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Abstract
Public capital investment plays an important role in long run growth through enhancing productivity and complementing the accumulation of private inputs. Under appropriate conditions, public capital could also have important implications for income distribution dynamics. When the credit market is imperfect and there are diminishing returns to private factors, income inequality is negatively related to economic growth. The dynamics of income distribution is determined by relative income shares of private input, wherever initial endowment differs among individuals. Therefore, if the provision of public capital has an effect on relative income shares of private inputs, then it will have an effect on income distribution dynamics. In this case, public capital once more becomes an important determinant of long-run growth through its indirect effect on income distribution. The paper studies this and other interesting issues with respect to public capital, income inequality and economic growth.
Key words: Income distribution, Public capital, economic Growth
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1 Introduction

Public capital, especially infrastructure, plays an important role in long run growth through enhancing productivity and complementing the accumulation of private inputs. Under appropriate conditions, public capital could also have important implications for income distribution dynamics. The evolution of income distribution is determined by relative income shares of private inputs, wherever initial endowment differs among individuals. Thus, if provision of public capital could have an impact on relative income shares of private inputs, then it would have impact on income distribution dynamics. When the credit market is imperfect (or the input market is missing) and there are diminishing returns to private factors, income inequality is negatively related to economic growth. In this case, public capital once more becomes an important determinant of long-run growth through its indirect effect on income distribution.

The main aim of this paper is to develop a joint theory of income inequality, public capital and economic growth. Thus, a two-sector model, with two public capitals, is developed, which analytically captures the possible interactions among the public inputs (and their associated congestion costs), income inequality dynamics, and growth, within an imperfect credit market scenario and using simple Cobb-Douglas production functions. As a prelude to the actual model, the next few paragraphs lay out the basic idea and intuition behind the new theory in the simplest possible setup.

Assume an economy where agents are heterogenous in terms of their initial wealth (human capital) but similar otherwise. If there is no trade in factor inputs (or, in other words, if access to credit is limited), investment opportunities depend on individuals' initial level of wealth. If production function faces diminishing returns to factor inputs, relatively poor individuals, who have relatively lower investment opportunities, would have high marginal productivity in production. This means initial wealth distribution also determines aggregate output that would be produced in this economy. Therefore, *ceteris paribus,* the more egalitarian (initial) wealth distribution is, the higher the aggregate production would be.

To be clear, consider a simple production function $y = Ah^{\alpha}$, where y and h are output and factor input (say human capital), respectively.[1] When $0 < \alpha < 1$, the production function faces diminishing return to h. To simplify aggregation, assume human capital is distributed log-

[1] In all the text, small and capital letters represent individual and aggregate (average) variables, respectively.

normally, i.e., $\ln h\tilde{\ } N(\mu,\ \sigma^2)$, and *initially differs* among individuals. Aggregate output is thus $Y = E[y] = AE[h^\alpha] = AH^\alpha \exp \frac{\sigma^2}{2}\alpha(\alpha-1)$.[2] Therefore, aggregate output is lower, when production function has a diminishing return ($0 < \alpha < 1$), for a greater income inequality level (denoted by a greater σ^2). In this case, the highest output (AH^α) is achieved by a society which is perfectly egalitarian ($\sigma^2 = 0$).[3] The intuition behind this result is that for a given average initial wealth, higher income inequality means more productive investment opportunities, which offer relatively higher return, would be forgone by poor households. Now, what would happen to income distribution in this economy through time? What determines income distribution dynamics?

Simply, income distribution evolves according to relative *private* factor income shares. When there are differences in initial endowment among households who are similar otherwise, the dynamics of income distributions depends on the degree to which households are able to exploit their relative initial advantages. The existence of any other complementary inputs in production have no effect on income distribution dynamics *unless* they alter relative factor income shares.

For instance, consider an economy similar to as described above, but now production has taken place with two complementary inputs, $y_t = A\,(h_t)^\alpha\,(X_t)^\theta$. X_t can be any complementary input in production (e.g., inelastic labour or public investment) but *similar* among individuals. Then, saving becomes $h_{t+1} = sy_t = sA\,(h_t)^\alpha\,(X_t)^\theta$, where s is exogenous saving rate. Income distribution is given by, a long story cut short, $var(\ln h_{t+1}) = \sigma^2_{t+1} = \alpha^2\sigma^2_t$. Therefore, what matters for income distribution dynamics is *only* the relative factor income share of the private input α.[4]

By the same token, if the provision of public capital as an additional input in production could affect private factor income shares, and there is no reason mentioned in the literature why it should not, then public capital becomes important for income inequality dynamics. This shall be done rather than by considering public capital as a non-rival, non-congested input where its service accrues homogeneously among individual households, but as rival congestible input where its importance varies among households. In this case, in the absence of a perfect credit

[2] See Appendix A for the aggregation.

[3] $Y = AH^\alpha$ is a production function we see in representative agent models. It can also be reached, in equilibrium, in heterogenous agent models, with a perfect credit (or input) market.

[4] Shown with endogenous saving later in the paper.

market, public capital could relax resource constraints of the poor, and brings a disproportional positive impact on the income of the poor, which in turn goes to reducing income inequality.

All these effects can be captured with a simple modification of our modelling of public capital on production function. For instance, once more, assume production takes place using private input h_t and public capital stock X_t. However, assume this time that not the whole public good X_t but a privately equivalent service of it x_t is used in individual households' production function. And, if production is $y_t = A\left(h_t\right)^{\alpha}\left(x_t\right)^{\theta}$ where $x_t \equiv \frac{X_t}{(h_t)^{\zeta}}$, then by substituting the latter on the production function, we obtain $y_t = A\left(h_t\right)^{\alpha-\theta\zeta}\left(X_t\right)^{\theta}$. The income distribution dynamics for this economy then becomes $\sigma_{t+1}^2 = (\alpha-\theta\zeta)^2\sigma_t^2$, which is improved when private factor income share is reduced by $\theta\zeta$. The simple specification $x_t \equiv \frac{X_t}{(h_t)^{\zeta}}$ captures the disproportionate importance of the public good X_t among households. Moreover, while aggregate, it consists of a congestion cost.

The above discussion demonstrates how to extend imperfect credit market theories in inequality and growth to public capital, inequality and growth, using the simplest possible setup. The next section presents the actual model, which, indeed, comprises many other interesting issues, in a more formal way. In the model we thus suppose an economy, populated by heterogenous agents, consists of two production sectors: human capital accumulation and goods production sector. In the former, human capital is generated using inputs from public and private resources while production technology is characterized by inter-generational spillover. Production in the goods sector takes place also using private and public inputs. The public inputs in both sectors are subjected to congestion. Moreover, the benefit accrues from using the public inputs is different among households. That is, relatively higher service from public capital is perceived by the poor than that of the rich households.

Within such setups, and using simple Cobb-Douglas (CD) production functions in both sectors, we show that the provision of public capital not only promotes growth but also improves income distribution dynamics. That is, the dynamics of income inequality not only depends on the magnitude of the elasticity of private inputs but also public inputs. The greater the elasticities of the public factors, the faster income inequality declines. Whereas income inequality is negatively related to economic growth, public capital thus helps to mitigate this effect. On the other hand, congestion costs related to the public factors in both sectors interact with inequality to hamper individual households' capital accumulation and goods production.

With respect to economic growth and public capital, some of the results show that growth is nonlinear with public spending in both the goods production and human capital accumulation sectors. Growth maximizing tax for public capital in human capital accumulation sector is equal to public capital elasticity in that sector times the output elasticity of human capital in the goods production sector, whereas the optimal tax (growth maximizing tax) for public spending in the goods production sector is equal to the public capital elasticity of output in the sector. However, this Barro-type result holds only when there is no inter-generational spillover on human capital accumulation. When inter-generational spillover on human capital accumulation prevails, the optimal tax for the public good in the goods production sector decreases at the elasticity of human capital in the human capital accumulation sector (the spillover parameter).

Moreover, congestion factors in both sectors play an important role in determining optimal taxes in the sectors. For instance, congestion factor in human capital accumulation sector lowers the optimal tax in its sector but raises the optimal tax on the goods production sector. But congestion cost related to the public capital in the goods production sector lowers the optimal tax in the human capital accumulation sector while it has no effect on the optimal tax in its own sector.

The study relates to three main strands of literature, which we extend along various dimensions. First, it relates to the large volume of literature dedicated to studying the relationship between public capital and economic growth. These studies investigate the effect of public capital on economic growth both analytically and empirically: conducted on an analytical level model with public capital as a *flow* or *stock* variable. For instance, Barro (1990), Turnovsky and Fisher (1995), Turnovsky (2000), Agénor (2005), Park (2006) and Park and Philippopoulos (2003) model public capital as a flow of public investment, similar to what we are doing here. By contrast, Futagami, Morita and Shibata (1993), Cassou and Lansing (1998), Rioja (1999), Turnovsky (1997; 2004), and Ziesemer (1990; 1995) among others treat public capital as a stock variable in their model.

This paper contributes to the literature by focusing on the analysis of two public capitals and economic growth in multiple sectors, simultaneously. Modelling two public factors at the same time in multiple sectors is not a bad idea. Public expenditure on primary schooling, basic research and health, which are important for accumulation of individuals' human capital, essentially coexists with other infrastructure services such as roads, airports and energy, which are primarily crucial for the production of firms. However, what is more important is the outcome arising from the interaction of the determinants of the macro-variables within the sectors. Some of these are mentioned above: Congestion cost in a sector determines the magnitude of optimal tax in another sector. Inter-generational spillover in a sector have effect on the level of growth maximizing taxes on another sector. Moreover, inequality interacts with congestion factors to determine individual households' capital accumulation and good production.

With the exception of Agénor (2005; 2008)**?**, the literature has mainly restricted public capital to be an input in only a single sector, either in the goods production or human capital accumulation sector.[5] For instance, Ziesemer (1990; 1995) model public capital as a factor which complements private human capital accumulation, while Barro (1990), Cassou and Lansing (1998), Park (2006) and Park and Philippopoulos (2003) among others use public spending as an input on the final goods sector. Similar to Agénor we model public capital as an input in both the final goods production and human capital accumulation sectors. However, unlike Agénor, we model the public factors in a heterogenous environment, and apply Lucas-type human capital production function, thus we are also able to analyse their implication for income distribution.

Second, the paper is related to literature that studies the relationship between public capital and income inequality. Recently, a growing number of empirical studies try to assess the impact of infrastructure on income inequality. For instance, Calderón and Serven (2004)**?**, Calderón and Chong (2004) and Lopez (2004) find that infrastructure reduces income inequality and enhances economic growth at the same time. World Bank (2003) and Estache (2003) argue infrastructure has a positive *disproportionate* impact on growth. OECD (2006) reports that "infrastructure is important for pro-poor growth".

However, few attempt to study their relationship analytically. Ferreira (1995) analysed it in a model with quite a complex setup where the accessibility of higher production activity requires minimum lumpy investment and hence, if the credit market is missing, depends solely on initial wealth distribution. A steady-state distribution is derived with three social classes: lower class workers, middle-class and upper-class entrepreneurs. The provision of public investment below some level might make the "government dependable" middle-class disappear, the argument goes, creating less equality of opportunity, as well as lower growth. The present paper is the first attempt to analytically capture the relationship between public capital and income inequality using a simple production function such as CD, but not, of course, without a

[5] Although Rioja (2005) also studies two public capitals in education and goods sector, he does not provide an analytical solution. Rather, he calibrates it to the Latin America data to solve it quantitatively.

constraint. The analytical tractability of CD comes with a cost of stringent restriction on relative factor shares, which makes the production function inconvenient for distribution analysis.[6]

The third strand of literature related to the present study deals with the dynamics of income inequality and long-run growth within an imperfect credit market scenario (e.g. Loury (1981), Galor and Zeira (1993), Banerjee and Newman (1993), Piketty (1997), Aghion and Bolton (1997), Aghion and Howit (1998), Aghion, Caroli, and García-Peñalosa (1999), and Benabou (1996; 2000; 2002)).[7] Our study complements their findings. For instance, Benabou (2000; 2002) showed, in his way of studying the effect of redistributive tax on income inequality and growth, that private factor income shares and family wealth determine income distribution dynamics and growth. We show here that public capital could also be an important determinant of income inequality dynamics.

The remainder of the paper is organized as follows. Section 2 provides the model. Section 3 is all about income distribution and public capital. Various macroeconomic aggregates that arise in the model and their dynamic behaviours are studied in section 4. Section 5 concludes.

2 The Model

2.1 Households and firms

There is a continuum of heterogenous households, $i \in [0,1]$. Each household i consists of an adult of generation t and a child of generation $t+1$. At the beginning, each adult of the initial generation is endowed with human capital h_0 and a public infrastructure G_0 which is shared among others. The distribution of income is assumed to take, initially, a known probability distribution of $\Gamma_0(.)$. Thus, the initial distribution is given and evolves over time at equilibrium.

Agents care about their consumption level and the human capital stock of their children. When young, they accumulate human capital using both private and public input. When adult, they use their human capital for final goods production. Government tax income with two fixed flat rate taxes, ψ and τ, in order to finance the public inputs, denoted by G_t and M_t, in the goods production and human capital accumulation sectors, respectively. Individuals allocate after tax income between current consumption c_t and children education e_t, while the

[6] In CD, relative factor income shares are fixed (constant) due to the constancy of elasticity of substitutions, which is, indeed, equal to one.

[7] In contrast to the first and the second strand of literatures, this literature does not focus on public capital.

latter represents private investment for human capital accumulation of the offspring. Preferences are logarithmic. Production functions are Cobb-Douglas.

A utility of an individual is thus defined as

$$\ln c_t + \beta \ln h_{t+1} \tag{1}$$

subject to

$$c_t + e_t = (1 - \tau - \psi) y_t \tag{2}$$

where y_t is income of an individual.

Human capital accumulation function for the offspring h_{t+1} is a function of parental human capital h_t, private educational investment e_t, and public service on the sector m_t,

$$h_{t+1} = B\left(h_t\right)^{\varepsilon}\left(m_t\right)^{\upsilon}\left(e_t\right)^{\eta} \tag{3}$$

The public service m_t represents a privatly-equivalent public input. We assume m_t to be proportional to the total public expenditure M_t on the sector, but inversely proportional to the individual's initial wealth (human capital) h_t. That is,

$$m_t = \frac{M_t}{\left(h_t\right)^{\zeta}} \tag{4}$$

where ζ represents the degree of disproportionate impact of public capital on an individuals' human capital accumulation. The case $\zeta = 0$ corresponds to an infrastructure service which is equally important (available) to each individual. But, the case $\zeta > 0$ implies a relatively higher service from public capital is perceived by poor than that of rich households during the human capital accumulation process. The latter is supported by a number of research documentation.

For instance, Ferreira (1995) argues infrastructure services could be more important for the poor due to their lack of ability to purchase their private substitute. Particularly when the credit market is imperfect, his argument goes, the reduction in public capital increases income inequality because this hurts the poor who are more dependent on the public provision of public capital than the rich, who have alternative private substitutes. As documented by Brenneman and Kerf (2002), many microeconomic studies also support this.[8]

[8] For instance, Leipziger, Fay and Yepes (2003) find an increase in water and sanitation reduces child mortality significantly. Jacoby (2000) argues that improvement in communication and road services could result in substantial benefits on average, much of it going to the poor.

Interesting enough, while aggregating, equation (4) captures *congestion cost*. In this case, ζ is called congestion factor, where the benchmark $\zeta = 0$ is a case of public capital with no congestion cost.[9] However, notice that in (4) public capital would be congested less than it is normally in representative agent models in the literature since

$$\int_0^1 \frac{M_t}{(h_t)^\zeta} d\Gamma_t(h_t) = \frac{M_t}{(H_t)^\zeta} \exp \frac{\sigma_t^2}{2} \left(\zeta^2 + \zeta\right) > \frac{M_t}{(H_t)^\zeta}$$

(See Appendix A for the aggregation).

In order to be in line with the literature, and make comparison possible, we prefer to avoid this effect. We can thus make congestion heterogeneity neutral by replacing (4) with

$$m_t = \frac{M_t}{(h_t)^\zeta \exp \frac{\sigma_t^2}{2} \left(\zeta^2 + \zeta\right)} = \frac{M_t}{(H_t)^\zeta} \tag{5}$$

Combining (3) and (5), the human capital accumulation function becomes

$$h_{t+1} = B\,(h_t)^\xi\,(M_t)^\upsilon\,(e_t)^\eta \exp\left(-\frac{\sigma_t^2}{2}\upsilon\left(\overline{\zeta}\right)\right) \tag{6}$$

where $0 < \varepsilon, \eta, \upsilon, \zeta < 1$; $0 \leq \xi \equiv (\varepsilon - \upsilon\zeta) < 1$ and $\overline{\zeta} \equiv \zeta^2 + \zeta$.

The last term in (6) arises due to the *simultaneous* effect of disaggregation and congestion and, it is the same for all individuals. It does not exist either in representative agent economy $\sigma_t^2 = 0$ or heterogenous economy without congestion cost $\zeta = 0$. In fact, it explicitly captures the negative spillover of the inequality-congestion interaction on individual household human capital accumulation.

2.2 The Firm

We assume each household owns a firm.[10] Aggregate output is thus the sum of individuals' production. We also assume individuals differ only in their initial human capital while human capital is lognormally distributed across agents: $\ln h_t \tilde{} N(\mu_t, \sigma_t^2)$.

Thus, the income of an agent of generation t is

[9] Traditionally, congestion is modelled in the literature as $M_t^c = \frac{M_t}{(H_t)^\zeta}$, where ζ represents the degree of congestion; H_t and M_t is aggregate private capital and public expenditure, respectively; and hence M_t^c represents public expenditure with congestion cost. See also Glomm and Ravikumar (1994).

[10] This assumption shuts off the input market, or it is another way of assuming credit market is imperfect.

$$y_t = A\left(h_t\right)^{\alpha}\left(g_t\right)^{\theta} \tag{7}$$

Once more we define the public service g_t, in the goods production sector, similar to (5)

$$g_t = \frac{G_t}{\left(h_t\right)^{\kappa}\exp\left(\frac{\sigma_t^2}{2}\theta\left(\kappa^2+\kappa\right)\right)} \tag{8}$$

Thus, g_t and κ, in the goods production sector, are counterparts of m_t and ζ, in the human capital accumulation sector, respectively; while (8) is a counterpart equation of (5), in the human capital accumulation sector. Therefore, g_t represents infrastructure service that is passed to individual households, and depends on infrastructure stock G_t in the goods production sector and initial human capital of the agent.

Combining (7) and (8), the production function for the goods production becomes

$$y_t = A\left(h_t\right)^{\omega}\left(G_t\right)^{\theta}\exp\left(-\frac{\sigma_t^2}{2}\theta\overline{\kappa}\right) \tag{9}$$

where $0 < \alpha, \theta, \kappa, \omega < 1$; $\omega \equiv (\alpha - \theta\kappa)$; and $\overline{\kappa} \equiv \kappa^2 + \kappa$.[11] The last term in (9) has the same interpretation with that of the last term in (6). Therefore, early on, from (6) and (9), the following proposition can be established:

Proposition 1 *Income inequality bears additional cost to household human capital accumulation and goods production when coupled with congestion.*

While aggregate, production Y_t is

$$Y_t = A\left(H_t\right)^{\omega}\left(G_t\right)^{\theta}\exp\left(\frac{\sigma_t^2}{2}\left(\omega(\omega-1)-\theta\overline{\kappa}\right)\right) \tag{10}$$

since $E\left[\left(h_t\right)^{\omega}\right] = \left(H_t\right)^{\omega}\exp\frac{\sigma_t^2}{2}\omega\left(\omega-1\right)$ (see appendix A); H_t is the aggregate (average) human capital.

According to (10), aggregate income is smaller in heterogenous economies than representative ones $\sigma_t^2 = 0$.

[11] The case $\alpha = \theta\kappa$ is not included since it is unlikely that goods would be produced using only public capital. As a matter of fact, roads do not produce by themselves.

2.3 Government

We assume government budget is at all times balanced:

$$I_t^g \equiv \int_0^1 y_t \psi d\Gamma_t(h_t) = Y_t \psi \tag{11}$$

$$M_t \equiv \int_0^1 y_t \tau d\Gamma_t(h_t) = Y_t \tau \tag{12}$$

Thus, government collects proportional taxes τ and ψ on output Y_t , to finance public expenditure I_t^g and M_t in the goods production and human capital accumulation sectors, respectively, while the accumulation of the public capital in the goods production sector follows the rule

$$G_{t+1} = I_t^g + G_t(1 - \delta^g) \tag{13}$$

where G_t and δ^g denote the public capital stock and depreciation in the goods production sector, respectively.

2.4 Competitive Equilibrium

According to the above descriptions, an individual of period t solves the following problem, which is derived by substituting (2) and (6) into (1),

$$\underset{e_t}{Max} \ln\left((1 - \tau - \psi)\, y_t - e_t\right) + \beta \ln B\,(h_t)^{\xi}\,(M_t)^{\upsilon}\,(e_t)^{\eta} \exp\left(-\frac{\sigma_t^2}{2}\upsilon\left(\overline{\zeta}\right)\right) \tag{14}$$

taking as given, τ, ψ, M_t, I_t^g and G_t.

The first order condition gives

$$e_t = a(1 - \tau - \psi) y_t \tag{15}$$

where $a = \frac{\beta\eta}{1+\beta\eta}$; (15) shows the agent's optimal saving as the function of her income. Notice that the saving rate is identical among individuals, due to logarithmic preferences, although rate of return on investment is different.[12]

To derive the individuals' human capital accumulation equation, which is associated to their optimal behavior, substitute (15) and (12) into (6), then use (9), to get

[12] In logarithmic utility function, inter-temporal elasticity of substitution is one, and consequently income effect exactly compensates substitution effect (See De La Croix and Michel 2002: pp. 13-14). In this case, individuals' saving rate is independent of the rate of return.

$$h_{t+1} = B\left(h_t\right)^{\xi}\left(Y_t\tau\right)^{\upsilon}\left(a(1-\tau-\psi)A\left(h_t\right)^{\omega}\left(G_t\right)^{\theta}\right)^{\eta}\exp\left(-\frac{\sigma_t^2}{2}\left(\theta\eta\overline{\kappa}+\upsilon\overline{\zeta}\right)\right)$$

Then, by substituting (10) into the above equation, we get the following difference equation,

$$\begin{aligned} h_{t+1} = {} & B\left(h_t\right)^{\xi}\left(A\left(H_t\right)^{\omega}\left(G_t\right)^{\theta}\tau\right)^{\upsilon}\left(a(1-\tau-\psi)A\left(h_t\right)^{\omega}\left(G_t\right)^{\theta}\right)^{\eta} \\ & \exp\left(\frac{\sigma_t^2}{2}\left(-\left(\theta\eta\overline{\kappa}+\upsilon\overline{\zeta}\right)+\upsilon\omega(\omega-1)-\upsilon\theta\overline{\kappa}\right)\right) \end{aligned}$$

$$\begin{aligned} = {} & BA^{\upsilon+\eta}\tau^{\upsilon}\left(h_t\right)^{\xi+\omega\eta}\left(G_t\right)^{\theta(\upsilon+\eta)}\left(H_t\right)^{\upsilon\omega} \\ & \left(a(1-\tau-\psi)\right)^{\eta}\exp\left(\frac{\sigma_t^2}{2}\left(\upsilon\omega(\omega-1)-\theta\overline{\kappa}(\eta+\upsilon)-\upsilon\overline{\zeta}\right)\right) \end{aligned} \quad (16)$$

According to (16), an individual's human capital accumulation is determined by human capital of his parent h_t, initial income distribution σ_t^2, aggregate public and private capital stock, H_t and G_t, in the economy, respectively. The negative effect of income inequality in individual household human capital production could not be a surprise. In fact, in the model, household human capital accumulation is a function of the provision of public capital M_t, which depends on the level of aggregate income Y_t. But Y_t has, in turn, a negative relationship with income inequality σ_t^2 due to credit market imperfection and the existence of diminishing returns to factors.

3 Income Distribution and Public Capital

From (16), we derive the following two difference equations, which characterize the evolution of capital accumulation and income distribution in the economy[13]

$$\begin{aligned} \mu_{t+1} \equiv {} & E\left[\ln h_{t+1}\right] = (\xi+\omega(\eta+\upsilon))\mu_t + (\upsilon+\eta)\ln A \\ & +\ln B + \theta(\upsilon+\eta)\ln G_t + \upsilon\ln\tau + \eta\ln a(1-\tau-\psi) \\ & +\frac{\sigma_t^2}{2}\left(\upsilon\omega^2-\theta\overline{\kappa}(\eta+\upsilon)-\upsilon\overline{\zeta}\right) \end{aligned} \quad (16')$$

$$\sigma_{t+1}^2 \equiv var\left[\ln h_{t+1}\right] = \left(\xi+\omega\eta\right)^2\sigma_t^2 \equiv \left(\varepsilon-\upsilon\zeta+(\alpha-\theta\kappa)\eta\right)^2\sigma_t^2 \quad (17)$$

[13]We use the fact that $E\left[\ln h_t\right] = \ln H_t - \frac{\sigma_t^2}{2}$ in deriving (16') (see Appendix A).

Since the distribution $\Gamma_t(h_t)$ is lognormal, and hence its log distribution is normal, the log distribution of $\Gamma_{t+1}(h_{t+1})$ is normal.

Equation (17) has a solution, $\sigma_t^2 = (\xi + \eta\omega)^{2t} \sigma_0^2$. Thus, steady state income distribution σ^2 takes a value of the initial distribution σ_0^2, 0 or ∞, depending on some conditions,

$$\sigma^2 = 0 \text{ if } \xi + \eta\omega < 1 \tag{18}$$

$$\sigma^2 = \sigma_0^2 \text{ if } \xi + \eta\omega = 1 \tag{19}$$

$$\sigma^2 \to \infty \text{ if } \xi + \eta\omega > 1 \tag{20}$$

Therefore, income inequality will decline through time and ultimately vanish for certain values on the parameters, $\xi + \omega\eta < 1$. The reason for the vanishing is that the heterogeneity, in this model, is only on individuals' initial wealth; agents are similar otherwise, in their ability, technology, etc. Thus, a diminishing return on net private accumulative factors, $\xi + \omega\eta < 1$, implies resource poor households are more productive than rich ones; consequently, it is inevitable for the poor to catch up with the rich in the long run.[14] Therefore, the model captures the possible role of public capital, and also family wealth on income inequality dynamics, in the short run. Particularly, (16') and (17) capture the intuition that differences in family wealth and the existence of public capital as an input for the production of goods and accumulation of human capital play important (but opposite) role in the persistence of income inequality. That is,

Proposition 2 *The existence of public factors in the goods production and human capital function, as rival congestible inputs, with a disproportionate impact on individual households, speeds up income distribution convergence, in the short run.*

Family wealth, similar to what is found by Benabou (2000; 2002), exasperates income inequality. More important is the parent's wealth, i.e., the larger is ε, for the accumulation of the offspring's human capital, the more income inequality persists. But, more important are the public capitals for the accumulation of human capital and production of goods, that is, the greater are θ and υ, also the larger their disproportionate impact on income, i.e., the greater are ζ and κ, the faster income inequality declines.

[14] See also Saint-Paul, G., and T. Verdier (1993) for a model where inequality dynamics is speeded up through public intervention.

How large is the magnitude of the elasticity of output of public capital is rather an empirical question. In the past, there were a number of instances where economists had estimated a significant large impact of public capital on output. For instance, Aschauer (1989; 2000) estimated a public capital output elasticity of 0.39 for the United States and about 0.3 for a set of developing countries. Kocherlakota and Yi (1996) also estimated with a marginal product of public capital that is well in excess of private capital. Although some economists think the estimates are too large to believe, there is more consensus recently among researchers on the significant and positive impact of public capital on output.[15]

To sum up, we have shown here, as we argued in the introductory section, that the dynamics of income distribution is governed by relative *private* input income shares (ε, η, and α). However, under certain conditions, provision of public capitals in the goods production and human capital accumulation sectors would have a positive role in income distribution dynamics, by altering relative private factor income shares. The impact of public capital on income inequality dynamics depends on the degree of its importance to private production, which is reflected on the magnitude of θ and υ. Moreover, whether provision of public capital is targeted towards the poor, which is also reflected on the magnitude of the parameters ζ and κ, is important to income distribution dynamics.

4 Growth, Inequality and Public Factors

4.1 Aggregate Capitals

To determine the remaining macro-variables, first, aggregate (16) in order to obtain the equation that characterizes the evolution of aggregate human capital,

$$\begin{gathered} H_{t+1} = BA^{\upsilon+\eta}\tau^{\upsilon}\left(G_t\right)^{\theta(\upsilon+\eta)}\left(H_t\right)^{\xi+(\eta+\upsilon)\omega}\left(a(1-\tau-\psi)\right)^{\eta} \\ \exp\left(\frac{\sigma_t^2}{2}\begin{pmatrix}(\xi+\omega\eta)(\xi+\omega\eta-1)-\upsilon\overline{\zeta} \\ +\upsilon\omega(\omega-1)-(\theta\overline{\kappa}(\eta+\upsilon))\end{pmatrix}\right) \end{gathered}$$

We use the fact that $E\left[\left(h_t\right)^{\xi+\omega\eta}\right] = \left(H_t\right)^{\xi+\omega\eta}\exp\left(\frac{\sigma_t^2}{2}(\xi+\omega\eta)(\xi+\omega\eta-1)\right)$ (see appendix A). By assuming constant returns to scale with respect to accumulative factors, in both human capital accumulation and the goods production sectors, i.e., $\eta+\upsilon=1$ and $\omega+\theta=1$, respectively, we can rewrite the above equation as

[15] See Gramlich (1994), Sturm et al. (1998) and Romp and de Haan (2005; 2007)**?** for a detailed survey of this literature.

$$H_{t+1} = BA\tau^{\upsilon} \left(G_t\right)^{\theta} \left(H_t\right)^{\xi+\omega} \left(a(1-\tau-\psi)\right)^{\eta} \Omega_t \tag{21}$$

where

$$\Omega_t = \exp\left(\frac{\sigma_t^2}{2}\left((\xi+\omega\eta)(\xi+\omega\eta-1)+\upsilon\omega(\omega-1)-\left(\theta\overline{\kappa}+\upsilon\overline{\zeta}\right)\right)\right)$$

Then, we easily derive the dynamic equations for the public capital in the goods production sector by substituting (11) into (13), using (10), and assuming a complete depreciation ($\delta^g = 1$)

$$G_{t+1} = \psi A \left(H_t\right)^{\omega} \left(G_t\right)^{\theta} \varGamma_t \tag{22}$$

where

$$\varGamma_t = \exp\left(\frac{\sigma_t^2}{2}\left(\omega(\omega-1)-\theta\overline{\kappa}\right)\right)$$

Equations (17), (21), and (22) describe the dynamics of the economy. Therefore, the growth rate of the economy is determined by these equations. Early on, the negative effect of income inequality on growth can be deduced from the relationship between income inequality and capital accumulation. Equations (21) and (22) state income inequality is detrimental for accumulation of public and private capital. In (21) and (22), the terms $\varGamma_t$, $\Omega_t \leq 1$.[16] The maximum values $\varGamma_t$, $\Omega_t = 1$ are reached when $\sigma_t^2 = 0$. Therefore, *cetris paribus*, the highest capital accumulation is realized when the society is perfectly egalitarian $\sigma_t^2 = 0$.

4.2 Dynamics and Steady State

The value that $\xi\,(\equiv \varepsilon - \upsilon\zeta)$ assumes is important in determining the long run behaviour of the system. First of all, recall that ε, the effect of family wealth on human capital accumulation of the offspring, and $\upsilon\zeta$, the disproportionate effect of the public capital on individual income, have opposite roles on income distribution dynamics. The former (the greater) makes income inequality persistent whereas the later (the greater) reduces it through time.[17]

When $\xi = 0$, i.e., the effect of family wealth on human capital accumulation of the offspring is equal to the disproportionate effect of the

[16]The proof is simple. First, recall the assumptions on the parameters (Section 2.1). Then, it is sufficient to assume a diminishing return on net private accumulative factors $(\xi + \omega\eta) < 1$, for the expressions inside the two large parenthesis in $\varGamma_t$ and Ω_t to be negative, $\frac{\sigma_t^2}{2}\,(.) < 0$.

[17]See (17) and the subsequent discussion in Section 3.

public capital on income distribution in the sector, the system behaves in the long run similar to a standard AK model. In steady state, the ratio $\frac{H}{G}$ is constant. That is, dividing (21) by (22), when $\sigma^2 = 0$,

$$\frac{H}{G} = Ba^{\eta}\tau^{\upsilon}\psi^{-1}(1-\tau-\psi)^{\eta}$$

Therefore, at equilibrium $\sigma^2 = 0$, the system is characterized by a continuum of steady state equilibria while each can be reached only if the system starts at equilibrium. Moreover, aggregate variables will be in a balanced growth path, where H, G and Y grow at the same rate.[18] And, the growth rate of the economy can be analytically determined at any point in time. However, in the short run, it exhibits transitional dynamics, unlike the textbook AK model, which arises from the existence of income inequality dynamics $\sigma_0^2 \neq 0$ in the model.

When $\xi > 0$, there exists a stable and unique global steady state where H, G and Y converge, where the steady state is saddle point stable. However, aggregate variables exhibit imbalance growth. While human capital grows faster than output, public capital grows at the same rate with the latter.

Propositions 3 and 5 below are related to the two different cases, $\xi = 0$ and $\xi > 0$, respectively.[19] But, first, in order to characterize the dynamics, we need to log-linearize the system, (21) and (22), near a local steady state point (H, G). That is,

$$\begin{aligned}(\ln H_{t+1} - \ln H) = {} & (\xi + \omega)(\ln H_t - \ln H) + \theta(\ln G_t - \ln G) \\ & +(\ln \sigma_t^2 - \ln \sigma^2)\ln \Omega \end{aligned} \tag{23}$$

$$\begin{aligned}(\ln G_{t+1} - \ln G) = {} & \omega(\ln H_t - \ln H) + \theta(\ln G_t - \ln G) \\ & + \left(\ln \sigma_t^2 - \ln \sigma^2\right)\ln F \end{aligned} \tag{24}$$

If we consider only equilibrium values of the income distribution which only exist, $\sigma^2 = \sigma_0^2$ or $\sigma^2 = 0$ (see (18)-(20)), then we will have $(\ln \sigma_t^2 - \ln \sigma^2)\,\sigma^2 = 0$. Thus, (23) and (24) will be simplified to

$$(\ln H_{t+1} - \ln H) = (\xi + \omega)(\ln H_t - \ln H) + \theta(\ln G_t - \ln G) \tag{25}$$
$$(\ln G_{t+1} - \ln G) = \omega(\ln H_t - \ln H) + \theta(\ln G_t - \ln G) \tag{26}$$

In matrix form, (25) and (26) become

[18] The variables without time subscript (H, G, Y and σ^2) denote steady state values.

[19] We exclude the case $\xi < 0$ because of its unlikeliness.

$$\begin{bmatrix} \ln H_{t+1} - \ln H \\ \ln G_{t+1} - \ln G \end{bmatrix} = \begin{bmatrix} (\xi + \omega) & \theta \\ \omega & \theta \end{bmatrix} \begin{bmatrix} \ln H_t - \ln H \\ \ln G_t - \ln G \end{bmatrix} \tag{27}$$

$$A = \begin{bmatrix} (\xi + \omega) & \theta \\ \omega & \theta \end{bmatrix} \tag{28}$$

where A is the Jacobian matrix.

In regard to the dynamic behaviour of the economy, for the case $\xi = 0$, we make the following proposition:

Proposition 3 *For $\xi = 0$ (or $\varepsilon = \upsilon\zeta$), the system is non-hyperbolic, i.e., one of the characteristic roots is a unit.*

Proof. The characteristic polynomial $P(\lambda)$ for the linear system is given by (25) and (26), recall our earlier assumption $\omega + \theta = 1$,

$$\begin{aligned} P(\lambda) &= \lambda^2 - Tr(A)\lambda + Det(A) \\ &= \lambda^2 - (\xi + 1)\lambda + \xi\theta \end{aligned} \tag{29}$$

Since $\xi = 0$, then $\lambda^2 - \lambda = 0$ and hence $\lambda_1 = 1$. ■

In this case, we can analytically derive the (AK type) growth rate γ_t at any point in time (see Appendix B for the derivation),

$$\gamma_{t+1} = \omega \ln \chi + \omega\eta \ln (1 - \tau - \psi) + \omega\upsilon \ln \tau + \theta \ln \psi + \Phi_t + \Delta_t \tag{30}$$

where

$$\Phi_t = -\omega \frac{\sigma_t^2}{2} \left(\upsilon\overline{\zeta} + \eta^2 \theta \overline{\kappa} \omega \right) < 0$$

$$\Delta_t = \omega^3 \frac{\sigma_t^2}{2} \eta(\omega\eta - 1) < 0$$

$$\chi = B a^\eta A^{\frac{1}{\omega}}$$

According to (30), both taxes τ and ψ and income distribution variable σ_t^2 are important for the economy's growth rate γ_t. The terms Δ_t and Φ_t capture the extent to which income inequality hampers economic growth during transition. While Φ_t represents the negative effect of inequality-congestion interaction on economic growth,[20] Δ_t captures the pure effect of income inequality on a heterogenous economy with a production function that exhibits diminishing returns to factors, and

[20] See proposition 2, and equations (6) and (9).

where there exists imperfect credit market. Whereas, the public capital is shown here mitigating this effect.[21]

Notice, if $\Delta_t = 0$ and $\Phi_t = 0$, i.e., $\sigma_t^2 = 0$, then growth rate of output $\gamma > 0$. But, for greater Δ_t and Φ_t (due to greater σ_t^2), the growth rate of output could be zero and even negative.

The relationship between the taxes used to finance the public capitals and long run growth is non-linear, in line with the literature. The growth maximizing taxes ($\psi_{g\max}$ and $\tau_{g\max}$), for the case $\xi = 0$, are derived by

$$\begin{aligned}
\frac{\partial \gamma}{\partial \psi} &= \frac{\theta}{\psi} - \frac{\eta(1-\theta)}{1-\tau-\psi} = 0 \\
&\Rightarrow \psi = \frac{\theta(1-\tau)}{\theta + \eta(1-\theta)} \\
\frac{\partial \gamma}{\partial \tau} &= \frac{\upsilon(1-\theta)}{\tau} - \frac{\eta(1-\theta)}{1-\tau-\psi} = 0 \\
&\Rightarrow \tau = (1-\psi)\upsilon
\end{aligned}$$

Recall the assumption on constant returns to scale on accumulative factors $\eta + \upsilon = 1$ and $\omega + \theta = 1$. Combining the above two, we obtain

$$\psi_{g\max} = \theta \tag{31}$$

$$\tau_{g\max} = \upsilon\omega (\equiv \upsilon(\alpha - \theta\kappa)) \tag{32}$$

The optimal tax for public capital in the goods production sector $\psi_{g\max}$ is equal to the share of public capital in the sector, similar to that found by Barro (1990) and others, while the growth maximizing tax for public capital in the human capital accumulation sector $\tau_{g\max}$ is equal to the share of the public capital in that sector times the net output elasticity of human capital. From the latter and (32), we have the following proposition:

Proposition 4 *Congestion cost κ related to public capital in the goods production sector lowers growth maximizing tax $\tau_{g\max}$ for the public capital in the human capital accumulation sector, through decreasing the human capital elasticity of output.*

The tax used to finance the public factor in human capital accumulation sector τ affects growth via its positive role in the accumulation of human capital, which in turn will be used for output production. However, human capital, in the model, has the side effect of congesting the

[21] Recall that $\omega = 1 - \theta$, where θ is the elasticity of output of the public capital.

public capital in the goods production sector. This leads to a negative relationship between the congestion factor on infrastructure in the goods production sector κ and its growth maximizing tax in the sector $\tau_{g\max}$.

With regard to the dynamic behaviour of the economy for the case $\xi > 0$, the following proposition is established:

Proposition 5 *For $\xi > 0$, the characteristic polynomial of the log-linearized system admits two positive roots, where only one root is stable. Given G_0 and H_0, there exists a unique solution to (25) and (26), which converges to (H, G). The path is monotonic and the steady state is saddle point stable.*

Proof. The characteristic polynomial for the Jacobian matrix of the linear system is given by (29)

$$P(\lambda) = \lambda^2 - (\xi + 1)\lambda + \xi\theta$$

Generally, when $|1 + Det(A)| < |Tr(A)|$, the steady state equilibrium is a saddle; there is one and only one real root which belongs to $(-1, 1)$.[22] But, we have $|1 + \xi\theta| < |1 + \xi|$. Moreover, since the product of the roots, which is equal to the determinant of the Jacobian matrix ($\lambda_1\lambda_2 = \xi\theta$), is positive, both roots have the same sign and hence positive. Therefore, the characteristic roots are positive, real and only one root is within a unit circle, $(0 < \lambda_1 < 1)$. Thus, given G_0 and H_0 (since both G_t and H_t are predetermined variables), the trajectory of the dynamic system is uniquely, locally, determined. The global analysis is established below using phase diagrams (Figures 1 & 2). ■

The graphical analysis can be done near the set of points where $\sigma_t^2 = 0$, for $\xi \neq 0$. Thus, from (21) and (22) we have

$$H_{t+1} = BA\,(H_t)^{\xi+\omega}\,(G_t)^{\theta}\,(a(1 - \tau - \psi))^{\eta}\,\tau^{\upsilon} \tag{33}$$
$$G_{t+1} = \psi A\,(H_t)^{\omega}\,(G_t)^{\theta} \tag{34}$$

To build the phase diagram, first we need to characterize the set of points where there is no change on the variables, for (33) and (34). That is, for (33) we solve $H_{t+1} = H_t$ for G_t and for (34) we solve $G_{t+1} = G_t$ for H_t, to get

$$G_t = \left(BA\,(a(1 - \tau - \psi))^{\eta}\,\tau^{\upsilon}\right)^{\frac{1}{\theta}}\,(H_t)^{\frac{\theta-\xi}{\theta}} \tag{35}$$
$$H_t = (\psi A)^{\frac{-1}{\omega}}\,(G_t)^{\frac{1-\theta}{\omega}} = (\psi A)^{\frac{-1}{\omega}}\,G_t \tag{36}$$

[22] See De la Croix and Michel (2002), A.3.4.

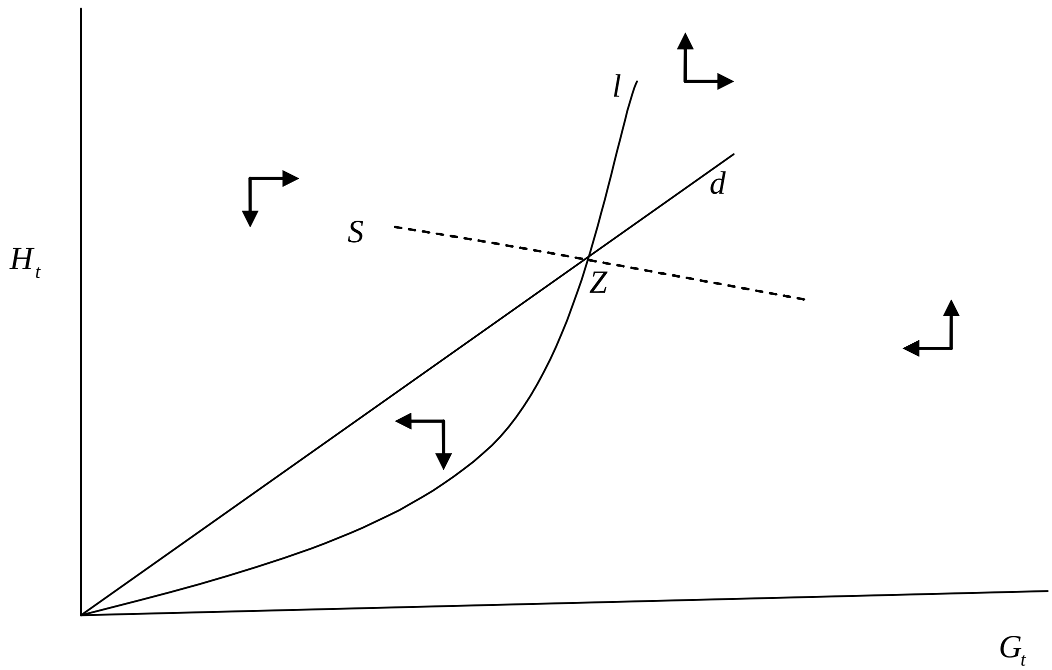

Figure 1: Phase diagram for a case $\theta > \xi$ or $0 < \frac{\theta-\xi}{\theta} < 1$. Together with the eigenvalues the steady state is globally stable.

The slope of the phase line (35) depends on the relative values of θ and ξ. If $\theta > \xi$, then $0 < \frac{\theta-\xi}{\theta} < 1$ and hence G_t is increasing at a decreasing rate in H_t, in space (H_t, G_t) (Figure 1). If $\theta < \xi$, then $\frac{\theta-\xi}{\theta} < 0$, and G_t is decreasing at an increasing rate in H_t (Figure 2). The curve (36) is easy to characterize. The phase line is a diagonal line, with slope $(\psi A)^{-\frac{1}{\omega}}$.

By combining (35) into (36), we obtain the equilibrium values where the two phase lines meet

$$G = \left(BA\left(a(1-\tau-\psi)\right)^{\eta}\tau^{\upsilon}\right)^{\frac{1}{\xi}}(\psi A)^{\frac{\xi-\theta}{\omega\xi}} \tag{37}$$

$$H = \left(BA\left(a(1-\tau-\psi)\right)^{\eta}\tau^{\upsilon}\right)^{\frac{1}{\xi}}(\psi A)^{\frac{-\theta}{\omega\xi}} \tag{38}$$

Figures 1 and 2 capture the qualitative feature of the model. Notice that although the slopes of the phase lines for H_t are different for the two cases, ($\theta < \xi$ and $\theta > \xi$), the steady state equilibrium loci Z remain the same. Moreover, the saddle path has a negative slope.

Once again by using the log-linearized system, we can characterize

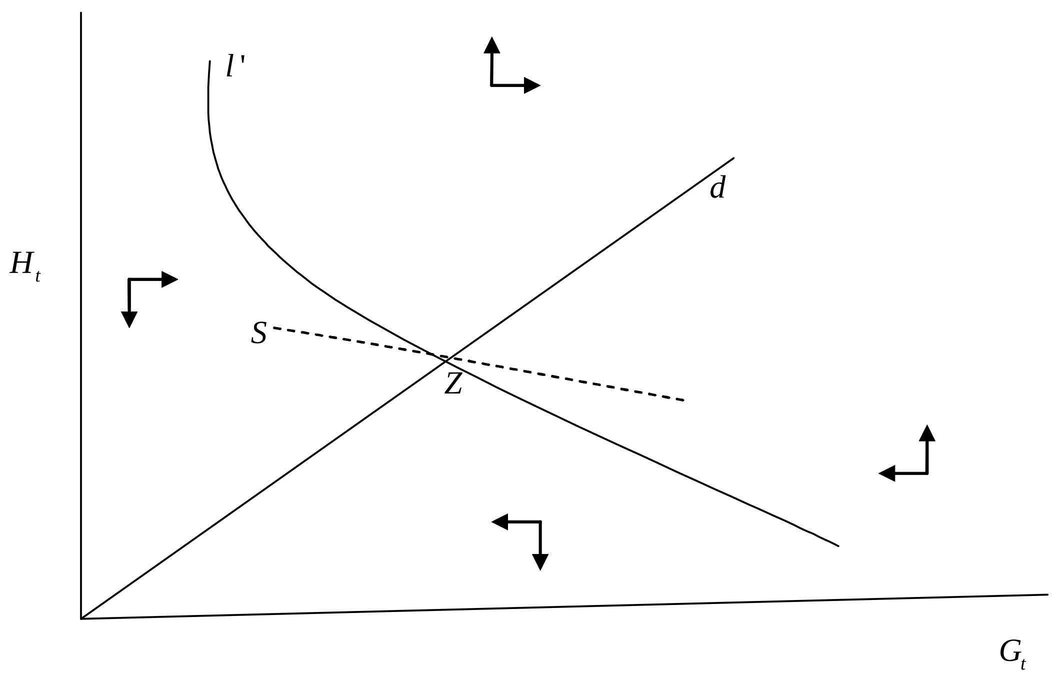

Figure 2: Phase diagram for a case $\theta < \xi$ or $\frac{\theta-\xi}{\theta} < 0$. The result is the same with that of Fig. 1 except here the phase line for equation (35) is downward slopping.

the growth maximizing tax rates ($\psi^*_{g\max}$ and $\tau^*_{g\max}$), for the case $\xi > 0$, near the steady state. The economy's growth rate near the steady state is defined, $\gamma = \ln Y_t - \ln Y$,

$$\gamma = \omega \left(\ln H_t - \ln H\right) + \theta \left(\ln G_t - \ln G\right) \tag{39}$$

Combining (26) and (39), we obtain

$$\gamma = \ln G_{t+1} - \ln G \tag{39'}$$

Alternatively, from (25) and (39), we can get

$$\ln H_{t+1} - \ln H = \xi \left(\ln H_t - \ln H\right) + \gamma \tag{39''}$$

From (39') and (39"), we see that there is an imbalance in the growth rate of macro-variables when $\xi > 0$. While output Y_t grows at the same rate with the public capital G_t, human capital H_t grows faster. To derive the optimal taxes, we substitute (34) and (37) in (39'), to obtain,

$$\gamma = \ln \psi A \left(H_t\right)^{\omega} \left(G_t\right)^{\theta} - \ln \left(BA \left(a(1-\tau-\psi)\right)^{\eta} \tau^{\upsilon}\right)^{\frac{1}{\xi}} \left(\psi A\right)^{\frac{\xi-\theta}{\omega\xi}}$$

By leaving out the superfluous variables and parameters, we can rewrite the last equation in an equivalent form

$$\max_{\psi,\tau} \gamma = \frac{\theta(1-\xi)}{\omega\xi} \ln \psi - \frac{\eta}{\xi} \ln(1-\tau-\psi) - \frac{\upsilon}{\xi} \ln \tau$$

Thus, the FOC is

$$\frac{\partial \gamma}{\partial \psi} = \frac{\theta(1-\xi)}{\omega\xi}\frac{1}{\psi} + \frac{\eta}{(1-\tau-\psi)\xi} = 0$$

Then, solving for ψ, we get

$$\psi = \frac{(1-\tau)\theta(1-\xi)}{\eta\omega + \theta(1-\xi)} \tag{40}$$

We do the same for the optimal tax in the human capital accumulation sector

$$\frac{\partial \gamma}{\partial \tau} = \frac{\eta}{(1-\tau-\psi)\xi} - \frac{\upsilon}{\xi\tau} = 0$$

$$\tau = (1-\psi)\upsilon \tag{41}$$

Solving (40) and (41) simultaneously we obtain the growth maximizing tax rate for the public capitals in the goods production sector, for the case $\xi > 0$,

$$\psi^*_{g\max} = \frac{\theta(1-\xi)}{1-\theta\xi} \quad (42)$$

and human capital accumulation sector

$$\tau^*_{g\max} = \frac{\omega\upsilon}{1-\xi(1-\omega)} \quad (43)$$

Thus, from (42) and (43), we make our last proposition:

Proposition 6 *First, when $\xi = 0$, equations (42) and (43) are equivalent to equation (31) and (32). Second, proposition 6 holds for both cases $\xi = 0$ and $\xi > 0$. Third, while $\tau^*_{g\max}$ is increasing at $\xi(\equiv \varepsilon - \upsilon\zeta)$, $\psi^*_{g\max}$ is decreasing at it. Therefore, the congestion factor ζ, of the public good in human accumulation sector, lowers the optimal tax on the sector but raises the optimal tax in the goods production sector.*

The existence of inter-generational spillover in the human capital accumulation sector, i.e., $\xi > 0$, increases the role of human capital in the economy. This is reflected by a positive relation between $\tau^*_{g\max}$ and ξ. On the other hand $\psi^*_{g\max}$ and ξ are inversely related when both have a similar role in the economy. According to Barro and Sala-I-Martin (1992)**?**, the tax rate ψ^* raises growth since the social rate of return on investment exceeds the private return, which reflects a spillover effect.

5 Conclusion

We studied public spending, in a two-sector economy populated with heterogenous agents, as a factor that both enhances productivity and promotes accumulation of human capital. We showed that public investment in both human capital accumulation and the goods production sectors have a net positive effect on long run growth. We documented the possible effects of congestion cost related to a public capital in a given sector on the magnitude of the optimal tax in its own sector as well as in the other sector. We also showed that the magnitudes of optimal taxes on both sectors depend on whether there is inter-generational spillover in the human capital accumulation sector. Moreover, we disclosed inequality-congestion interaction and the consequent damage on individual households' production and accumulation of capital. We were also able to capture the negative effect of income inequality on economic growth.

More importantly, we showed infrastructure development in both sectors could improve income inequality dynamics, and hence could promote economic growth, once more, through an indirect effect of mitigating the negative influence of income inequality on economic growth.

Therefore, in line with recent empirical findings (Calderón and Serven 2004?, Calderón and Chong 2004, Lopez 2004, amongst many), we conclude that under appropriate conditions, infrastructure could promote pro-poor growth (i.e., loosely defined as an increase in growth and reduction in income inequality). In particular, with public investment, especially infrastructure, which targets the poor (or, at least, which is accessible to the poor), not only would the economic pie grow but also a larger slice would pass to the poor. That makes a wise investment on productive public good an area that belongs to the win-win type of policies.

Appendices

A Aggregation

The logarithm of a variable with lognormal distribution will have a normal distribution (and vice versa). A normal distribution preserves under linear transformation (Greene 2003, appendix B).**?** We use these facts and other important relations between lognormal and normal distribution to study the evolution of income distribution in our model.

Since we assume a lognormal distribution for individual's initial human capital, i.e., $\ln h_t \tilde{} N(\mu_t, \sigma_t^2)$, we have the following relation

$$\begin{aligned} \ln E[h_t] &= E[\ln h_t] + \frac{\sigma_t^2}{2} \\ &\Longleftrightarrow E[\ln h_t] = \ln E[h_t] - \frac{\sigma_t^2}{2} \equiv \ln H_t - \frac{\sigma_t^2}{2} \end{aligned} \quad \text{(A1)}$$

since $E(h_t) \equiv H_t$

We derive $E\left[(h_t)^{\omega}\right] = (H_t)^{\omega} \exp\left(\frac{\sigma_t^2}{2}\omega\left(\omega-1\right)\right)$, for instance, in equation (10), using the above facts. If h_t is a lognormal distribution then $(h_t)^{\omega}$ is also a lognormal distribution, thus, according to (A1),

$$\begin{aligned} \ln E[(h_t)^{\omega}] &= E[\ln (h_t)^{\omega}] + \frac{1}{2} var[\ln (h_t)^{\omega}] \\ &= E[\omega \ln h_t] + \frac{1}{2} var[\omega \ln h_t] \\ &= \omega\left(\ln H_t - \frac{\sigma_t^2}{2}\right) + \omega^2 \frac{\sigma_t^2}{2} \\ &= \omega \ln H_t + \omega\left(\omega - 1\right)\frac{\sigma_t^2}{2} \end{aligned} \quad \text{(A2)}$$

$$E[(h_t)^{\omega}] = (H_t)^{\omega} \exp\left(\frac{\sigma_t^2}{2}\omega\left(\omega-1\right)\right) \quad \text{(A3)}$$

To derive $E\left[(h_t)^{\xi+\omega\eta}\right] = (H_t)^{\xi+\omega\eta} \exp\left(\frac{\sigma_t^2}{2}(\xi+\omega\eta)(\xi+\omega\eta-1)\right)$ for equation (21) follow similar steps as above.

B The Growth Rate

For the case $\xi = 0$, growth rate γ_t can be derived as follows. Since

$$\gamma_{t+1} = \ln Y_{t+1} - \ln Y_t \quad \text{(B1)}$$

From (10) and (B1), we have

$$\gamma_{t+1} = \omega\left(\ln H_{t+1} - \ln H_t\right) + \theta\left(\ln G_{t+1} - \ln G_t\right) + \frac{\left(\sigma_{t+1}^2 - \sigma_t^2\right)}{2}\left(\omega(\omega-1) - \theta\overline{\kappa}\right) \tag{B2}$$

By substituting (21) and (22), and using (17), in (B2), we obtain

$$\begin{aligned}\gamma_{t+1} =& \,\omega\left(\begin{array}{c}\ln BA\tau^{\upsilon}\left(H_t\right)^{\omega}\left(G_t\right)^{\theta}\left(a(1-\tau-\psi)\right)^{\eta} \\ \exp\left(\frac{\sigma_t^2}{2}\left(\omega\eta(\omega\eta-1) + \upsilon\omega(\omega-1) - (\upsilon\overline{\zeta} + \theta\overline{\kappa}\right)\right) - \ln H_t\end{array}\right) \\ &+\theta\left(\ln\psi A\left(H_t\right)^{\omega}\left(G_t\right)^{\theta}\exp\left(\frac{\sigma_t^2}{2}\left(\omega(\omega-1) - \theta\overline{\kappa}\right)\right) - \ln G_t\right) \\ &+\frac{\sigma_t^2}{2}\left(\omega(\omega-1) - \theta\overline{\kappa}\right)\left((\omega\eta)^2 - 1\right)\end{aligned}$$

Alternatively,

$$\begin{aligned}\gamma_{t+1} =& \,\omega\ln BA\tau^{\upsilon}\left(H_t\right)^{\omega-1}\left(G_t\right)^{\theta}\left(a(1-\tau-\psi)\right)^{\eta} \\ &+\theta\ln\psi A\left(H_t\right)^{\omega}\left(G_t\right)^{\theta-1} + \omega\frac{\sigma_t^2}{2}\left(\omega\eta(\omega\eta-1) + \upsilon\omega(\omega-1) - \left(\upsilon\overline{\zeta} + \theta\overline{\kappa}\right)\right) \\ &+\theta\frac{\sigma_t^2}{2}\left(\omega(\omega-1) - \theta\overline{\kappa}\right) + \frac{\sigma_t^2}{2}\left(\omega(\omega-1) - \theta\overline{\kappa}\right)\left((\omega\eta)^2 - 1\right)\end{aligned} \tag{B3}$$

Then, simplifying (B3), while applying $\omega + \theta = 1$ and $\upsilon + \eta = 1$ repeatedly, we get equation (30)

$$\gamma_{t+1} = \omega\ln Ba^{\eta}A^{\frac{1}{\omega}} + \omega\eta\ln\left(1-\tau-\psi\right) + \omega\upsilon\ln\tau + \theta\ln\psi + \Phi_t + \Delta_t \tag{30}$$

where

$$\begin{aligned}\Phi_t &= \frac{\sigma_t^2}{2}\left(-(\upsilon\overline{\zeta} + \theta\overline{\kappa})\omega - \theta^2\overline{\kappa} - \theta\overline{\kappa}\left((\omega\eta)^2 - 1\right)\right) \\ &= -\omega\frac{\sigma_t^2}{2}\left(\upsilon\overline{\zeta} + \eta^2\theta\overline{\kappa}\omega\right) < 0\end{aligned}$$

and

$$
\begin{aligned}
\Delta_t &= \frac{\sigma_t^2}{2}\omega \begin{pmatrix} \omega\eta(\omega\eta - 1) + \upsilon\omega(\omega - 1) \\ +\theta(\omega - 1) + (\omega - 1)\left((\omega\eta)^2 - 1\right) \end{pmatrix} \\
&= \frac{\sigma_t^2}{2}\omega \begin{pmatrix} \omega\eta(\omega\eta - 1) + \omega\upsilon(\omega - 1) \\ +(\omega - 1)\left((\omega\eta)^2 - \omega\right) \end{pmatrix} \\
&= \frac{\sigma_t^2}{2}\omega \begin{pmatrix} \omega\eta(\omega\eta - 1) + \\ (\omega - 1)\left((\omega\eta)^2 + \omega\upsilon - \omega\right) \end{pmatrix} \\
&= \frac{\sigma_t^2}{2}\omega^2 \left(-\theta\left(\omega\eta^2 + \upsilon\right) + \eta(\omega\eta - 1) + \theta\right)
\end{aligned}
$$

or,

$$
\Delta_t = \omega^3 \frac{\sigma_t^2}{2}\eta(\omega\eta - 1).
$$

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