phase-diagram in Figure 1. A further plot holds the development over time of the present value of utility per capita (called *PVUPC*) along the transition path until the moment it hits the  $\varepsilon$ -region. The final plot shows the growth rate of  $W$  (called *GW*) along the transition path. In all plots, the graph associated with the lowest value of the parameter range is dotted. The central value graph is solid, and the highest value in the parameter range is associated with the striped graph. It should be noted that in the plot of  $Z$  against  $t$  (but also against  $\tau$ ) there is a horizontal at  $Z=1$  (that corresponds to the steady state value of  $Z$ ). The other horizontals in the plot of  $Z$  against  $t$  are associated with the initial values of  $Z$  as given by  $Z_0 = 1/B$ . The horizontals in the plot of  $GW$  against  $t$  are the steady state values of  $GW$  corresponding to each individual parameter vector concerned.

In Figure 2, the results for a change in the elasticity of substitution between varieties in the utility function have been depicted. Note that  $\sigma$  also equals the (absolute value of the) price-elasticity of demand. A higher value of  $\sigma$  would therefore lower profit margins, and hence would enable a community to sustain a lower number of varieties (see eq. 16), *ceteris paribus*. We see that a relatively low price elasticity of demand raises the transition path for  $Z$ .

Higher values of  $\sigma$  lower the transition path relative to the central value path. However, it should be noted, that the paths do actually intersect. This is most clearly seen in the plot for  $\tau$ . In addition to this, it can be seen from the position of the end-points of the transition paths in the plot for  $W$ , that the length of the transition path for a low value of  $\sigma$  is higher than that of the central value for  $\sigma$ . The same holds for the endpoint for the high-value of  $\sigma$ -path, suggesting the relevance of intertemporal trade-offs in a setting like this. For, as apparent from the plot of  $GW$ , the growth rate of  $W$  is very close to its steady state value from the beginning in the low  $\sigma$  case, while it is very low to start with in the high  $\sigma$  case but ends higher than in the low  $\sigma$  case. From the plot holding the outcomes for  $PVUPC$ , it follows immediately that a lower value of  $\sigma$  is relatively good news for the consumers that are all connected. There is some bad news as well, since the (steady state) growth rate  $GW$  is the lowest of the three. It follows from a comparison of the plots for  $PVUPC$  and  $GW$ , that having a high  $GW$  doesn't have to be a good thing *per se*. While the central value for  $\sigma$  does generate the highest  $GW$  during the transition and in the steady state, the present value of utility per head is below that of the low  $\sigma$  case at all times.

Figure 3 shows the results for variations in  $\theta$ , where  $1/\theta$  is the intertemporal elasticity of substitution between (the utilities derived from) consumption at different moments in time. As with the variations in  $\sigma$ , we find that the central value of  $\theta$  run generates the highest growth rates, whereas the rest of the results are reversed, that is to say the highest value of  $\theta$  now generates a steady state growth rate that falls below the one for the lowest value of  $\theta$ . This is what one would expect, since a higher value of  $\theta$  implies a lower value of the intertemporal elasticity of substitution and hence a higher willingness to give up resources now in exchange for higher returns in the future. We see therefore that the transition path for  $\tau$  for the low  $\theta$ -case is indeed above the path for the high  $\theta$ -case at the end of the transition period and in the steady state itself. Also, it should be noted that a lower value of the intertemporal elasticity of substitution (hence a higher value of  $\theta$ ) would tend to cause a more uniform distribution of consumption over time, and hence a lower dispersion in transitional growth rates and lower steady state growth. This is exactly what can be observed from the plot of  $GW$ : the transitional growth dispersion falls as  $\theta$  increases. However, the steady state value of  $GW$  is a hump-

shaped function of  $\theta$  around its central value, as apparent from eq. (24), where the numerator of (24.B) is linear in  $\theta$ , while the denominator is a quadratic function of  $\theta$ .

Figure 4 shows the results for variations in  $\alpha$ , the marginal production cost of each variety. As stated before, variations in  $\alpha$  only affect the transitional dynamics, hence the levels of  $W$ , but not the steady state growth rate  $GW$ . Raising  $\alpha$  from its low value to its high value leads to a shortening of the transition period on the one hand and to a rise in the transitional growth rate, while leaving the steady state growth rate untouched. Interestingly, higher values of  $\alpha$  raise the relative contribution of variety to utility as compared to the contribution of quantity to utility. Hence for high values of  $\alpha$ , the incentive to increase the number of available varieties through raising connectivity increase as well. Consequently, we find higher transitional growth rates as  $\alpha$  increases. Higher values of  $\alpha$  also shift down the time path for the present value of utility per capita ( $PVUPC$ ) as one would expect. This is because higher marginal production cost, would, for a given level of resources reduce (ex-ante) profits, and hence the number of varieties that can be sustained by a community. At the same time, for a given number of varieties, the volume of each variety must go down as well, reducing per capita utility on both accounts (cf. eq. (17)). Note that, even though the time-paths  $Z(t)$  and  $\tau(t)$  are clearly influenced, they are affected to exactly the same extent so that  $Z(t)$  plotted against  $\tau(t)$  for all values of  $t$  remains exactly where it was. This is easy to understand, since a change in  $\alpha$  (or  $\beta$ ) would only affect the value of  $Z_0$ , i.e. the position of the horizontal  $Z=Z_0=1/B$  in Figure 1. Hence, when Integrating Backward, we would still follow the same trajectory along the stable arm from the steady state and down to the 'old' value for  $Z_0$ , and then we would have to extend the stable arm from that point upto the 'new' value of  $Z_0$  (assuming the latter is below the former).

The results are qualitatively similar for variations in  $\beta$ , i.e. unit transportation cost in the sense that only transitional dynamics are affected and not the steady state, see Figure 5. However, now we find that an increasing value of  $\beta$  will lead to lower transitional growth and longer transition periods, whereas a rising value in  $\alpha$  would tend to have the opposite effect. Still, the effect on utility per head goes in the same direction as for variations in  $\alpha$ . The reason for the different impacts of variations in  $\alpha$  and  $\beta$  on utility per capita can be found in equation

(17). As  $\beta$  is associated with transportation costs, its impact on utility per capita becomes bigger the larger the number of connected communities is. Hence if  $\beta$  rises, the direct impact on per capita utility will be negative, but that impact can be mitigated to some extent by reducing the rate at which  $W$  grows.

Figure 6 shows the results for variations in  $\delta$ . These are relatively spectacular. Note that the (labour-) costs of extending the number of connected communities (by building ground stations for communication and transportation infrastructure) depend inversely on  $\delta$ . Hence low values of  $\delta$  imply high costs of extending the number of connected communities, and we consequently see that the duration of the transition period falls as  $\delta$  increases. We also see that the growth rate of  $W$ , i.e.  $GW$ , is positively affected, and quite significantly so, if  $\delta$  increases. As a rise in  $\delta$  implies lower cost for creating new connections and the resources allocated to making those new connections actually go up. This follows from the fact that the steady state values of  $\tau$  increase as  $\delta$  rises. But even though  $\tau$  increases, a rising value of  $\delta$  has a positive net effect on the present value of per capita utility.

Figure 7 shows how the model reacts to variations in the rate of discount  $\mu$ . Basically, the results are opposite to those of variations in  $\delta$ , as a rise in  $\mu$  would disfavor the execution of activities which return would be in the future (like indeed extending the number of connected communities). Consequently we find lower growth in  $W$  and longer transitions as  $\mu$  rises. As a consequence, the time-path for utility per capita also shifts downward as  $\mu$  increases.

Figure 8 shows what happens for variations in the size of the communities. Increases in size would allow more varieties to be produced, before profits are squeezed to zero due to free entry. This means that the returns to connecting additional communities go up. Consequently, we observe a rise in both transitional and steady state growth as  $\bar{L}$  increases, while the length of the transition period decreases. Because of the increasing returns to making new connections, we see that the resources allocated to doing that also increase for rising values of  $\bar{L}$ .

Finally, Figure 9 shows what happens for variations in the fixed cost per variety: virtually nothing. As  $\bar{l}$  only enters the per capita utility function in a multiplicative fashion, it follows

that only the level of utility per capita will be affected, but not the way in which the relative contribution of  $W$  to per capita utility changes over time. Consequently, there will be no reason to change anything in the time-path for  $W$ , and so all plots remain the same except for the per capita utility plot. Obviously, per capita utility falls if  $\bar{l}$  increases, as each community can now support a lower number of varieties.

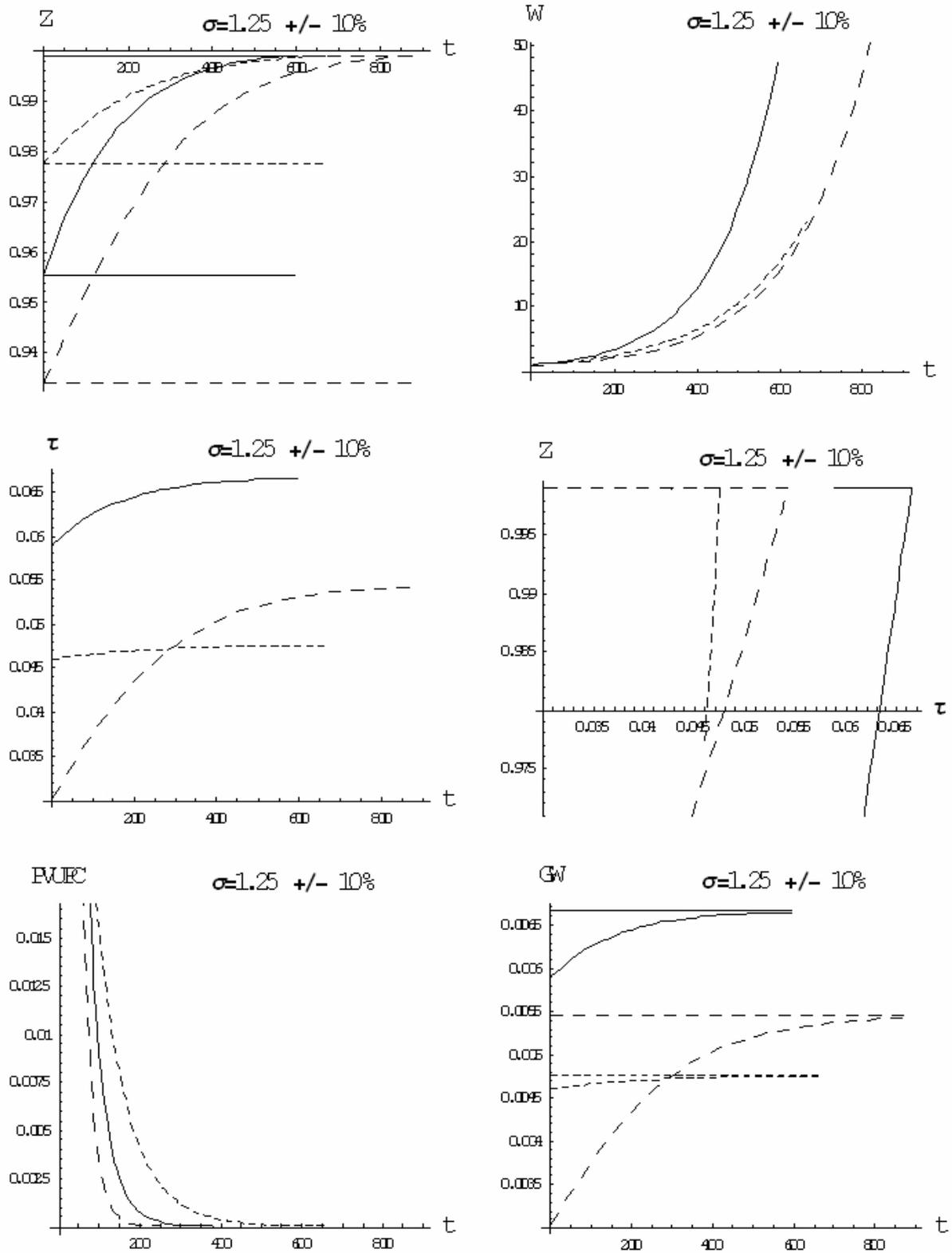


Figure 2. Sensitivity Results for Variations in  $\sigma$

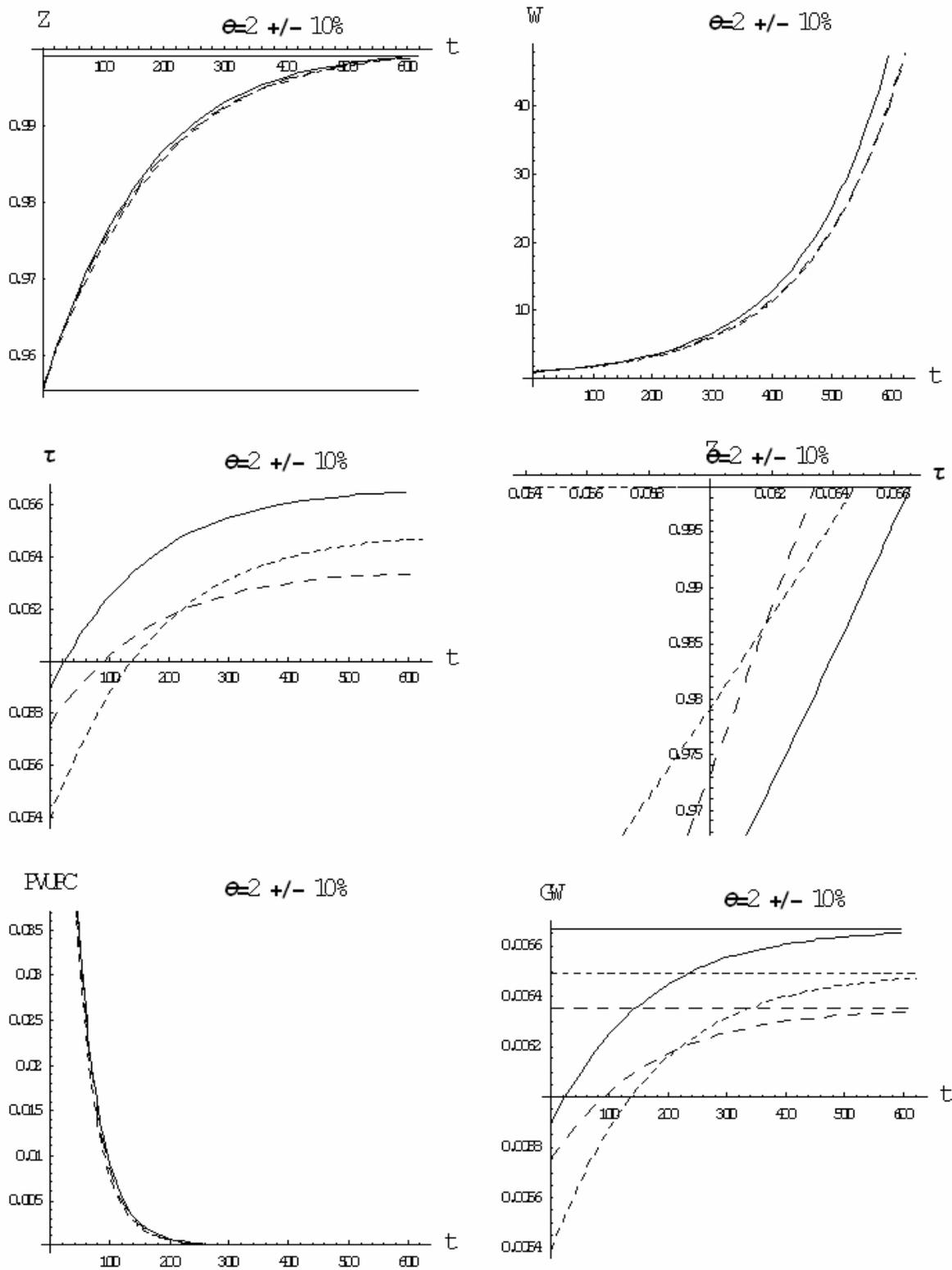


Figure 3. Sensitivity Results for Variations in  $\theta$ .

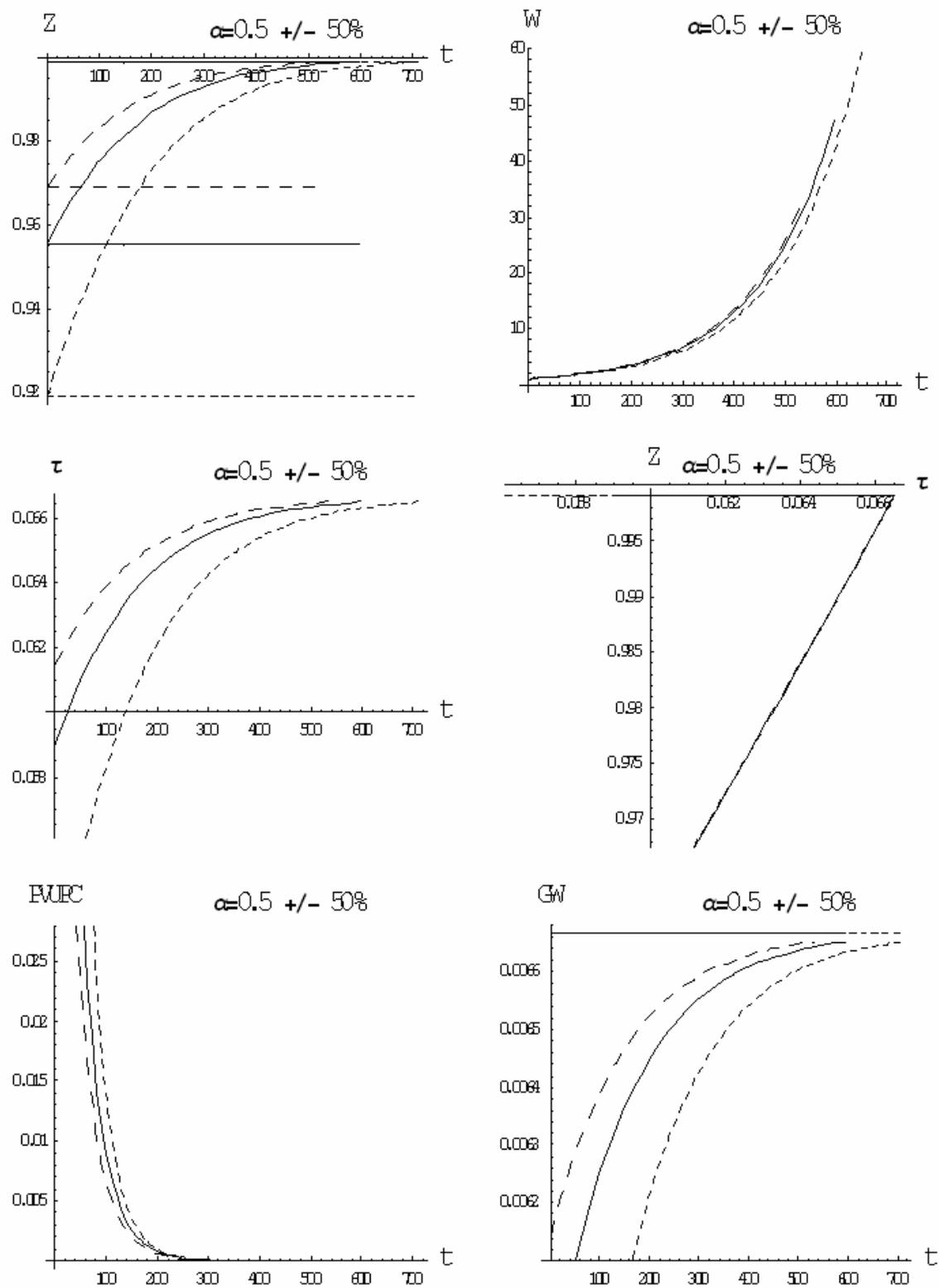


Figure 4. Sensitivity Results for Variations in  $\alpha$ .

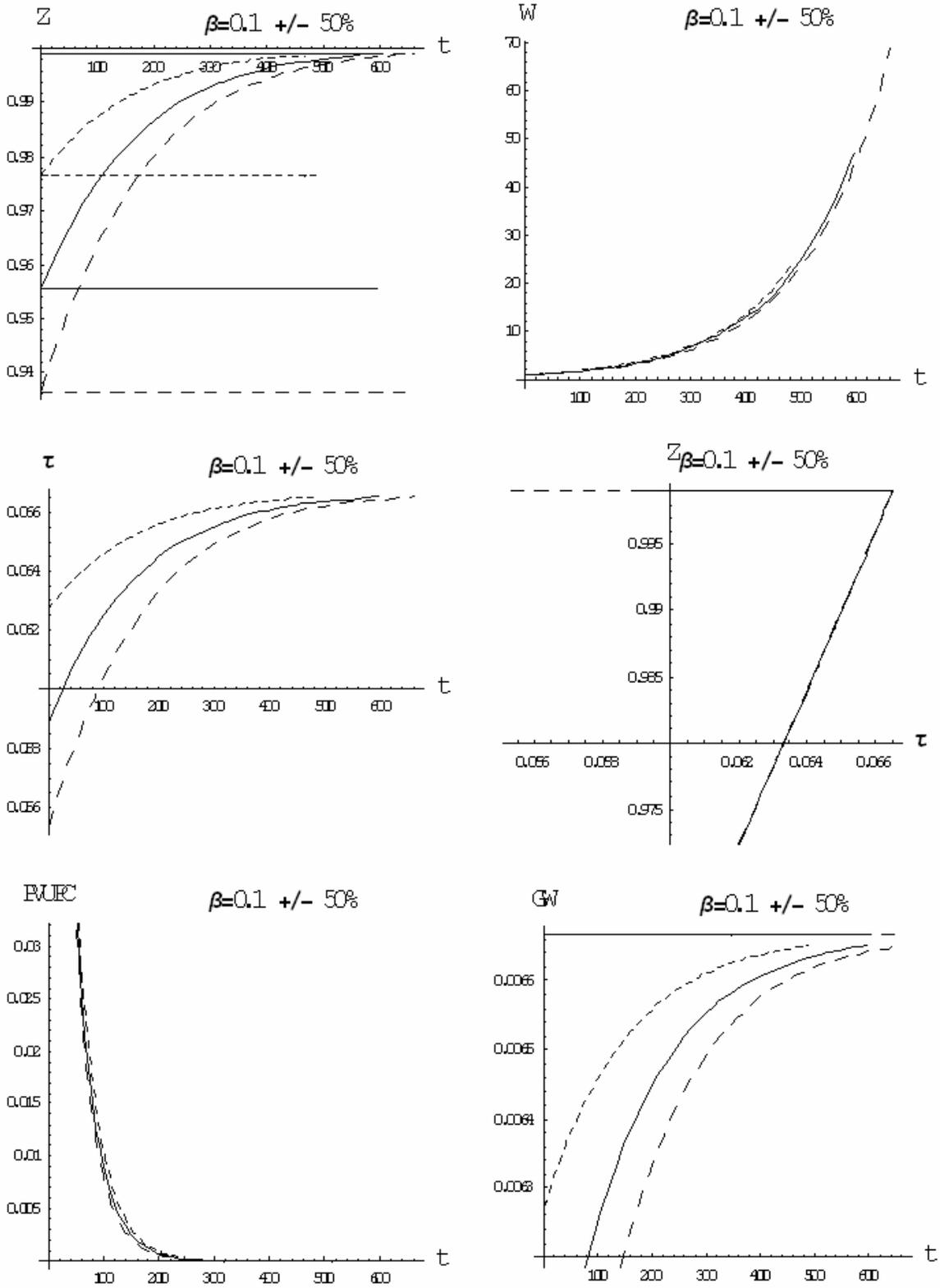


Figure 5. Sensitivity Results for Variations in  $\beta$ .

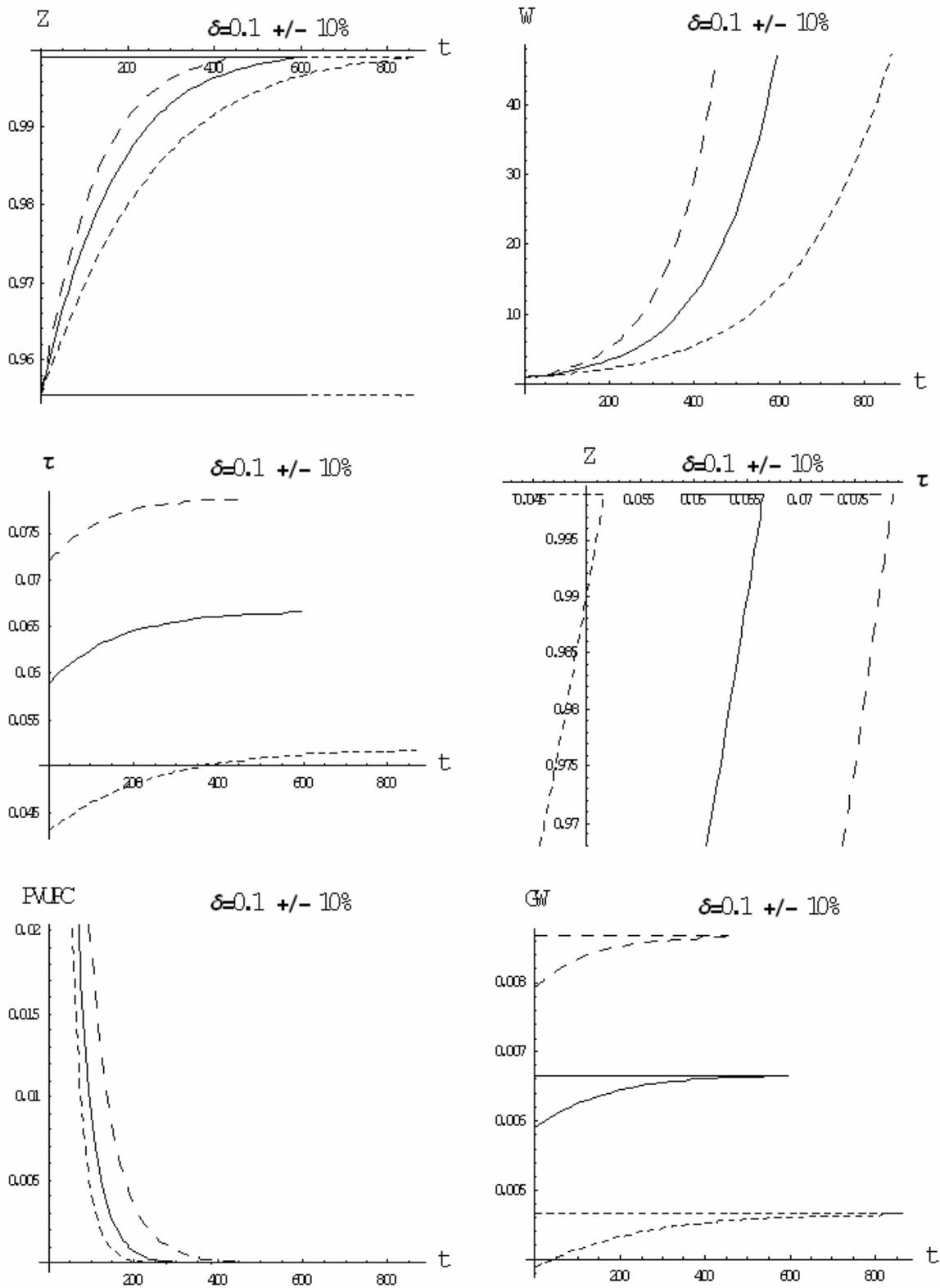


Figure 6. Sensitivity Results for Variations in  $\delta$ .

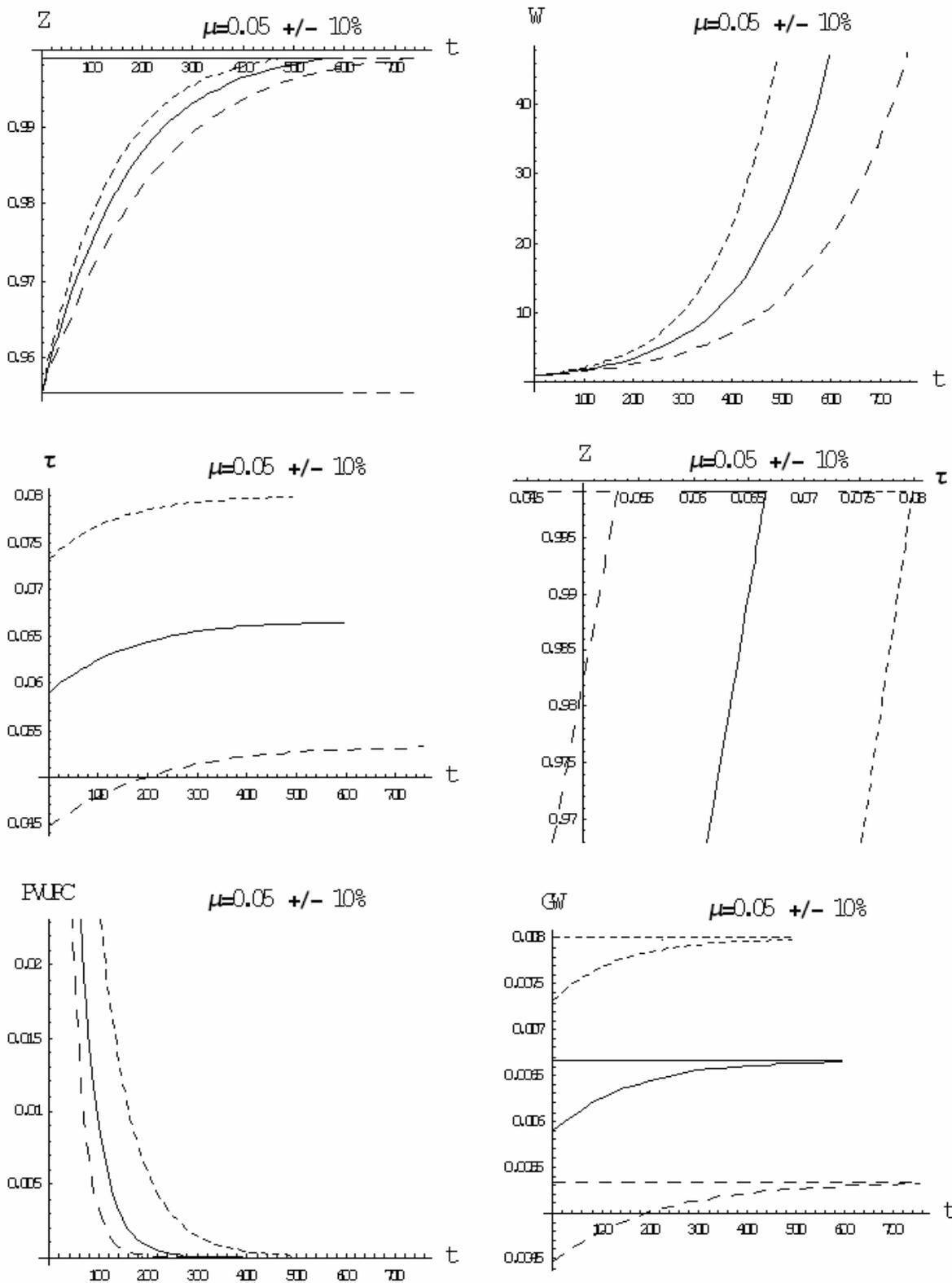


Figure 7. Sensitivity Results for Variations in  $\mu$ .

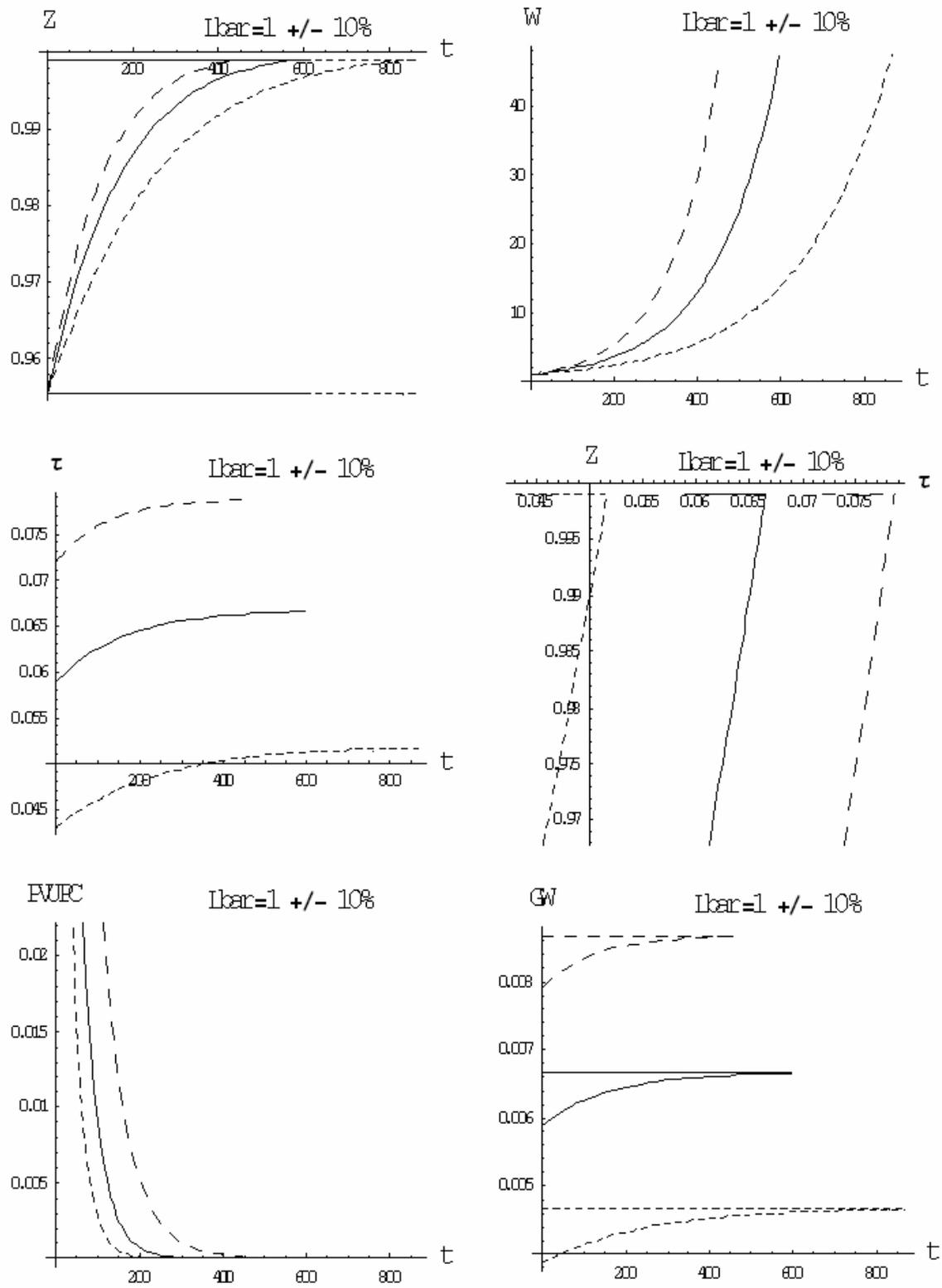


Figure 8. Sensitivity Results for Variations in  $\bar{L}$ .

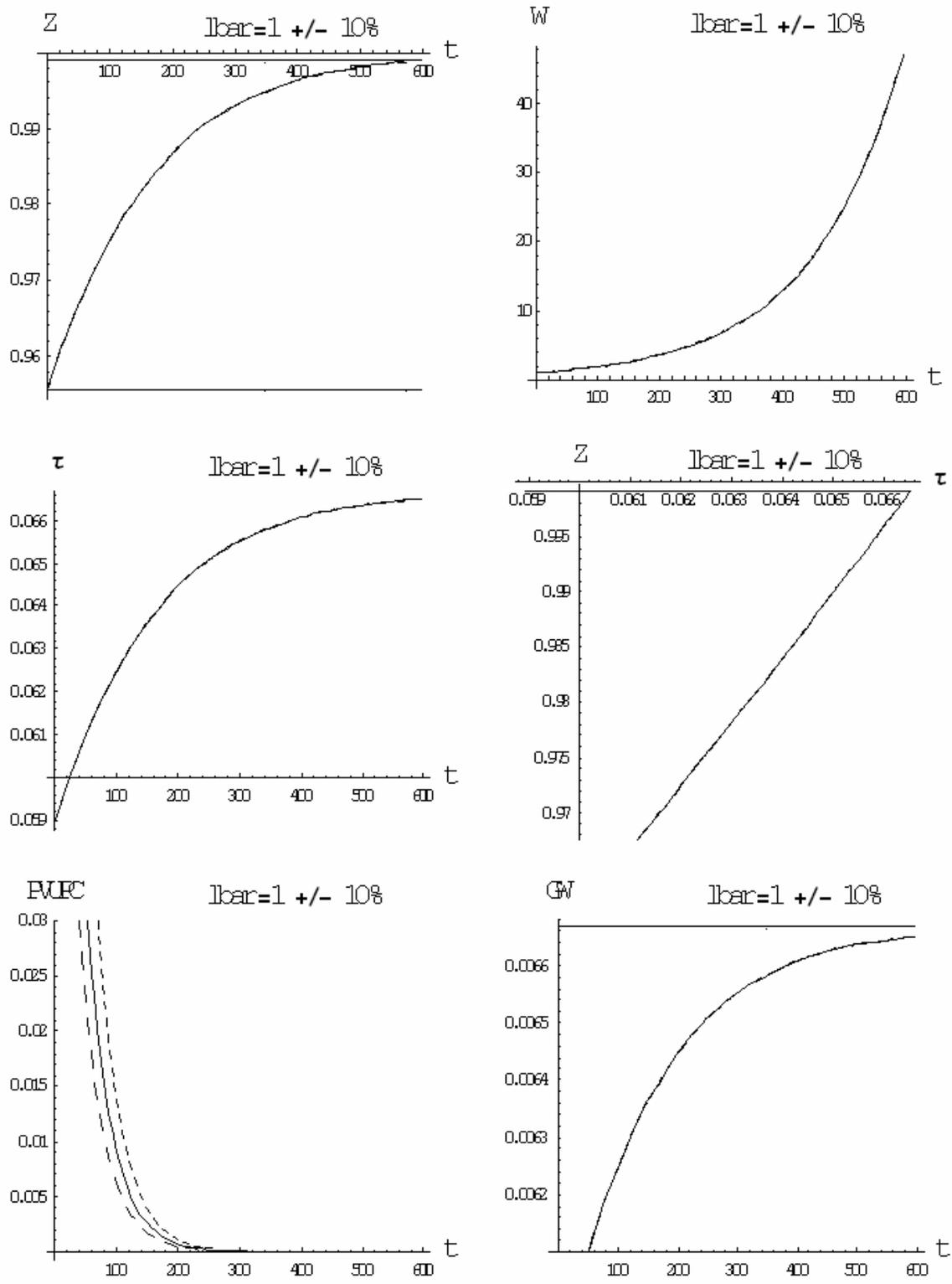


Figure 9. Sensitivity Results for Variations in  $\bar{l}$ .

#### 4. Concluding Remarks

In the previous section we have shown that reductions in production costs but also in transport and communication costs have an immediate effect on the rate at which communities would want to become connected and trade with each other. We have shown that increasing production costs lead to faster transitions, while increasing transportation costs lengthen the transition towards the steady state. Both types of costs do not affect the steady state as such, but since the transition period is quite long, their different impact on transitional growth points at transportation and communication cost reductions in particular as a suitable vehicle to speed up growth.

The largest effect in growth-terms, however, can be observed for the case of reductions in the cost of making new connections. That has a relatively significant impact on both the steady state growth rate AND on transitional growth, while reducing the transitional period equally significantly. The same goes, *mutatis mutandis*, for changes in the rate of discount. Communities with lower rates of discount would have a higher incentive to connect/become integrated with 'the rest of the world'. This also goes the other way around; if the rate of discount in some community is low, the rest of the world has a relatively strong incentive to become connected to that community, since that community would be more willing to share in the common burden of maintaining and extending current and future connections.

We also showed that the population size of the communities (to be) connected strongly determines both the steady state growth rate and the transitional growth rates of all connected communities. The larger the communities are, the stronger growth will be, pointing towards a positive scale effect that arises out of the nature of the communication and transportation network itself, rather than out of a 'knife-edge' assumption about the functional form of the production function underlying the process of connecting communities. To some extent then, this scale-effect can be considered to be more 'real' than the one present in Romer (1990) or Aghion and Howitt (1992), for example. But even in the presence of a positive scale-effect, communities that are lagging behind in educational terms, would probably not be able to produce as many varieties as other communities of similar size but with a higher average level

of education would be able to do. This would severely limit the benefits for other communities from being connected with low level education communities. It follows that to make such connections worthwhile for every community concerned, differences in educational levels shouldn't be too large. This again points to education as an important determinant of the growth performance of an economy, through its impact not just on the quality of labor *per se* (as in the Lucas (1988) model), but on the size of the sub-spectrum of varieties that could be produced depending on average levels of educational attainment.

For now, we have to leave an extension of our model in which we formally integrate investment in the level of education of a community as an additional determinant of a community's growth potential through its impact on that communities' attractiveness to other communities as a potential trading partner for future research.

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