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PRODUCT LIFECYCLES AND SKILL-BIASED TECHNICAL CHANGE*

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Abstract:

The labour market position of low skilled workers has deteriorated dramatically over the 80s and early 90s. A well-known manifestation of this deterioration is the rise in relative high-skilled wages observed in the United States. It is a well-documented fact that demand shifts underlie this deterioration and several studies indicate that technical change is a likely candidate to explain such relative demand shifts. It is, however, still not very well understood why technological change would be biased persistently against low skilled workers when their relative labour costs are dropping. Modern economics considers technological change to be endogenous and the result of rational decisions taken by economically constrained agents. This paper presents a model that provides two possible answers to this puzzle. The bias can be caused by the deliberate development of skilled labour complementary technologies under certain conditions. If these conditions are met, the bias and the resulting labour demand shift is a permanent one and so are the consequences for relative wages and employment levels. A slightly different specification with more realistic assumptions, however, can still explain the observed shift in demand. When I assume that the production of new products is inherently skill biased because uncertainty is high and flexibility required in the initial stages of the product life cycle, then the model can generate the shift in response to the introduction of a new general-purpose technology, for example the IT-revolution. Once new products mature and process innovations are made can low skilled workers be involved in the production of such products. As such the model introduces some old ideas into a new debate.

Keywords: Skill-Bias, Endogenous Growth, Product-Lifecycle

JEL-Classification: J21; J29; J31; O15; O33

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INTRODUCTION

The labour market position of low skilled workers has deteriorated dramatically over the 80s and early 90s. The most well known manifestation of the drop in relative demand is the rise in relative wages that low-skilled American workers endured. Autor-Katz-Krueger (1998) show that although the supply of college graduates has increased by 3% annually between 1970 and 1995 relative to non-college graduates, their wages increased by over 25% relative to those of high-school graduates over the same period. Acemoglu (2002b) shows that over that period relative wages first went down (1970-1979) then up (1981-1995). Figure 1 shows relative employment, relative supply and relative wages for that period. Although relative demand may have remained stable during the 70s, the simultaneous increase in relative supply, employment *and* wages implies the demand curve must have shifted outward in the 80s.

Similar trends in demand have been observed throughout the OECD and beyond.[1] Berman and Machin (2001) survey the abundant empirical evidence and conclude that a consensus is forming on the underlying causes of these developments. Because many authors have found robust positive correlations between the use of new technologies and the skill intensity of production, the shift in relative labour demand has been attributed to the introduction of technologies that require or favour the employment of skilled labour. This is known as the Skill-Biased Technical Change (SBTC) hypothesis.

Over the past decades, however, the growth-theoretical economists have come to the understanding that technical change does not fall like manna from heaven and therefore its characteristics are not God-given but man-made. They claim that the generation of innovations is a painstaking, resource consuming economic activity that is subject to rational decision-making on behalf of economic agents. This implies a puzzling contrast with the attribution of relative demand shifts to Skill-Biased Technical Change, since what incentives exist to develop Skill-Biased technologies when relative wage developments clearly favour technologies that require low or unskilled labour?

Only a few economists have set out to explain what might have caused the apparently strong skill biases in technical change in the early 80s.[2] Acemoglu (1998, 2002a, 2002b) shows that in models with endogenous innovation, technical change can be biased towards a factor in response to a positive supply shock. Such a shock was indeed observed in the US in

[1] Although the resulting wage divergence seems to be typical only for Anglo-Saxon countries (US, UK, New Zealand and Canada).
[2] And indeed, as Acemoglu (2002a) argues, throughout the 20th century.

the early 70s. In his model a positive supply shock makes the development and introduction of skilled labour using technologies more worthwhile because the fixed costs of developing such technologies could be spread over a larger number of potential users. Initially high skilled wages drop but the shift towards skilled labour using technology develops a dynamic of its own and in the medium run endogenously biased technical change can cause relative wages to rise above their initial levels. I label this the Strong Market Size Effect (SMSE).[3] In Acemoglu (2002b) it is shown that this effect exists when high and low skilled labour are gross-substitutes and the model predicts that through endogenously biased technical change the deterioration in the low skilled labour market position is a permanent and steady state result. It will be shown below, however, that this result depends crucially upon rather restrictive assumptions made on parameter values.

As an alternative or complementary explanation Aghion (2001) introduced Major Technical Change (MTC) in the Aghion and Howitt (1992) model of endogenous innovation. In that model it is the random arrival of a new General Purpose Technology (GPT), in this case computers, that causes the entire economy to go into a stage of experimentation and implementation, which is a high skilled intensive activity. As the new technology is used, however, knowledge accumulates and gradually the stage of experimentation ends and relative demand returns to normal. In this model, the deterioration in low skilled labour market perspectives over the 80s is a temporary phenomenon and the model predicts a return to normal relative wage levels as the GPT 'ages'. The Aghion (2001) model, however, assumes new GPTs to age automatically and effortlessly and hardly compares to the analytical rigour and elegance of the Acemoglu model.

In this paper I will present and analyse a model in which Aghion's MTC-idea is analysed more formally and in such a way that a comparison to Acemoglu is relatively straightforward. To model the idea of a technology that 'ages', however, it is useful to take a few steps back in the literature. Aghion's idea is not new to growth theory. Krugman (1979) already presented a model in which he assumed that new products and technologies require high skilled labour to produce them in the initial stages of their existence.[4] Applying this principle to a GPT only implies that it affects a large part of the economy at once. In that model knowledge about the process or product accumulates and diffuses and less educated

[3] As opposed to the weak market size effect, which implies that technical change is biased towards the abundant factor but not to the extent that it's relative price increases. The weak market size effect implies that in the long run the elasticity of substitution is larger than in the short run due to endogenous technology responses. The demand elasticity, however, remains negative.
[4] As indeed did Nelson and Phelps (1966), Vernon (1966) and several others before him.

workers - Krugman positioned them in the South - can effectively compete and take over production. He took his idea from Vernon (1966), who labelled it the Product Life Cycle (PLC). Grossman and Helpman (1991a) presented a model that introduces an endogenous innovation driven PLC in the basic Krugman (1979) framework using by now standard modelling techniques from growth theory.[5]

Although the PLC-literature primarily aimed at explaining the international pattern of specialisation, the idea of a PLC can also be applied in a closed economy context to operationalise the 'aging' idea. If we assume that products can be produced by high skilled labour only in the first stage and further (process) R&D is required to enable low skilled workers to produce them in the second, we have a simple two-stage PLC. The "birth"- and "transition"-rates can easily be endogenised as in Grossman and Helpman (1991a).[6] Because R&D now has two options, developing high skilled complementary 'new' products, or low skilled complementary processes for existing products, comparisons can be made to Acemoglu (2002b).

In earlier publications together with Adriaan van Zon; Van Zon and Sanders (2000, 2001), the basic model for this paper was developed. In this paper I present an improved version of our reinterpretation and extension of the Krugman (1979) framework and provide the link to models presented in Acemoglu (1998, 2002a, 2002b). The mathematical structure of the model used is very similar to existing PLC and innovation driven endogenous growth models but the interpretation of the results is rather different. To those familiar the product life cycle literature, the modelling structures used in this paper are well known. For them the novelty in this paper is in their application to a different problem. Those familiar with the literature on endogenous skill biases in technical change will appreciate the potential of dynamic product life cycle models to create endogenous biases in labour demand.

Section 1 presents the basic structure of the model and section 2 discusses the existence and properties of the steady state under various specifications. In section 3 some numerical simulation experiments are presented to show how the model responds to the introduction of a GPT and a shift in relative supply.

[5] Other references for extensions of the Krugman framework include Dollar (1986), Jensen and Thursby (1986, 1987) and Grossman and Helpman (1989, 1990).

[6] Krugman (1979) and the references above refer to the "innovation"- and "imitation"-rates respectively.

SECTION 1

In the model presented below there are four groups of economic agents, consumers, producers, researchers and developers. Consumers consume the final output produced by producers, who require a patent that is provided by the researchers and may or may not use a production process that the developers have designed for them. In the remainder of this section their decision-problems are formalised. The next section will analyse the equilibrium.

The Consumers

Consumers consume at every instant a range of products to maximise a love-of-variety utility function and save the remainder of their income to purchase risk free assets, given an intertemporal budget constraint. Their problem:

$$\max_{c_{it}} U = \int_t^{\infty} e^{-\rho(\tau-t)} \log\left[\left(\sum_{i=0}^{n(\tau)} c_i(\tau)^{\alpha}\right)^{\frac{1}{\alpha}}\right] d\tau \quad \text{where } 0 \le \alpha \le 1 \qquad (1)$$

$$\text{s.t.} \qquad \int_t^{\infty} e^{(r(\tau)-r(t))} \sum_{i=0}^{n(\tau)} c_i(\tau) p_i(\tau) d\tau \le \int_t^{\infty} e^{-(r(\tau)-r(t))} Y(\tau) d\tau + A(t)$$

where ρ is the discount factor, n the number of varieties, i identifies one variety, c is the amount consumed and p is the price, r is the interest rate, Y is factor income and A the value of assets.[7] Intertemporal maximisation yields[8]:

$$\dot{C}(t)/C(t) = \dot{r}(t) - \rho \qquad \text{where} \quad C(t) \equiv \sum_{i=0}^{n(t)} c_i(t) p_i(t) \qquad (2)$$

[7] Note that by assuming 0<α<1 the elasticity of substitution between goods is larger than 1. This implies that since goods are assumed to be produced exclusively by one type of labour later on, high and low skilled labour are assumed to be gross-substitutes as in Acemoglu (2002b). I do not exclude the possibility of a market size effect on the demand side. See Acemoglu (2002b) for a detailed discussion.
[8] See for example Grossman and Helpman (1989), (1991a).

whereas the intratemporal maximisation yields *n* iso-elastic demand equations[9]:

$$c_i = \frac{p_i^{\frac{1}{\alpha-1}}}{P^{\frac{\alpha}{\alpha-1}}} \qquad \text{where } P \equiv \left(\sum_{i=0}^{n} p_i^{\frac{\alpha}{\alpha-1}} \right)^{\frac{\alpha-1}{\alpha}} \tag{3}$$

The Producers

All varieties *i* are assumed to be produced by monopolists who set their prices to maximise profits. In addition assume that a linear production technology with labour only is available to them. These producers then face the problem:

$$\max_{p_i} \pi_i = c_i p_i - w_i l_i \tag{4}$$

s.t. $\mu_i l_i = c_i$ and (3)

where w_i represents the wage of the type of labour used in producing variety *i*. The first order conditions yield the standard Amoroso-Robinson result that prices are set as a mark-up over marginal costs - i.c. wages[10]:

$$p_i = \frac{1}{\alpha \mu_i} w_i \tag{5}$$

Substituting (5) into the demand equation and using the production function yields the implied labour demand equations for an individual variety:

$$l_i = \left(\frac{1}{\alpha \mu_i} \frac{w_i}{P^{\alpha}} \right)^{\frac{1}{\alpha-1}} \tag{6}$$

[9] Dropping time arguments to save on notation and normalising the expenditure on consumption, *C*, to 1 at every point in time yields (3).

[10] Note that for large *n* we can assume $\frac{dP}{dp_i} \approx 0$

Substituting (5) into (4) yields that the flow of rents is given by:

$$\pi_i = (1-\alpha)\left(\frac{1}{\alpha\mu_i}\frac{w_i}{P}\right)^{\frac{\alpha}{\alpha-1}} \tag{7}$$

which is positive as long as α<1. Note also that the flow of profits is negatively related to the wage and positively to the overall price index.

Labour Demand

If we distinguish the varieties only by the type of labour that produces them we can order and split up the range of goods in a range n_H for which $i\in(n_l, n]$, such that $n_H \equiv n\text{-}n_l$. And a number of low skilled intensive products, n_L for which $i\in[0, n_l]$, such that $n_L \equiv n_l\text{-}0 = n_l$. For simplicity I assume that within these ranges the products are perfectly symmetric. This implies the price index, *P*, is a weighted average of the two wage rates:

$$P = \frac{1}{\alpha}\left(\sum_{i=1}^{n}\frac{1}{\mu_i}w_i^{\frac{\alpha}{\alpha-1}}\right)^{\frac{\alpha-1}{\alpha}} = \frac{1}{\alpha}\left(\frac{n_L}{\mu_L}w_L^{\frac{\alpha}{\alpha-1}} + \frac{n_H}{\mu_H}w_H^{\frac{\alpha}{\alpha-1}}\right)^{\frac{\alpha-1}{\alpha}} \tag{8}$$

and one can than derive the aggregate demand for high and low skilled labour for production by using (8) in (7) and multiplying by n_H and n_L respectively[11]:

$$H_P = \frac{\alpha}{w_H}\frac{X}{1+X} \qquad L_P = \frac{\alpha}{w_L}\frac{1}{1+X} \qquad X \equiv z\left(\frac{w_H}{w_L}\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{\mu_H}{\mu_L}\right)^{\frac{\alpha}{1-\alpha}} \tag{9}$$

where $z=n_H/n_L$. Two simple labour market clearing conditions could be used to solve the model at this point, and relative wages would be determined entirely in terms of the distribution of final output over the variety ranges, represented by $z=n_H/n_L$.

[11] The symmetry assumption is often used in Ethier- or Dixit-Stiglitz-type specifications of production and utility. See for example Grossman and Helpman (1991a, 1991b), Krugman (1979) and Acemoglu (2002a).

Solving relative demand given z could be regarded as the short run relative labour demand curve:

$$\frac{H_P}{L_P} = z\left(\frac{w_H}{w_L}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu_H}{\mu_L}\right)^{\frac{\alpha}{1-\alpha}} \tag{10}$$

where the short run elasticity of substitution, $\sigma_{SR}=1/(1-\alpha)$, which is larger than 1.[12] In the long run, however, z will respond to the incentives to develop products for either labour type, which, in an innovation driven growth model follow from the rents that can be extracted due to patent protection.

The Demand for Innovations

In the model outlined above an increase in the ranges n_H and n_L would represent skilled and unskilled labour using, skill biased innovations respectively.[13] Since varieties are assumed to be imperfect substitutes in consumption, the demand for such new varieties is always positive and patent protection implies the potential buyers of an innovation are willing to pay the R&D sector for their innovative efforts.[14] Before we proceed with the innovation process itself it is useful to analyse that willingness to pay, the demand for innovations, in more detail, since it is the relative willingness to pay that will ultimately induce the R&D sector to produce one rather than the other type of innovations. I follow traditional innovation driven growth models and assume that the value of a new patent on a product $n+1$ is equal to the expected discounted flow of rents that that patent generates:

$$V_{n+1}(t) = E_t\left[\int_t^{\infty} e^{-(r(\tau)-r(t))}\pi_{n+1}(\tau)d\tau\right] \tag{11}$$

[12] This is important to note since Acemoglu (2002a) shows that the strong market size effect can only occur if that condition is fulfilled.
[13] Skill bias is defined as increasing the marginal productivity of one factor relative to another for given factor intensities. For any given skill intensity of aggregate production an increase in n_H reduces the amount of high skilled labour available to all n_H producers, causing the average marginal product of high skilled workers to rise.
[14] Obviously only when successful but I shall assume for simplicity that R&D is a deterministic process below.

The patent can be regarded as an asset yielding a return in the future, which implies that consumers or entrepreneurs are willing to invest in them once created.[15] Assuming that a steady state equilibrium exists and distinguishing varieties only by the type of labour they require for production one obtains:

$$V_j(T) = \pi_j(T) E_T\left[\int_T^{\infty} e^{(G_{\pi j}(t)-r(t))(t-T)} dt\right] = \frac{\pi_j(T)}{\rho - G_{\pi_j}} \tag{12}$$

where $G_{\pi j}$ is the steady state growth rate of profits when producing with labour type *j*.[16] Using (12), (7) and (10) to substitute for relative wages at this point yields a similar straightforward demand structure for innovations as in Acemoglu (2002b):

$$\frac{V_H}{V_L} = \frac{\pi_H}{\pi_L} = z^{-\alpha}\left(\frac{\mu_H}{\mu_L}\right)^{\alpha}\left(\frac{H_P}{L_P}\right)^{\alpha} \tag{13}$$

where it can be verified that the incentives to bias innovations towards a factor rather than another are shifted towards the more abundant factor for a given *z*. If, for example, the supply of high skilled workers rises and we assume market clearing, this induces R&D firms to reallocate resources towards the invention of new n_H-goods. However, as they generate such innovations, *z* will rise, which has a negative impact on the incentive. If *z* rises in proportion to relative supply, (13) and (10) show that relative wages and incentives do not change. If *z* changes less or more than proportional than relative wages will fall or rise permanently in response to the shock. The latter case would correspond to the Strong Market Size Effect, the former to the Weak Market Size Effect or Say's Law in disguise.

The Supply of Innovations

What happens to relative wages in the long run, therefore, is crucially determined by how *z* responds to incentives and depends on the supply of innovations and more notable the

[15] I will not model the asset market explicitly since it this will not affect results and adds little to the understanding of the mechanisms that are of primary interest. It is sufficient to note at this point that the utility function in (1) implies that consumers are willing to postpone consumption today as long as the return (more than) compensates them in the future, which implies they would be willing to hold such assets.
[16] Which is 0 due to the fact that in the steady state *relative* wages are constant. It can be verified in equation (8) that this implies they grow at the same rate as *P* and (7) shows this implies the growth rate of profits is 0.

knowledge spillovers that are assumed to exist. Consider two rather general innovation production functions:

$$\dot{n}_H(t) = \mu_R n_H(t)^{\eta} n_L(t)^{1-\eta} S_R(t) \text{ and } \dot{n}_L(t) = \mu_D n_L(t)^{\varphi} n_H(t)^{1-\varphi} S_D(t) \quad (14)$$ [17]

where I have specified knowledge spillovers as a Cobb-Douglas aggregate of the innovations of both types made in the past.[18] S is an inelastically supplied R&D specific factor. This implies the marginal value product must always be equal in both types of innovation and using (12) one obtains:

$$\frac{d\dot{n}_H}{dS_H} * V_H = \frac{d\dot{n}_L}{dS_L} * V_L \quad \text{and} \quad z^{SS} = \left(\frac{\mu_H H_P}{\mu_L L_P}\right)^{\frac{-\alpha}{1-\alpha-\eta-\varphi}} \left(\frac{\mu_R}{\mu_D}\right)^{\frac{1}{1-\alpha-\eta-\varphi}} \quad (15)$$

In a steady state equilibrium z is constant, hence the growth rate of n_H must equal that of n_L which implies the equilibrium allocation of R&D resources is given by:

$$\frac{S_R}{S_D} = \left(\frac{\mu_R}{\mu_D}\right)^{\frac{1+\alpha}{1-\alpha-\eta-\varphi}} \left(\frac{\mu_H H_P}{\mu_L L_P}\right)^{\frac{-\alpha(2-\eta-\varphi)}{1-\alpha-\eta-\varphi}} \quad (16)$$

Substitution of (15) in (10) and solving for relative labour demand yields the long run steady state relative demand for labour in production:

$$\frac{H_P}{L_P} = \left(\frac{\mu_H}{\mu_L}\right)^{\frac{-\alpha(\eta+\varphi)}{(1-\alpha)(1-\eta-\varphi)}} \left(\frac{\mu_R}{\mu_D}\right)^{\frac{1}{1-\eta-\varphi}} \left(\frac{w_H}{w_L}\right)^{\frac{-(1-\alpha-\eta-\varphi)}{(1-\alpha)(1-\eta-\varphi)}} \quad (17)$$

The long run elasticity of demand is now positive, i.e. we obtain an upward sloping long run demand curve and the Strong Market Size Effect for a limited range of parameter values.

Acemoglu (2002b) presents some evidence that the short run elasticity of substitution between high and low skilled labour in aggregate production lies between 1.4 and 2, implying an α of between 0.3 and 0.5.

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[17] I will drop time arguments in the remainder of the paper to save on notation.

[18] This specification is only a slight generalisation of Acemoglu (2002b), who considers only strictly symmetric Cobb-Douglas spillovers.

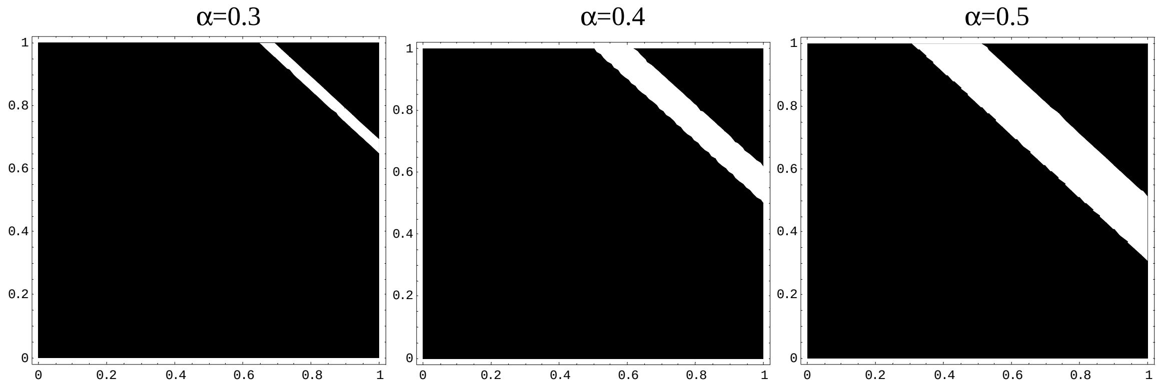


Figure 1: Spillover specifications and the Strong Market Size Effect

Figure 1 shows in η, φ- space for what values the long run demand curve is upward sloping (white areas) for α=0.3, 0.4 and 0.5.[19] It is clear from these graphs that, although the possibility cannot be excluded a priori, it requires a rather strict set of constraints on the spillover effects.[20] The scope for the SMSE becomes even smaller when we assume that high skilled labour, besides producing output, is also required to produce innovations.

To see this I will replace (14) by: $\dot{n}_H = \mu_R n_H{}^{\eta} n_L{}^{1-\eta} H_R$ and $\dot{n}_L = \mu_D n_L{}^{\varphi} n_H{}^{1-\varphi} H_D$ and use $H_D + H_R + H_P = H$ as the aggregate labour market clearing condition. Since the bulk of R&D expenditures is on labour and typically that labour is relatively high skilled this extension would make the model more realistic. The extension, however, comes at the cost of losing the analytical solution and will take too much space to discuss it as such. In the next paragraphs I will first specify the innovation production functions further. The long run equilibrium is then derived and analysed in section 2 and discussed in section 3 under the numerical simulations.

Although under this specification the Strong Market Size Effect does not occur, the Weak version is still present and it can be shown that the spillover parameters are again crucial in determining the long run response of relative labour demand. As Acemoglu (2002b) points out, there is little empirical evidence to guide us in setting these parameters, but the concept of a life-cycle can provide a strong and intuitively appealing way to proceed with the specification of the innovation supply functions.

[19] Acemoglu (2002b) only considers the parameter combinations η, φ- that lie on the 45 degree line.
[20] In addition it can be shown that the switch in sign from + to – when moving in north-easterly direction implies a switch from a long run elasticity of +∞ to -∞, i.e. the long run relative demand curve is vertical (as is supply) and an equilibrium is no longer guaranteed.

The Life Cycle

At this point it I introduce the idea of an endogenous life cycle. Suppose that devoting high skilled labour to research yields an increase in the *total* range of products, *n*, due to *product innovation*. When smart people put their minds to it they will occasionally come up with a new design. When the design can be patented and the patent holder earns positive rents, as is the case in the model developed above, that activity would be worthwhile and the future rents can finance the current investment.

Upon introduction a product is by definition in the early stage of its life cycle. In that stage the design has many problems. It needs frequent adjustments to customer demands, market changes, production problems and weaknesses in the design. I follow Aghion (2001) and assume that to deal with such problems the production in the early stage requires relatively high skilled workers.[21] I stylise that idea in line with the model above by assuming that early stage products, in range n_H, are produced using high skilled labour only.

Then, as the product matures and competition from substitutes intensifies, it becomes worthwhile to develop the tools and machinery that would allow less costly low skilled workers to produce such an early stage variety. In the tradition of endogenous technical change models that activity requires the allocation high skilled labour to Development away from Research or Production. Once the developers succeed in making this *process* innovation, the product enters its mature stage in the product life cycle and production is taken over by lower skilled personnel and machinery. For simplicity I assume therefore that mature products are produced entirely with low skilled workers. Note, however, that development is by definition product specific and therefore no patent protection is required to make this activity economically viable. This also implies that current investments must be financed out of *additional* future rent flows that result from the cost savings. The value of a low skilled labour using process innovation is then given not given by (12) but by:

$$V_D(t) = \int_t^{\infty} e^{-(r(\tau)-r(t))}\pi_L(\tau)d\tau - \int_t^{\infty} e^{-(r(\tau)-r(t))}\pi_H(\tau)d\tau \tag{18}$$

[21] The idea that high skilled labour is required in the early stages of the product cycle is at the heart of the product life cycle literature and was already voiced as early as 1965 by Hirsh (1965).

Hence we have distinguished two types of labour, low and high skilled, of which the former produces mature products whereas the latter produces new goods but also invents new products and processes. Note that in any positive growth equilibrium in which development is undertaken, the wages of the high skilled must exceed those of the low skilled by a factor sufficient to induce the development of tools, machines and procedures that mature previously new products. To see this point one should to realise that a near zero wage differential implies a near zero cost reduction, which is always insufficient to cover the significant positive development costs. In this simple, two types of labour, two types of R&D and two types of final output-world the basic mechanism of endogenous cycle induced biases in relative labour demand can be illustrated.

Research and Development in the Product Life Cycle

As was mentioned above, the specification of the research and development production functions ultimately determines the impact of the market size effect. By casting the model in a life cycle framework we can analyse an intuitively plausible specification of these functions. I follow Grossman and Helpman (1991a) and assume that the knowledge used in generating new products is proportional to the number of product innovations made in the past and available to all research workers at no costs.[22] Choosing units accordingly we have:

$$\dot{n} = n\mu_R H_R^{\beta} \qquad 0>\beta>1 \qquad (19)$$

where H_R as before refers to the total amount of high skilled labour devoted to product variety expanding research.[23] Although in the PLC specification the entire n-range ($n=n_H+n_L$) is used as the relevant range one could compare the knowledge spillover assumed here to the generalised Acemoglu specification in (14) with $\eta = 1/2$.

Regarding process innovation, however, I do not follow Grossman and Helpman (1991a), who assumed that the relevant knowledge stock for developers is proportional to the

[22] In Acemoglu (2001) terminology this would be 'state-dependence'.
[23] Note that I have assumed diminishing returns to labour in R&D. This is done to obtain a concave innovation possibilities frontier, necessary to determine the labour allocation over research and development out of equilibrium. Constant returns to labour implies a linear IPF and out of equilibrium a corner solution is always chosen. This is a nuisance in numerical simulations. I will consider β=1 later on as a special case.

number of previously developed processes.[24] [25] Hirsh (1956) and Aghion (2001) both assume that experimentation and early stage production generates the knowledge necessary for the development of the process. It therefore seems natural to assume that the relevant knowledge stock for the development of new processes is proportional to the time the product has been produced by high skilled workers. Again by suitable choice of units this can be set equal to n_H, the range of goods for which such processes can but have not yet been developed.[26] We obtain:

$$\dot{n}_L = n_H \mu_D H_D^{\beta} \qquad 0>\beta>1 \qquad (20)$$

where H_D refers high skilled labour allocated towards low skilled labour enabling process development. Both Researchers and Developers take all prices and wages as given. Note also that the presence of n_H in (20) ensures that processes are never developed for non-existing products.[27] In analogy to the generalised Acemoglu model presented above I use φ=0.

The next step in solving the model is to set the high skilled wage equal to the marginal value products in R&D. R&D-arbitrage implies:

$$w_H(t) = \beta\mu_R n(t) V_R(t) H_R(t)^{\beta-1} = \beta\mu_D n_H(t) V_D(t) H_D(t)^{\beta-1} \qquad (21)$$

These equations are easily rewritten as standard labour demand curves for R&D.

[24] Remember, however, that they were referring to a situation where Southern imitators imitated Northern products, in which case the absence of knowledge spillovers can perhaps be justified.
[25] Other differences are less fundamental. One could for example introduce diminishing returns to knowledge accumulation as in Jones (1995) or constant returns to labour as in Grossman and Helpman (1991a) and Acemoglu (2002a), without changing steady state results qualitatively.
[26] The implicit assumption being that new processes are either developed for the oldest "new" products or there is a random allocation over the existing "new" products such that the expected "waiting time" is constant and proportional to the number of "new" products. This assumption is more problematic out of steady state equilibrium, since then "waiting time" is not proportional to n_H. I thank Huub Meijers for raising this point. Still the product specificity of process innovations makes the spillover of previously developed processes less likely. Experimentation with the above-presented more general Cobb-Douglas specification of spillovers showed that the results do not change qualitatively.
[27] Following Grossman and Helpman (1991a) at this stage would leave that possibility open in principle (although not in equilibrium) since their formulation allows Southern engineers to imitate at a constant (larger) rate than product innovation in the North. At some point that means that non-existing varieties are imitated.

By substituting for innovation values, profits and prices we obtain:[28]

$$H_R = \left(\frac{(1-\alpha)}{\alpha} \beta \frac{\mu_R}{r - \overline{G}_{\pi_H}} \frac{(1+z)}{z} \right)^{\frac{1}{1-\beta}} \left(\frac{\alpha}{w_H} \frac{X}{1+X} \right)^{\frac{1}{1-\beta}} \tag{22}$$

$$H_D = \left(\frac{(1-\alpha)}{\alpha} \beta \frac{\mu_D}{r - \overline{G}_{\pi_H}} \left(\left(\frac{w_H}{w_L} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\mu_H}{\mu_L} \right)^{\frac{\alpha}{\alpha-1}} \frac{r - \overline{G}_{\pi_H}}{r - \overline{G}_{\pi_L}} - 1 \right) \right)^{\frac{1}{1-\beta}} \left(\frac{\alpha}{w_H} \frac{X}{1+X} \right)^{\frac{1}{1-\beta}} \tag{23}$$

where it should be noted that the latter terms are simple powers of high skilled labour demand in production. The high skilled labour market equilibrium requires:

$$H_R + H_D + H_P = H^* \tag{24}$$

Together with a labour market clearing condition for the low skilled labour market:

$$L_P = L^* \tag{25}$$

with the production demand equations and the definition of X in (9), equations (22), (23), (24) and (25) form a system of 7 equations in 7 variables: w_L, w_H, X, H_R, H_D, H_P and L_P. This system has no closed form solution as I have indicated above but can be solved numerically and it is easy to verify that the short-run equilibrium is conditional on the value of z. The next section will show that a long-run positive growth equilibrium also exists, in which z and the 7 variables above are constant, while the price index decreases at a constant rate, increasing the value of constant wages and expenditure levels.

[28] See the appendix

SECTION 2

In this section I will prove that the endogenous life cycle model presented in section 1 has a unique, stable steady state equilibrium and analyse its properties. The long run equilibrium we are looking for has positive and stable amounts of labour allocated to the various uses. The amount of labour allocated to mature products production is constant by equation (25). This implies by equation (10) that in the steady state:

$$\frac{w_H}{w_L} = L^{*1-\alpha} H_P{}^{\alpha-1} \left(\frac{\mu_H}{\mu_L} \right)^{\alpha} z^{1-\alpha} \tag{26}$$

must hold. I label this relationship the Product Market Arbitrage (PMA) condition since it relates relative wages to the relative employment levels in final output production. It is obvious and intuitively plausible that for high relative wages the relative aggregate level of employment in new products is low, whereas a large relative size of the new products sector, a high z, drives up relative demand for a given relative wage, due to increasing marginal productivity at the individual variety level. Using (22) and (23) we can also derive that in the steady state[29]:

$$\frac{w_H}{w_L} = \frac{\mu_H}{\mu_L} \left(\frac{1+z}{z} \frac{\mu_R}{\mu_D} \left(\frac{H_R}{H_D} \right)^{\beta-1} + 1 \right)^{\frac{1-\alpha}{\alpha}} \tag{27}$$

must hold. I label this the R&D Arbitrage (RDA) condition since it relates the relative wages to the relative employment levels in Research and Development. Here too the intuition is clear. A high relative wage increases the profitability in the mature goods production and hence the R&D sector shifts towards Development. For a given relative wage an increase in z increases the cross-spill-over of knowledge, making Development the relatively more productive branch of R&D and hence a reallocation of high skilled labour towards development occurs. It can be verified in (26) that a stable aggregate employment level in the new goods sector is consistent with a stable relative wage only for a stable value of z. (27) implies that under those conditions the relative employment in the Research and Development

[29] Where I have also used the fact that in a steady state the growth rates of mature and new goods producers' profits are equal. See the Appendix.

is also constant. The requirement that *z* is constant provides us with additional information on the allocation of high skilled labour over R&D in the steady state. The constancy of *z* implies through (19) and (20) that:

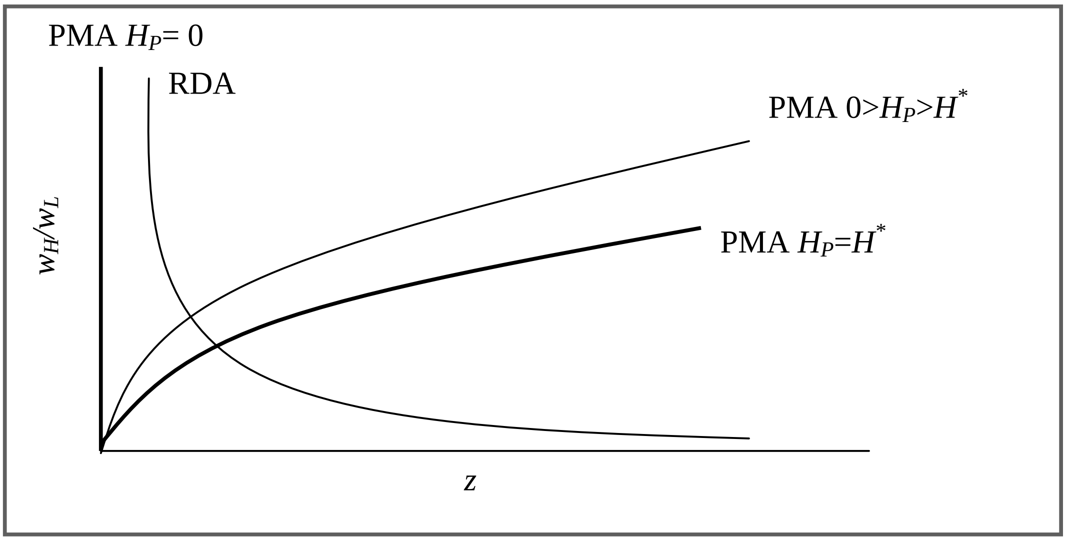
$$H_R = \left(\frac{\mu_D}{\mu_R} z\right)^{\frac{1}{\beta}} H_D \tag{28}$$

Adding this equation to those derived in section 1 we now have a system of 8 equations in 8 variables and *z* can be calculated for the steady state. Substituting (28) into (27) yields the steady state RDA:

$$\frac{w_H}{w_L} = \frac{\mu_H}{\mu_L}\left(\frac{1+z}{z^{1/\beta}}\left(\frac{\mu_R}{\mu_D}\right)^{1/\beta} + 1\right)^{\frac{1-\alpha}{\alpha}} \tag{29}$$

In Figure 1 both the PMA and the RDA condition have been drawn. From (29) it is immediately clear that the RDA-curve is downward sloping and has a vertical and horizontal asymptote at 0.

Figure 1: Conditional Equilibria



The intuition is that as *z* rises, the productivity of development rises. To keep *z* stable at the higher level, however, the allocation of labour over research and development must remain stable and hence to compensate for the higher productivity, a drop in relative wages is required to decrease the value of process development.

The PMA-curve is a concave upwards-sloping line for any given level of H_P. At higher z the marginal productivity of high skilled workers rises since the number of workers per variety falls and a rise in relative wages is required to choke off the increased relative demand.

It can be verified that in a steady state equilibrium there is a positive relationship between z and H_P, that is described by equating (29) to (26) and solving for H_P.[30] Using this relationship in (10) yields the relation between z and the high skilled wage level that must hold in the steady state. I label this relationship the Production Wage Level (PWL) condition. In the steady state this condition traces out a downward-sloping curve in w_H, z –space. An increase in the steady state z requires a drop in the wage level because less labour is required in Development to stabilise z due to the increased cross-spillover. Wages drop, profitability rises and employment is increased in the production sector but the increased profitability and reduction in high skilled wages also increases the employment of Researchers, which is necessary to sustain the higher z.

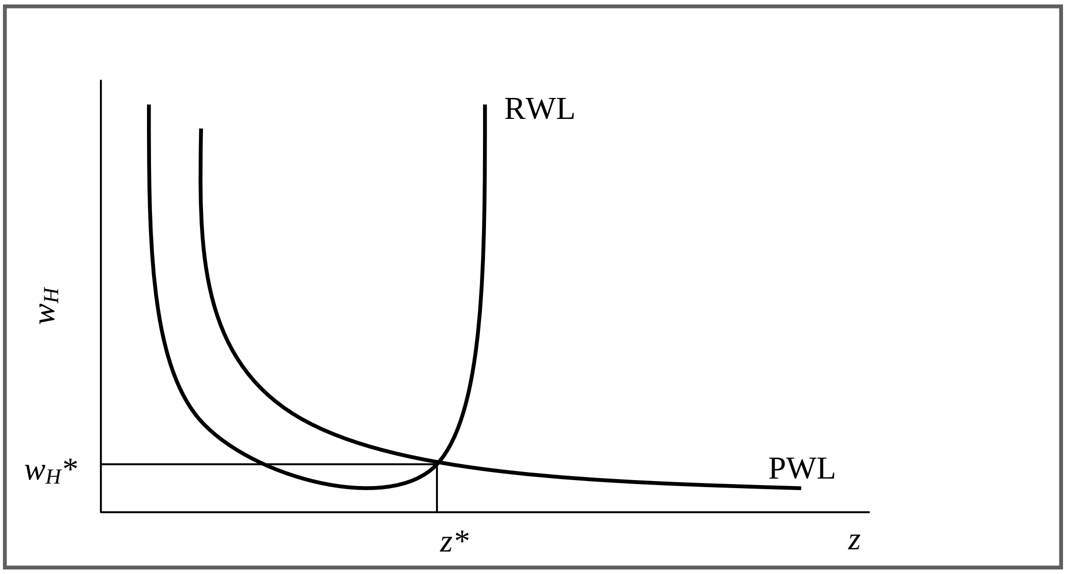


Figure 2:
The Steady State

Using (24) and (28) we can use this relationship to write H_R as a function of z alone. Then (22), after substituting for the G_π's and H_R yields a second relation between w_H and z that I

[30] The closed form solution is in the Appendix.

label the Research Wage Level (RWL) condition. The derivations are in the Appendix and the RWL-curve is also plotted in Figure 2.[31]

The RWL-curve is U-shaped due to vertical asymptotes at z=0 and some finite z. The intuition behind this result is less obvious. At z close to 0 a higher steady state level of z implies a small increase in the marginal productivity of high skilled workers in production because they are spread over few varieties. Hence production employment increases a little at given wages and a little more at reduced wages. This leaves only slightly less labour available for R&D. The higher level of z, however, makes Development proportionally more productive and hence of the slightly less R&D labour available a larger share must be allocated towards Research to maintain the same growth rates for n_L and n_H. At very low levels of z this effect dominates and the high skilled wage falls to induce the Research and Production sector to absorb the redundant Development workers. At higher levels of z the demand in the Production sector will absorb much more high skilled labour and for the Research sector to have the necessary labour available to maintain growth rates the same will require a higher wage level to induce the Production sector to release labour into Research.

Graphical analysis of the model for all possible parameter constellations shows that the equilibrium is unique and there is only one point of intersection in figure 2. In addition the concavity of the innovation possibilities frontier I have assumed precludes a corner solution in which only one type of innovation occurs at a finite non-zero relative wages.

Properties of the Steady State

We can now study the properties of the steady state equilibrium by analysing how exogenous shocks affect the long run equilibrium z and the variables of interest here, the relative wages. Equation (29) relates the steady state relative wage to z and shows that the relative wage is negative in z. The intuition for this result is that a higher steady state z implies that development is very productive in the steady state due to large spillovers. To offset this incentive the low skilled wages must be relatively high to depress mature product rents. For the same reason the relative productivity of production and research tend to affect the relative wage positively in the steady state. The full comparative static effects of shifts in parameters, however, also depend on the response of the steady state z to such shifts. Table 1 shows how

[31] In the plot the PWL and RWL lines represent the PWL and RWL conditions after dividing both by $X/(1+X)$ which is constant in the steady state.

the steady state z and the relative wages will respond to shifts in exogenous variables and parameters.[32]

Increases in high skilled labour supply, the main factor in the Strong Market Size Explanation, is unambiguously negatively related to relative wages in the steady state. Acemoglu's (2002b) result disappeared as was mentioned before due to the competition of R&D over skilled labour, but also because the life cycle structure precludes the spillovers the Acemoglu model requires for the effect to exist. The Weak effect does exist as z responds positively to the shift, implying that the relative wage does not fall as much in the long run due to the endogenous technology response. Acemoglu (2002b) refers to this as the LeChatelier Principle and it also applies here.

Table 1: Comparative Statics in the Steady State

Parameter or Variable	Effect on PWL	Effect on RWL	$\frac{dz^*}{dx}$	$\frac{d\frac{w_H}{w_L}}{dx}$
H*	No effect	High asymptote shifts right, minimum drops	+	-
L*	Shift down and to left	High asymptote shifts left, minimum rises	-	+
μ_R/μ_D	Shift up and to right	Both asymptotes shift right, minimum drops	+	?
μ_H/μ_L	Shifts up and to right	High asymptote shifts right, minimum drops	+	?
α	Counter-clockwise	High asymptote shifts right, minimum rises	-	?
β	Shifts up and to right	High asymptote shifts right, minimum indeterminate	-	?
r	No effect	Minimum drops	+	-
μ_R	No effect	Minimum rises	-	+

Increases in r would also decrease the steady state relative wages. The reason is that higher interest rates and therefore discount rates reduce R&D activity and hence increase the supply of labour to production.

The level of productivity in R&D and the supply of low skilled labour increase the relative wage of high skilled workers in equilibrium, as might be expected. Since all these changes increase the relative marginal product of high skilled workers. The ambiguous effects are harder to explain intuitively since they typically involve a shift in both curves.

Of all the possible shocks considered above, however, only the increase in R&D productivity would be an acceptable candidate for explaining labour market developments

[32] These claims were made on the basis of numerical graphical analysis in Mathematica.

over recent decades. In light of the observed higher relative skilled labour supply, however, it would have to be a dramatic and permanent shock to explain the hike in relative wages as a steady state phenomenon. Although the introduction of IT-technology might indeed have increased the productivity of research by opening up a whole new range of new products and services, this productivity effect is, according to life cycle models, at best a temporary phenomenon. A numerical simulation experiment analysed in the next section is therefore the best instrument to show that a temporary shock on productivity can have significant temporary wage effects in the transition back to the initial steady state.

SECTION 3

In the previous section I showed a unique and stable steady state exists and is characterised among other things by a stable relative wage.[33] It was shown that the parameter changes required to generate wage divergence as a steady state phenomenon, are rather unlikely to have occurred. Alternatively, as Aghion (2001) proposed, one could interpret the 80s and 90s as a period of transition, a temporary deviation from steady state due to an incidental shock. In this section some numerical simulation results are presented graphically to gain additional insights into the short run dynamic behaviour of the model. In these simulations the effects of temporary shocks and the adjustment to long-term steady state equilibrium are immediately visible.

To programme the simulation I have assumed that the expected growth rates of profits are set adaptively and are equal to the average growth rates over the past 2 periods. This simplifying assumption is required to make the model converge but implies also that the true steady state will never be fully reached. In the steady state, as was shown in the appendix, the growth rate of new and mature production rents will be equal and constant. By setting them equal to a moving average, the average will of course converge to that constant level but never reaches it. The base run parameters were set in accordance with Acemoglu's (2002a) "back of the envelope" calculations and correspond reasonably well with available empirical evidence. For those parameters where empirical evidence was lacking, extensive sensitivity tests were performed and the model is calibrated to yield a steady state growth rate in utility of 0.3%. Table 2 presents the starting values, exogenous variables and parameters for the baseline simulation run.

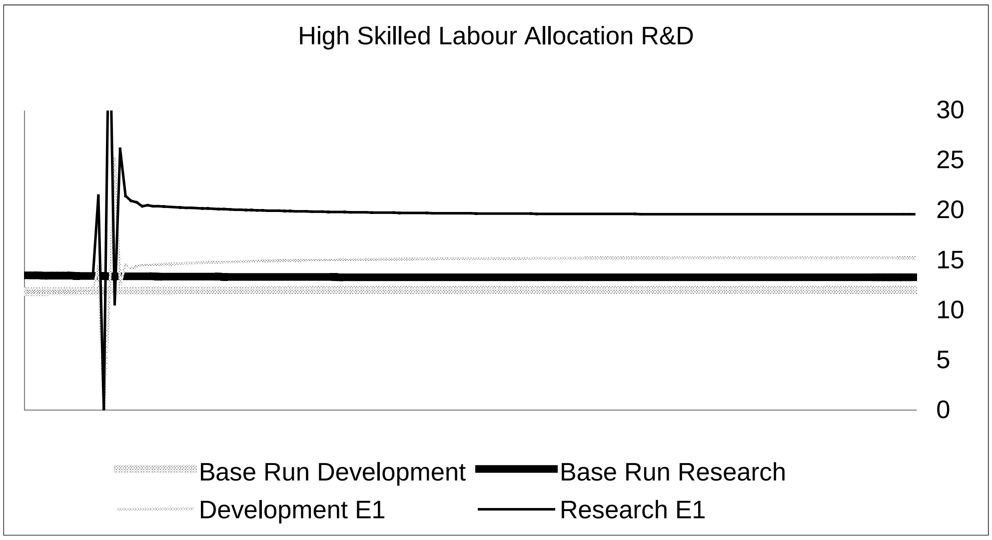
[33] And by implication stable relative demand since relative supply is exogenous and inelastic I all these models, so demand determines relative employment.

Table 2: Base Run Variables, Parameters and Staring Values

EXOGENOUS VARIABLES	
H^*	300
L^*	1000
r	0.1
PARAMETERS	
α	0.6
β	0.4
μ_H	1
μ_L	1
μ_R	0.002
μ_D	0.005
STARTING VALUES	
n_0	13
n_{H0}	3
n_{L0}	10

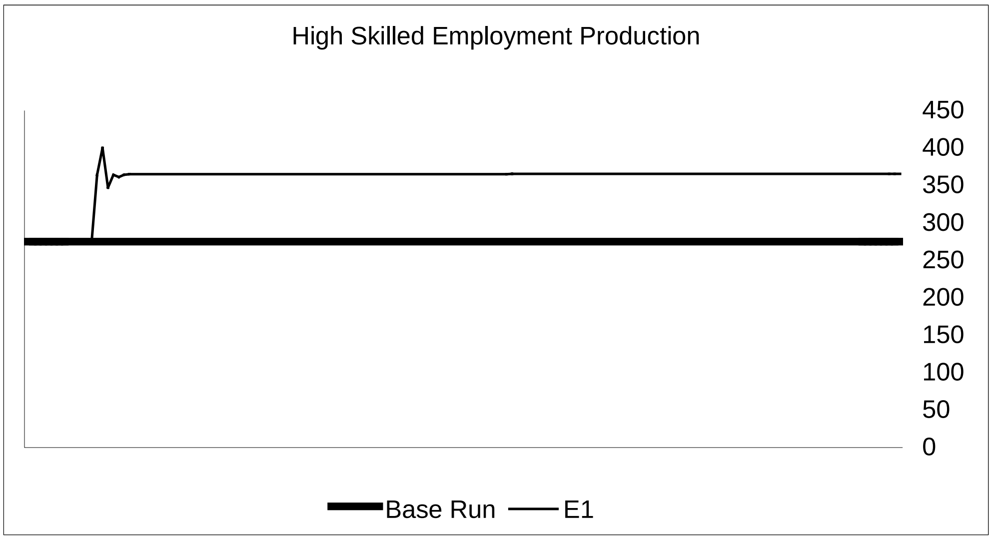
Two experiments were analysed. In treatment run 1 I illustrate the effect of an exogenous permanent increase in high skilled labour supply. It can be verified that the model does not exhibit wage divergence in short or long run.

Figure 3.1



Employment rises in both Research and Development, cf. figure 3.1, increasing both innovation rates. The product innovation rate will increase faster, which partially offsets the downward wage pressure.

Figure 3.2



As was noted in the previous section, however, the increased supply leads to a drop in steady state relative wages, cf. figure 3.3 even though there is a permanently higher n_H/n_L , cf. figure 3.4. Supply does create its own demand but not to the extent that prices rise as in Acemoglu.

Figure 3.3

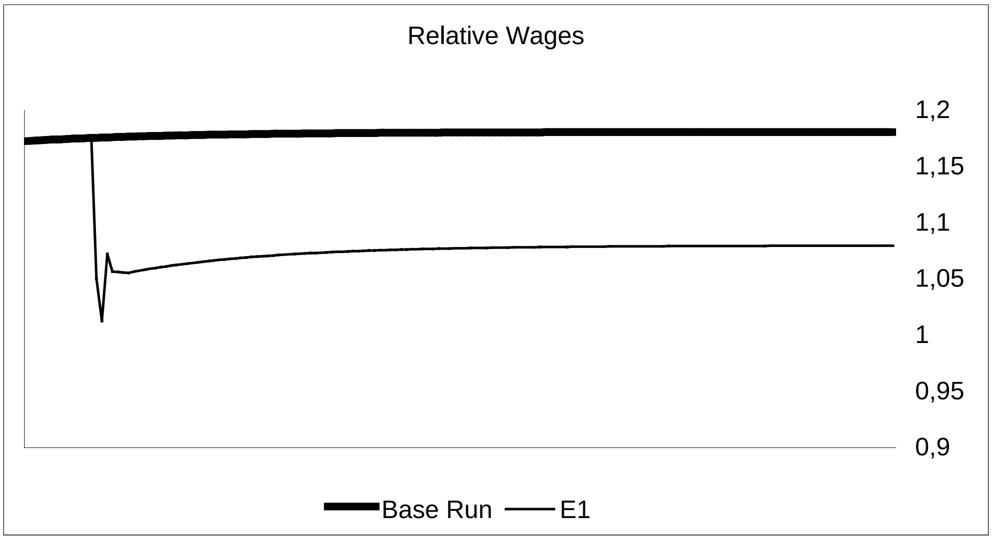
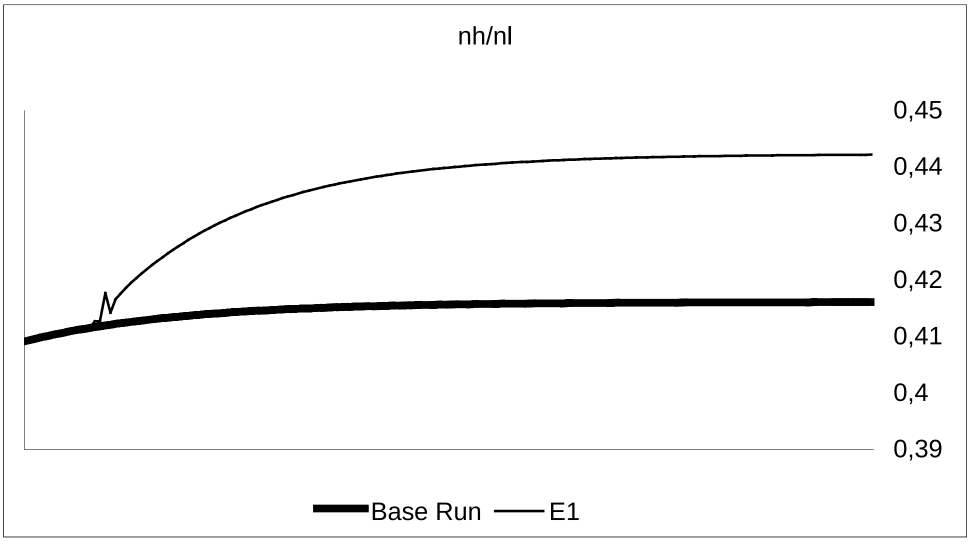


Figure 3.4



In treatment run 2 I illustrate the alternative, a version of the Major Technical Change Explanation formulated by Aghion (2001). The relative productivity of research was increased by 10% in period 200. In this model such a shock could represent the introduction of a new General Purpose Technology such as ICT. Such a GPT would open up a whole new range of potentially useful applications making the quest for new products an easier one. Such a shock results in the rapid introduction of many new products.

As more and more applications are introduced, however, the possibilities for further product innovations become harder to find and the productivity effect withers away. To illustrate this I have assumed the productivity returns to the pre-shock level in period 210.[34] Figures 3.5-3.8 show how relative wages, the allocation of high skilled labour and the composition of final output over the two types respond in this experiment.

The initial effect of the productivity shock is to pull more workers into Research, driving up the relative size of the new goods range. The cross-spill-over effect compensates for the relative productivity loss in Development and employment there remains rather stable-cf. Figure 3.5. The additional labour is therefore drawn away from final output production-cf. Figure 3.6. The increased relative number of new varieties, -cf. Figure 3.7, adds to the reduction in available labour per variety and hence wages rise relative to the low skilled labour using sector-cf. Figure 3.8. Note how the model predicts a hump-shaped response in relative wages.

[34] Of course in reality productivity would rise fast and then gradually fall back to its original level. A similar response pattern would emerge, however.

Figure 3.5:

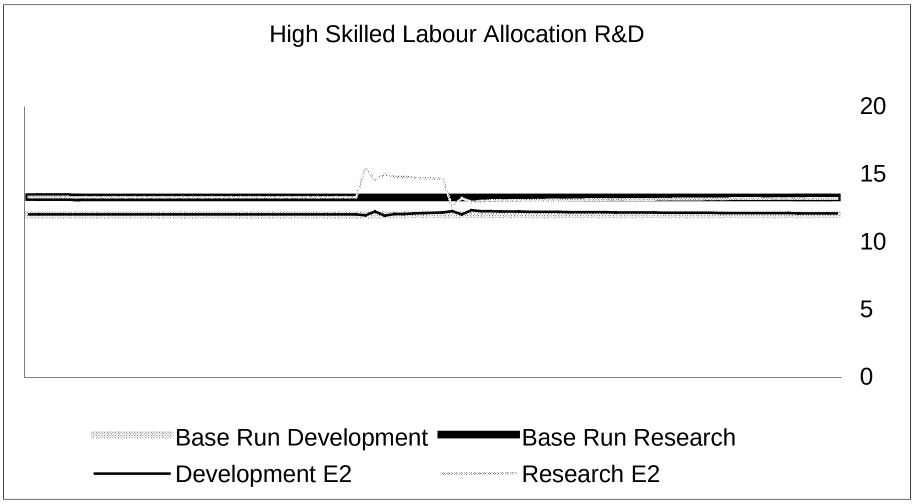


Figure 3.6:

Figure 3.7:

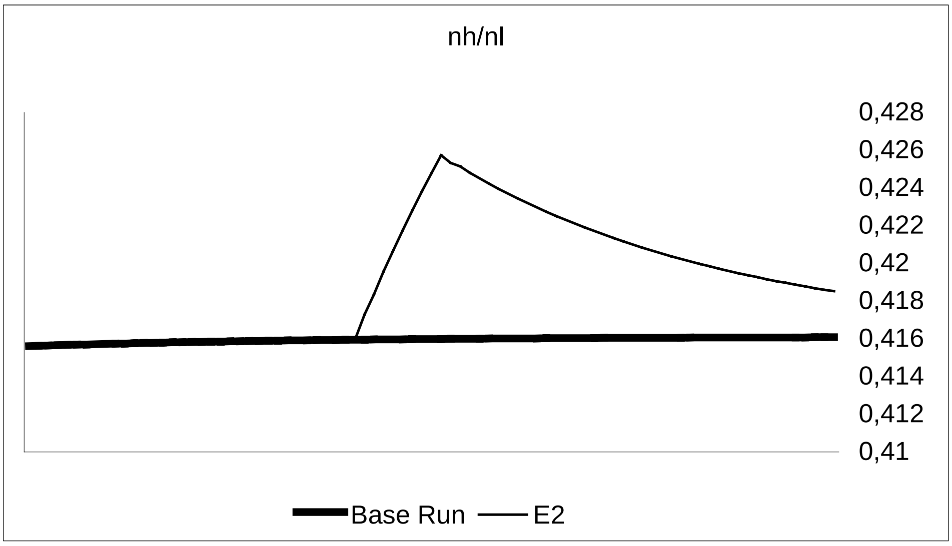
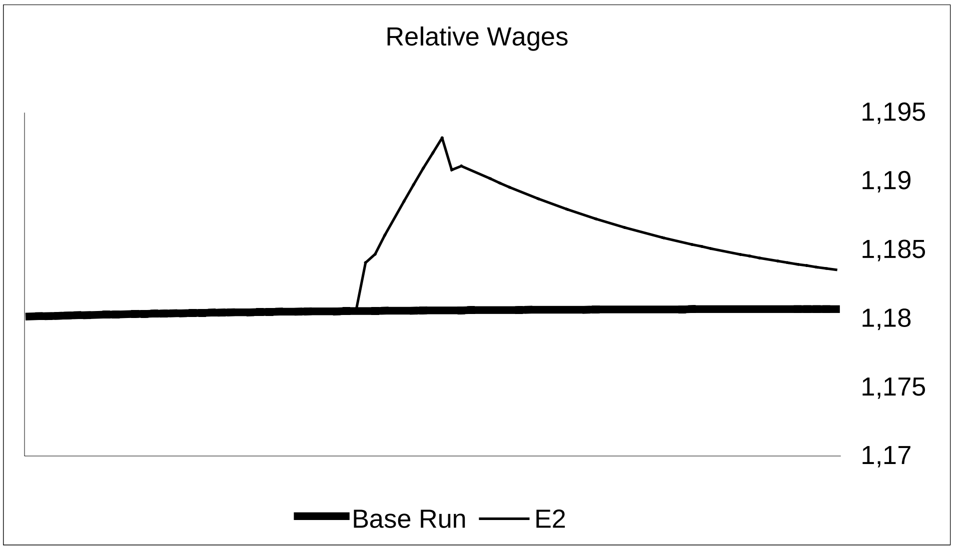


Figure 3.8:



CONCLUDING REMARKS

In this paper I have investigated the potential of a product life cycle model in providing an explanation for the skill biased technical change that has shifted labour demand towards high skilled labour over the past two decades. Combining various insights and modelling techniques I find that a temporary positive shock on the productivity of product innovations is the only reasonable candidate (in the model) for explaining the observed labour market trends.

It was argued that such a shock could occur when a new General Purpose Technology is introduced. Such a GPT would open up a wide range of potential product innovations to be exploited by the research sector. Since new products are skilled labour intensive, relative wages rise. As opportunities are developed, however, the productivity shock should be reversed and the model returns to its original steady state. Wage divergence is then temporary because the knowledge generated in the production of these new products spills over to the generation of low skilled intensive production processes and increasing relative wages reduce the value of further product innovations. The emphasis in R&D shifts (back) towards cost reducing process innovations and the relative wages return to their pre-shock levels in the long run.

As such the model revitalises the literature of product life cycles in international trade and uses the concepts and techniques developed there to understand a paradox that exists when we confront the intuitions developed in endogenous R&D driven growth theory with the observed persistent skill biases despite rising relative wages.

A useful extension of the model would be to generalise the knowledge spillover structure. In principle it is also possible to reproduce the Acemoglu (2002a) market size effect when the production structure is reformulated to allow for high and low skilled labour to be gross-substitutes. Under such a specification an increase in high skilled labour supply would yield a long run positive effect on relative wages. The productivity shock analysed here would also cause permanent effects if relative supply were allowed to respond endogenously. The model analysed in this paper, however, should be regarded as the optimistic extreme, where full spillover and gross-complementarity ensure that such effects do not emerge. The only plausible candidate for explaining the observed labour market trends left then emerges in the form of a GPT that temporarily raises one type of R&D’s productivity. A more general model would allow me to identify exactly how both explanations interact.

APPENDIX

Derivation of Research and Development labour demand equations

If we assume that our agents form expectations on the growth rate of profits we have:[35]

$$V_i(T) = \pi_i(T) E_T\left[\int_T^{\infty} e^{(G_{\pi_i}(t) - r(t))(t-T)} dt\right] = \frac{\pi_i(T)}{\rho - G_{\pi_i}} \quad \text{(A1)}$$

where:

$$G_{\pi i}(t) = E_t\left[\int_T^{\infty} \frac{d\log \pi_i(t)}{dt} dt\right] \quad \text{(A2)}$$

is the average expected growth rate in rents.

The discounted expected flow of rents in (A1) is exactly what the potential entrants are willing to pay for a product design. Once they buy the patent they can immediately start making profits. The flow of rents they receive is given by the maximand in (3) after substitution of quantities and prices we find (6):

$$\pi_i = (1-\alpha)\left(\frac{1}{\alpha\mu_i}\frac{w_i}{P}\right)^{\frac{\alpha}{\alpha-1}} \quad \text{(6)}$$

Substituting for the price index *P* and using the definition of *X* we find:

$$\begin{aligned} \pi_H &= \frac{1-\alpha}{n_H}\frac{X}{1+X} \\ \pi_L &= \frac{1-\alpha}{n_L}\frac{1}{1+X} \end{aligned} \quad \text{(A3)}$$

[35] Note that this simplification is only allowed if $\rho \geq G_{\pi}$, , which is the case for all $\rho \geq 0$ as I will show shortly.

Plugging these expressions into (A1) and subsequently into (14) and solving for H_R and H_D respectively yields equations (15) and (16).

Profit Growth Rates in the Steady State

From equation (A3) and the result that relative wages and z are constant in the steady state we can immediately obtain that X is constant in the steady state and the growth rate of profits must equal minus the growth rate of n_H and n_L for new and mature products respectively. Hence the rate of interest will always exceed the growth rate of profits. We have also seen that in a steady state these growth rates are equal and therefore so are the profit erosion rates and the rate at which future rent flows are discounted to value a new product or process.

The Steady State

Setting (19) equal to (22) yields:

$$H_P = L^* z \left(\frac{\mu_H}{\mu_L}\right)^{-1} \left(\frac{1+z}{z^{1/\beta}} \left(\frac{\mu_R}{\mu_D}\right)^{1/\beta} + 1\right)^{\frac{-1}{\alpha}} \quad \text{(A4)}$$

which can be used in (9) and solved for w_H to yield:

$$w_H = \frac{\alpha}{L^*} \frac{1}{z} \frac{X}{1+X} \frac{\mu_H}{\mu_L} \left(\frac{1+z}{z^{1/\beta}} \left(\frac{\mu_R}{\mu_D}\right)^{1/\beta} + 1\right)^{\frac{1}{\alpha}} \quad \text{(A5)}$$

This is equation PWL. Subtracting (A4) from (17) and using (21) to substitute for H_D in (17) and solving for H_R yields:

$$H_R = \frac{H^* - L^* z \left(\frac{\mu_H}{\mu_L}\right)^{-1} \left(\frac{1+z}{z^{1/\beta}} \left(\frac{\mu_R}{\mu_D}\right)^{1/\beta} + 1\right)^{\frac{-1}{\alpha}}}{1 + z^{-1/\beta} \left(\frac{\mu_R}{\mu_D}\right)^{1/\beta}} \quad \text{(A6)}$$

Now we rewrite (15) for the steady state using the fact that the growth rate of profits in the steady state is equal to minus the growth rate of n_H, which is equal to the growth rate of n, that is implicitly given as a function of H_R in (12):

$$G_{\pi_H} = -\frac{\dot{n}_H}{n_H} = -\frac{\dot{n}}{n} = -\mu_R {H_R}^{\beta} \qquad \text{(A7)}$$

Which implies that, as was stated above, the growth rate of profits is always smaller than r. Solving (15) for the high skilled wage using (A6) and (A7) yields:

$$w_H = (1-\alpha)\frac{\beta}{H_R}\frac{X}{1+X}\frac{1+z}{z}\frac{1}{\frac{r}{\mu_R}{H_R}^{-\beta}+1} \qquad \text{(A8)}$$

Substituting for H_R in (A8) using (A6) then yields the RWL:

$$w_H = \frac{\beta(1-\alpha)\frac{X}{1+X}\frac{1+z}{z}\left(1+z^{-1/\beta}\left(\frac{\mu_R}{\mu_D}\right)^{1/\beta}\right)}{H^* - L^* z\left(\frac{\mu_H}{\mu_L}\right)^{-1}\left(\frac{1+z}{z^{1/\beta}}\left(\frac{\mu_R}{\mu_D}\right)^{1/\beta}+1\right)^{\frac{-1}{\alpha}}}\left(\left(\frac{\frac{r}{\mu_R}\left(1+z^{-1/\beta}\left(\frac{\mu_R}{\mu_D}\right)^{1/\beta}\right)}{H^* - L^* z\left(\frac{\mu_H}{\mu_L}\right)^{-1}\left(\frac{1+z}{z^{1/\beta}}\left(\frac{\mu_R}{\mu_D}\right)^{1/\beta}+1\right)^{\frac{-1}{\alpha}}}\right)^{\beta}+1\right)^{-1}$$

Observe that the term in X will fall out when we equate (A8) and (A5), leaving an equation with z as the only variable on both sides. Mathematica was used to numerically establish the shape of the PWL and RWL, resulting in a robust picture presented as figure 2.

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