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Inertia, Interaction and Clustering in Demand[*]

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Abstract

We present a discrete choice model of consumption that incorporates two empirically validated aspects of consumer behaviour: inertia in consumption and interaction among consumers. We specify the interaction structure as a regular lattice with consumers interacting only with immediate neighbours. We investigate the equilibrium behaviour of the resulting system and show analytically that for a large range of initial conditions clustering in economic behaviour emerges and persists indefinitely. Short-run behaviour of the model is investigated numerically. This exercise indicates that equilibrium properties of the system can predict a short-run behaviour of the model quite accurately.

Key Words: *Clustering, Interaction, Habits, Consumer choice*
JEL codes: *D11, D83, C65.*

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Introduction

One of the challenges in modelling consumer behaviour lies on the observation that consumption is in many ways a social activity. This has been observed both in the context of bandwagon behaviour or conspicuous consumption (Liebenstein 1950; Smith 1776; Veblen 1899), but also in the context of learning to consume (Witt 2001). Consumers often face incomplete information both about what is available, and how to get the most out of the goods they consume. In both cases agents rely on friends and neighbours as sources of information. In addition, though, consumers appear to form habits (Guariglia and Rossi 2002), depending on rules of thumb and past behaviour to guide future choices.

In this paper we model the dynamics of individual consumer behaviour and analyze its implications for the distribution of the demand for goods over a social space. There are empirical studies of this issue, reporting on the impact of social space on demand (e.g. Birke and Swann 2006), but those papers tend to explain their results entirely through network externalities. In this paper we use more general constructs and show that the network structure of social interactions can be reflected in demand. Key to the consumer's decision-making, and thus to the dynamics of demand, is the consumer's on-going, or repeated evaluation of her alternatives. In our model valuations are based on two things: the consumer's own consumption history; and the consumer's neighbours' consumption histories. Consumers repeatedly decide which products to buy, and learning by consuming increases the future valuation of a product for a consumer. Consumers also routinely interact with their neighbours and exchange information about products on the market. Based on these two distinct information streams consumers update their valuations for each product and in response (possibly) change their behaviour.

From this starting point we model two aspects of consumer behaviour: inertia in consumption; and local influence of peers through interaction. The model can be interpreted in two ways.[1] One is to say that there is an imperfect informational structure in the economy and consumers are aware of that fact. They try to reduce uncertainty in the decision process (Jacoby et al. 1994) by using two sources of information. One is the information they receive through own experience. As consumers have the better understanding of the value of the goods they have already consumed, consuming the same good avoids possible disappointment. The other is the information they receive from their social networks about the available goods. Information gathered from "friends" can similarly reduce the risk of disappointment.

The second interpretation of the two parts of consumption dynamics would be that people form habits for the goods they consume, but that there is also an interdependence in the utilities of nearby consumers. With regard to habit formation, we assume that in the consumption process a consumer forms some special skills for using the product and as a result receives higher utility every time she consumes the same product. Interdependencies arise because people get higher utility if their consumption bundles are similar to those of their neighbours. This is similar to the effect of a "peer group" discussed by Bordieu (1984) and addressed in a formal model of consumption by Cowan et al. (1997).

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[1] Throughout the paper we use these two interpretations interchangeably.

We analyze the long-run (equilibrium) dynamics of a population of consumers subject to these two forces and show that spatial clustering in economic behaviour emerges as a stable, long run equilibrium pattern for a large set of initial conditions. Additionally though, analysis of the short-run behaviour indicates that equilibrium properties of present complex system can predict the short-run dynamics of the model quite accurately.

The remainder of the paper is organized as follows. The first section briefly reviews related literature. The second section presents the model. In the third section we present the analysis of the long- and short-run behaviour of the model. The fourth section presents one particular extension to the model. The last section of the paper concludes.

1 Literature

Of central interest here is information. Economists have long known that an assumption of perfect information was a strong one, and it has been relaxed in a variety of contexts. Early theoretical relaxations of the perfect information structure were applied to market organization (see Rothschild 1973 for a survey), credit rationing (e.g. Jaffee and Russel 1976; Stiglitz and Weiss 1981) as well as to a general consumer behaviour (e.g. Nelson 1970). But more recently, the consumer's lack of and need for different types of information have been studied more closely. For example, uncertainty about prices is discussed by Galeotti (2004), who examines the welfare implications of search costs when the distribution of prices is unknown. Similarly to the model presented in this paper, Samuelson (2004) models interdependency among consumers. There, consumers observe the actions of relatively successful consumers and use that information to impute which actions are likely to be good for themselves. In that model consumers are differentially successful, and information flow consists only of agents observing each others' actions. By contrast, in the model we develop below, agents are successful in optimizing at each step (given their current information), and have a richer information flow in that they pass to each other opinions about the values of all goods. Another important distinction between these two models is that Samuelson models the decision of "how much" to consume, while we model the decision "what" to consume. Our consumers act in complex environment and use information communicated to them in deciding on which product to buy, unlike Samuelson's consumers who are deciding on the consumption budget based on the information available to them.

In any situation in which information is imperfect, information acquisition can be valuable. Research in both marketing and psychology stresses the immense importance of information collection for the consumer decision process (Bettman 1971), in that it permits consumers to make better (in the sense of utility-increasing) choices. A large empirical literature shows that people tend to collect information through many different sources, such as the media, sellers or other consumers. However, in a seminal work, Hansen (1972) shows that information received from peers through social networks, is the dominant source of knowledge about goods considering both the information's reliability and its ability to affect the receiver. Thus, if one wants to understand the influence of external information on consumer decisions, it seems reasonable to

concentrate on information coming from peers, rather than from any other external source. So, while not denying the importance of other sources of more general external information, in this paper we focus on socially localized peer effects.

The view that agents use both internal and external sources of information in making decisions is not new in economics and has been applied to related fields. For example, information cascade models (two canonical papers being Banerjee 1992; and Bikhchandani et al. 1992) consider a population of agents, sequentially making decisions using both public and private information. The interest there is the conditions under which public information can overwhelm private, and the possibility of that creating a sub-optimal (aggregate) outcome. In a certain sense, the model presented in this paper is also an information cascade model, but it differs from the conventional models in two ways. First, consumers make repeated choices. This allows us to study the effects of the change in internal information driven by the consumption process itself. Second, cascade models agents receive information only about other agents' actions. In our model they receive (somewhat subjective, though higher bandwidth) information not only about the current actions of others, but also about available options not chosen. Thus, information about *any* particular option, even if it is not being taken by any agent, can form a cascade as it flows within the population. Agents use information (which may or may not be cascading) about each of the options to make a choice for one of them.

A second literature that relates closely has to do with habit formation. Habit formation in consumption was discussed early by Duesenberry (1949) and Brown (1952). These approaches are concerned with the formation of the general habit of consuming, meaning that people form habits to consume in general, rather than the habit of consuming some particular good. More recently, habit formation has been rigorously incorporated into consumer decision models by Abel (1990), Constantinides (1990) and others. These models have been extensively used to explain equity premium and risk-free rate puzzles (Constanides 1990; Otrok et. al. 2002) as well as the stylized fact that higher growth rates lead to the higher savings rates (Carrol and Weil 2000). By contrast to the formation of the general habit of consuming the present paper is concerned with habit formation for isolated products. These are the habits that people develop themselves through the consumption process.

One good, and well-studied example would be eating habits. Smith (2004), for example, drawing on empirical literature from a wide variety of behavioural and hard sciences, shows that people acquire very strong eating habits that persist for a long period. He refers here not to the habit of eating generally, but to habits regarding particular foods. He also shows that people are more likely to consume products that they see other people consuming, which is a basic assumption of our model.

The marketing and psychology literatures referred to above have shown that friends and neighbours are an important source of information. The "externalities in consumption" literature has a similar feature in that externalities are often seen as (spatially or socially) limited in scope. The possibility that interactions can be localized in various dimensions has been raised in other contexts. Scheinkman and Woodford (1994), or Weisbuch and Battiston (2007), for example, examine non-market interactions between consumers and producers; Eshel et al. 1996, and

Cowan et al. (1997) look at interactions among consumers. In general, interactions generate feedback loops that affect the decisions of the economic agents. But as noted by Glaeser and Scheinkman (2000) the structure of those interactions can make a significant differences both for the sorts of equilibria that emerge and for the dynamics leading to them. In particular, they show that when interactions are local the economy generates richer dynamic possibilities, having multiple equilibria and the possibility of moving from one equilibrium to another.

More contextualized work on interactions shows that they can explain certain interesting phenomena in economics or other social sciences, such as the standardization process (e.g. Arthur 1989; Cowan 1991; Eshel et al.1998), waves in consumption across the population classes (Cowan et al. 2004), or contagious justice (Alexander and Skyrms 1999).

2 The model

The model we develop here can be seen as a repeated discrete choice model in which consumers' evaluations of goods are determined by internal and external information sources.

Consider an economy inhabited by a large, finite number (S) of agents, indexed by s. Each is a single consumer faced with the same fixed, finite set of substitute goods, indexed by n. In each period, each consumer consumers one unit of one good. The consumption choice is based on the consumer's "valuation" of the goods.

The valuation a consumer ascribes to a given good is the maximum price she is willing to pay for it. Using very basic consumer theory, the utility a consumer derives from consuming a good will be the difference between its valuation and price that she pays. We define $v_{n;t}^s$ at the net valuation consumer s ascribes to good n at time period t.

We adopt a standard discrete choice approach (Andersen et. al. 1992) and assume that each consumer buys one and only one product each time period. Under this assumption the utility of individual agent can be written as

$$U_t^s = v_{n^*,t}^s, \tag{1}$$

n^* is the good consumed by consumer s in period t. We assume that consumers are unable to deliberately manipulate the choices of their neighbours, and so do not choose "strategically", but rather simply maximize instantaneous utility. Under this setup, utility maximization implies that in each period the consumer chooses $n_t^{s*} = \arg\max(v_{n;t}^s)$.

What we seek to model here is the dynamics of product purchases as they respond to changes in the valuations of consumers of the goods available on the market. Following the discussion in the introduction, we assume that valuation is derived from information of two types: internal and external. So we can write:

$$v_{n;t}^s = f(x_{n;t}^s, y_{n;t}^s), \tag{2}$$

where $x_{n;t}^s$ is determined by own consumption history, and $y_{n;t}^s$ by the consumption history of other members of the same social group as consumer s.

Both parts of the valuation are subject to change over time: $x_{n;t}$ is subject to change due to habit formation (which results in inertia in consumption) and $y_{n;t}$ is subject to change due to local interaction (because of information exchange or network externalities). Assume that $f(\cdot)$ is additive, and write the dynamics of v_n^s as[2]

$$\Delta v_n^s = \Delta x_n^s + \Delta y_n^s. \tag{3}$$

To model interaction among consumers we assume that every consumer has a fixed social location and a fixed neighbourhood. A neighbourhood is the set $(\mathcal{H}^s)$ of other agents with whom an agent (s) interacts directly. In this context, interaction is tantamount to information exchange. Each information exchange consists of two agents revealing to each other their private evaluations of each of the goods. The information revealed is assumed to be "convincing" in the sense that the post-exchange valuations of each of the two agents partially converge. Hence, this exchange process can be expressed simply in terms of the dynamics of beliefs of a single agent, s, following her exchanges with all of her neighbours, i:

$$\Delta y_n^s = \sum_{i \in \mathcal{H}^s} \frac{\mu}{|\mathcal{H}^s|}(v_n^i - v_n^s), \tag{4}$$

where $|\mathcal{H}^s|$ is the cardinality of the set $\mathcal{H}^s$ (number of neighbours of agent s), and μ ($\in [0, 1]$) is the intensity of interaction. We assume that all products are substitutes and there are no *ex ante* systematic differences among consumers, so interaction intensity is the same across all the goods and agents.

For concreteness, assume that consumers are located on a one-dimensional, regular, periodic lattice such that the distance between any two agents corresponds to the social distance between them, and the distance between immediate neighbours is constant across all the population. In this case we can define the neighbourhood of an agent $(\mathcal{H}^s)$ simply by specifying the number of agents (H^s) with whom this consumer interacts on the left and on the right. Then $|\mathcal{H}^s| = 2H^s$.

If we assume neighbourhood size to be equal across the population, that is $H^s = H \ \forall s$, we can write

$$\Delta y_n^s = \frac{\mu}{2H} \sum_{h=1}^{H} \left[(v_n^{s+h} - v_n^s) + (v_n^{s-h} - v_n^s) \right], \tag{5}$$

where s can be interpreted as a "serial number" of an agent, or her address (consequently, $s + 1$ and $s - 1$ are her immediate neighbours to the right and left respectively).

Re-arranging, (5) can be rewritten as

$$\Delta y_n^s = \frac{\mu}{2H} \left[\sum_{h=1}^{H} (v_n^{s+h} + v_n^{s-h}) - 2H v_n^s \right]. \tag{6}$$

Valuations are also influenced by habit formation.[3] Habits are formed only for goods that

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are consumed. Thus, Δx_n^s is equal to zero for the goods that are not consumed in a given period and is equal to some positive value for the good that has been consumed:

$$\Delta x_n^s = \begin{cases} \zeta & \text{if } n = n_t^{s*} \\ 0 & \text{otherwise,} \end{cases} \quad (7)$$

where $\zeta \ (> 0)$ is a constant and n_t^{s*} is a product that agent s has purchased in period t.

To summarize the model we can make explicit the sequence of consumers' actions. At the start of each period every agent decides which good to consume. After purchase she consumes it and forms habits for it. At the end of the period each agent meets all of her neighbours and passes to them all the information (that is, her valuations of all goods) that she possesses. Based on the information communicated to them by neighbours all agents adjust their valuations of all goods.

We are interested in whether this kind of behaviour has implications for the social geography of demand; more precisely, whether any specific patterns emerge in the long-run. Essentially we ask whether one can determine anything about the consumption basket of a consumer by looking at the consumption baskets of her neighbours.

3 Equilibrium analysis

In this section we analyse long-run equilibria of the model. It is not possible to solve the model as presented in section 2, so in the process of solution we make two modifications. First, we assume that the habit formation process can be well-approximated (at least in the region of interest) by a linear function. Second, we re-write the model as continuous in time and space.

Linearization. Above, equation (7) shows habit formation: a consumer forms habits only for the good he consumes, and the effect on her valuation takes place in discrete jumps. This describes a path dependent process. This is problematic, as analysis of the system at any point in time requires analyst's knowledge of the whole history of the system. However, employing a standard way of modeling expected product choices, allows us to approximate the dynamics of (7) with a Markov process (Andersen et al. 1992). We model the choice of the consumers as a conventional discrete choice, where it is based on probabilities: agent s chooses good n with probability $p_{n;t}^s$ at time period t. In this case, the law of motion in equation (7) becomes:

$$\Delta x_n^s = \begin{cases} \zeta & \text{with probability } p_{n;t}^s \\ 0 & \text{with probability } 1 - p_{n;t}^s. \end{cases} \quad (8)$$

Further, $p_{n;t}^s$ will be a function of the vector of valuations for the agent s at period t. Thus we can write $p_{n;t}^s = p_n(\mathbf{V}_t^s)$, where $\mathbf{V}_t^s$ is the vector of valuations. Then the expected change in valuation due to habit formation, $x_{n;t}^s$, can be written as:

$$E\left(\Delta x_n^s\right) = \zeta p_n(\mathbf{V}_t^s). \quad (9)$$

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product, rather than formation of a general habit to consume.

The choice probability for a product n depends on valuations of all the products. However, it is reasonable to assume that the contribution of changes in valuations of products other than n are of second order significance. This is easy to see if we consider the effects of an increase in the valuation of good n. This will increase its purchase probability by Δp_n. This will also decrease the purchase probabilities of all the other products, each by Δp_j. As probabilities are normalized it should be the case that $|\Delta p_n| = \sum_{j \neq i} |\Delta p_j|$. If we have relatively large number of products in the economy, it will in general be true that $\Delta p_n \gg \Delta p_j$, $\forall j \neq n$. Thus a change in the valuation of one good will cause the change in its purchase probability. It will also cause the changes in purchase probabilities of other goods, but the size of each of these changes will be considerably smaller. Therefore we impose a restriction on our probability function: it has to satisfy the following relation

$$\left| \frac{\partial p_n}{\partial v_n} \right| \gg \left| \frac{\partial p_n}{\partial v_j} \right|, \tag{10}$$

$\forall j \neq n$.

Consider the linearization of function $p_n(\mathbf{V}_t^s)$. If the requirement (10) is satisfied, as a first approximation, we can disregard the effects of lower orders of magnitude and write a linearized function as $p_n(\mathbf{V}_t^s) \approx \gamma v_{n;t}^s$. This permits us to write the expected change in $x_{n;t}^s$ as

$$\Delta x_n^s = \alpha v_n^s, \tag{11}$$

where α ($= \gamma \zeta$) can be interpreted as the rate of habit formation.[4]

Substituting equation (11), allows us to write the system specified in section 2 as

$$\Delta v_n^s = \alpha v_n^s + \frac{\mu}{2H} \left[\sum_{h=1}^{H} (v_n^{s+h} + v_n^{s-h}) - 2H v_n^s \right]. \tag{12}$$

From (12) it is clear that the law of motion of valuation for every good for any agent depends on the agent's own valuation of that good, and on the valuations of the agent's neighbours of that same good.

For the demonstration of the solution to the system, assume that each agent has exactly two neighbours ($H = 1$), and that there are only two goods available on the market ($N = 2$).[5] The model reduces to a system of S pairs of equations of the form

$$\Delta v_1^s = \alpha v_1^s + \frac{\mu}{2} \left(v_1^{s+1} + v_1^{s-1} - 2v_1^s \right) \tag{13}$$

$$\Delta v_2^s = \alpha v_2^s + \frac{\mu}{2} \left(v_2^{s+1} + v_2^{s-1} - 2v_2^s \right), \tag{14}$$

where $s = 1, 2, 3, \ldots, S$.

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[4]In what follows we drop the expectation sign, although it should be remembered that all the discussion in this section is about the expected values of the variables.

[5]Both of these assumptions are relaxed at a later stage in propositions 5 and 6.

Continuous time and space. We seek to obtain the solution to the system given by (13) - (14). In the two-good system, what drives the dynamics at any point in time is the difference in the probabilities that each of the goods is chosen (by each consumer). We can thus re-write the system in terms of the difference in valuations of the two goods. Define the valuation difference $z^s = v_1^s - v_2^s$ and rewrite the system (13)-(14) as

$$\Delta z^s = \alpha z^s + \frac{\mu}{2}\left(z^{s+1} + z^{s-1} - 2z^s\right). \tag{15}$$

Next step is to approximate the discrete system (15) with its continuous counterpart. To do this we define a new variable δ which is the distance between two neighbouring consumers on the circle. Using δ we can rewrite equation (15) in continuous time and space

$$\frac{\partial z(s)}{\partial t} = \alpha z(s) + \frac{\mu}{2}\left(z(s+\delta) + z(s-\delta) - 2z(s)\right). \tag{16}$$

Then we can make a second order Taylor approximation in space around s for the terms $z(s+\delta)$ and $z(s-\delta)$. This will result in

$$z(s+\delta) \approx z(s) + \delta\frac{\partial z(s)}{\partial s} + \frac{\delta^2}{2}\frac{\partial^2 z(s)}{\partial s^2} \tag{17}$$

and

$$z(s-\delta) \approx z(s) - \delta\frac{\partial z(s)}{\partial s} + \frac{\delta^2}{2}\frac{\partial^2 z(s)}{\partial s^2}. \tag{18}$$

Substituting equations (17) and (18) into equation (16) collapses our system into one partial differential equation

$$\frac{\partial z}{\partial t} = \alpha z + \tilde{\mu}\frac{\partial^2 z}{\partial s^2}, \tag{19}$$

where $\tilde{\mu} = \mu\delta^2/2$.

In the following section we investigate the long run equilibrium behaviour of the system (19). Some insights to the behaviour of the model in the short-run will be provided in section 4.

3.1 Distribution of behaviour over space

It simplifies the analysis to separate the dynamics of $z(s;t)$ into the dynamics of the average over the population – $\bar{z}(t)$; and the dynamics of the deviations from this average – $\tilde{z}(s;t) = z(s;t) - \bar{z}(t)$.

Proposition 1. *The cross-agent average of valuation-differences ($\bar{z}_t$) evolves according to*

$$\bar{z}(t) = e^{\alpha t}\bar{z}(0).$$

Proof. In the continuous case the average over space can be defined as $\bar{z} = (1/S)\int_0^S z\,ds$. This

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implies that

$$\frac{\partial \bar{z}}{\partial t} = \frac{1}{S} \int_0^S \frac{\partial z}{\partial t} ds.$$

Then, using equation (19) we can write

$$\frac{\partial \bar{z}}{\partial t} = \alpha \frac{1}{S} \int_0^S z\, ds + \tilde{\mu} \frac{1}{S} \int_0^S \frac{\partial^2 z}{\partial s^2} ds. \tag{20}$$

As space in our system is a periodic lattice the second summand in equation (20) is zero.[6] Then, using the definition of average again we can write equation (20) as

$$\frac{\partial \bar{z}}{\partial t} = \alpha \bar{z}. \tag{21}$$

This is an ordinary differential equation with the solution described in the proposition. $\square$

Proposition 2. *With time, deviations of valuation-differences ($\tilde{z}_t^s$) in system (19), converge to*

$$\tilde{z}(s;t) = e^{\sigma t} \cos\left(k\frac{2\pi}{l}s\right) \tilde{z}(0;0),$$

where l is the length of the circle on which consumers are placed, while σ is the amplitude growth rate and $k (\in \mathbb{Z}_+)$ is the frequency of the sinusoid $\tilde{z}$.[7]

The comprehensive proof of this proposition can be found in Turing (1952); here we give the basic intuition. The general solution to differential equations of this type can be represented as the (possibly infinite) sum of exponential functions of the form Ae^{bt}, where A and b are (possibly complex) coefficients. The real part of each summand in the solution can be represented as the dynamic sinusoid (in our case around the lattice on which consumers are located). The real part of each b will be the growth rate of the amplitude of the corresponding sinusoid. As a result, as $t \to \infty$ one summand will dominate all the others. This will be the term with the largest real part of b. Consequently the dynamics of the solution will converge to one sinusoid.

Proposition 3. *The amplitude growth rate of the dominant sinusoid of system (19) is*

$$\sigma = \alpha - \tilde{\mu}k^2 \left(\frac{2\pi}{l}\right)^2.$$

Proof. From proposition 1 and 2, we know that $z(s;t) = e^{\alpha t}\bar{z}(0) + e^{\sigma t}\cos\left(k\frac{2\pi}{l}s\right)\tilde{z}(0;0)$. Substituting this into equation (19) and noticing that $\partial^2 \cos(\beta x)/\partial x^2 = -\beta^2 \cos(\beta x)$, allows us to

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[6]To see more easily why the second summand is zero, one can discuss the discrete case and thus use equation (15) instead of equation (19). In the discrete case the second summand is $\sum_s \left((z^{s+1} - z^s) - (z^s - z^{s-1})\right)$. As consumers are indexed by s around a circle, it is obvious that this sum is zero.

[7]Note that as consumers are located on a periodic lattice, the identity of agent zero is arbitrary, and thus can be placed anywhere on the circle. To write down proposition 2 we have set label 0 such that $s_0 = \arg\max_{x \in \left[0, \frac{l}{k}\right]} \cos\left(k\frac{2\pi}{l}x\right)$, which effectively means that we label agents such that the sinusoid identified in proposition 2 reaches its maximum at the agent number zero.

solve for σ. □

Propositions 1 through 3 fully characterizes the solution to the system (19). Following subsections investigate the implications of the solution.

3.2 Temporal stability of clustering

In order describe the behaviour of the model in equilibrium we have to combine the results of propositions 1 and 2. For making interpretations of the results transparent, it is useful to go back to the discrete space and time. Thus, we move back to treat s as the serial number of an agent.[8] This makes $\tilde{\mu} = \mu/2$ and $l = S$. In this case we can write the complete solution to our system as

$$z_t^s = e^{\alpha t}\bar{z}_0 + e^{\sigma t}\cos\left(k\frac{2\pi}{S}s\right)\tilde{z}_0^0. \tag{22}$$

where

$$\sigma = \alpha - 2\mu\frac{\pi^2}{S^2}k^2 \tag{23}$$

Equation (22) determines the value of the difference in valuations (z) for every agent for every $t \gg 0$. The distribution of z along the circle has the form of a wave in space around the average, which points to the fact that in some neighbourhoods $\tilde{z}$ is positive, while in some other neighbourhoods it is negative (this is easiest so see if we assume that $\bar{z}_0 = 0$). This means that some neighbourhoods are more likely to buy one product, while some other neighbourhoods are more likely to buy the other with a gradual transition between them. Thus the general result is that the clustering in demand is an emergent property of our system.[9]

Our concern in this section is whether any observed clustering is persistent over time. Consider the case where $\bar{z}_t \neq 0$, $\exists t \geq 0$. That is, at some point in time one of the products is perceived as superior on average. In this case propositions 1 and 2 have an important corollary:

Corollary 1. *If $\exists t$ such that $\bar{z}_t \neq 0$, then as $t \to \infty$, $v_i^s > v_j^s \ \forall s$ and thus in equilibrium there will be one cluster of size S in the economy.*

Proof. Consider the situation when $\bar{z}_t > 0$. Define $z^{min} \equiv \min_s (z^s)$ as the valuation difference of an agent with the lowest z.

Case 1: $z^{min} > 0$. This implies that $\forall s \ z^s > 0$, thus there is one cluster of size S. This is a stable pattern as both forces (interaction and habit formation) work to reinforce it.

Case 2: $z^{min} < 0$. In this case some of the consumers prefer the relatively "inferior" product.
Case 2a: $\sigma < 0$. Proposition 2 tells us that if $\sigma < 0$, with time, the amplitude of the wave goes to zero, which implies that $\forall s \ z^s = \bar{z}$. This, together with proposition 1, results in $z^s > 0 \ \forall s$ as $t \to \infty$.

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[8]This effectively means that we fix $\delta = 1$. This move does not undermine the results of propositions 1 through 3. Moving back to consumer addresses is convenient for relating parameters in the solution to the parameters of the model.

[9]Our model can be applied to any type of economic behaviour that involves the choice among exclusive options at a constant cost. Thus clustering in this system will be a property of not only demand but of any similar economic activity.

Case 2b: $\sigma > 0$. From proposition 2 we know that the amplitude of the wave around the average increases at rate σ. At the same time, propsition 1 suggests that the average over agents of the valuation-difference rises at the rate α. Therefore z^{min} is rising at the rate $\alpha - \sigma$. Equation (23) establishes that this rate is positive.[10] $\alpha - \sigma > 0$ ensures that as $t \to \infty$, $z^{min} > 0$. $z^{min} > 0$ implies that $\forall s \; z^s > 0$. Thus case 2b with certainty collapses into case 1 at some point in time.

These intuitions hold for the situation when $\bar{z}_t < 0$.[11] $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

In relation to market structure, we can have another corollary:

Corollary 2. *If $\exists t$ such that $\bar{z}_t \neq 0$, as $t \to \infty$, $v_i^s - v_j^s \to \infty \; \forall s$ and in equilibrium everybody will purchase only one of the products.*

Proof. Proof of corollary 1 directly implies not only that $v_i^s > v_j^s \; \forall s$ in equilibrium, but also that $v_i^s - v_j^s \to \infty$, which on its own implies that as long as the choice probability function is a positive monotonic mapping of valuations to choice probabilities, the probability of any agent purchasing product i converges to 1. $\qquad\qquad\qquad\qquad\qquad\qquad\square$

Thus, $\bar{z}_t \neq 0$ is a relatively trivial case, and implies that ultimately only one product is consumed in the population, no matter the dynamics of the deviations from the average, and that clustering is a stable pattern.

Far more interesting is the case in which $\forall t \; \bar{z}_t = 0$, which permits both products to co-exist on the market indefinitely. Intuitively the stability of the cluster should depend on its size. For example, if one individual constitutes a cluster she is susceptible to influence from both her neighbours, both proponents of the choice contrary to hers. This cluster is less likely to be stable than a larger cluster where most of the members of the cluster (the ones away from its boundaries) receive information that reinforces their choices. Thus, there should be some minimum cluster size for which clustering will be persistent. When $\forall t \; \bar{z}_t = 0$ we know that behaviour of the system is governed by the pattern sine wave, which implies that all the clusters are of an equal size in the equilibrium.

Proposition 4. *In system (19), if $\forall t \; \bar{z}_t = 0$, clustering in demand is stable if and only if the pattern wave of the system results in the clusters of size $c \geq \underline{c} = \frac{\pi}{\sqrt{2}} \sqrt{\frac{\mu}{\alpha}}$.*

Proof. From equation (22) it can be readily seen that, when $\bar{z}_t = 0 \; \forall t$, temporal stability of clustering depends on the sign of σ. If $\sigma < 0$, as $t \to \infty$, $z^s \to 0 \; \forall s$, which implies that $v_1^s \to v_2^s \; \forall s$. This means that valuations of products converge, so in the case of probabilistic purchases every agent decides on her purchase by tossing a (fair) coin. This, clearly, will result in no clustering pattern.

However, if $\sigma > 0$ the amplitude of the pattern wave increases exponentially with time, thus clustering becomes more and more pronounced. If $\sigma = 0$, the amplitude of the wave does not change with time, and clustering is still stable.

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[10]Unless $\mu = 0$, which is not a very interesting case as it implies no social influence. In this case the existing consumption pattern is reinforced indefinitely.

[11]This proof can be easily generalized to a multiproduct case.

Given the parameters of the model, the sign of σ depends on the frequency of the wave in the initial condition. We can pin down the critical frequency of the pattern wave (k), for which clustering will be stable, by simply solving $\alpha - \mu k^2 \frac{2\pi^2}{S^2} = 0$, for k. This results in $\bar{k} = \frac{S}{\pi}\sqrt{\frac{\alpha}{2\mu}}$. And $k \leq \bar{k}$ ensures that $\sigma \geq 0$. The inverse of the frequency is the wave length, and the size of the cluster is half of the wave length. Since the size of the economy is S, the size of the cluster(s) is $S/(2k)$. Thus, given $\bar{k}$, we can find the size of the smallest cluster that will persist over time: $\underline{c} = \frac{\pi}{\sqrt{2}}\sqrt{\frac{\mu}{\alpha}}$. Any pattern wave exhibiting clusters larger than $\underline{c}$, would ensure $\sigma \geq 0$, and thus will result in stable clustering. $\qquad\square$

The important property of the minimum stable cluster size is that it does not depend on the size of the economy. However, as σ depends on S, a larger economy (*ceteris paribus*) increases the likelihood that the pattern wave of the system will support clusters of any given size c, thus it also increases the likelihood of clustering. We also point out that the minimum stable cluster size depends on the ratio of two parameters, habit formation and information transmission: μ/α.

The analysis so far has assumed that there were two goods ($N = 2$) and each agent has 2 neighbours ($H = 1$ on either side). It is also interesting whether these two variables have any influence on minimum stable cluster size.

Proposition 5. *In the case of arbitrary neighbourhood size $2H$ minimum sustainable cluster size is*

$$\underline{c}_H = \frac{\pi}{2\sqrt{3}}\sqrt{2H^2 + 3H + 1}\sqrt{\frac{\mu}{\alpha}}.$$

Proposition 6. *In the case of a multi-product environment, N being the number of products, minimum sustainable cluster size is $\underline{c}_N = \underline{c} = \frac{\pi}{\sqrt{2}}\sqrt{\frac{\mu}{\alpha}}$.*

Proofs of propositions 5 and 6 can be found in the appendix.

From proposition 6, it is obvious that an increase in the number of products does not affect the stability properties of the system. However, proposition 5 implies that as neighbourhoods grow in size so does the minimum sustainable cluster. The intuition is that a larger neighbourhood facilitates the information diffusion process: each agent receives information from relatively distant agents. This works to homogenize the information structure across the population, and so works against small clusters.

We can analyze how minimum sustainable cluster size changes with enlargement of the neighbourhood. It is obvious from proposition 5 that $\underline{c}_{H+1} - \underline{c}_H$ is increasing with H. Moreover, it turns out that

$$\lim_{H \to \infty} \left(\underline{c}_{H+1} - \underline{c}_H\right) = \frac{\pi}{\sqrt{6}}\sqrt{\frac{\mu}{\alpha}}. \tag{24}$$

Equation 24 implies that for any value of μ/α, minimum sustainable cluster size increases linearly with the size of the neighbourhood, as long as H is sufficiently high.

4　Short-run analysis

Analysis of the model in section 3 characterizes its long-run, equilibrium dynamics. However, as those are asymptotic results, which might take a long time to emerge, short run behaviour of

the system is also worth investigating. In particular how do clusters emerge and develop? What is the relation between the average cluster size and the parameters of the model? In this section we address these questions numerically.

Recall that the discrete nature of habit formation posed a problem for the mathematical analysis of the model. To address issues of tractability we assumed probabilistic purchases, and linearized the probability. With numerical simulations we are not so constrained and we can directly analyze the original model. However in order to ensure that the simulation and analytic results are in general agreement, initially we present the results for the linearized model as analyzed in sections 3.1 and 3.2.

4.1   Linearized model

We set the number of goods to $N = 10$;[12] and the population size to $S = 100$. The population is located on a one-dimensional periodic lattice, so the neighbours of agent 1 are agents 2 and 100. The specific parameters for habit formation, α and interaction μ are $\alpha = 0.0005$ and $\mu = 0.01$. Finally, each agent has one neighbour on either side, $H = 1$.[13] To read the figures below, agents are arrayed along the abscissa, remembering that the axis is a circle, so the right-most and left-most agent are neighbours. Time is read on the ordinate, from the initial period, $t = 0$ to the final period, $t = 2000$. Each good is assigned a different shade of gray. The ordering of the goods, and therefore the shades of gray, is arbitrary. At each point in time the choice (or the good with the highest valuation) for each agent is shown by the colour corresponding to that good.

For completeness, we show not only probabilistic purchases (driven by valuations), but also actual purchaces. Thus we have to specify the function mapping valuation to the probability of choice. Here we simply adopt the multinomial logit, from discrete choice theory:

$$p_n(V_t^s) = \frac{e^{v_{n;t}^s}}{\sum\limits_{i \in \mathbf{N}} e^{v_{i;t}^s}}, \tag{25}$$

where $\mathbf{N}$ is the set of available products. Note that $\partial p_n / \partial v_n = p_n(1 - p_n)$ and that $\partial p_n / \partial v_j = -p_n p_j, \ \forall j \neq n$. As in multi-product case $|p_n(1 - p_n)| \gg |-p_n p_j| \ (\Rightarrow 1 - p_n \gg p_j)$ is true, probability function (25) satisfies the requirement (10).

Figure 1 shows the dynamics of the most preferred products and actual purchases in a representative run of the linearized model with random initial conditions: for each agent-product pair a $v_{n;0}^s$ is drawn from the uniform distribution over the interval $[0, 20]$.[14] As one can see the clustering pattern in "most preferred goods" is clearly identifiable after just a few periods. The same pattern is replicated (although with some noise) by the actual purchase. Actual choices differ from the preferred good only due to the probabilistic choice function (equation

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[12]We expand the number of goods for reasons of generality. According to proposition 6, this does not affect the stability of the system.

[13]Note that for this constellation of the parameters $\bar{k} \approx 5.04$ and $\underline{c} \approx 9.93$, as derived above.

[14]Changing the uniform distribution to other standard symmetric distributions does not change the numerical results.
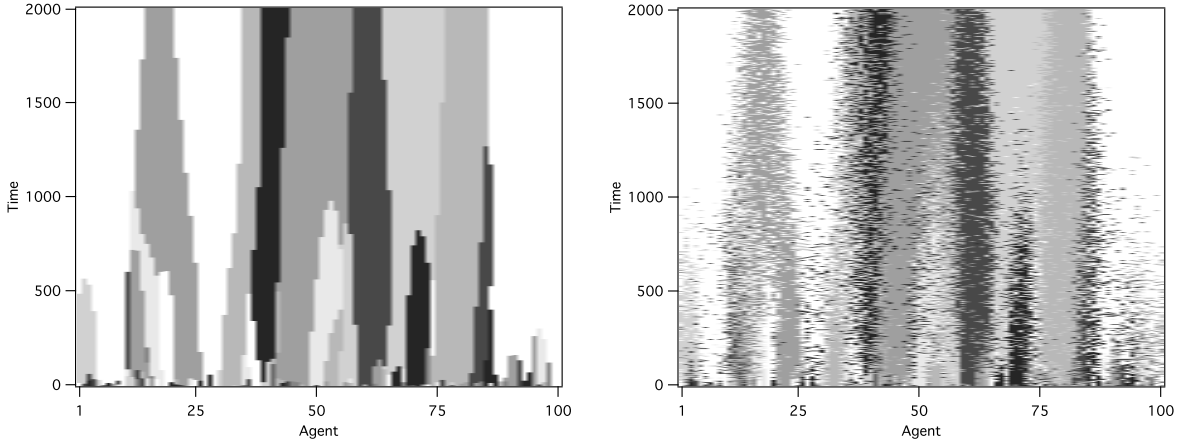
Figure 1: Most preferred products (left), and actual purchases (right) in the linearized model.

(25)). This difference is especially marked near the borders of a region, since here agents receive contradictory information about products, which tends to reduce the difference between their valuations of the most preferred good and other goods. This makes the probability choice function relatively flat for agents near the borders of clusters, and choices less correlated with those of their neighbours.

We must point out that clustering patterns identified in Figure 1 are only meta-stable. The reason is that stability of the multiple clusters requires $\bar{z}_t = 0 \ \forall t$ (corollary 1), the multi-product equivalent of which is $\bar{v}_{i;t} = \bar{v}_{j;t} \ \forall i, j, t$. Although this requirement can be imposed on the system while simulating the linearized model, it can not be guaranteed for the original model. (In fact there will always be some finite time at which mean valuations of two goods will differ: $\exists t < \infty$ such that $\bar{v}_{i;t} \neq \bar{v}_{j;t}$.) To make the examples comparable we do not impose the $\bar{v}_{i;t} = \bar{v}_{j;t} \ \forall i, j, t$ constraint on simulation of the linearized model either. Thus we know that the equilibrium of all our runs is the state which results in only one cluster (corollary 1). However, our experiment shows that in the short-run multiple clusters emerge and persist for relatively long periods.

4.2 Original model

Recall that there are two major differences between the linearized and the original model. One is that in the original model there are no probabilistic purchases, thus utility maximization implies that each consumer purchases her most preferred product in each period. Another is that in this case we have a habit formation *step* ζ instead of a habit formation *rate* α. We know that $\zeta = \alpha/\gamma$ where γ is the constant coming from the linearization of the choice probabilities. Unfortunately there is no way to pin down the value of γ. For this reason we cannot make a judgment about the relative magnitudes of α and ζ (apart from the fact that they are proportional), and thus the choice of the value of ζ is somehow arbitrary. We choose $\zeta = 0.01$ and use the same values for all other parameters as in the previous run. The result of the representative run of the original model is presented in figure 2. As one can see the clustering in purchases is clearly visible and relatively stable.

These numerical exercises also permit us to make a comment about what revealed preferences
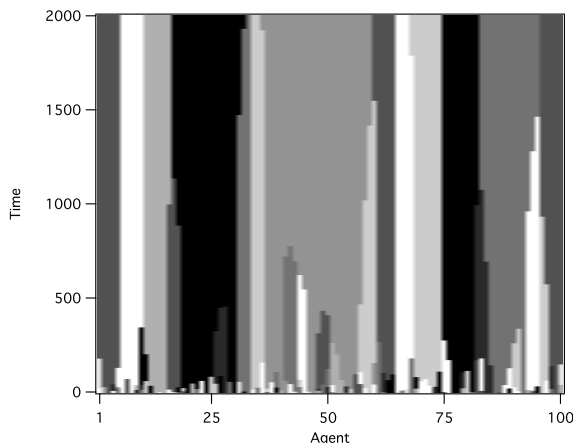
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Figure 2: Most preferred products and actual purchases in the original model.

cannot reveal. Revealed preferences give us information only about the most preferred product, namely which it is, and completely neglect the story that is going on in the background. By this we refer to the fact that agents do have preferences over, and information about the goods they do not in fact consume. Without acknowledging the importance of those "unexpressed" preferences it is difficult to understand a sudden change in consumption which is not simply imitating neighbours. This is something that is possible in our approach, and in fact is observed in figure 2 as well as in figure 1.[15] We observe several cases of an agent adopting a new good which neither she nor her neighbours have consumed in the past. The explanation lies in the fact that an agent close to the border of a cluster can receive contradictory signals. Consider the following simple example. Agent $s-1$ ranks good 1 first and good 3 last; agent $s+1$ ranks good 3 first and good 1 last. Both agents, though, rank good 2 second. It is clear that agent s, based on her external information, could easily rank good 2 before either 1 or 3. If the high rankings of good 2 by $s-1$ and $s+1$ have emerged (due to information received by their neighbours) at roughly the same time, agent s can then switch to good 2, regardless of what he was doing in the past. This explains the emergence and growth of such neighbourhoods in our framework. Thus, our model is consistent not only with shrinking and disapearance of smaller clusters, but also with the emergence and growth of new ones.

As clustering in these simulations is only meta-stable, average cluster size should be steadily increasing over time until it reaches the equilibrium size $\bar{c} = S$. It is interesting to see how the rate of increase depends on the parameters of the model. As the amplitude growth rate, σ, of the dominant wave controls the speed of convergence to the equilibrium, intuitively it should also control the growth rate of the average cluster. Besides it's partial dependency on initial conditions (due to k, which is the frequency of the dominant wave as determined by initial conditions), σ also depends on habit formation, α, interaction intensity, μ and population size, S. Or in the case of the original model on ζ, μ and S. Figure 3 shows the dynamics of the average

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[15]In the figure 2 the best example of this sort is agent 62 at period 70, who switches to consuming a product never consumed in her neighbourhood before. In the left panel of figure 1 a similar pronounced example is agent 39 at period 50, who is the pioneer of a new product consumption in her neighbourhood. In both cases products introduced survive and spread.
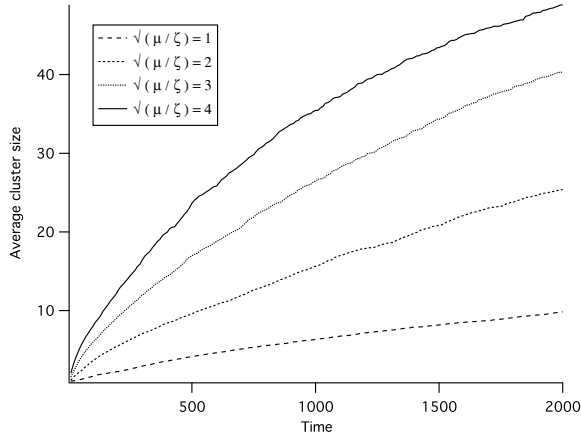
Figure 3: Dynamics of average cluster size for different parameter constellations in the original model.

cluster size under different parameter constellations. As the effects of these parameters are similar and it is only their joint effect which is important, we omit the size of the economy from the analysis and report the results for the different values of $\sqrt{\mu/\zeta}$ (for the sake of compatibility with the later results presented in figure 4). Here we present the average cluster size further averaged over 500 simulations. As expected[16] higher $\sqrt{\mu/\zeta}$ implies a higher rate of increase of average cluster size.

But equilibrium analysis can also be exploited to predict the behaviour of the system in the short-run. First we examine average cluster size in the equilibrium of linearized model. In equilibrium we have either one cluster of size S (corollary 1), or we have many clusters of the same size, with some minimum possible cluster size $\underline{c}$ (proposition 4). Thus the size of the representative cluster is bounded by $\underline{c}$ and S. The realized cluster size depends of course on the initial conditions, so we discuss expected cluster size given some distribution of initial conditions.

How expected cluster size scales with $\underline{c}$ (assuming a fixed population size S), depends on how initial conditions map to equilibrium outcomes. Proposition 4 suggests that in the linear model minimum sustainable cluster size will be $\underline{c} = \frac{\pi}{\sqrt{2}}\sqrt{\frac{\mu}{\alpha}}$. Without a formal proof, the law of large numbers suggests that expected cluster size should be roughly the average of the minimum and maximum cluster sizes. Thus mean cluster size should scale linearly with $\underline{c} = \frac{\pi}{\sqrt{2}}\sqrt{\frac{\mu}{\alpha}}$. Similarly, in the original, non-linearized, model, since $\zeta \propto \alpha$, it would follow that the mean cluster size is proportional to $\sqrt{\mu/\zeta}$.

To examine whether this intuition carries over to describe clustering behaviour in the short run we make 500 runs of both the linearized and the original model for 200 equally spread values of $\sqrt{\mu/\alpha}$ and $\sqrt{\mu/\zeta}$ in interval $(0, 4]$ and show the results in figure 4. Here we present the average cluster size at different points in time averaged over the 500 simulations. These are essentially the same plots as in figure 3 but with a different abscissa. A sense of time in these plots comes from the differences between curves along the ordinate for each point on abscissa.

The results indicate that average cluster size increases with time (which we have already seen

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[16]Recall from equation 23 that $\sigma = \alpha - 2\mu\frac{\pi^2}{S^2}k^2$.
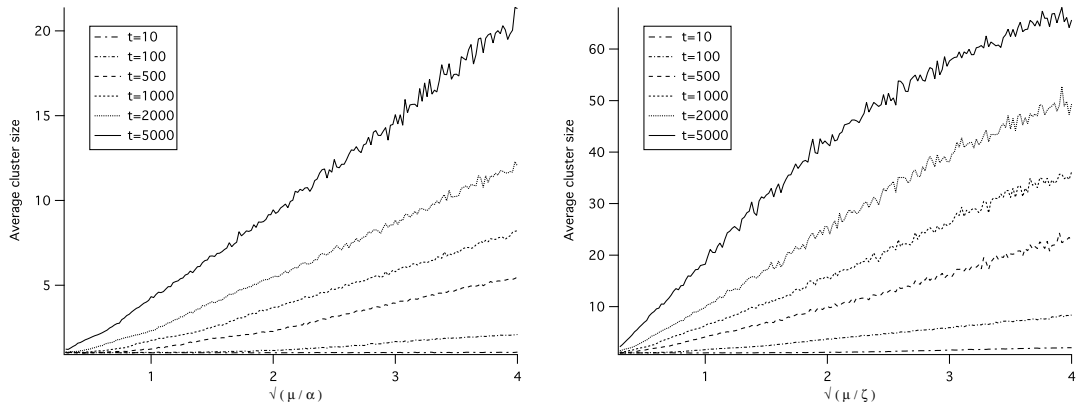
Figure 4: Average neighbourhood size in the linearized model (left) and in the original model (right).

in figure 3). They also indicate that the linear relationship predicted above for equilibrium state is also present in the transition to equilibrium, at least in the linearized model. However, in the original model, we observe a linear relationship during the early periods, but this disappears as the system gets close to the equilibrium in which average (over agents) valuations of the goods differ. We can conclude from this that the linearized version of the model is a very good approximation of the original model except for the near-equilibrium dynamics.

The reason for this discrepancy between the original and linearized models close to equilibrium is that the linear model exaggerates the effect of habit formation when valuations are sufficiently high. To see this recall that in the original model habit formation parameter ζ is additive to the valuation and is constant (equation 7). Consequently, as valuations increase the relative habit formation effect will decrease. However, in the linear model, the contribution of habit formation depends linearly on the *level* of valuation (equation 11). Thus the change in valuations does not change the relative size of the habit formation effect. As valuations are monotonically increasing in time, this is seen in the difference between the two panels of figure 4 at high values of t.

We have two effects in this model: habit formation, which drives the evolution of the average valuations in the economy, and information exchange, which controls the idiosyncratic deviations from this average. These effects are completely separable in the solution (22), and each dominates the dynamics under different conditions. When the average difference between the valuations of the two goods becomes large (infinity in the limit) the information exchange effect becomes negligible (proposition 1), and habit formation dominates. However, if this difference is small (zero in the limit), habit formation is weak, and the dynamics are driven by the information exchange effect (proposition 2). Because we start from random initial conditions, ensuring that $\bar{v}_{i;0} \approx \bar{v}_{j;0} \ \forall i, j$, the effect of habit formation will initially be small, and the dynamics will be driven by information exchange. In this case, the linear model provides a good approximation. However, after a sufficiently long time, the dynamics come to be dominated by habit formation, and as the linearized model exaggerates its effect, we get distortion in the picture: with the linearized model predicting a linear relationship (left panel), while the original model shows

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a sub-linear relationship (right panel). This distortion persists until the system reaches the equilibrium implied by the corollary 1.

5 An extension to the model

The model discussed in the previous section assumes a very specific structure for habit formation, which governs the movement of the average of the valuation differences ($\bar{z}$). This system implies that consumers' valuations increase without bound with consumers' experience. Thus the average of the deviations either stays at zero forever or increases without bound (proposition 1). Although mathematically convenient, this assumption is not very realistic. It is more plausible that habits can be formed to a certain point, but no further. In this case the average of valuation differences ($\bar{z}_t$) will be bounded. For understanding the implications of this extension we return to the two good case of the linearized model, but the results generalize straight-forwardly.

Because the solution to our model is separable into the average and deviations from the average, it is possible to incorporate a finite bound on the average valuation difference. Unfortunately in this case it is not feasible to pin down the exact relation between σ and parameters of the model. However, we can characterize the set of possible equilibrium states.

When we impose a bound on average valuations, the discrete version of the solution[17] to the system (19) becomes

$$z_t^s = \bar{z} + e^{\sigma t} \cos\left(k\frac{2\pi}{S}s\right)\tilde{z}_0^0. \tag{26}$$

where $\bar{z}$ ($\neq 0$)[18] is the equilibrium level of the average valuation difference. The solution here differs from the unbounded case only in the first term: in the unbounded case, the first term can grow without bound eventually dominating the solution, whereas when the valuations are bounded, this term converges to a constant.

In the model with bounds on valuations, there are three regimes, characterised by the sign of σ, the growth rate of the dominant sinusoid. The three regimes exhibit qualitatively different behaviour with respect to clustering.

$\sigma > 0$: In case when amplitude growth rate of the dominant sinusoid is positive ($\sigma > 0$),[19] qualitative results with respect to clustering do not differ from the baseline model (proposition 4): distinct clusters emerge and are stable, with the share of consumers preferring a certain product being equal across products.

For the other two cases adding bounds to average valuations changes the qualitative results of the baseline model discussed in section 3.

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[17]To recall, the discrete version of the solution to the unbounded model is $z_t^s = e^{\alpha t}\bar{z}_0 + e^{\sigma t}\cos\left(k\frac{2\pi}{S}s\right)\tilde{z}_0^0$.

[18]The case $\bar{z} = 0$ is equivalent to the case $\bar{z}_t = 0 \ \forall t$, implying that the bounds make no difference to the solution. This case was discussed above in section 3.2.

[19]Recall that with time the constant part of the solution (26), $\bar{z}$, is dominated by one wave, as its amplitude goes to infinity. The effect is that z_t^s converges to $e^{\sigma t}\cos\left(k\frac{2\pi}{S}s\right)\tilde{z}_0^0$.
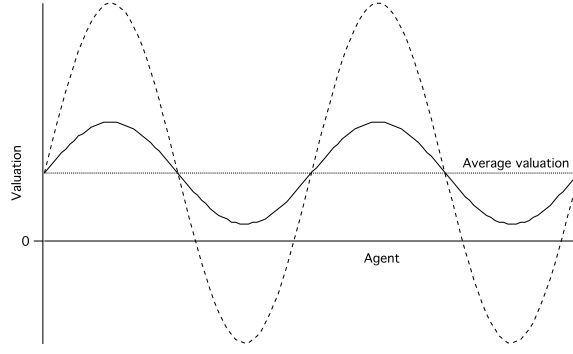
Figure 5: Difference between the cases with one stable cluster (solid line) and multiple different sized stable clusters (dashed line) when amplitude of the dominant sinusoid does not change.

$\sigma < 0$: Where the amplitude growth rate of the dominant sinusoid is negative, we know that even the dominant sinusoid vanishes in equilibrium. This implies that, in the limit, the second term of (26) vanishes, and for each agent, valuation difference collapses to the average: $z_t^s \to \bar{z}$. In the baseline model this would imply either no clustering (if $\bar{z}_0 = 0$), or one cluster with everybody purchasing the same product in equilibrium (if $\bar{z}_0 \neq 0$).

However, in extended case no clustering is not an option (as $\bar{z} \neq 0$). In this case every consumer's valuation difference converges to $\bar{z}$, thus there will be one big cluster. But, again unlike the baseline model, none of the products will attain 100 percent of the market in equilibrium, as choice probability of the dominant product will be bounded by some value below 1.

$\sigma = 0$: When the growth rate of the amplitude of the dominant sinusoid is zero, clustering is stable, as it is in the baseline model, but here the bounds imply a richer set of possible outcomes. In general there are two types of possibilities, illustrated in Figure 5. If the amplitude of the dominant sinusoid is small relative to the average difference in valuations ($|\bar{z}|$), then all agents prefer one product over the other, though the strength of preference varies (solid line). A single cluster emerges in preferences, but again, similar to the case when $\sigma < 0$, no product attains 100 percent of the market. If, however, the the amplitude is relatively large, we have stable clustering with multiple clusters with different sizes (dashed line in Figure 5). The relationship between the value of $\bar{z}/\tilde{z}_0^0$, and the frequency of the wave, determines the size of the clusters and share of the individuals preferring one product over another. More precisely, we can state

Proposition 7. *If $\bar{z} \neq 0$, $\sigma = 0$ and $\tilde{z}_0^0 < |\bar{z}|$, the share of consumers preferring one of the products will be $\frac{s_1 - s_2}{S} k$, where s_1 and s_2 are the solutions to*

$$s = \frac{S}{2\pi k} \arccos\left(-\frac{\bar{z}}{\tilde{z}_0^0}\right),$$

such that $s_1 \geq s_2$ and $s_1, s_2 \in (0; S/k]$.

Proof. $\tilde{z}_0^0 < |\bar{z}|$ guarantees that the equilibrium wave crosses the abscissa, thus for some consumers $z^s > 0$, while for others $z^s < 0$. We have to find the share of one of these groups of

consumers. For this we have to solve the equation

$$\bar{z} + \cos\left(k\frac{2\pi}{S}s\right)\tilde{z}_0^0 = 0 \tag{27}$$

for s. This results in

$$s = \frac{S}{2\pi k}\arccos\left(-\frac{\bar{z}}{\tilde{z}_0^0}\right).$$

Denote the two solutions on one cycle of the wave with s_1 and s_2 ($s_1, s_2 \in (0; S/k]$) and order them such that $s_1 \geq s_2$. This implies that within each cycle, $s_1 - s_2$ agents have a valuation difference of one sign (and comprise one cluster), $S/k - (s_1 - s_2)$ the other. Thus $\frac{s_1-s_2}{S}k$ will be the share of one kind of agents in the whole population. $\qquad\square$

This extension of the model results in richer equilibrium patterns which allow for stable clustering patterns with clusters of different sizes co-existing in the equilibrium, whereas in "unbounded" version of the model, all clusters were of the same size in equilibrium. This extension could explain the existence of temporally stable geographical or social beighbourhoods of different sizes engaging in similar activities (e.g. voting for the same party).

6 Concluding remarks

In this paper we have argued that interaction with peers over social networks can have important effects on the social distribution of demand. This external force, together with internal forces such as inertia, generate rich demand dynamics for markets containing goods that are close substitutes. Information diffusion through fixed social networks naturally generates clustering in demand: some neighbourhoods collectively prefer one good over another, while other neighbourhoods do the reverse. But depending on the characteristics of the society, this pattern can be either fragile or stable. In essence, several parallel informational cascades can result in persistent spacial distributions, where clearly identified neighbourhoods have higher concentrations of one particular type of information (information about one product), or to put it differently, where the peaks of different positive informational cascades (Hirshleifer 1993) are located in different places in social space.

It worth noting that stable clustering phenomena can also be obtained with simpler models. For example one can model consumers as celular automata, who are basing their decisions on purely neighbours' current states (for example Greenberg and Hastings, 1978; or for an economics application, Cowan and Miller, 1998). Our model differs from these specifications in two ways: firstly, we can discuss the importance of communication intensity, which is impossible in celular automata and secondly, in our model consumers exchange information about the merits of (all) the products with their friends instead of just observing their consumption baskets. This is particularly important as we can analyze deeper structures than revealed preferences.

In the present paper we model the dynamics of valuations through local interactions. There has been an interest in the literature about the difference between local and global information flows, or interaction more generally (Ellison 1993; Brock and Durlauf 2000; Glaeser and

Scheinkman 2000). This issue can be addressed in our model by looking at its behaviour as neighbourhoods become very large ($H \to S/2$). Increasing the neighbourhood size (H) puts an upward pressure on minimum stable cluster size $\underline{c}$ (see proposition 5), and for larger region of parameter space pushes it above the threshold ($\underline{c} > S/2$) beyond which clustering is unstable in the long run (in case when the differences between average valuations are zero).[20] Thus, in line with Glaeser and Scheinkman (2000), our model demonstrates that local interactions result in richer and more complex dynamics than do global interactions.

The model presented in this paper can be applied not only to choices between substitute products, but also to any mutually exclusive decision. For example the valuations in this paper can be easily interpreted as the level of satisfaction one gets from voting for a certain party. Similarly, the difference between valuations (in case of the two product model) can be interpreted as the satisfaction from various economic and social behaviour (e.g. bribery or other forms of criminal activity). In this respect the present model which can not only explain the emergence of the (geographical) clusters in which similar behaviours prevail but also provide the conditions under which this phenomenon will be temporally stable.

Appendix

Appendix A. Proof of proposition 5.

Proof. Consider the case of arbitrary neighbourhood size of $2H$. In this case after assuming that the distance between two neighbouring consumers is δ and considering the two-good case, continuous version of equation (12) can be rewritten as

$$\frac{\partial z(s)}{\partial t} = \alpha z(s) + \frac{\mu}{2H} \left[\int_{-H}^{H} z(s + \delta h) dh - 2H z(s) \right]. \tag{28}$$

Using second order taylor approximation we can rewrite the part of (28) under the integral as

$$\int_{-H}^{H} z(s) dh + \int_{-H}^{H} \delta h \frac{\partial z(s)}{\partial s} dh + \int_{-H}^{H} \frac{\delta^2 h^2}{2} \frac{\partial^2 z(s)}{\partial s^2} dh.$$

Which, after integration of first two summands, is equal to

$$2H z(s) + 0 + \frac{\delta^2}{2} \frac{\partial^2 z(s)}{\partial s^2} \int_{-H}^{H} h^2 dh.$$

To obtain more accurate values for smaller neighbourhood size, we go back to discrete space and replace the integral in expression above with the sum of squares of integer values.

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[20]For example, in the small economy that we have simulated ($S = 100$), $H = 49$ implies that the speed of habituation, α, must be roughly 80 times higher than the influence of neighbours, μ, in order the system to be stable for the largest possible cluster ($\underline{c} = S/2$)

Substituting this result back to (28) yields

$$\frac{\partial z(s)}{\partial t} = \alpha z(s) + \frac{\mu \delta^2}{4H} \sum_{h=-H}^{H} h^2 \frac{\partial^2 z(s)}{\partial s^2}.$$

Thus, it follows that the only modification that this generalization brings to the system can be captured by the definition of $\tilde{\mu}$ in the text being changed to

$$\tilde{\mu} = \frac{\mu \delta^2}{4H} \sum_{h=-H}^{H} h^2. \tag{29}$$

Going back to consumer addresses ($\delta = 1$), using new definition of $\tilde{\mu}$, and the identity $\sum_{n=1}^{x} n^2 = \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{6}$ we can rewrite equation (23) as

$$\sigma_H = \alpha - 2\mu \left(k\frac{\pi}{l}\right)^2 \left(\frac{H^2}{3} + \frac{H}{2} + \frac{1}{6}\right), \tag{30}$$

which results in

$$\bar{k}_H = \frac{S}{\pi}\sqrt{\alpha / \left(2\mu \left(\frac{H^2}{3} + \frac{H}{2} + \frac{1}{6}\right)\right)}, \tag{31}$$

and further in

$$\underline{c}_H = \frac{\pi}{2\sqrt{3}}\sqrt{2H^2 + 3H + 1}\sqrt{\frac{\mu}{\alpha}}. \tag{32}$$

$\square$

Appendix B. Proof of proposition 6.

Proof. Consider the case of the arbitrary number of products (N) being available on the market, but each consumer still communicating with only immediate neighbours ($H = 1$). Continuous counterpart of equation (12) after applying a Taylor approximation procedure looks as follows

$$\frac{\partial v_n(s;t)}{\partial t} = \alpha v_n(s;t) + \tilde{\mu}\frac{\partial^2 v_n(s;t)}{\partial s^2}. \tag{33}$$

Define two $N \times N$ dimensional diagonal matrices: one $\mathbf{A}$ with only α's on the diagonal, the other $\tilde{\mathbf{M}}$ with $\tilde{\mu}$'s on the diagonal, and three vectors, $\mathbf{V}$ which is the vector of v_ns, $\partial \mathbf{V}/\partial t$ and $\partial^2 \mathbf{V}/\partial s^2$ which contain first derivatives with time and second derivatives with space, the system defined in (33) can be written in a matrix form

$$\frac{\partial \mathbf{V}}{\partial t} = A\mathbf{V} + \tilde{\mathbf{M}}\frac{\partial^2 \mathbf{V}}{\partial s^2}. \tag{34}$$

The pattern wave solution to (34) is

$$\mathbf{V} = e^{\alpha t}\bar{\mathbf{V}}_0 + e^{\sigma t + ik\frac{2\pi}{l}s}\tilde{\mathbf{V}}_0^0, \tag{35}$$

where $\bar{\mathbf{V}}_0$ and $\tilde{\mathbf{V}}_0^0$ are vectors of initial values and just like in the paper agents are reindexed in a way that the wave reaches maximum at agent zero. The real part of (35) can be written as

$$\mathbf{V} = e^{\alpha t}\bar{\mathbf{V}}_0 + e^{\sigma t}\cos\left(k\frac{2\pi}{l}s\right)\tilde{\mathbf{V}}_0^0,$$

which is the same as the combination of propositions 1 and 2.

For the analysis of the stability of the system we again need to determine σ. Doing the same trick as in the paper (taking the first derivative with time and the second derivative with space and plugging back to the original equation), we get the following expression

$$(\mathbf{A} - \mathbf{B})\mathbf{V}_0^0 = 0, \tag{36}$$

where $\mathbf{A}$ is the same matrix of coefficients, while $\mathbf{B}$ is a new diagonal matrix, which has $\tilde{\mu}w^2 + \sigma$ terms everywhere on the main diagonal. So we get a new $N \times N$ dimensional diagonal matrix of a form

$$\begin{pmatrix} \alpha - \tilde{\mu}w^2 - \sigma & 0 & \cdots & 0 \\ 0 & \alpha - \tilde{\mu}w^2 - \sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha - \tilde{\mu}w^2 - \sigma \end{pmatrix}, \tag{37}$$

determinant of which has to vanish for the nontrivial solution of the system. The determinant of the matrix above is easy to calculate: the determinant of a diagonal matrix is the product of its diagonal entries, so

$$Det = (\alpha - \tilde{\mu}w^2 - \sigma)^N. \tag{38}$$

Equating the determinant to zero and plugging the definition of w gives the opportunity to solve for σ

$$\sigma = \alpha - \tilde{\mu}k^2\left(\frac{2\pi}{l}\right)^2, \tag{39}$$

which is the same as the solution obtained for the $N = 2$ case. Thus, this system, of course, has N solutions but all of them are given by (39). As a result $\bar{k}_N = \bar{k}$ and $\underline{c}_N = \underline{c}$. $\qquad\square$

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