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**Stochastic environmental effects, demographic variation,  
and economic growth**

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# Stochastic environmental effects, demographic variation, and economic growth\*

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## Abstract

We consider a stochastic environment to study interactions among pollution growth, demographic changes, and economic growth. Drawing on the empirical findings of slow convergence patterns of pollution shocks (viz., with a long-memory), we build an analytical framework where stochastic environmental feedback effects on population changes are reflected upon aggregate economic growth. Long-memory in economic growth, in our model, is shown to arise due to the inherent stochasticity in environmental and demographic system. Empirical results for a set of developed and developing countries generally support our conjecture. Simulation experiment is carried out to lend additional support to this claim.

*JEL* Classification codes: C13, J11, O47

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# 1 Introduction

Decades long theoretical and empirical scrutiny by economists, environmental scientists, and demographers have offered us ample reasons to believe that demographic system, environmental quality and economic growth of a country are inextricably linked.<sup>1</sup> Some recent examples include Kelley and Schmidt (2001) for demography and economic growth nexus and Maler and Vincent (2005) for environment and economic growth. Indeed, it has been rightly felt that economic growth measured in terms of only volume of production cannot proxy the improvement in welfare of a nation. It is rather necessary for relevant scientific and socio-economic-ethical reasons that the measure of economic growth includes environmental quality as a determining variable. Intense debates have occurred therefore on the choice of a maximum deterrent level of environmental pollution (see for instance, the arguments of Kyoto Protocol and Environmental Kuznets Curve). The academic investigation to this effect have mostly been limited within the celebrated scope of economic growth models linking environmental quality changes and pollution (see e.g., Henderson and Millimet, 2007, Millimet et al., 2003, Xepapadeas, 2003). Econometric illustration have approximated and/or banked upon the constructed theoretical reasonings.

Consequences and implications of a wide range of growth models have been examined in the last two decades by introducing environment directly either as an input of production or indirectly as one of the inputs critically depending upon environmental qualities. For instance, technological changes can be assumed to depend on environment or labor productivity depending upon the level of environmental pollution (Boserup, 1981; Boucekine et al., 2007). A striking aspect of these research is that environmental quality or pollution and population growth are assumed to be non-stochastic and/or stationary and exogenous in the model.<sup>2</sup> In fact, stationary assumption rules out the possibility of any stochastic shock affecting the future trajectory of the growth of variables and therefore the role of shocks in the long-run growth of the economy in conventional models is seriously undermined. However, recent theoretical and empirical studies provide evidence, for example, that population growth, contrary to the conventional assumption of being a stationary variable, can be characterized by a stochastic process with non-stationary features. Furthermore, assumption of a stochastic data generation mechanism for a variable in fact allows us to consider some degree of imperfection of the system that generates it. This is indeed a viable and desirable assumption as no economic system is perfect and therefore allowing the degree of imperfection in the model actually approximates the realistic situations.

Changes in environmental quality depend on two factors. First, there is a natural rate of degradation of the environment and second there is human intervention. Both environment and population act and interact in such a way that one can be taken to be endogenously determined due to the changes in the level of other. Irrespective of the direction and causality of effect,

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<sup>1</sup>Theoretical modeling in this direction have found robust explanation in Dasgupta (1981) and Dasgupta and Heal (1991). Expected future scenarios of environmental degradation are routinely presented by different national and international agencies, especially the Inter-Governmental Panel on Climate Change (IPCC). The recent Nobel Peace prize to IPCC speaks volume of the genuine concern of environmental degradation and the future of mankind.

<sup>2</sup>Generally, environmental stochasticity refers to temporal fluctuations in the probability of mortality and the reproductive rate of all individuals in a population in the same fashion. The impact of environmental stochasticity is roughly the same for small and large populations. It therefore constitutes an important risk of population decline in all populations regardless of their abundance at a given location.

changes in environmental quality can be assumed to be stochastic in nature in the sense that the changes are not completely predictable and that future changes in the quality would very much depend upon the current as well as the past changes that occurred. History dependence property play a pivotal role in perceiving environmental and/or economic growth as stochastic with long-term memory. Recent literature in economic growth suggests that some components of the aggregate activity follows a stochastic trend - the long-run path of the macroeconomy is permanently affected by contemporary events (Nelson and Plosser, 1982).<sup>3</sup> This is in fact a natural justification for the imperfections and incompleteness underlying any economic system.

Statistically, the effect of contemporaneous events on the economic system is characterized by unit root process or broadly by the presence of persistence. In fact, the parsimonious way to express the persistence of fluctuations is a significant stylized fact about macroeconomy, which as Durlauf (1989) puts, is a natural implication of dynamic coordination failure and is therefore consistent with much macroeconomic theory. Understanding the degree of fluctuations in an aggregate variable therefore, lends an assessment exercise of the potential role ‘coordination failure’ as a source of fluctuation. Then it may be conjectured that degree of imperfection and ‘coordination’ failure among the interacting agents (in this case different countries) would result in more imperfection and would ultimately characterize environment more in terms of stochastic changes. Nature’s intention is seldom predictable and so is the pattern of human intervention and impact on environment. This motivates the paper’s objective to model environmental changes in a stochastic setting and weigh their impacts on economic and population growth.

Frontier research have occurred in recent years by robustly explaining economic growth variation with respect to (stochastic) demographic changes (e.g., Birdsall et al., 2001). Independent theoretical contributions have flowed over the years my modeling environment and economic growth, while econometric research have been limited to treating environment and economic growth system in a stationary milieu. Since environment, economy and demographic systems are naturally bound by intense and complex interactions - which to a reasonable extent arise out of the direction and extent feedback effects - the presence and persistence of stochastic shocks in one of the systems, (i.e., environment, population or economy) permeate the environment-demography-economic growth (EDG for short) system as a whole. Non-stationary nature of population (viz., Gil-Alana, 2003; Mishra, 2008) and economic growths (e.g., Michelacci and Zaffaroni, 2000) have been characterized in recent studies. However, modeling environment in a non-stationary setting is rather sparse, especially embedding the effects of non-convergent shocks in the explanation of demographic and economic growth volatilities. In this paper, we intend to exploit the stochastic feedback effects from environment and model EDG system in a non-stationary setting.

The paper is structured as follows. In section 2 we provide a summary of the state-of-the-art literature on environmental effects of economic growth based on their deterministic and stochastic characters. Section 3 describes the model and provide supporting evidence of a long-memory in pollution growth before an analytical framework is designed in section 4. Section 5 presents our simulation model and analysis of the model developed in Section 3. Finally Section

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<sup>3</sup>It refers to the fact that not only are trend-cycle decompositions extremely difficult (that is, the same structural stochastic elements affect both the underlying time series), but that in addition, the feedback mechanisms from current activity to long-run growth render the traditional distinction meaningless.

6 summarizes our main findings and provides analysis of our results in purview of development objectives.

## 2 A helicopter tour of the literature

Environment, economic growth and demography have been subject to intense research in their individual domains in more than a century or so. However, alleviating a common research agenda by combining the three for future policies have been undertaken only recently. Due to the existence of the vast literature in EDG it is not possible to present their implications in a single scientific piece. However, keeping in mind the objective of this paper, we summarize below the conclusions of EDG literature by classifying them into deterministic and stochastic characters.

### 2.1 Deterministic setting

By deterministic setting we refer to the fact that there exists a stationary relationship between environment, demographic system and economic growth. Shocks to one of the systems in the EDG, as assumed by stationarity, while might affect the aggregate system due to inevitable correlatedness, they are not allowed to affect the long-run growth and are certain to converge in the long run. Moreover, it is also assumed that the transition from one state of the economy, demographic system and the environment to the other is stationary and that it is without long-lasting perturbations.

In fact, majority of the extant EDG literature is based on the formulation of deterministic setting. Ample reasons follow this convention. One of the main reasons concerns the methodological complexity involved and more so providing an empirically testable model for measurement and analysis. Furthermore, the subject of stochastic economic and environmental growth is the least trodden area by researchers as pure interpretation of stochasticities have been unequivocally limited to the area of pure science. The fact that any evolving entity could be subject to stochastic shocks and that the profile of the shock can extend beyond short-run dynamics, was seldom recognized in EDG model. For many years the trend in macroeconomics has been towards models which are both explicitly stochastic and explicitly dynamic. With these models, researchers seek to replicate and explain observable properties of the major economic time series. One manifestation of this trend towards stochastic dynamic modeling has been increasing use of the inherently dynamic models developed in the field of economic growth. However, we summarize below the findings and implications of research that dealt with stationary treatment of a shock arising in the EDG framework in that any shock arising in the model will ultimately, and more possibly very rapidly, taper off thus leaving the long-run growth consequences in tact.

Traditionally, demographic variables required a particular attention since population is recognized as one of the main causes of environmental pollution, especially for local environment (see, e.g., Ehrlich and Ehrlich, 1981 and Dasgupta, 1995).<sup>4</sup> According to Malthus (1798), an increasing population presents a significant nutritional demand, which creates pressures on agriculture. The quality of arable land is then degraded by intensive exploitation. Consequently,

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<sup>4</sup>See Panayotou (2000a,b) and Robinson and Srinivasan (1997) for a review of the literature regarding population growth, the economic development, and the environment.

the marginal productivity of labour decreases, and due to food insufficiencies, the growth rate of the population drops. The population is stabilized on low levels of income and bad environmental quality. Moreover, according to the World Bank (1992), demographic growth has induced an increasing demand for goods, services and basic provisions, which has an impact on the environment and exerts a pressure on natural resources. Therefore, an increase in population might pose a direct threat to local environment and reduce its assimilation capacity. It should be noted that the impact of the population on the environment can be modified by economic growth and the state of technology (Cropper and Griffiths, 1994). For example, an increase of income might change the demand of energy towards sources other than fuelwood.

In the same way, water quality is improved. The adoption of modern technology in agriculture reduces the necessity to convert forests into arable land since it makes intensive agriculture possible. The controversy over whether rapid population growth in the countries of the South or high consumption in those of the North is to be held accountable for global environmental degradation is usually modeled by  $I = PAT$  identity, which implies that environmental impact (I) can be thought of as a product of population (P), affluence (A), and technological efficiency (T) first introduced by Ehrlich and Holdren (1971). Which one of the three factors act (upon) less on others actually defines the weight of the problem - i.e., whether it is demography-pull environmental problem or excess consumption-push environmental problem. Irrespective of the weight of importance, it is true that there is always a complex feedback mechanism involved in the EDG process and defining the exact structure and nature of the process would actually solve the riddle on the demography-pull or consumption-push environmental problems. Keeping in mind the  $I = PAT$  identity, we summarize below some of the important theoretical and empirical findings related to EDG model.

Among important research approaches in the study of population and pollution, two fundamentally different approaches have been used, viz., simulation models<sup>5</sup> and statistical<sup>6</sup> models (Cramer, 2002). While most research, in both approaches, have been carried out at the national or global level, Cramer (2002) performs a local level analysis on the impact of population growth on pollution and considered the possibility of reciprocal causality and feedback.

The effect of population on deforestation was studied by Postel (1984), Allen and Barnes (1985), the Food and Agriculture Organization (FAO, 1993), Cropper and Griffiths (1994), Shafik (1994), Koop and Tole (1999) and Bhattarai and Hammig (2001), among others. Allen and Barnes (1985) stress that, in developing countries, a high demographic growth rate is associated with a significant rate of deforestation. The FAO (1993) suggests that the relationship between forest surface and total surface is a logistic function of population density. This implies that the deforestation rate depends on both the density and the growth rate of population. Postel (1984) notes that poverty is a principal cause of deforestation. Rural population density is also recognized by Cropper and Griffiths (1994) as a determining factor of deforestation in Africa. Nguyen van and Azomahou (2006) use a panel data set of 59 developing countries over two

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<sup>5</sup>The simplest simulation models use population projections and data on per capita emissions on pollutants. Prospective projections compare the emissions that would occur due to no population growth to the emissions that would occur with population growth (Birdsall, 1992; Meyerson, 1998).

<sup>6</sup>This approach requires identification of all relevant variables and the functional forms by which they are linked to pollution. Over the years sophisticated models have been developed to link pollution to its sources and to climate and meteorology.



decades and study the deforestation process employing both parametric and semi-parametric models. The authors study the environmental Kuznets Curve (EKC) and found no evidence of EKC and also found that political institution failures may worsen the deforestation process in developing countries, which provides rather a direct evidence to an unusual source of armed conflict in the region. Similarly, Azomahou et al. (2005) employed nonparametric panel approach to study economic development and CO<sub>2</sub> emissions during 1960-1996, and found the existence of structural stability of the relationship and that this relationship is upward sloping.

In the study of Koop and Tole (1999), population (population density and the growth rate of population) does not have a significant effect on the rate of deforestation. Bhattarai and Hammig (2001) use a panel data model with fixed effects on a sample of 66 countries over the period 1972-1991 and showed that the density of rural population and the growth rate of population have very different impacts on deforestation in Latin America and Africa. However, the effect of rural population density is more significant.

Chu and Yu (2002) provide a succinct survey of the literature investigating how population dynamics cause decline in biodiversity. The authors have outlined several important reasons concerning why bio-diversity decline is particularly a serious concern for sustainable economic growth today. Direct population pressure on land use, path-dependence and lock-in in the process of economic development, international coordination in a global environment, the difficulty of valuing bio-diversity decline, and the problem in assessing the benefit to future generations, are some of the factors responsible.<sup>7</sup>

The models of population dynamics and its linkages to environment can be traced first to Malthusian idea which characterizes the following density dependent model:

$$N_{t+1} = G(N_t)N_t \tag{1}$$

where  $N_t$  is the population size in period  $t$  and  $G$  is an adjustment coefficient connecting  $N_t$  and  $N_{t+1}$ .  $G$  being a function of  $N_t$  implies that the dynamic adjustment of the state variable is affected by the current population size  $N_t$ , which is a standard approach to specifying carrying capacity constraint. Verhulst (1838) provided a more specific way of characterizing the constraint of population growth in the form of a logistic equation as:

$$\frac{\Delta N_t}{N_t} = r \left( \frac{C_t - N_t}{C_t} \right) \tag{2}$$

where  $r$  is a positive constant, and the carrying capacity  $C_t$  appears explicitly on the right hand side of the equation. It is straightforward to see that the larger the carrying capacity  $C_t$ , the higher the population growth rate. According to Renshaw (1991), most theoretical models characterizing the relationship between population and environment are extensions of (2). Seidl and Tisdell (1999) pointed out that Malthus seemed to have put human beings and other species on the same level and ignored role of institutions in human development. Indeed, as Solow (1956) and Romer (1990) respectively theorized that in the past two hundred years the historically passive interaction between environment and population has been gradually replaced

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<sup>7</sup>It is now recognized that the surge of rapid population growth has intensified the problem of *environmental scarcity* with multiple socio-economic-political dimensions. Due to severity of environmental scarcity in less developed countries (LDCs), recent literature argue that this scarcity is increasingly promoting armed conflicts in LDCs.

by the active accumulation of capital and continuous innovation of new technologies. One of the conclusions drawn from various growth models is that both per capita income and population size can grow infinitely in the steady state, implying that there is essentially no carrying capacity constraint for population.<sup>8</sup>

Macro models describing the interactions between population and environment are rare, and the models of the relationship between population and biodiversity decline are even rarer except, for instance, the following population-environment interaction model proposed by Nerlove (1991):

$$Z_t = g(Z_{t-1}, N_{t-1}) \tag{3}$$

and

$$\frac{N_t}{N_{t-1}} = h(Z_{t-1}) \tag{4}$$

where  $Z_t$  denotes the state of environmental degradation. Equation (3) indicates that, other than the impact of population size on environmental quality, the latter also has an autoregressive property. Equation 4 indicates that the population growth is dependent on the degree of environmental degradation. Using (3) and (4), Nerlove showed that multiple steady-states are associated with these two equations, and that government policies such as taxes and subsidies can be used to achieve a desirable steady-state. In another line of research Prskawetz et al. (1995) studied bifurcation models with population, resources, and economy as components. The authors showed that slow processes of degradation could quite suddenly cumulate to produce chaotic/catastrophic changes. Their model was simply a mathematical possibility of sudden environmental changes rather than on the empirical context of environmental feedback.

Bateman (2004) provides a nice summary of the ingeniously crafted contributions in honor of Karl-Goran Maler edited in the volume: *Economic Theory for the Environment: Essays in Honor of Karl-Goran Maler*. The theoretical contributions by Maler that started as early as 1974 has been adequately qualified by contributions from diverse range of theoretical research such as control of pollution, bio-diversity, sustainability, fertility and mortality effects and household behavior, etc. From statistical perspective, some stimulating studies exist, viz., Jeon and Sickles (2004). The authors study the role of environmental factors in growth accounting and thereby directly contribute to the literature in seeking direct role of environmental factors in economic growth. Using a directional distance function method, Jeon and Sickles (2004) analyze productivity growth and explicitly evaluate the role that undesirable outputs of the economy, viz., CO<sub>2</sub> and other greenhouse gases, have on the frontier production process.

## 2.2 Stochastic setting

Since stochasticity has its general root in probabilistic methods, we will use this convention in this paper as well. An additional aspect we would like to consider is the time series behaviour of economic-demographic-environmental system. Studies of EDG model in stochastic setting

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<sup>8</sup>This conclusion, however, seems to be at odds in the ecological literature as there has to be a supporting limit for human population on earth unless of course human generations seek to settle in other planets!

are rather sparse<sup>9</sup> except some modest contributions from Zhang et al., (2006) and Soretz (2003). By introducing pollution into the utility function and the productive function, Zhang et al. (2006) prove under appropriate macroeconomic equilibrium conditions that the equilibrium levels of the main economic indexes are uniquely determined by the model parameters. Soretz (2003) investigates the impact of pollution and abatement policy within a stochastic endogenous growth model. Environmental care is provided by the government and financed through income taxation and government bonds. Due to environmental preferences and partial perception of the individual's impact on pollution, government debt influences equilibrium growth. Hence, there is an additional growth effect of income taxation due to portfolio adjustment. It is shown that the optimal income tax rate decreases with the perception of the influence of individuals on aggregate capital. In contrast, the impact of environmental preferences and uncertainty on optimal environmental policy is ambiguous.

Mishra (2008) investigated the relationship between stochastic demographic system and economic growth and analytically showed that long-memory in economic growth can result from the long-memory properties of demographic system. The author also empirically demonstrated that the variation in the cross-country growth is accounted for by stochastic demographic shocks in the past four decades. The converse was also shown to be true establishing that stochastic demographic system has also been significantly affected by stochastic economic growth variation over the last four decades. Although Mishra (2008) provided new insights into the source of economic growth variations which emerge from stochastic demographic dynamics, the inevitable effect of environment, which could act as instrumental in propagating the demographic shocks in the economic growth system, was missing. Similarly Zhang et al. (2006) and Soretz (2003) also investigated the EDG model in a different framework where the time-profile of evolution of the respective systems did not arise. Some studies in population biology introduce multi-state markov model to study stochastic character of the demographic process,<sup>10</sup> however we do not follow this convention in this study.

In the following section, we provide a stochastic long-memory framework for EDG model where shocks' persistence properties can be investigated and their effects on each other can be studied.

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<sup>9</sup>A scenario very closely considered in IPCC forecasting is a probabilistic method building some alternative confidence intervals and calibrating it with the economic scenarios. While this is a real sign of moving out from the deterministic setting, it is not yet fully stochastic, because, a stochastic system requires embedding history dependence properties and analyzing the system with a possible coordination failure theory. That has been the case with output persistence (see Durlauf, 1991 and 1989 for example) where the presence of unit root in output was explained to be an outcome of coordination failure and occurrence of multiple equilibria among firms' economic activities.

<sup>10</sup>It is also possible to build a stochastic environmental-demographic and economic system using a non-stationary Markovian transition mechanism, where the transition of the system from one state to the other is non-stationary. Utilizing multi-state Markov model, it is possible to model environment, demography and economic growth with a feature where endogenous shifts occur in the economy and the endogenous shifts are due to environmental and demographic changes. In this paper, we do not directly address this channel which is preserved for future research.

### 3 Model

While persistence of shocks in economic growth is forthrightly real, it is rather difficult to outline a unified source. An aggregate variable, like output, is bound to inherit pitfalls of several co-moving components. However, it is sometimes useful to characterize the most important ones which are supposed to have direct consequence on the aggregate. We will therefore stress on the demographic and environmental side of output growth.

An appropriate theoretical model and empirical justification can provide the base-work on which stringent and binding policy on environmental regulations can be imposed. To this end, it is necessary to go beyond the orthodox approach. A more flexible formulation is needed where the extent of stochasticities in one of systems in EDG can directly appear as an explanatory or explained variable in the other. As a first step to such modeling we provide below a long-memory framework for EDG system where the effects of slow convergent shocks can be analyzed.

#### 3.1 Set-up

We specify a model which accounts for the negative effects of accumulating pollution on the supply side through the reduction of labour productivity and population growth by environmental pollution. Let us consider an aggregate Cobb-Douglas production function

$$Y(t) = K(t)^\alpha [A(t)N(t)]^{1-\alpha}, 1 < \alpha < 0 \quad (5)$$

with the standard notations:  $Y(t)$  the output at time  $t$ ,  $K(t)$  the level of physical capital,  $N(t)$  the labour input,  $A(t)$  the labour augmenting technical progress. Then,  $A(t)N(t)$  is the effective labour. We denote  $g(E)$  the growth rate of labour augmenting technical progress which depends on environmental quality  $E$ , with  $g'(E) < 0$ . This translates the negative effects of accumulated pollution on labour productivity. The labour input is governed by the growth rate of population,  $n(E)$  so that

$$N(t) = (1 + n(E))N(t-1) \quad (6)$$

where  $n(E)$  represents the growth rate of population that also depends on environmental quality. We assume that  $n'(E) < 0$ . Both  $g(E)$  and  $n(E)$  imply that we can endogenize technical progress and population growth in terms of environmental quality.

As we focus on population and environmental quality (where population exerts some pressure on environment via consumption, production), the latter can be viewed as a physical good. Following Aghion and Howitt (1998, Chap.5), we assume that there is an upper limit to environmental quality, denoted by  $E_{\max}$ . We measure  $E(t)$  as the difference between the actual quality and this upper limit. Thus, environmental quality will always be negative. The equation of motion of environmental quality is given by

$$\dot{E}(t) = -\theta E(t) - \psi n(E) \quad (7)$$

where  $\theta E(t) > 0$  in (7) indicates the maximum potential rate of recovery of the environment, and  $\psi > 0$  measures the environmental damage following from demographic pressures. Furthermore, from sustainable economic perspective, we assume that environmental quality also has a lower

limit,  $E_{\min}$  referred to as catastrophic. This, implies that the optimal growth path, if it exists, will be constrained as

$$E_{\min} \leq E \leq 0. \quad (8)$$

The function  $A(t)$  describing the level of labour augmenting technical progress specified as

$$A(t) = \Omega(t)E(t)^{-\beta}, \beta > 0. \quad (9)$$

This formulation has two parts. The first part like Solow (1956) and Swan (1956) represents some exogenous portion of technical progress:  $\Omega(t) = \Omega(0)e^{\mu t}$  where  $\mu$  is its constant rate of growth. The second part implies that as  $t \rightarrow 1$ , and  $E(t) \rightarrow \infty$  negatively (recall that  $E(t)$  is always negative),  $A(t) \rightarrow 0$ . This means that more environmental pollution will lead labour productivity to decline increasingly more slowly and thus approaches zero only asymptotically. It can be easily seen from relations (7) and (9) that

$$g(E) = \frac{\dot{A}(t)}{A(t)} = \mu - \beta \frac{\dot{E}}{E} = \mu + \beta\theta + \frac{\psi n(E)}{E} \quad (10)$$

from which one observes that  $g'(E) < 0$  if  $n(E) = n$ . Since  $n(E)$  is not constant and incorporates some stochastic feature, we study their properties next.

### 3.2 Stochastic population and environmental linkage

This section provides a formulation for stochastic features in the demographic and environmental linkage. In real surveys, such as those on environmental pollution, the observed data have been shown by scientists to contain long memory characteristics. In particular, the time series of pollution usually show ‘persistence’ in the sense that their correlation functions decrease to zero at a much slower rate than the exponential rate implied by a short memory time series. The implications of the presence of long-memory in environmental pollution can be due to the possible and persistent coordination failure among economic decision makers on arriving at an agreeable and low-pollution strategies. Excessive emphasis on production volumes in some countries provides a leeway for higher pollution and while this is not restrained by others, the failure of policy coordination results in persistence of environmental pollution. Thus, the evidence of long-memory in pollution (if any) would reflect upon the coordination failure problems among policy makers and the existence of the resultant multiple equilibria in the world economy.

In time series setting, properties of long-memory (see Bailey, 1991 for a succinct survey) are studied for the evolving pattern of environment and demographic system. Later these characteristics are embedded in the economic growth framework. For the moment, we describe below the stochastic environmental equation using long-memory features. By empirically investigating the long-run equilibrium relation between environment and economic growth in a long-memory setting, we then build up the EDG model in a stochastic long-memory framework and investigate if stochasticities in economic growth is caused by stochastic environment and demographic system.

In (7), we assume that  $\dot{E}_t \equiv E_t - E_{t-1}$  and that  $LE_t = E_{t-1}$  where  $L$  is the lag operator such that  $\dot{E}$  can now be written as  $(1-L)^d E_t$  where  $d$  is the integration parameter. Since we are interested in the changes in the environmental quality, the fractional difference representation

of  $E_t$  as in  $(1 - L)^d E_t$  justifies our purpose because when standard formula for changes in environmental quality occurs for the limiting case of  $d = 1$ . The imposition of fractional difference in fact juxtaposed more flexibility concerning the nature of change of environmental quality. Assuming that Eq. 7 is now described as

$$(1 - L)^d E_t = \epsilon_t \quad (11)$$

where  $\epsilon_t \sim (0, \sigma^2)$  is a gaussian fractional noise. The assumption of real  $d$  values, combined with the filter  $(1 - L)$ , displays various memory characteristics of  $E_t$ . Usually, this can be known by looking at the following binomial expansion of  $(1 - L)^d$ :

$$(1 - L)^d = \sum_0^{\infty} h_j L^j = 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 - \dots \quad (12)$$

$h_0 \equiv 1$ ,  $L^j$  is backward operator  $j$  times, and  $h_j \equiv (1/j!)(d+j-1)(d+j-2)(d+j-3) \cdots (d+1)(d)$ . It may be noted from the above that the coefficient of lagged  $E_t$  provides the rate of declining weights. However, based on the non-integer values and sign of  $d$ , the following memory properties are observed.

With  $d = 0$  in the above, the process exhibits ‘short memory’ as the autocorrelations in this case is summable and decay fairly rapidly so that a shock has only a temporary effect completely disappearing in the long run. Long memory and persistence is observed for  $d > 0$ . In this case, the shock affects the historical trajectory of the series. However, greater is the magnitude of  $d$ , stronger is the memory and shock persistence. For  $d \in (0, 0.5)$ , the series is covariance stationary and the autocorrelations take much longer time to die out. When  $d \in [0.5, 1)$ , the series is a mean reverting long-memory and non-stationary process. This implies even though remote shocks affect the present value of the series, this will tend to the value of its mean in the long run. For  $-1/2 < d < 0$  the process is known to be fractionally over-differenced. In this case, there is still short memory with summable autocovariances, but the autocovariance sequence sums to 0 over  $(-\infty, +\infty)$ . For  $d < -1/2$  the series is covariance stationary but not invertible. And finally, when  $d \geq 1$  the series is nonstationary and exhibits ‘perfect memory’ or ‘infinite memory’. There is no unconditional mean defined for the series in this case. The process defined by this value of  $d$  is non-stationary and non-mean reverting. In this case, the mean of the series has no measured impact on the future values of the process. Important to note that for  $0.5 \leq d < 1$ , there is no variance, so the existence of the mean would need to be established in each case. There is a median, however. So this case may be described by ‘median reversion’. The results are summarised in Table 1.

Table 1: Fractional components and their interpretation

$d$	Interpretation
0	: Short-memory population growth, log population is $I(1)$
1	: Non-stationary population growth, log population is $I(2)$
$< 0, 0.5 >$	: Long-memory population growth, log population is $I(d+1)$

### 3.3 Empirical facts

#### 3.3.1 Data characteristics

We present here the characteristics of the population, pollution and real GDP data and analyze the trend of these variables over time. For pollution, we have univariate time series for five OECD countries and five non-OECD countries chosen according to the highest volume of CO<sub>2</sub> emissions. The national CO<sub>2</sub> emission per capita series, measured in metric tons, were provided by the *Carbon Dioxide Information Analysis Center (CDIAC)* of the *Oak Ridge National Laboratory* (see Marland et al., 2004). The real GDP per capita series, measured in thousand constant dollars in 2001 international prices, were extracted from the *Penn World Table 6.1* (Summer and Heston, 2005). The aggregate population data has been collected from Maddison (2004) which gives the mid-year population.

Annual data, 1870-2003, for a set of developed and developing countries pollution growth rates, measured by  $g_E = \ln(E_t/E_{t-1})$ , where  $E_t$  is total CO<sub>2</sub> emissions, have been assembled from the CDIAC Data base. The logarithmic plots of CO<sub>2</sub> emissions and total population for OECD and non-OECD countries are provided in Figures 1 and 2. The spectral density<sup>11</sup> plots of CO<sub>2</sub> emissions and population for the two sets of countries are presented in Figures 5-16. From the spectral plots it is clear that CO<sub>2</sub> emissions for both OECD and non-OECD countries have steadily grown over the decades - non-OECD countries depict more fluctuations than OECD countries. The spectral density estimates vindicate the fact that there are higher number of ordinates near zero frequencies than at higher frequencies indicating that the series are non-stationary, or that some stochastic shocks (trends) have governed the evolution of their process (both for CO<sub>2</sub> and total population). The real GDP plots for OECD (Figure 3) and Non-OECD (Figure 4) countries show steady trends. Fluctuations occur at different periods but overall OECD countries display similar growth trend, where Non-OECD countries characterize differential trends.

#### 3.3.2 Stochastic long-memory and long-run equilibrium features

In Tables 2 and 3 below we report the estimates of the long memory parameter,  $d$ , the magnitudes of which indicate the relative rates of convergence of shocks to the long-run mean-values over time. The estimation has been performed using Kim and Phillips' (2000) modified log-periodogram regression method (MLPR). The MLPR method is a modified version<sup>12</sup> of the following Geweke and Porter-Hudak (GPH, 1983) log periodogram regression:

$$\ln[I_n(\lambda_\zeta)] = -2d \ln|1 - e^{i\lambda_\zeta}| + \ln(f_u(\lambda_\zeta)) + \eta_j \quad (13)$$

where the periodogram ordinates of population growth (left hand side of the equation) are

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<sup>11</sup>A stationary series can be decomposed in cyclical components with different frequencies and amplitudes. The spectral density gives a graphical representation of this. It is symmetric around 0, and only graphed for  $[0, p]$  (the horizontal axis in the PcGive graphs is scaled by  $p$ , and given as  $[0, 1]$ ). The spectral density consists of a weighted sum of the autocorrelations, using the Parzen window as the weighting function. The truncation parameter  $m$  can be set (the larger  $m$ , the less smooth the spectrum becomes, but the lower the bias). A white-noise series has a flat spectrum.

<sup>12</sup>See appendix for detailed description.

regressed over the spectral representation of the error term and the transformation of  $(1 - L)^d$  in the frequency domain. The ordinates are evaluated at the fundamental frequencies  $\zeta = 1, \dots, \nu$ . Kim and Phillips (2000) note that (13) is a moment condition and not a data generating mechanism. The modified GPH, i.e., the MLPR is given as<sup>13</sup>:

$$\ln(I_V(\lambda_\zeta)) = \alpha - d \ln|1 - e^{i\lambda_\zeta}|^2 + u(\lambda_\zeta) \quad (14)$$

in which the periodogram ordinates,  $\ln(I_n(\lambda_\zeta))$  are replaced by  $\ln(I_V(\lambda_\zeta)) = V_n(\lambda_\zeta)V_P(\lambda_\zeta)^*$  with  $\alpha = \ln(f_u(0))$  and  $u(\lambda_\zeta) = \ln[I_n(\lambda_\zeta)/f_n(\lambda_\zeta)] + \ln(f_u(\lambda_\zeta)/f_u(0))$ . Note that  $V_n(\lambda_\zeta)V_P(\lambda_\zeta)^*$  is the discrete fourier transform and is to be used in the regression instead of  $\ln(I_V(\lambda_\zeta))$ .

A practical problem is the choice of  $\nu$ , the number of periodogram ordinates to be used in the regression. Geweke and Porter-Hudak (GPH, 1983) suggests that the optimal  $\nu = T^\alpha$  where  $\alpha = 1/2$  and  $T$  is the sample size. The choice involves a tradeoff that may be described as follows. The smaller the bandwidth, the less likely the estimate of  $d$  is contaminated by higher frequency dynamics, i.e., the short-memory. However, at the same time smaller bandwidth leads to smaller sample size and less reliable estimates. As in the case of GPH method, the smaller value of  $\alpha$  (as in  $\nu = T^\alpha$ ) implies the smaller number of harmonic ordinates (i.e., the smaller bandwidth) will be used for the estimation of  $d$ . Generally, in empirical analysis, preference is given to increasing the value of  $\alpha$  to check for the consistency of the estimate of  $d$  although simulation experiments can confirm the validity of the selection. For our purpose, we have used  $\alpha = 0.60$  through  $\alpha = 0.80$  to estimate  $d$ . We choose<sup>14</sup>  $\alpha = 0.7$  based on a Monte Carlo simulation experiment (see table below) where we have minimum bias for that bandwidth.<sup>15</sup>

Tables 2 and 3 we test the null hypothesis of short-memory against the alternative of long-memory. From the  $d$  estimates, we find clear evidence of long-memory for non-OECD countries where estimated  $d$  are significantly greater than  $1/2$ . The  $d$  estimates are however smaller than 1 indicating the long-memory persistence with the possibility of convergent shocks in the long-run. For OECD countries' CO<sub>2</sub> emissions are less persistent than non-OECD countries as we analyze the estimates over different bandwidths. Stationary long-memory features are observed for both sets of countries for CO<sub>2</sub> emissions and non-stationary long-memory or highly persistent demographic shocks are observed for some non-OECD countries (where estimated  $d$  is higher than 0.5). To strengthen our conjecture of stochastic environmental effects on economic growth, we have also performed a fractional cointegration test between environment, economic growth and demographic growth. The results are presented in Table 5 in the appendix. Theoretically, fractional cointegration between environmental quality and economic growth would imply that, although there exists a long-run relationship between the two, the equilibrium errors exhibit slow reversion to zero, i.e., that the error correction (F-ECM in the table 5) term possesses long-memory and so the deviations from the equilibrium are permanent. We have performed analysis in an fractionally integrated ARMA environment (ARFIMA) which has been estimated up to order  $(2, d, 2)$ . The reported order of *ARFIMA* has been chosen by comparing the Akaike Information Criterion (AIC) and looking at their highest likelihood. As revealed by our estimation,

<sup>13</sup>For details refer to Kim and Phillips, 2000

<sup>14</sup>The estimates of  $d$  for other bandwidth are available with the authors though we have not reported in the main text due to space limitation.

<sup>15</sup>Davidson's (2007) TSM software is used to carry out the simulation experiment which is built for the GPH model.



there is a clear evidence of long-run fractional cointegrating relationship between population and pollution growth and due to this, it can be assumed that high degree of stochasticity in either demographic system and/or environment will induce instability in the other due to their reactionary and feedback effects. The above results motivates us to model EDG in a stochastic framework with persistent properties (i.e., with long-term memory). This is described in the next section.

Table 2: Modified log-periodogram estimation of the long-memory parameter for CO<sub>2</sub> emissions

$T^\tau$	$\tau = 0.5$	$\tau = 0.55$	$\tau = 0.60$	$\tau = 0.65$	$\tau = 0.70$	$\tau = 0.75$	$\tau = 0.80$
Periodogram Ordinates	11	14	18	24	30	39	50
<b>OECD</b>							
USA	0.416 (0.215)	0.269 (0.172)	0.204 (0.151)	0.096 (0.131)	0.203 (0.118)	0.162 (0.112)	0.104 (0.052)
Japan	0.160 (0.066)	0.211 (0.056)	0.331 (0.113)	0.312 (0.094)	0.200 (0.083)	0.211 (0.075)	0.244 (0.064)
UK	0.052 (0.187)	0.101 (0.144)	-0.029 (0.146)	-0.340 (0.147)	-0.461 (0.124)	-0.484 (0.104)	-0.478 (0.085)
France	-0.515 (0.335)	-0.446 (0.250)	-0.397 (0.189)	-0.163 (0.175)	-0.089 (0.142)	-0.119 (0.120)	-0.152 (0.103)
Canada	0.703 (0.130)	0.453 (0.153)	0.324 (0.129)	0.307 (0.106)	0.173 (0.101)	0.251 (0.133)	0.169 (0.113)
<b>Non-OECD</b>							
China	0.706 (0.134)	0.519 (0.154)	0.529 (0.131)	0.514 (0.137)	0.583 (0.126)	0.619 (0.105)	0.625 (0.085)
India	0.387 (0.314)	0.658 (0.298)	0.795 (0.264)	0.503 (0.218)	0.544 (0.179)	0.280 (0.173)	0.241 (0.157)
Brazil	0.373 (0.271)	0.321 (0.215)	0.455 (0.182)	0.508 (0.138)	0.547 (0.123)	0.538 (0.101)	0.611 (0.125)
South Africa	0.589 (0.296)	0.969 (0.379)	0.751 (0.304)	0.731 (0.235)	0.593 (0.203)	0.404 (0.179)	0.327 (0.145)
Mexico	0.117 (0.294)	0.108 (0.235)	0.267 (0.215)	0.501 (0.179)	0.550 (0.157)	0.476 (0.130)	0.399 (0.107)

Table 3: Modified log-periodogram estimation of the long-memory parameter for total population

$T^\tau$	$\tau = 0.5$	$\tau = 0.55$	$\tau = 0.60$	$\tau = 0.65$	$\tau = 0.70$	$\tau = 0.75$	$\tau = 0.80$
Periodogram Ordinates	11	14	18	24	30	39	50
<b>OECD</b>							
USA	0.962 (0.171)	0.925 (0.131)	0.982 (0.116)	1.027 (0.115)	1.046 (0.117)	0.968 (0.102)	1.003 (0.090)
Japan	0.696 (0.240)	0.720 (0.179)	0.581 (0.145)	0.430 (0.117)	0.574 (0.132)	0.708 (0.110)	0.664 (0.088)
UK	0.504 (0.371)	0.414 (0.276)	0.315 (0.208)	0.320 (0.150)	0.262 (0.122)	0.195 (0.097)	0.137 (0.078)
France	0.125 (0.152)	0.540 (0.266)	0.494 (0.240)	0.494 (0.176)	0.607 (0.152)	0.575 (0.120)	0.526 (0.103)
Canada	1.087 (0.311)	1.077 (0.268)	0.843 (0.223)	0.658 (0.171)	0.788 (0.153)	0.728 (0.127)	0.703 (0.106)
<b>Non-OECD</b>							
China	0.405 (0.267)	0.528 (0.224)	0.534 (0.175)	0.476 (0.127)	0.581 (0.117)	0.670 (0.109)	0.813 (0.108)
India	0.118 (0.072)	0.110 (0.060)	0.119 (0.047)	0.116 (0.035)	0.112 (0.028)	0.089 (0.023)	0.077 (0.019)
Brazil	1.169 (0.213)	1.092 (0.198)	0.978 (0.164)	0.880 (0.131)	0.855 (0.118)	0.773 (0.099)	0.700 (0.081)
South Africa	0.880 (0.239)	0.921 (0.201)	0.984 (0.153)	1.009 (0.115)	1.029 (0.116)	1.016 (0.098)	1.037 (0.085)
Mexico	0.978 (0.509)	0.686 (0.435)	0.707 (0.359)	0.535 (0.264)	0.392 (0.213)	0.466 (0.185)	0.357 (0.148)

Table 4: Monte Carlo simulation for choice of bandwidth

Bandwidth	Estimated bias	Significance	RMSE bias
$\tau=0.60$	0.018	3.03	0.019
$\tau=0.65$	0.021	2.86	0.023
$\tau=0.70$	0.014	2.20	0.015
$\tau=0.75$	0.015	2.47	0.016
$\tau=0.8$	0.017	2.83	0.018

## 4 Stochasticity in environment and economic growth

Recall that given the assumption underlying the environmental equation (described as a stochastic long memory process) we can now formalize how the effects of the stochasticities are translated into the following economic growth consequences:

$$(1 - L)^d E_t = -\theta E_t - \psi n_t \quad (15)$$

then

$$E_t = -\psi n_t \left[ (1 - L)^d + \theta \right]^{-1} \quad (16)$$

Observe that (7) is conceptualized to be a function of the level of environmental quality and the evolution of population, viz., the population growth in the model. Here, we assume that

demographic pressure impacts upon environmental quality and this consequently affects population growth via mortality and fertility changes. Thus the equation accommodates the feedback effect from demographic process to environmental quality. Moreover,  $E_t$  can be described by

$$(1 - L)^d \Phi(L) E_t = \Theta(L) \epsilon_t. \quad (17)$$

$L$  is the lag operator as defined before and  $\Phi(L) = (1 + \phi_1 L + \dots + \phi_p L^p)$  and  $\Theta(L) = (1 - \vartheta_1 L - \dots - \vartheta_q L^q)$  are *AR* and *MA* polynomials respectively. Following the expressions in (9) and (13), we re-write the population growth equation as a function of stochastic environment:

$$n_t = - \left( E_t \left[ \psi (1 - L)^d + \theta \right]^{-1} \right) \quad (18)$$

$$\equiv - \left( E_t \psi \left[ \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1)} \Gamma(-d) L^j + \theta \right]^{-1} \right) \quad (19)$$

Now since, population in the production function is described by (5) that is the level of population depends on the growth of population,  $n_t$ . Moreover, the investment,  $I_t$  and capital stock equations are described as

$$K_{t+1} = (1 - \delta) K_t + I_t \quad (20)$$

In the above equation, capital stock is assumed to decline at a constant rate of  $\delta$  ( $0 < \delta < 1$ ) per period. Given that  $s$  is the fraction of  $Y$  to be invested, then

$$I_t = s Y_t \quad (21)$$

And consumption is defined according to

$$C_t = (1 - s) Y_t \quad (22)$$

Now since  $Y_t$  is described by 1 as above, then inducting the growth of population equation (15) in the production function we have.

$$Y_t = K_t^\alpha \left[ A_t \left( 1 - E_t \left[ \psi \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1) \Gamma(-d)} L^j + \theta \right]^{-1} N_{t-1} \right) \right]^{1-\alpha} \quad (23)$$

We will use this modified production function along with savings and consumption function with an assumed depreciation rate of capital stock and proportion of labor and capital for simulation.

Now, given the formulation as above if we assume that environmental quality is governed by the mean of the process, i.e.,

$$E_t = \bar{E} + \epsilon_t \quad (24)$$

then the spectral density of  $E_t$  is written as

$$f_E(\lambda) = \frac{\sigma_\epsilon^2}{2\pi} \quad (25)$$

However, if a stochastic shock persists in  $E_t$  or in its growth, i.e., if  $(1 - L)^d E_t = \epsilon_t$ , then the spectrum is governed by the stochastic memory in the environmental equation:

$$f_E(\lambda) = \left| 1 - e^{i\lambda} \right|^{-2d} f_\epsilon(\lambda) = \left| 2 \sin \frac{\lambda}{2} \right|^{-2d} f_\epsilon(\lambda) \quad (26)$$

where  $f_\epsilon(\lambda)$  denotes the spectral density of the error term.

**Proposition 1** *Under the assumption of the environmental and economic growth system described in Eqs. 5-7, the long memory in output growth,  $y_t$ , can be represented by the long memory in the growth of environmental quality and population growth.*

*Proof.* Here we outline the sketch of the proof. Empirical verification has already proven that demography-push led environmental problems have inflicted substantial fluctuations to economic growth. However, the analytical expression of our proof will follow from the composition of the economic growth equation which has components of population growth and environmental quality. As defined above, the long memory in the growth of output can be written as the growth of output, then

$$(1 - L)^d y_t = u_t \quad (27)$$

with the usual restrictions of  $d$  on the real line. A natural way to present whether, say  $y_t$  is a short-memory or a long memory process, is to know the shape of the spectral density of  $y_t$ .

If  $y_t$  is described by  $y_t = \bar{y} + u_t$ , i.e., the process is independently distributed around the mean, then the spectral density of  $y_t$  is  $f_y(\lambda) = \frac{\sigma_u^2}{2\pi}$ . If shocks persist in  $y_t$  and is characterized in long memory setting, then  $y_t$  follows  $(1 - L)^d y_t = u_t$  with the spectral density  $f_y(\lambda) = |1 - e^{i\lambda}|^{-2d} f_u(\lambda) = |2\sin(\frac{\lambda}{2})|^{-2d} f_u(\lambda)$ , where  $f_u(\lambda)$  is the spectral density of the error term. Now following the definitions above, since we can define the persistent properties of  $E_t$  (growth of environment) and  $n_t$  (growth of population) in terms of long-memory process, then the possible source of long-memory in  $y_t$  can be expressed as a product of the stochastic long-memory components from  $E_t$  and  $n_t$ . Therefore, the long-memory in  $y_t$ , can be expressed as a product of  $f_n(\lambda)$  and  $f_E(\lambda)$ , such that  $f_y(\lambda) = [(|1 - e^{i\lambda}|^{-2d} f_u(\lambda))(|1 - e^{i\lambda}|^{-2d} f_\epsilon(\lambda))]$ , where  $u_t$  and  $\epsilon_t$  are the *iid* error processes of population growth and environment. Thus the likelihood of a possible stochastic shock in the output growth equation can be expressed as the joint likelihood of the stochastic shocks from demographic and environmental system. In a way, we can express the variance in output growth as the sum of the variance of environment and demographic system and the covariance between them. Thus,

$$\text{Var}[y_t] = \text{Var}[N_t] + \text{Var}[E_t] + \text{Cov}[E_t, N_t] \quad (28)$$

As  $t \rightarrow \infty$ , the contribution of  $E_t$  and  $N_t$  to the variance of  $y_t$  increases and under deterministic assumption there would be steady state equilibrium. However, under alternative assumption of stochasticity, both  $E_t$  and  $N_t$  tend to experience heavy spurts and stochastic growth and can be presented by spectral variance of their respective shocks which would contribute to the total variance of output.  $\square$

**Proposition 2** *Given that the memory structure, represented by  $d$ , has shock convergent properties and possesses limiting distribution, then it is possible to achieve  $g'(E) < 0$ .*

*Proof.* Recall that in Eq. 10 ( given as  $g(E) = \frac{A(t)}{A(t)} = \mu - \beta \frac{\dot{E}}{E} = \mu + \beta\theta + \frac{\psi n(E)}{E}$ ),  $g'(E) < 0$  if  $n(E) = n$ . Now since,  $n(E)$  is stochastic and is given by Eq. 18, where  $(1 - L)^d$  is given by the power series expansion:  $(1 - L)^d = \sum_0^\infty h_j L^j = 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 - \dots$  as in 12, then  $g'(E)$  is achieved only when  $d$  has convergent properties in the sense that  $d \in (0, 0.5)$ . Since when  $d \leq 1/2$ , the population growth series is covariance stationary with summable

autocorrelation function and which is positive, then (local) stability is achieved for the growth system. Moreover, under the assumption that  $n(E) = n$ , that is under deterministic assumption, population growth can at least be zero and will be positive normally. This implies that even under stochastic formulation, the sum of stochastic shocks when add to a deterministic constant, the change in environmental quality ( $g'(E)$ ) will be negative. The intuition is that as long as  $n(E)$  is a non-linear function and the sign of which depends on the functional form and so on the magnitude of different values,  $g'(E)$  may not be negative. However, as long as stochastic shocks accumulate and has convergent properties such that  $0 < d \leq 1/2$ , the numerator still can be positive and therefore  $g'(E) < 0$ .  $\square$

**Proposition 3** *Under the assumption that  $n(E)$  is stochastic and is characterized by stochastic memories,  $n'(E) < 0$  only if  $0 < d < 1/2$ .*

*Proof.* The proof of this claim follows from elucidation of Proof 2. The idea is that stochastic population growth does not necessarily pertain to constant and positive numerator as in Eq. 10. The conditions under which  $n'(E) < 0$  can be given by the convergence properties of the  $n(E)$  function under stochastic memory properties.  $n'(E) < 0$  implies that a unit change in  $E$  will induce a negative response from population growth. The converse is also true. To prove the proposition assume that  $d$  lies in the region  $(1/2, 1)$  and second  $d \geq 1$ , then the autocorrelation function is not summable and the series exhibit non-stationary long-memory. There is no convergence of shocks to the mean value, thus the growth of the population series has no constant value in the numerator. Only when  $0 < d < 1/2$ ,  $(1 - L)^d = \sum_0^\infty h_j L^j = 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 - \dots$  has finite sum and has a positive constant on the numerator thus giving rise to  $n'(E) < 0$ . Local stability and long-run convergence (to steady state) occurs only when the stochastic  $E_t$  and  $n_t$  have  $(1 - L)^d$  with  $d \in (0, 0.5)$ . Chaotic EDG system occurs when  $d \geq 1$  due to the high sensitivity of the system to their initial values and high non-linearity due to propagation of shocks.  $\square$

## 5 Simulation experiments

The simulation experiment carried out in this section aims to lend additional support for the long-memory dynamics of Economy-Demography-Environment framework. The simulation economy closely follows the modified Solow-Swan growth model by embedding stochastic long-memory characters. Our idea is to show that as stochastic shocks move from convergent to high degree of non-convergence, i.e., as  $d$  moves from 0 to 1, the response of the economy and the environment over time becomes fairly stochastic in nature. Different regimes of change for economy and environment arise due to shifting patterns of stochastic memory from convergence to high degree of non-convergence. Our standard Solow-Swan economy has global assumption about labor and capital usage in production (that is, 2/3rd and 1/3rd) in line with many empirical research. The depreciation rate has been kept constant following the tradition of constant scrapping rule. Time dimension is set to 50 years so that the effect of long memory can be gauged over five decades.

From Figures 17-21, we observe interesting features of the response of environment to long-memory shocks. Essentially, three distinct patterns can be noted. For instance, we observe

that depending on the value of  $d$ , there is a change of regime starting from approximately year 15-16 (cf. figures 17 and 20). So, there are basically two regimes for the first type. Figures 18-19, 21 and 23 do show a different pattern (one regime with no change). The third pattern is observed in Figure 22 where now, two dates matter, year 8 and year 12. There are a total of three phases: before year 8, between year 8 and year 12, and after year 12. Moreover, for this third pattern, in each phase, the place of curves varies depending on  $d$ . Thus, before year 8, the curve for 'd=1' is above all curves. Between year 8 and year 12, the curve for 'd=1' is below all curves, and after year 12, we still obtain the same curve ranking as in phase 1 (before year 8). So, here, we have in total 3 regimes. So, generally, we can say that there exists a chaotical dynamic structure depending on the sensitivity of the initial values.

From the above, it appears that response of output and environment to long memory shock varies with the level of  $d$  values. The output growth equation which embeds stochastic features of environment and population growth responds to the long-memory shock in expected pattern: that stochastic shocks to environment and demography would result in stochasticity in output growth in the long-run. From environmental perspective we observed that the stochasticity of pollution growth, broadly the environmental system grows over time. That is what we observe for the output figures. Regimes changes are also presented in Figures 23-25 where distinction is made between one regime with no regime change. To summarize, the simulation experiment carried out for a modified Solow-Swan economy shows that economy-demography and environmental interactions are exceedingly complex and that the persistence of stochastic shocks in one system easily filters into the others, accelerating over time and pressing the system to behave in a stochastic pattern in the long-run. Different regime changes observed in our experiment due to variations in the magnitudes of the long-memory parameter indicate how initial distribution can change the growth profile while stochastic shocks move from convergence to non-convergence. Different economic and thus environmental systems would thus be observed depending on how fast  $d$  moves from 0 to 1 and how initial values change.

## 6 Conclusion

This paper explored the dynamics of economy-demography-environmental system in a stochastic environment. Specific emphasis was laid on introducing long-memory characteristics of population and pollution growth and mapping out their impacts on economic growth. The source of stochasticity in our model was perceived to arise from small perturbations as well as large scale changes arising in the economic and demographic system. Theoretical construct of long-memory demography-environment and economic growth was motivated by our empirical findings of the high degree of persistence in these systems. We used the long-memory framework to demonstrate how different rates of shock convergence occurred in these systems and how they were shown to be cointegrated under these settings. Our analytical model combined the time series properties of population, output and pollution and suggested a mechanism to show how stochastic shocks would arise in output in the long run when the output is a function of environmental and demographic parameters. Simulation experiment for a modified Solow-Swan economy provided additional support to our proposition that rate of convergence of demographic and environmental shocks would determine the rate of convergence of output shocks.

An interaction model of economy, demography and environment was simulated while allowing initial distribution and long-memory parameter to vary in magnitudes. Regime changes in economy and environment were observed to be consequences of the presence of persistence in environment and demographic system. Our results weigh some important policy perspectives, for instance controlling for stochastic shocks in environment and demography first before suggesting stringent measures to curb volatility and shocks in output growth over time. Moreover, a coordinated policy program can also help in controlling the proliferation of growth shocks.

## References

- Aghion, P. and P. Howitt (1998), *Endogenous Growth Theory*, MIT Press, Cambridge, Mass.
- Agiakloglou, C., P. Newbold and M. Wohar (1993), "Bias in an Estimator of the Fractional Difference Parameter," *Journal of Time Series Analysis*, 14, 235-246.
- Allen, J. C., and D. F. Barnes (1985), "The causes of deforestation in developing countries," *Annals of the Association of American Geographers* 75, pp. 163-184.
- Azomahou, T., Laisney, F., Nguyen Van, P., 2006, "Economic development and CO<sub>2</sub> emissions: A nonparametric panel approach", *Journal of Public Economics* 90, pp. 1347-1363.
- Bateman, I. I. (2004), "Review of economic theory for the environment: Essays in honor of Karl Goran Maler," *The Economic Journal* 114, pp. 539-541.
- Bhattarai, M., and M. Hammig (2001), "Institutions and the Environmental Kuznets Curve for Deforestation: A Cross-Country Analysis for Latin America, Africa and Asia," *World Development* 29, pp. 995-1010.
- Birdsall, N. (1992), "Another look at population and global warming", Population, health, and nutrition policy research working paper, WPS 1020, World Bank, Washington, DC.
- Boserup, E. (1981), *Population and Technological Change: A Study of Long-Term Trends*, University of Chicago, Chicago.
- Boucekkine, R., D. de la Croix and D. Peeters (2007), "Early literacy achievements, population density, and the transition to modern growth", *The Journal of the European Economic Association*, 5, 183-226, 2007
- Cheung, Y.W. (1993), "Long Memory in Foreign-Exchange Rates," *Journal of Business & Statistics*, Vol. 11, No.1, 93-101.
- Chu, C.Y.C. and R.R. Yu, (2002), "Population dynamics and the decline in biodiversity: A survey of the literature," *Population and Development Review* 28, pp. 126-143.
- Cramer, J. C., (2002), "Population growth and local air pollution: Methods, models and results", In: Population and Environment: Methods of Analysis, *Population and Development Review*. W. Lutz et al (Eds), New York: Pop. Council 28, 22-52.
- Crenshaw, E., A. Ameen, and M. Christenson (1997), "Population Dynamics and Economic Development: Age Specific Population Growth and Economic Growth in Developing Countries, 1965 to 1990," *American Sociological Review* 62(6):974-984.
- Cropper, M. and C. Griffiths (1994), "The interaction of population growth and environmental quality," *The American Economic Review* 84, pp. 250-254.
- Dasgupta, P. (1995), "The population problem: Theory and evidence," *Journal of Economic Literature*, 33, pp. 1879-1902.
- Davidson, J. (2007), Time Series Modeling Version 4.25, <http://www.timeseriesmodeling.com/>



- Durlauf, S. (1989), "Output persistence, economic structure, and the choice of stabilisation policy," *Brooking Papers on Economic Activity*, 20(2), 69-136.
- Durlauf, S. N. (1991), "Multiple Equilibria and Persistence in Aggregate Fluctuations," *American Economic Review*, American Economic Association, vol. 81(2), 70-74.
- Ehrlich, P. R., and A. H. Ehrlich (1981), *Extinction: the Causes and Consequences of the Disappearance of Species*. Random House, New York.
- Ehrlich, P.R. and J.P. Holdren (1971), "Impact of population growth", *Science*, 171: 1212-1217
- FAO (1993), "Forest Ressources Assessment 1990: Tropical Countries," Forestry Paper no. 112, Food and Agriculture Organization, Rome.
- Fox, R. and M. S. Taqqu (1986), "Large-sample properties of parameter estimates for strongly dependent stationary Gaussian time series," *The Annals of Statistics*, 14, 517-532.
- Geweke, J. and S. Porter-Hudak (1983), "The estimation and application of long memory time series models," *Journal of Time Series Analysis*, 4, 221-238.
- Gil-Alana, L.A. (2003), "A fractional integration analysis of the population in some OECD countries," *Journal of Applied Statistics*, 30(10), 1-13.
- Henderson, D.J., D.L., Millimet (2007), "Pollution abatement costs and foreign direct investment inflows to U.S. states: A nonparametric reassessment," *The Review of Economics and Statistics* 89, pp. 178-183.
- Hurvich, M. C., R. Deo, and J. Brodsky (1998), "The Mean Squared Error of Geweke and Porter-Hudak's Estimator of the Memory Parameter of a Long-Memory Time Series," *Journal of Time Series*, 19, 19-46.
- Jeon, B.M and R. C. Sickles (2004), "The role of environmental factors in growth accounting," *Journal of Applied Econometrics* 19, pp. 567-591.
- Kelley, A.C. and R.M. Schmidt (2001), "Economic and Demographic Change: A Synthesis of Models, Findings and Perspectives," in 'Birdsall N, Kelley A. C, Sinding, S (eds) *Demography Matters: Population Change, Economic Growth and Poverty in the Developing World*.' Oxford University Press, 2001.
- Kim, C.S. and P.C.B. Phillips (1999), "Log Periodogram Regression: The Nonstationary Case," Working Paper, Yale University.
- Kim, C.S. and P.C.B. Phillips, (2000), "Modified Log Periodogram Regression," Working Paper, Yale University.
- Koop, G., and L. Tole (1999), "Is There an Environmental Kuznets Curve for Deforestation?," *Journal of Development Economics* 58, pp. 231-244.
- Maddison, A. (2004), *Total Economy Database* from Groningen Growth and Development Centre. <http://www.ggdc.net/index-dseries.html>.

- Maler, K-G. and J. R. Vincent (2005), *Handbook of Environmental Economics*. Volume 3. Economywide and International Environmental Issues. Amsterdam: Elsevier, North-Holland.
- G. Marland, T.A. Boden, and R.J. Andres (2004), "Global, Regional, and National Fossil Fuel CO<sub>2</sub> Emissions", Carbon Dioxide Information Analysis Center, Environmental Sciences Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6335, U.S.A.
- Meyerson, F.A.B. (1998), "Population, carbon emissions, and global warming: the forgotten relationship at Kyoto", *Population and Development Review*, 24(1), 115-130.
- Malthus, T. R. (1798), *An Essay on the Principles of Population*. St. Martins Press (Reprint 1966), Macmillan.
- Michelacci, C and P. Zaffaroni (2000), "(Fractional) Beta convergence," *Journal of Monetary Economics*, 45, 129-153.
- Millimet, D.L., J.A., List, T., Stengos (2003), "The environmental Kuznets curve: Real progress or misspecified models?," *The Review of Economics and Statistics* 85, pp. 1038-1047.
- Mishra, T. (2008), "Stochastic demographic dynamics and economic growth," *Historical Social Research*, 33(4), 9-187.
- Nelson, C.R. and C.I. Plosser (1982), "Trends and random walks in macroeconomic time series," *Journal of Monetary Economics*, 10, 139-162.
- Nerlove, M. (1991) "Von Thuenen's Model of the Dual Economy," *Journal of Economics* , 54: 97-123, 1991.
- Panayotou, T. (2000a), "Economic growth and the environment," CID working paper no. 56, Harvard University.
- Panayotou, T. (2000b), "Population and environment," in *The International Yearbook of Environmental and Resource Economics 2000/2001: A Survey of Current Issues*, ed. by T. Tietenberg, and H. Folmer, chap. 4. Edward Elgar, Cheltenham.
- Phillips, P.C.B.(1999a), "Discrete Fourier Transforms of Fractional Processes," Unpublished working paper No. 1243, Cowles Foundation for Research in Economics, Yale University. <http://cowles.econ.yale.edu/P/cd/d12a/d1243.pdf>
- Phillips, P.C.B. (1999b), "Unit Root Log Periodogram Regression," Unpublished working paper No. 1244, Cowles Foundation for Research in Economics, Yale University. <http://cowles.econ.yale.edu/P/cd/d12a/d1244.pdf>.
- Pommeret, A. and W. T. Smith, (2005), "Fertility, volatility, and growth," *Economics Letters* 87, pp. 347-353.
- Postel, S. (1984), "Protecting Forests," in *State of the World*, ed. by Brown L.R. W.W. Norton & Co., New York.
- Prskawetz, A. and G. Feichtinger (1995), "Endogenous Population Growth may Imply Chaos," *Journal of Population Economics*, 8(1):59-80

- Nguyen Van, P. and T. Azomahou, (2006), "Nonlinearities and heterogeneity in environmental quality: An empirical analysis of deforestation," *Journal of Development Economics* 84, pp. 291-309.
- Renshaw, E. (1991), "Modelling biological populations in space and time", Cambridge Univ. Press, New York, N. Y.
- Robinson, J. A., and T. N. Srinivasan (1997), "Long-Term Consequences of Population Growth: Technological Change, Natural Resources, and the Environment," in *Handbook of Population and Family Economics*, ed. by M. R. Rosenzweig, and O. Stark, chap. 21. North-Holland, Elsevier Science, Amsterdam.
- Romer, P.M. (1990), "Endogenous Technological Change," *Journal of Political Economy*, 98, pp. S71-S102.
- Shafik, N. (1994), "Economic Development and Environmental Quality: An Econometric Analysis," *Oxford Economic Papers* 46, pp. 757-773.
- Seidl I. and C.A. Tisdell (1999), "Carrying capacity reconsidered : from Malthus' population theory of cultural carrying capacity", *Ecological Economics*, 31, 395-408.
- Solow, R.M. (1956), "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics* 70(1), 65-94.
- Soretz, S., (2003), "Stochastic pollution and environmental care in an endogenous growth model," *The Manchester School*, 71, 448-469.
- Sowell, F. (1992), "Maximum likelihood estimation of stationary univariate fractionally integrated time series models," *Journal of Econometrics*, 53, 165-188.
- Summers, R. and A. Heston. (1994), "Data update 5.5," Computer Diskette based on The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988. Cambridge, MA: National Bureau of Economic Research.
- Swan, T. W. (1956), "Economic Growth and Capital Accumulation", *Economic Record*, 32, 334-361.
- Zhang, X., S. Hu and H. Wang (2006), "A stochastic growth model with environmental pollution," *Journal of Systems Science and Complexity* 19, pp. 414-422.
- Verhulst, P. F. (1838). "Notice sur la loi que la population poursuit dans son accroissement", *Correspondance mathmatique et physique* 10:113121.
- Whittle, P. (1951), *Hypothesis Testing in Time Series Analysis*, Uppsala, Almqvist and Wiksells Boktryckeri AB.
- World Bank (1992), *World Development Report 1992: Development and the Environment*. Oxford University Press, New York.
- Xepapadeas, A., (2003), "Economic growth and the environment," in the *Handbook of Environmental Economics*, edited by Karl-Gran Mler and Jeffrey Vincent.

# Figures and Appendix

Figure 1: Logarithmic plots of CO<sub>2</sub> emissions and population: OECD countries

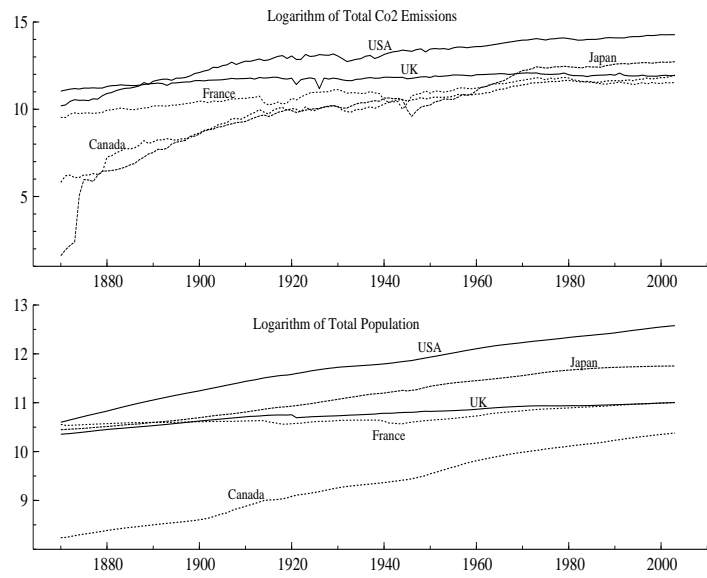


Figure 2: Logarithmic plots of CO<sub>2</sub> emissions and population: Non-OECD countries

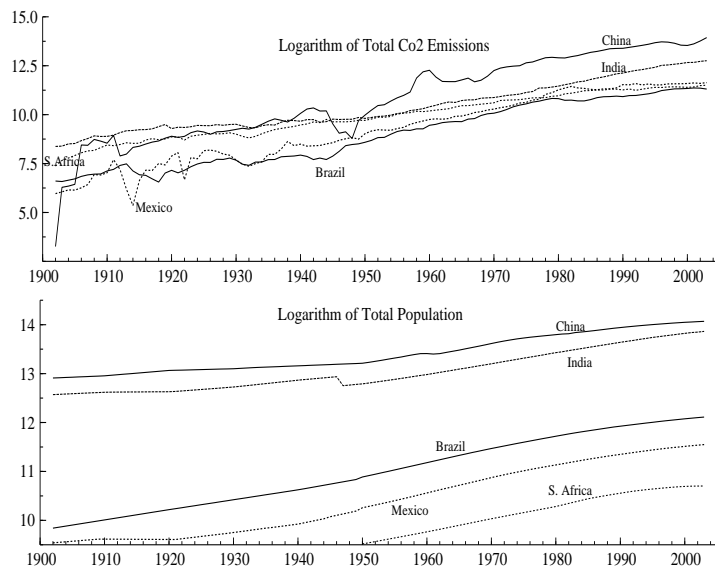


Figure 3: Logarithmic plots of real GDP: OECD countries

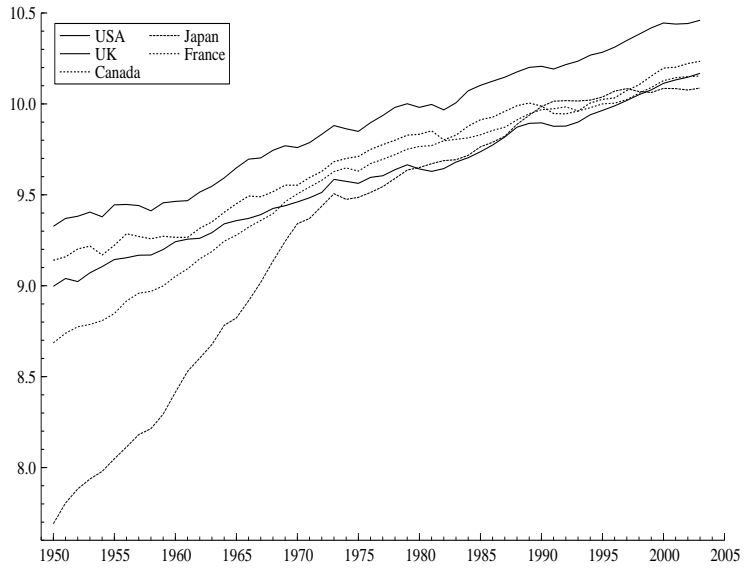


Figure 4: Logarithmic plots of real GDP: Non-OECD countries

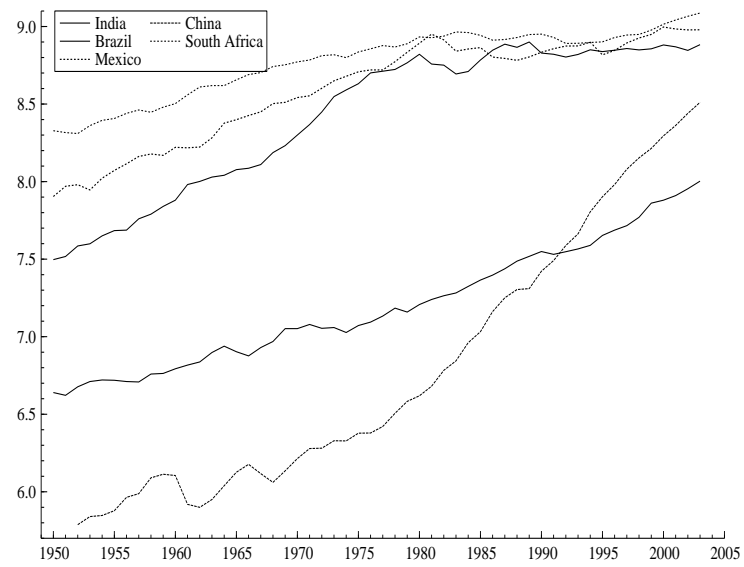


Figure 5: Spectral plots of CO<sub>2</sub> emissions for OECD countries: USA [left], UK [right].

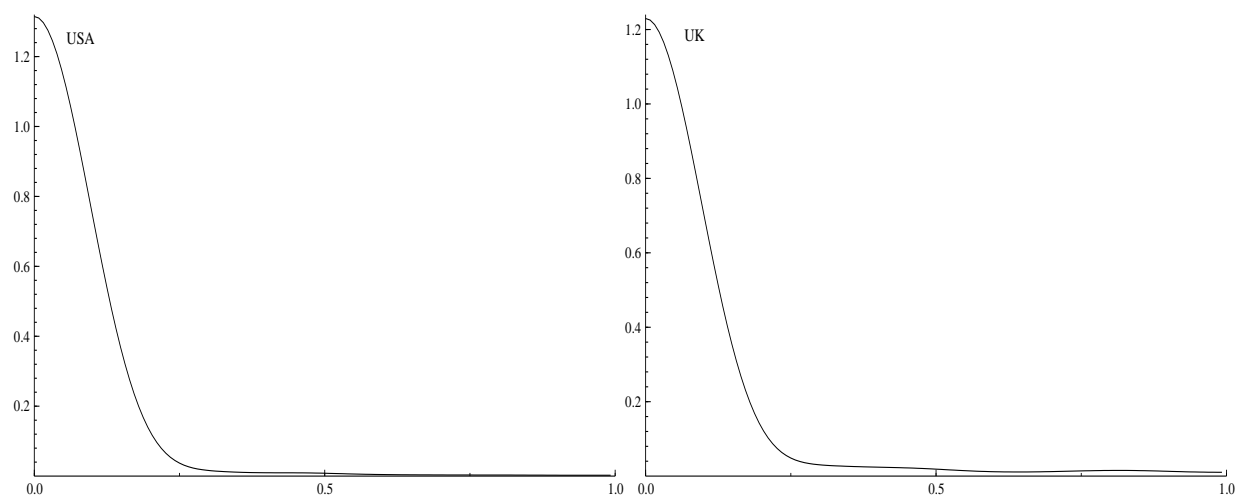


Figure 6: Spectral plots of CO<sub>2</sub> emissions for OECD countries: France [left], Japan [right].

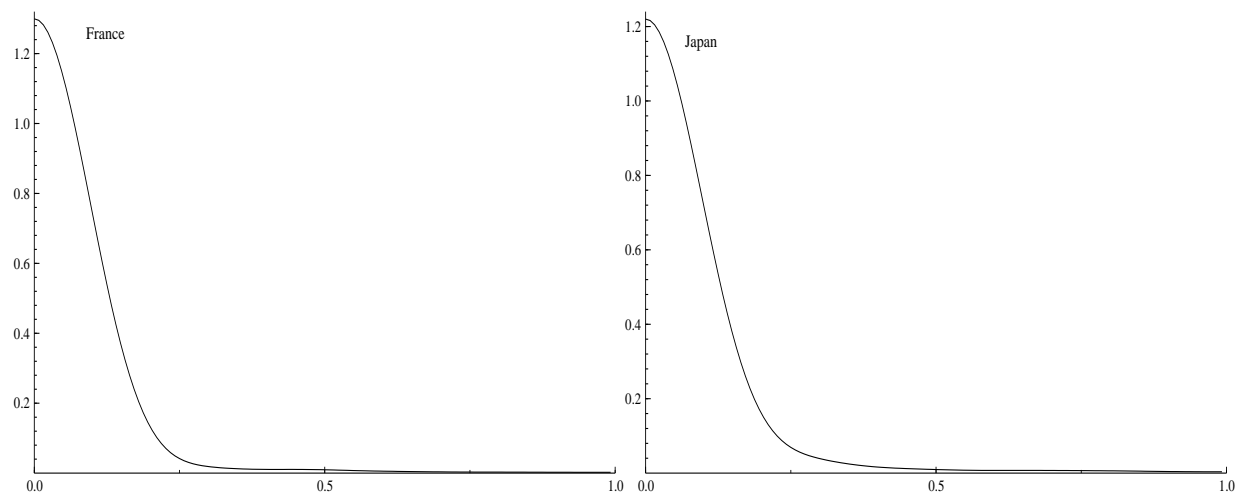


Figure 7: Spectral plots of CO<sub>2</sub> emissions for OECD countries: Canada

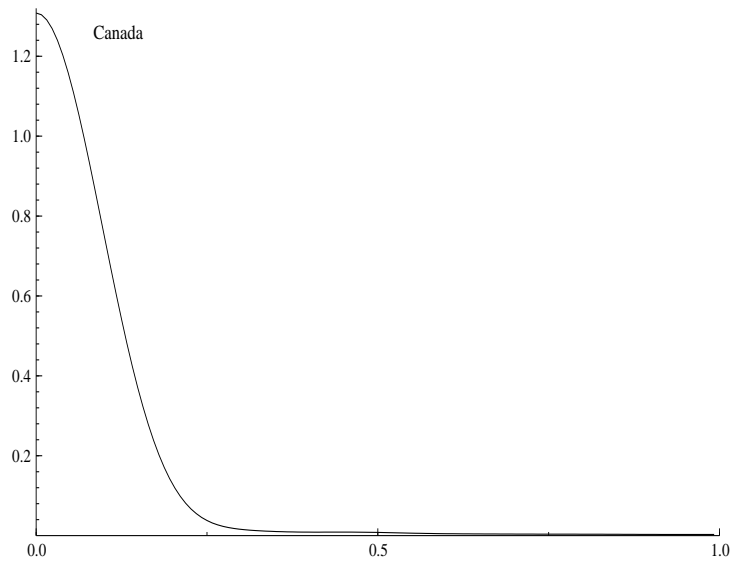


Figure 8: Spectral plots of CO<sub>2</sub> emissions for Non-OECD countries: India [left], China [right].

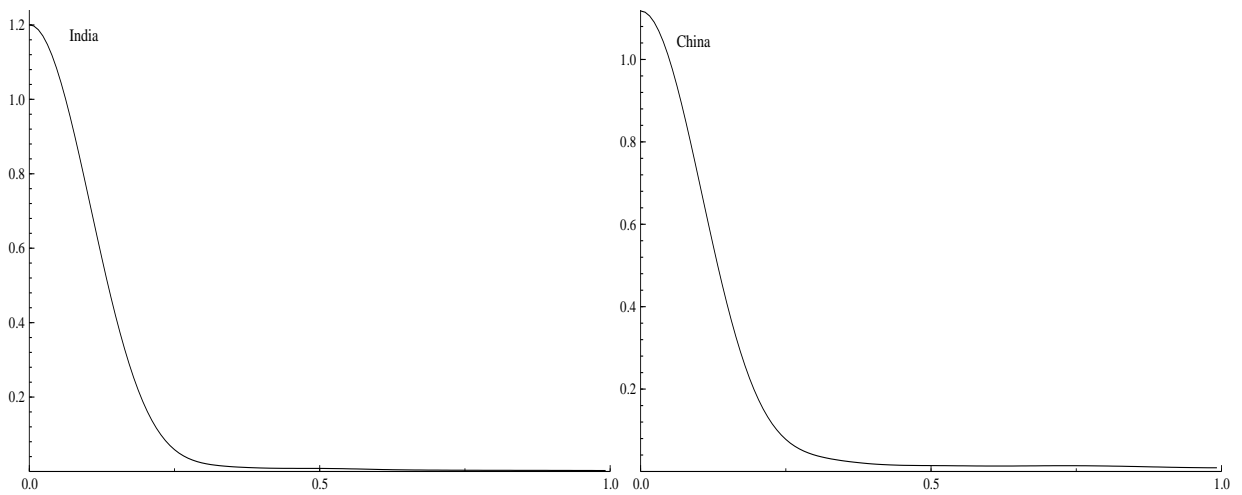


Figure 9: Spectral plots of CO<sub>2</sub> emissions for Non-OECD countries: Brazil [left], Mexico [right].

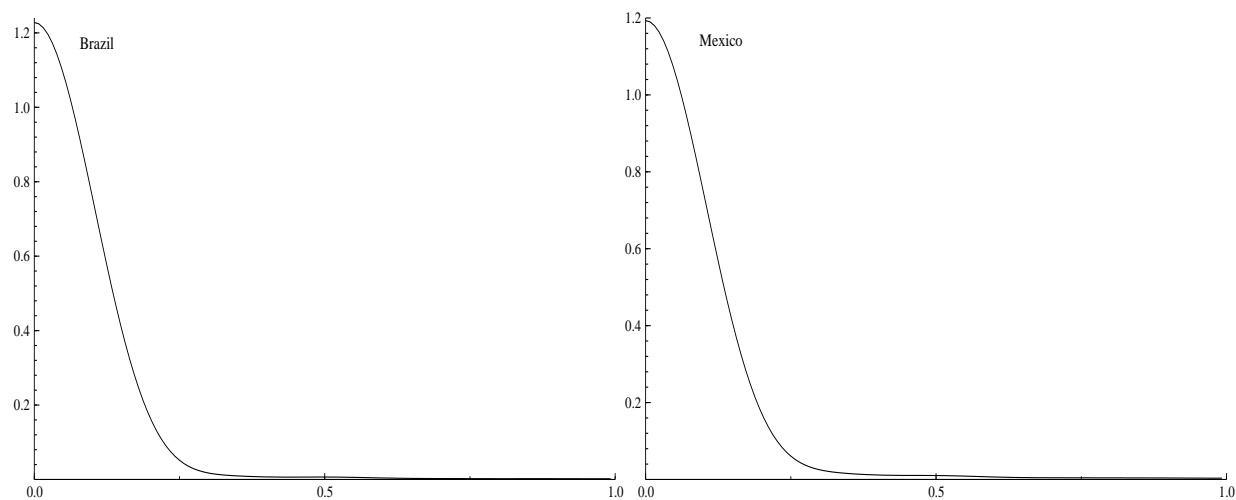


Figure 10: Spectral plots of CO<sub>2</sub> emissions for Non-OECD countries: South Africa

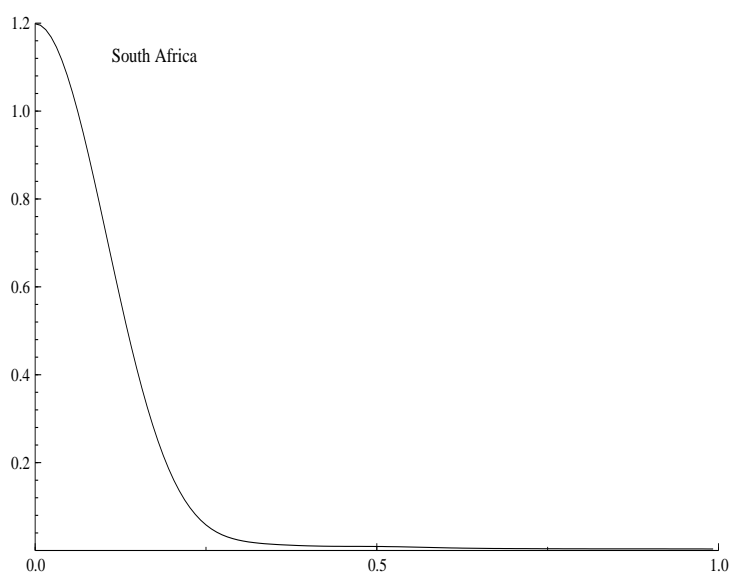




Figure 11: Spectral plots of total population for OECD countries: USA [left], UK [right].

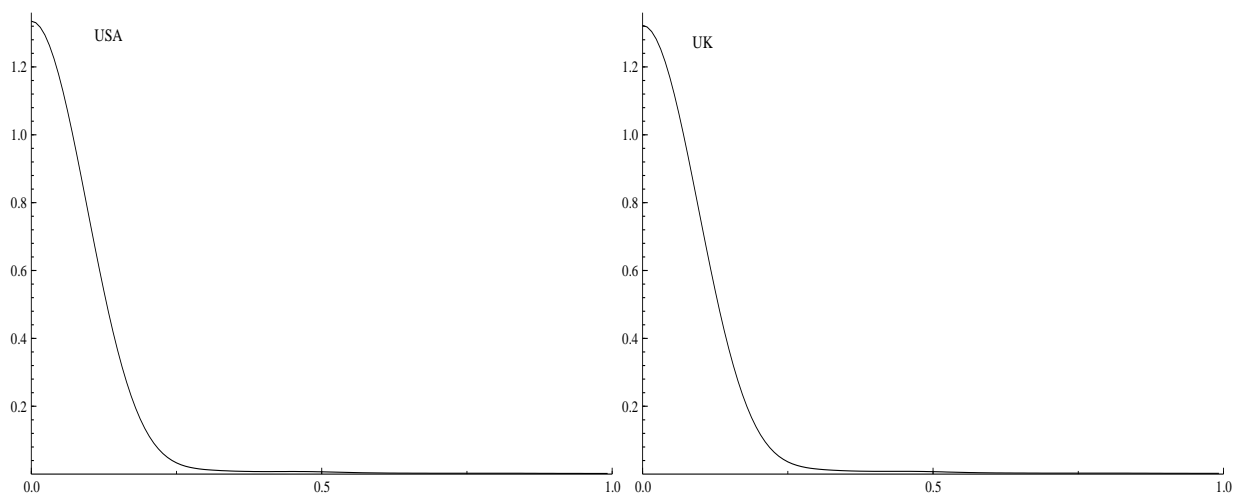


Figure 12: Spectral plots of total population for OECD countries: France [left], Japan [right].

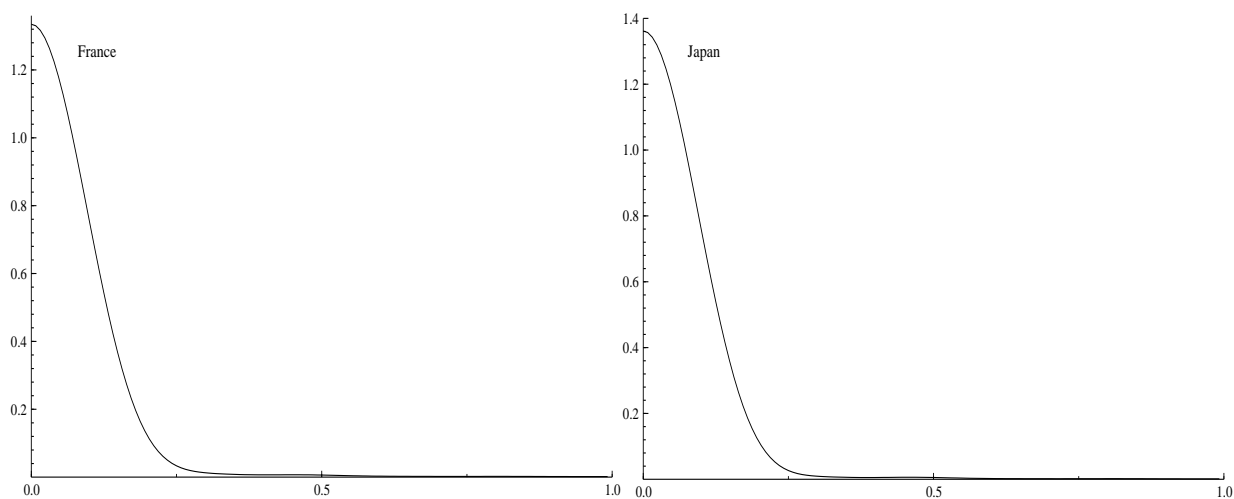


Figure 13: Spectral plots of total population for OECD countries: Canada

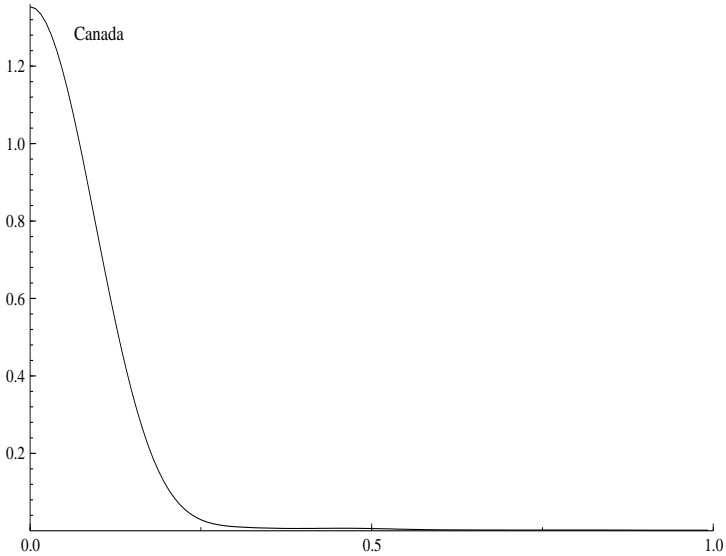


Figure 14: Spectral plots of total population for Non-OECD countries: India [left], China [right].

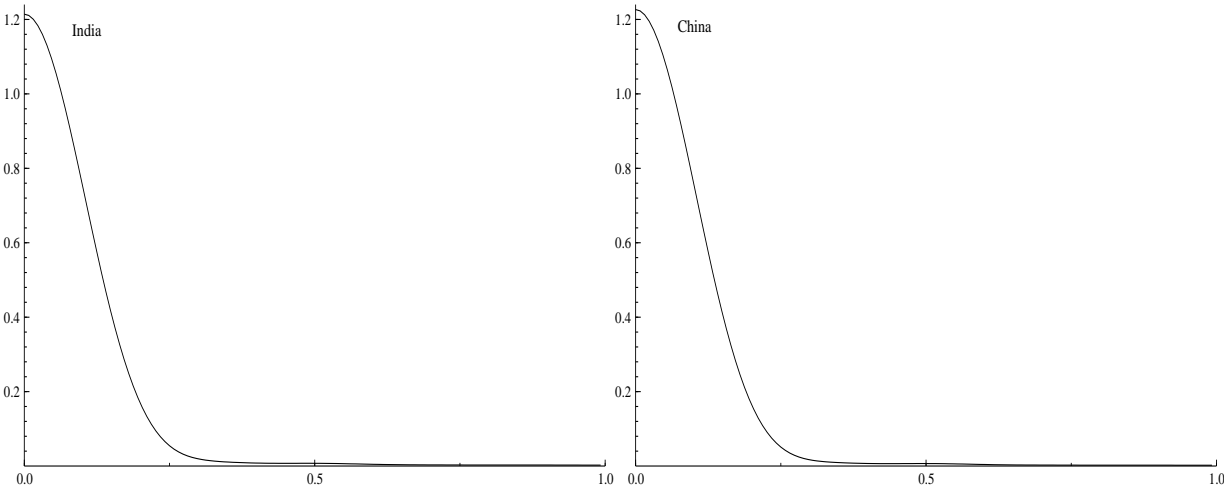


Figure 15: Spectral plots of total population for Non-OECD countries: Brazil [left], Mexico [right].

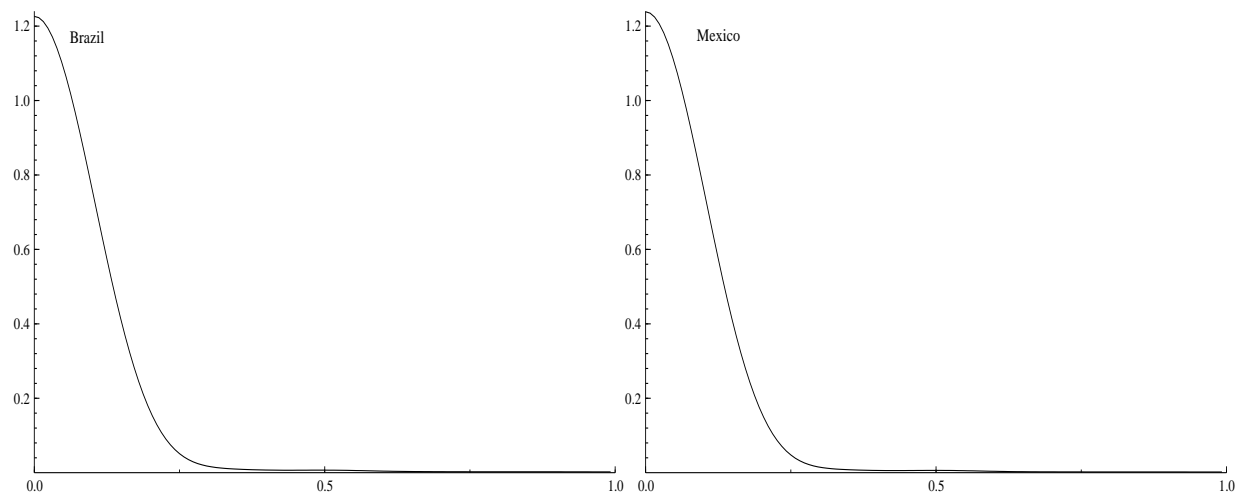
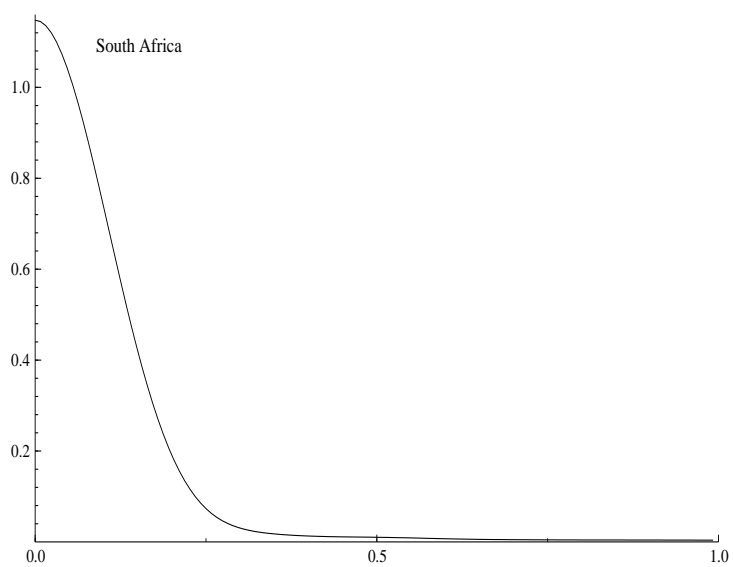


Figure 16: Spectral plots of total population for Non-OECD countries: South Africa



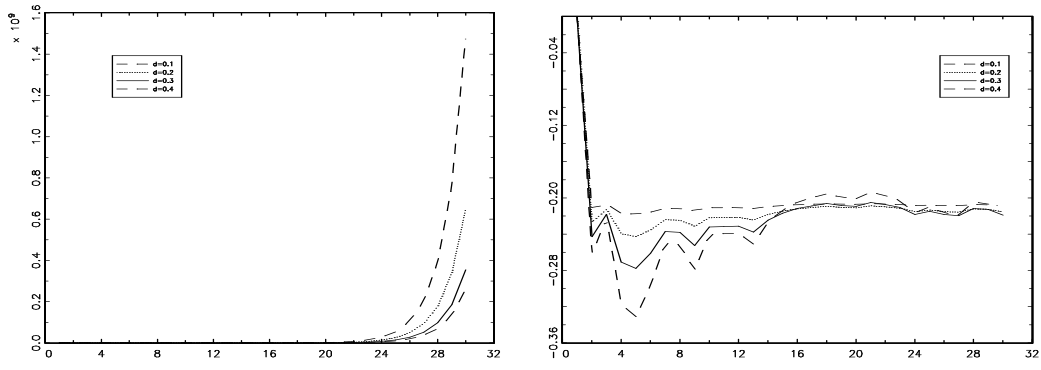


Figure 17: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

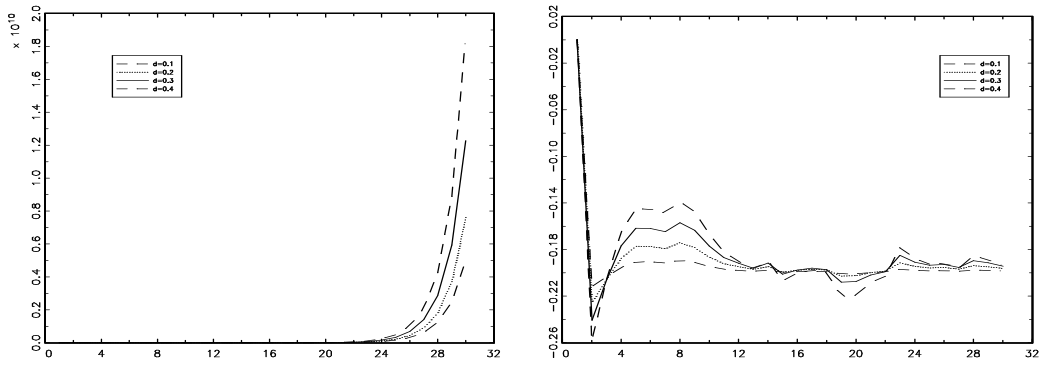


Figure 18: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

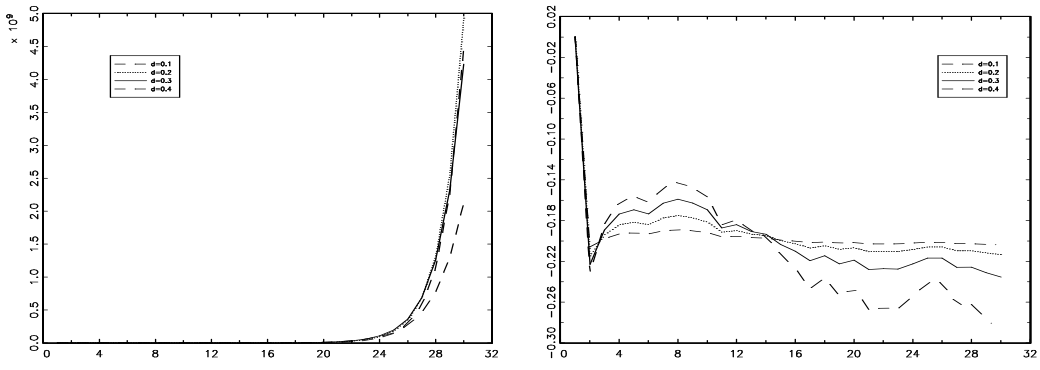


Figure 19: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

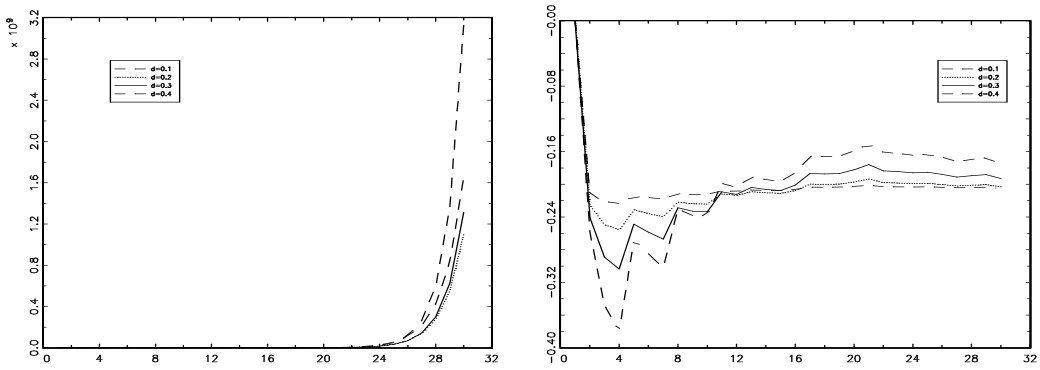


Figure 20: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

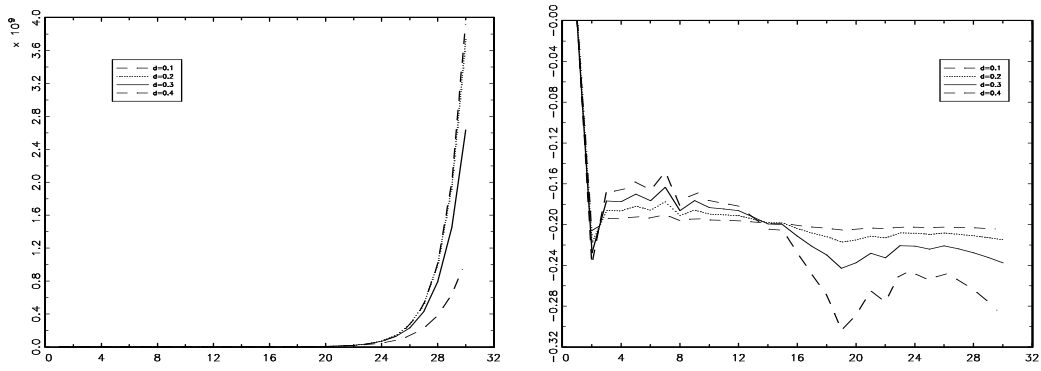


Figure 21: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

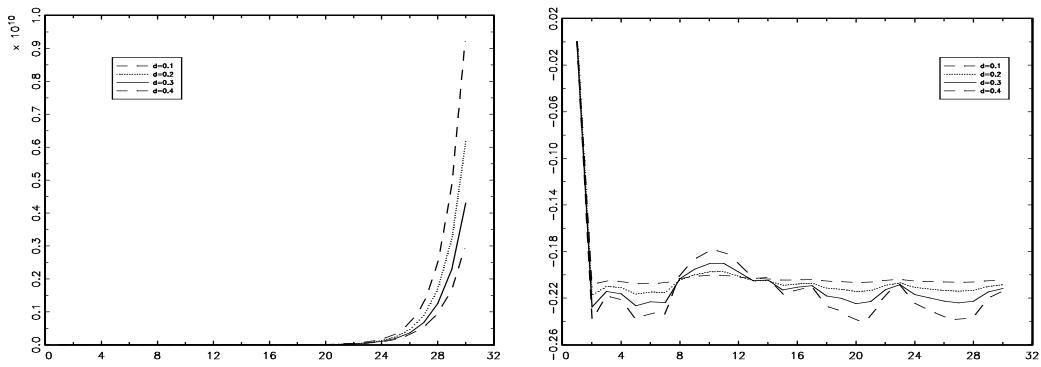


Figure 22: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

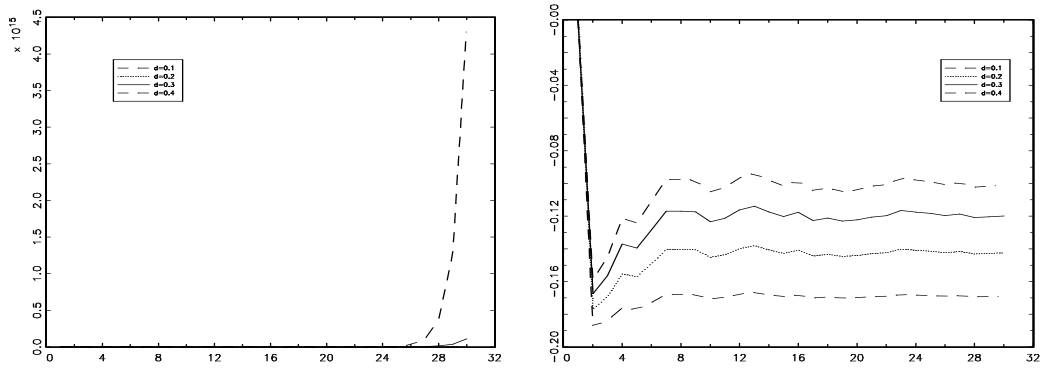


Figure 23: Simulation of the dynamics of the environment demographic growth system: two regimes. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

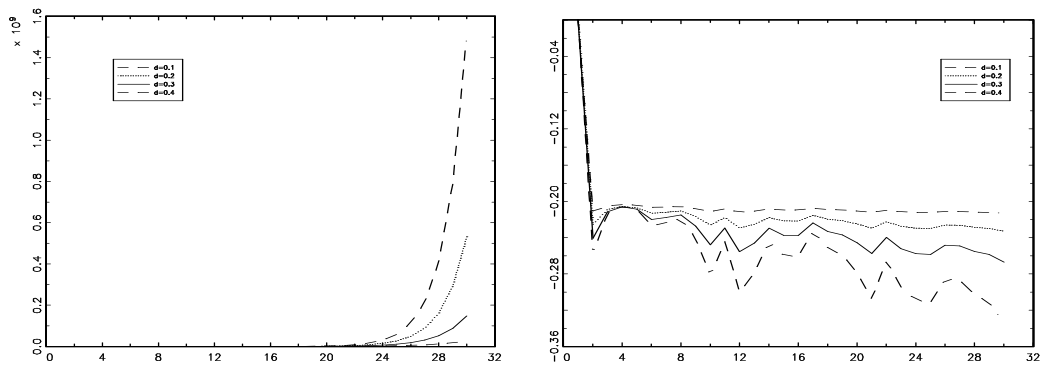


Figure 24: Simulation of the dynamics of the environment demographic growth system: three regimes. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

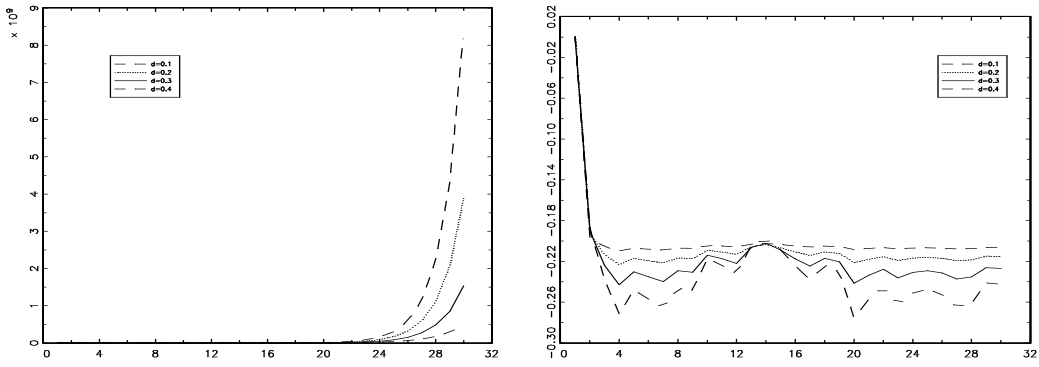


Figure 25: Simulation of the dynamics of the environment demographic growth system: one regime with no regime change. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

## Appendix: Fractional cointegration results

Table 5: Fractional cointegration results for population and CO<sub>2</sub> emissions

Countries	F-ECM (d1)	d3	Eq.Relation (1+d1-d3)
<b>OECD</b>			
USA (ARFIMA (2,1+d,2))	0.013 (0.005)	0.172 (0.086)	(1+d1-d3)
Japan (ARFIMA (2,1+d,2))	0.013 (0.007)	0.771 (0.056)	(1+d1-d3)
UK (ARFIMA (0,1+d,0))	-0.004 (0.002)	0.613 (0.075)	(1+d1-d3)
France (ARFIMA (1,1+d,2))	-0.011 (0.004)	0.325 (0.097)	(1+d1-d3)
Canada (ARFIMA (2,1+d,0))	-0.003 (0.001)	0.768 (0.040)	(1+d1-d3)
<b>Non-OECD</b>			
China (ARFIMA (1,1+d,0))	-0.001 (0.001)	0.362 (0.436)	(1+d1-d3)
India (ARFIMA (0,1+d,2))	-0.010 (0.002)	-1.185 (0.155)	(1+d1-d3)
Brazil (ARFIMA (2,1+d,2))	0.001 (0.002)	0.205 (0.059)	(1+d1-d3)
South Africa (ARFIMA (2,1+d,0))	0.007 (0.006)	0.171 (0.125)	(1+d1-d3)
Mexico (ARFIMA (1,1+d,0))	-0.020 (0.004)	1.029 (0.1.705)	(1+d1-d3)



## Appendix: modified log-periodogram regression of the estimation of $d$

We describe here the estimation method of  $d$ , the memory parameter in the regression. Among several approaches of estimating  $d$ , notable among them are the parametric approach by Sowell (1992)(exact maximum likelihood estimator) and the approximate Whittle estimator due to Whittle (1951), and Fox and Taqqu (1986). In semi-parametric class, Geweke and Porter-Hudak estimator (GPH, 1983), and Gaussian semi-parametric estimator due to Robinson (1995) are extensively used in the literature. For the empirical investigation, we use the modified log periodogram regression (LPR) method developed by Phillips (1999a, 1999b). Agiaklglou et al. (1993), Cheung (1993) and Hurvich et al. (1998) argue that GPH has severe small-sample bias and very inefficient if the results are likely to be contaminated by possible short-memory parameters (i.e.,  $AR$  and  $MA$  parameters). They also argue that the estimator is not invariant to first differencing so that there might be an over-differencing issue. Moreover, distinguishing unit-root behavior from fractional integration may be problematic, because the GPH estimator is inconsistent against  $d > 1$  alternatives. This weakness of the GPH estimator has been addressed by Kim and Phillips' (2000) Modified Log Periodogram Regression method (MLPR), in which the dependent variable is modified to reflect the distribution of  $d$  under the null hypothesis that  $d = 1$ . This is in fact the modified version of GPH (1983). The estimator gives rise to a test statistic for  $d = 1$ , which is a standard normal variate under the null.

GPH is a standard estimation technique for  $d$ , however, the modifications suggested to GPH method needs to be explained. Specifically, we briefly explain the difference between the two by first defining the discrete Fourier transform (DFT) of the time series, and pointing out how the dependent variable, viz., the periodogram, is modified in case of MLPR. To elucidate the problem, recall Eq.1, and simplify the DGP of the process  $E_t$  as  $(1 - L)^d E_t = u_t$  for convenience. Note that the stationary component of  $u_t$  is a linear process of the form:  $u_t = C(L)\epsilon_t = \sum_{j=0}^{\infty} c_j \epsilon_{t-j}$ ,  $\sum_{j=0}^{\infty} j|c_j| < \infty$ ,  $C(1) \neq 0$  for all  $t$  and with  $\epsilon_t = iid(0, \sigma^2)$  with finite fourth moments. Under this assumption, the spectrum of  $u_t$  is  $f_u(\lambda) = \frac{\sigma^2}{2\pi} |\sum_{j=0}^{\infty} c_j e^{ij\lambda}|^2$ . Then, the spectrum of  $E_t$  can be defined as

$$f_E(\lambda) = |1 - e^{i\lambda}|^{-2d} f_u(\lambda) \quad (29)$$

where  $E_t$  is stationary, i.e.,  $|d| < 1/2$ . This is also the analogue of the spectrum in the nonstationary case when  $d \geq 1/2$ . Taking logs of Eq. 29 produces:

$$\ln f_E(\lambda) = -2d \ln(|1 - e^{i\lambda}|) + \ln f_u(\lambda). \quad (30)$$

GPH (1983) propose that  $d$  can be estimated from the above by a linear log periodogram regression, where  $f_E(\lambda)$  is replaced by the periodogram ordinates,  $I_E(\lambda)$  evaluated at the fundamental frequencies  $\lambda_\zeta = \frac{2\pi\zeta}{n}$ ,  $n = 0, 1, \dots, n - 1$ . Here  $I_E(\lambda_\zeta) = F_E(\lambda_\zeta)F_E(\lambda_\zeta)^*$ ,  $F_E(\lambda_\zeta)$  is the dft  $F_E(\lambda_\zeta) = \frac{1}{\sqrt{2\pi n}} \sum_{t=1}^n E_t e^{it\lambda_\zeta}$  of  $E_t$  and  $F^*$  is the complex conjugate of  $F$ . Substituting these information in Eq. 30 now becomes

$$\ln[I_E(\lambda_\zeta)] = -2d \ln|1 - e^{i\lambda_\zeta}| + \ln(f_u(\lambda_\zeta)) + \eta_j \quad (31)$$

where  $\eta_j = \ln[I_E(\lambda_\zeta)/f_E(\lambda_\zeta)]$ . Note that  $f_u$  is continuous, and therefore  $f_u(\lambda_\zeta)$  is effectively constant for frequencies in a shrinking band around the origin (Phillips, 1999a), thus enabling

a linear least square regression of  $\ln[I_E(\lambda_\zeta)]$  on  $\ln|1 - e^{i\lambda_\zeta}|$  over frequencies  $\zeta = 1, \dots, \nu$ . The regression gives rise to the GPH estimation of  $d$ , where asymptotically  $\hat{d}_{GPH} \sim N(d, \frac{\pi^2}{6})$ . This method has been extensively used in practice due to ease of handling. However, Phillips (1999a) note that Eq. 30 is a moment condition and not a data generating mechanism, and the analysis of this regression estimator is complicated while characterising the asymptotic behavior of the dft  $F_P(\lambda_\zeta)$  which is central to determining the properties of the regression residual  $\eta_j$  in Eq. 31.

Phillips (1999a) suggested modification in the GPH method by properly demonstrating the asymptotic properties of dft. Moreover, the modified GPH, known as MLPR provides a wider range for both the stationary and nonstationary areas such as  $d \geq 0$  and for *AR* and *MA* errors and test for nonstationarity. The author argues that since we usually have no prior information about the order of integration,  $d$ , hence it is instructive to cover a wide range plausible parameter values of  $d$ . The biggest concern in GPH technique is that little is known about the short memory component of  $u_t$  in our DGP and that its spectrum  $f_u(\lambda)$  is treated nonparametrically. In log periodogram regression, viz., in GPH, this is accomplished by working with the dft  $F_E(\lambda_\zeta)$  of the data  $E_t$  over a set of  $\nu$  frequencies  $\lambda_\zeta = \frac{2\pi\zeta}{n} : \zeta = 1, \dots, \nu$  that shrink slowly to origin as the sample size  $n \rightarrow \infty$  by virtue of a condition on  $\nu$  of the type  $\frac{\nu}{n} \rightarrow 0$ . As  $d \rightarrow 1$ , the dft  $F_E(\lambda_\zeta)$  behaves differently due to the effects of leakage in semiparametric estimation of  $d$ . Therefore it needs modification. For  $d \in (1/2, 1)$ , Phillips (1999a) derives the dft of  $F_E(\lambda_\zeta)$  as

$$F_E(\lambda_\zeta) = (1 - e^{i\lambda_\zeta})^{-d} F_u(\lambda_\zeta) - \frac{e^{i\lambda_\zeta}}{1 - e^{i\lambda_\zeta}} \frac{E_n}{\sqrt{2\pi n}} + o_p \left( \frac{(1 - e^{i\lambda_\zeta})^{1-d}}{\zeta^{1-d}} \right) \quad (32)$$

where  $\frac{\zeta}{n} + \frac{n^\alpha}{\zeta} \rightarrow 0$  as  $n \rightarrow \infty$ , for some  $\alpha \in (1/2, 1)$ . The asymptotic behavior of  $F_E(\lambda_\zeta)$  is dominated by the first two terms in Eq. 32, however as  $d \rightarrow 1$  the importance of the second term in Eq. 32 is reflected. Apparently, it is necessary to correct the dft  $F_E(\lambda_\zeta)$  by adding the correction term represented by the known form of expression in Eq. 32. For log periodogram regression this amounts to using the quantity

$$V_E(\lambda_\zeta) = F_E(\lambda_\zeta) + \frac{e^{i\lambda_\zeta}}{1 - e^{i\lambda_\zeta}} \frac{E_n}{\sqrt{2\pi n}} \quad (33)$$

in place of  $F_E(\lambda_\zeta)$  in the regression. Thus in stead of the usual least square regression (over  $\zeta = 1, \dots, \nu$ )

$$\ln(I_E(\lambda_\zeta)) = \hat{\alpha} - \hat{d} \ln |1 - e^{i\lambda_\zeta}|^2 + error \quad (34)$$

which is motivated by the form of the moment equation in the frequency domain, the argument above suggests the linear least square regression

$$\ln(I_V(\lambda_\zeta)) = \tilde{\alpha} - \tilde{d} \ln |1 - e^{i\lambda_\zeta}|^2 + error \quad (35)$$

in which the periodogram ordinates,  $\ln(I_E(\lambda_\zeta))$  are replaced by  $\ln(I_V(\lambda_\zeta)) = V_E(\lambda_\zeta)V_E(\lambda_\zeta)^*$ . This is known as modified log periodogram regression *a la* Phillips. Thus in place of the ‘regression model’

$$\ln(I_E(\lambda_\zeta)) = \alpha - d \ln |1 - e^{i\lambda_\zeta}|^2 + u(\lambda_\zeta), \quad (36)$$

with  $\alpha = \ln(f_u(0))$  and  $u(\lambda_\zeta) = \ln[I_E(\lambda_\zeta)/f_E(\lambda_\zeta)] + \ln(f_u(\lambda_\zeta)/f_u(0))$ , as in Eq.31, we now have from Eq. 32

$$I_V(\lambda_\zeta) = |1 - e^{i\lambda_\zeta}|^{-2d} I_u(\lambda_\zeta) \left| \left[ 1 + o_p \left( \frac{1}{\nu^{1-d}} \right) \right] \right|^2 \quad (37)$$

The new regression (Eq. 35) seems likely to be most useful in cases where nonstationarity is suspected, especially when  $d > 1$ . As in the case of GPH, the distribution of  $\hat{d}_{MLPR} \sim N(d, \frac{\pi^2}{24\nu})$ . Among several advantages in the modified LPR method, Phillips notes that it modifies the periodogram ordinates to find the correct form of the data generating process for the discrete fourier transforms (DFT) which is simple and involves no unknown parameters. Moreover, consistency can be obtained under weaker conditions without assuming distributional forms - which is a big advantage in comparison to GPH.

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