MERIT-Infonomics Research Memorandum series

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(revised version 11-08-2003)

2002-006

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Mutual Illusions and Financing New Technologies: Two-Sided Informational Cascades

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July 28, 2003

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Abstract

A model in which agents on both sides of the market are subject to informational cascades is examined. In an uncertain environment with asymmetric information agents tend to be overoptimistic about the state of the world, a result which fits with empirical evidence on financing new technologies. This overoptimism based on mutual illusions makes the system vulnerable to two-sided bubbles, and may be one of the reasons behind 'dot com' crash.

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JEL Codes: D83, G14, O31.
Keywords: Information, Cascades, New technologies, Venture capital.
Acknowledgements: We would like to thank the participants of the seminar at MERIT and DRUID 2002 conference in Aalborg, Denmark for their comments.

1 Introduction

Many authors writing on human behavior have noticed that individual decisions are often influenced by decisions made by others. They have documented a number of situations in which individuals prefer to follow the 'crowd', while their own feelings are against it. There is a range of different social mechanisms which may cause conformist behavior of individuals such as punishment of deviators [1], positive payoff externalities [2] and so on. It is also possible that herding arises as consequence of the bounded rationality of individuals [3].

In the last decade there has been a surge of interest in a particular kind of mechanism behind conformist behavior, which can explain *voluntary rational* 'herding'. After the seminal works of Bikhchandani, Hirshleifer and Welch [4] (BHW henceforth) and of Banerjee [5] this social phenomenon is often referred to as an 'informational cascade'.

The basic idea of informational cascades is that in certain environments where private information can be revealed only through individual actions because of information externalities, truly rational agents may find it optimal to follow the choice of others, rejecting their own information. Perhaps the most striking feature of informational cascades is that when there is noise in private information there is always a positive probability that the overall outcome will be suboptimal, i.e. agents will form a cascade in which they reject optimal actions in favor of inferior ones regardless of their private information. Therefore, information structures that are vulnerable to information cascades in this way can have negative effects on social welfare.

The original BHW model has been extended in a number of ways and the robustness of the model with respect to changes in assumptions has been examined. The informational cascade framework has been used to explain a wide range of social phenomena such as fads, fashions, medical (mal) practice, collapse of political regimes among the others (see [6] for a review). There are also some important applications of this kind of model in economics and finance. Welch [7] applied informational cascades to the IPO market and explains underpricing incentives of issuers. Avery and Zemsky [8] argued that short-run mispricing on financial markets may be a consequence of investors' herd behavior.

Most of the informational cascade models assume that the agents forming a cascade are the same either with respect to the sort of information available to them, or with respect to roles they play, or both. For instance, in Avery and Zemsky's model of financial market investors have different roles: some of them are sellers and others are buyers of assets, but the sort of information available to agents is essentially the same.

However, there is no *a priori* reason to suppose that the agents on the two sides of the market have identical information sets, and modelling some situations may require us to take into account that two sides of the market having access to different information.

Consider financing new technology. It is widely recognized today that one of the main problems of external financing in new high-tech industries is information asymmetry between firms developing new technologies and their potential investors [9]. On the one hand, financial institutions and individual investors often do not have enough expertise to judge 'state-of-the-art' technologies. On the other hand, firms working in new industries, especially new small ventures which mainly contribute to development of 'at-the-edge' technologies, have problems with evaluating both market and financial potential for the products they are developing. It seems quite reasonable to assume that investors have better knowledge of the *market perspectives* for new technologies, while firms know the *technology* with which they are working. This is an example of the situation in which information sets available to the opposite sides of the market are different.

There is also no *a priori* reason to believe that only one side of the market is subject to information cascades. In our example of financing new technology the process of acquiring information about the technology and about the market for new products is costly, and therefore has commercial value. As a consequence, agents have incentives not to share their private information and so spillovers of this kind of knowledge are limited, at least in the short term. Nevertheless, their actions, or 'outcomes' arising from the actions, in many cases are observable and it may create conditions necessary for informational cascades. This argument must be valid for both entrepreneurs and venture capitalists. Hence we may expect informational cascades on the two sides.

In this paper we examine a simple setting of two-sided informational cascades and show that taking into account both sides of the market with different information sets may generate interesting learning dynamics on both sides of the market and emphasise the conclusion about suboptimality of some information structures.

2 The model

Actors There are two populations: potential investors, who, for convenience, we call venture capitalists (VCs) and the population of entrepreneurs.

States of the World Success of a project is determined by two factors: by technology itself (e.g. quality of the product if the project involves product innovation, productivity gains if it is about process innovation), and by the market prospects for the new technology (e.g. its prospective demand). As in BHW we assume the state of technology, E, and the state of market, V, be binary variables: $E \in \{h, l\}$, $V \in \{H, L\}$, with equal prior probabilities of 1/2.

Payoffs There is a market place, where an entrepreneur meets with a venture capitalist to discuss the potential project. If any of the sides chooses to *Decline* the deal, the negotiations fail, no project takes place, and both sides stay with their reservation values, which without loss of generality are set to be zero.

If the negotiations have been successful, and project starts, then the payoffs to the contracting sides are determined by the state of the world. If both technology and market are high ($E = h$, $V = H$), the project will have success, and both parties will gain from the project. When one of the sides, say market, is low, but the technology has a great potential ($E = h$, $V = L$), then the project may still be successful. To achieve this success the entrepreneur will have to work hard, and his payoff in this case is below his reservation value, while the investor in this case will be a 'free-rider', and his payoff exceeds the reservation value. Similarly, if the technology is mediocre, but the market is high ($E = h$, $V = L$), the venture capitalist will have to put in more effort to ensure the success. In the case of both sides being mediocre ($E = l$, $V = L$), both parties have payoffs below their reservation values. We also assume that once the venture started, it is not in the interest of the agents to disrupt it. [1]

[1] For example, that might be the case if an immediate liquidation of the project would severely damage their reputation, and in this way the cost of liquidation exceeds their losses from continuation with the project.

Table 2 summarizes the payoffs. The values are chosen so that from the both sides payoffs are the same as in the BHW model.

Table 1: Payoffs (EP, VC)

	$V=H$	$V=L$
$E=h$	(1, 1)	(-1, 1)
$E=l$	(1,-1)	(-1,-1)

The agents choose their actions maximizing expected payoffs, which are based on public information and their private signals. In the case of a 'draw' i.e. when the expected payoffs from both actions are the same, we assume that agents trust more to their own intuition, and take the decision according to their private signals.[2]

Information structure We assume that entrepreneurs observe the state of technology E, while venture capitalists observe the market prospects for the new technology V; but neither do entrepreneurs know V, nor do venture capitalists know E. That is, entrepreneurs and venture capitalists have different information sets.

We model agents' subjective 'guesses' about the state of the other side of the market as private signals of limited precision. The t-th entrepreneur observes a conditionally independent identically distributed signal $v \in \{H, L\}$ about state of V, and the t-th venture capitalist gets a signal $e \in \{h, l\}$ about E. Tables 2 and 3 describe the signal probabilities ($p, q > 1/2$).

Each period t=1,2,... a pair of agents meets. The project goes ahead only if both sides agree to participate. Thus, the outcome of the negotiations is *Proceed*, or *Not Proceed.*

We consider two information structures that differ in the scope of the public information about the past. In previous-action-observable action (POA) model, as in BHW, the information about **actions** (*Agree* or *Decline*) chosen by all agents in the past is available to public, while in previous-outcome-observable action (POO) model only information about **outcomes** (*Proceed* or *Not Proceed*) of the negotiations becomes public.

[2]This kind of tie-breaking rule was employed by Anderson and Holt in their experimental study of informational cascades [10].

Table 2: Signal probabilities for entrepreneurs

	$\Pr(v = H \mid V)$	$\Pr(v = L \mid V)$
$V = H$	p	$1 - p$
$V = L$	$1 - p$	p

Table 3: Signal probabilities for venture capitalists

	$\Pr(e = h \mid E)$	$\Pr(e = l \mid E)$
$E = h$	q	$1 - q$
$E = l$	$1 - q$	q

As one might expect, and it will be shown in the next section, POA model is essentially the same as BHW. Therefore, comparison of the two models may provide us an idea of how the limitations of the public history affect the probability of 'incorrect herding'.

3 Analysis

Our analysis proceeds as follows. First, we examine a one period game with exogenously-given agents' beliefs. Then we will turn to how the beliefs in the multiperiod setting are formed under Bayesian learning in POO model. The section concludes with the general set up for the multiperiod POO model.

However, before we start to examine POO model, we will discuss POA model which is almost the same as the (POA) model in BHW. Later, when we will discuss the results of our simulations for the POO model, the POA model will be used as a benchmark.

3.1 Model with observable actions

The analyis for the 1-period game that will be developed in the next section can be applied to the model with publicly observable actions. However, analysis of POA can be made with much more simpler means by analogy with BHW.

Notice that the payoff to an agent is entirely determined by the state of the other side, and does not depend on the state of the world on his side, as seen in the payoff matrix, Table 1. Entrepreneurs only care about V, and venture capitalists are interested only in E. It follows that once an agent, say an entrepreneur, has an opportunity to observe actions of the entrepreneurs that are driven by their feelings about V, and the actions chosen by the other side has no value for him since venture capitalists' actions depends on venture capitalists beliefs about E which is of no interest for an entrepreneur. Due to this in POA model we have two sides that lock into one-sided cascades independently.

The only thing we have to be cautious about is that a cascade in actions does not necessarily mean a cascade in outcomes, which is our primary interest. While a DOWN cascade on any side (as it is defined in BHW) always results in an infinite negative series in the outcomes, an UP cascade on one side may not result in a never ending series of positive outcomes yet, since the other side may be declining the offers. Therefore, we can say that a DOWN cascade in two-sided POA model happens when either or both of the sides rejects the deals regardless of signals. Two-sided UP cascade happens when *both* sides agree whatever is their private information.

One can easily find that as in BHW one of the sides starts a cascade when the agents on this side receive two (or three) similar signals in row.[3] If signals are negative a DOWN cascade emerges, if they are positive an UP cascade arises. Two signals of different signs cancel each out, and the following agent finds himself in the same situation as the agent two periods before him, e.g. if $v_0 = H$, $v_1 = L$, then the third entrepreneur has the same prior belief about V as the first one. The probability for entrepreneurs to be locked in UP cascade is

$$\Pr(EP \ in \ UP \ cascade) = \frac{p_V^2}{1 - 2p_V + 2p_V^2}.$$

where $p_V = \Pr(v = H|V)$ (Table 2). Similarly for venture capitalists' UP cascade

$$\Pr(VC \ in \ UP \ cascade) = \frac{q_E^2}{1 - 2q_E + 2q_E^2}.$$

where $q_E = \Pr(e = h|E)$ (Table 3).

[3] This is given the tie-breaking rule we use. BHW use different tie-breaking rule and have somewhat different results including the formula for the probability of a cascade.

Since a two-sided UP cascade is nothing more than two simulataneous independent UP cascades on each side, the probibility to end up in two-sided UP cascade is

$$\Pr(\textit{two-sided UP cascade}) = \frac{p_V^2 q_E^2}{(1 - 2q_E + 2q_E^2)(1 - 2p_V + 2p_V^2)}. \tag{1}$$

since the system in BHW model converges to one of the cascades with probability 1, the probability of two-sided DOWN cascade is

$$\Pr(\textit{two-sided DOWN cascade}) = 1 - \frac{p_V^2 q_E^2}{(1 - 2q_E + 2q_E^2)(1 - 2p_V + 2p_V^2)}. \tag{2}$$

The ease with which we manage to examine the POA model comes from the payoff matrix that we choose to be similar to payoffs in BHW model, and from the fact that before an information cascade arises agents can infer (from the actions) the private signals their predecessors received.

The latter is not the case in the POO model. From a negative outcome an outside observer cannot infer actions and therefore the private signals that the pair received.

3.2 Model with observable outcomes

3.2.1 One-period game

Consider a pair of entrepreneur and venture capitalist who are to decide upon setting up a project. For the moment we assume that their beliefs (based on their private signals and the public history of previous negotiations) are exogenously given.

We use a static Bayesian game with 4 types of players on each side to analyse agents' decisions. The type of a player in this game is determined by the state of the market on his side (E for entrepreneurs, V for venture capitalists), and the private signal he receives (v or e).

Let P_{Ev} be the probability that the market is in a high state ($V = H$), given that the state of the technology is E, and he has received signal v (i.e. his type is Ev). Similarly, Q_{Ve} is venture capitalist's belief that $E = h$, when the state of the market for the new technology is V and the private signal about technology is e. We require rather natural conditions on the beliefs: $P_{EH} \geq P_{EL}$, $Q_{Vh} \geq Q_{Vl}$, i.e. a positive signal strengthens (at least does not weaken) one's belief that the other side is in the favourable state. The probability for a venture capitalist to get positive signal conditional on that the state of the technology is E will be denoted as q_E, and the conditional probability for the entrepreneur to receive a positive signal is p_V.

The signals are independent, hence, for example, the belief of an entrepreneur of type Ev that he is playing with the venture capitalist of type Lh is $(1 - P_{Ev})q_E$. A venture capitalist's belief that his opponent belongs to the type hL is $Q_{Ve}(1 - p_V)$, given that VC's type is Ve and so on.

Each of player has a choice between two actions (*Agree* or *Decline*). The normal form of the game is presented in Table 4 (letters A and D stand for players' actions: *Agree* or *Decline*).

Table 4: 1-period game in the normal form

<table>
<tr><td colspan="3" rowspan="3">(EP, VC)</td><td colspan="4">P_{Ev}</td><td colspan="4">$1 - P_{Ev}$</td></tr>
<tr><td colspan="2">$q_E P_{Ev}$
Hh</td><td colspan="2">$(1 - q_E)P_{Ev}$
Hl</td><td colspan="2">$q_E(1 - P_{Ev})$
Lh</td><td colspan="2">$(1 - q_E)(1 - P_{Ev})$
Ll</td></tr>
<tr><td>A</td><td>D</td><td>A</td><td>D</td><td>A</td><td>D</td><td>A</td><td>D</td></tr>
<tr><td rowspan="4">Q_{Ve}</td><td rowspan="2">$p_V Q_{Ve}$
hH</td><td>A</td><td>(1, 1)</td><td>0</td><td>(1, 1)</td><td>0</td><td>(-1,1)</td><td>0</td><td>(-1, 1)</td><td>0</td></tr>
<tr><td>D</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td rowspan="2">$(1 - p_V)Q_{Ve}$
hL</td><td>A</td><td>(1, 1)</td><td>0</td><td>(1, 1)</td><td>0</td><td>(-1,1)</td><td>0</td><td>(-1, 1)</td><td>0</td></tr>
<tr><td>D</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td rowspan="4">$1 - Q_{Ve}$</td><td rowspan="2">$p_V(1 - Q_{Ve})$
lH</td><td>A</td><td>(1,-1)</td><td>0</td><td>(1,-1)</td><td>0</td><td>(-1,-1)</td><td>0</td><td>(-1,-1)</td><td>0</td></tr>
<tr><td>D</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td rowspan="2">$(1 - p_V)(1 - Q_{Ve})$
lL</td><td>A</td><td>(1,-1)</td><td>0</td><td>(1,-1)</td><td>0</td><td>(-1,-1)</td><td>0</td><td>(-1,-1)</td><td>0</td></tr>
<tr><td>D</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>

Now we will examine what are (Bayes-Nash) equilibria in this game for different sets of beliefs. We will limit our analysis, considering only equilibria in the pure strategies.

Equilibria Player's strategy (decision rule) in a Bayesian game is a set of the actions for all types of the player. There are 16 strategies available for each player in our game. To denote them we will extend notations of Table 4.

In this notation the entrepreneur's strategy is a vector $(a_{hH}, a_{hL}, a_{lH}, a_{lL})$ where $a_{hH} \in \{Agree,\ Decline\}$ is the action played when an entrepreneur is of type hH, a_{hL} if he is of type hL and so on. Similarly vector $(b_{Hh}, b_{Hl}, b_{Lh}, b_{Ll})$ stands for venture capitalist's strategy such that a venture capitalist chooses action b_{Hh} when he is of type is Hh and so on.

A Bayes-Nash equilibrium (in pure strategies) for a Bayesian game is the strategy profile such that each of the players chooses a best response to the conditional distribution of his opponents' strategies *for each type that he may belong to.*

Table 5 presents payoffs to each type of entrepreneur. Since an agent gets zero payoff if he *Decline*s, we need only consider payoffs to the action *Agree* when played against different strategies of his opponent. For every type of the entrepreneur if the expected payoff of *Agree* is negative he must play *Decline*, and *Agree* whenever it is positive. In the case of a 'draw' (i.e. zero expected payoff),according to our tie-breaking rule, we assume that one follows his private signal: *Agree* if the signal is favourable, ans *Decline* otherwise.

Similarly, Table 6 presents VC's expected payoffs if he plays *Agree* against different strategies of his opponent.

Examining of Tables 5 and 6 one might note that,

- Strategies $(*, *, D, A)$ and $(D, A, *, *)$, where '$*$' stands either action *Agree* or *Decline*, should not be played in an equilibrium (which is rather natural: why would one *Decline* the deal when he receives a favourable signal, and nevertheless *Agree* if the signal were negative?). Indeed, by our assumption about beliefs: $P_{EH} \geq P_{EL}$ and $Q_{Vh} \geq Q_{Vl}$. Therefore according to Tables 5 and 6 the expected payoff of *Agree* to an agent, say, to an entrepreneur of type EH is greater or equal, than to one of type EL. Thus that if an entrepreneur of type EL plays *Agree*, so does the one of type EH. In the case of $P_{EH} = P_{EL}$ or $Q_{Vh} = Q_{Vl}$, the tie-breaking rule applies.[4]
- For any set of beliefs there will be 'status quo' equilibria ($DD**$,$DDDD$) and ($DDDD$,$DD**$). Indeed, on the one hand, if my opponent is playing $DDDD$, I would get a zero payoff whatever strategy I will use. On the other hand, if I am playing $DD**$, then the payoff of my opponent is negative (or zero) whatever is his

[4] To compress the notation, in what follows we will express the strategy vector as a 4-digit string of As, Ds, and $*$s where $*$ is a place holder meaning that either A or D could be played.

type, and would therefore be better off refraining from the deal.

- Whenever there is non-zero probability that the other side is in a favourable state, strategy $AAAA$ is the only best response to $A * DD$. Otherwise, one should play $ADAD$ or $ADDD$ in response to $A * DD$.

Note, that although 'status quo' equilibria exist for any set of beliefs, whenever the beliefs are so that there is another equilibrium, hold, 'status quo' equilibria are not *regular*, that is they are very sensitive to small perturbations of the payoffs. For this reason we will not consider those equilibria in the following analisis.

For the similar reason we will ignore equilibria ($ADAD$,$A*DD$) and ($A*DD$,$ADAD$). Those equilibria require that one of the sides has to be sure that the other side of the market is weak. Hence whatever startegy the other side is playing the expected payoff is non-positive. Due to that fact, those equilibria are not stable with respect to 'trembling hand': if the other side is playing $**A*$ or $***A$ instead of $**DD$ with any small but non-zero probability, then the expected payoff is negative and the player should *Decline* regardless to his signal.

Taking into account these remarks there are only 20 possible equilibria in our game that may result in a positive outcome. The conditions on the beliefs necessary for these equilibria are listed in the Table 9. We will asume that once the conditions for any of these equilibria hold this equilibrium will be played (if there are several possibile equilibria, players will toss a coin). If neither of the conditions hold then an equilibrium with one of the sides playing $DDDD$ strategy will be played (the result would be a negative informational cascade).

So far, we have treated agents' beliefs as exogenously given. Now we will discuss how the beliefs are formed, but before that we will redefine conditions for equilibria (Table 9) in the terms of 'likelihood ratios'.

Likelihood ratios Let P_E^t be entrepreneur's belief that $V = H$ *prior* to the private signal, given that the state of the technology $E \in \{h, l\}$. Similarly, Q_V^t will be venture capitalist's belief that $E = h$ before he receives his private signal, conditional on V. We can define "likelihood ratios" A_t, B_t, C_t, and D_t as

$$P_h^t = \frac{1}{1+A_t}, \qquad Q_H^t = \frac{1}{1+C_t},$$
$$P_l^t = \frac{1}{1+B_t}, \qquad Q_L^t = \frac{1}{1+D_t}.$$

At time $t = 0$, by assumption $P_h^0 = P_l^0 = Q_H^0 = Q_L^0 = \frac{1}{2}$, Therefore, initial values for the likelihood ratios are $A_0 = B_0 = C_0 = D_0 = 1$.

Let us denote history up to time t as I_t. According to Bayes' formula we can write P_h^t as

$$\Pr(V = H|I_t) = \frac{\Pr(I_t|V = H)\Pr(V = H)}{\Pr(I_t|V = H)\Pr(V = H) + \Pr(I_t|V = L)\Pr(V = L)} = \frac{1}{1 + \frac{\Pr(I_t|V=L)\Pr(V=L)}{\Pr(I_t|V=H)\Pr(V=H)}}$$

Given that at time $t = 0$, priors $\Pr(V = H)$ and $\Pr(V = L)$ are equal it follows that

$$P_h^t = \frac{1}{1 + \frac{\Pr(I_t|V=L)}{\Pr(I_t|V=H)}}.$$

Comparing this expression with the definition of A_t we find that

$$A_t = \frac{\Pr(I_t|V = L)}{\Pr(I_t|V = H)}.$$

in other words, A_t describes how well the history I_t can be explained with two possible alternatives $V = L$ or $V = H$.

Remark: From the definition of A_t one can also find that

$$A_t = \frac{1 - P_h^t}{P_h^t} \equiv \frac{1 - \Pr(V = H|I_t)}{\Pr(V = H|I_t)} = \frac{\Pr(V = L|I_t)}{\Pr(V = H|I_t)}.$$

Posteriors in likelihood ratios Let $E = h$. Consider an entrepreneur, his prior is P_h^t, and the corresponding likelihood ratio A_t. Suppose that he receives private signal $v = H$, i.e. entrepreneur's type is hH. According to Bayes' formula the posterior is

$$P_{hH}^t \equiv \Pr(V = H|I_t, v = H) = \frac{\Pr(v = H|I_t, V = H)\Pr(V = H|I_t)}{\Pr(v = H|I_t, V = H)\Pr(V = H|I_t) + \Pr(v = H|I_t, V = L)\Pr(V = L|I_t)} = \frac{1}{1 + \frac{\Pr(v=H|I_t,V=L)\Pr(V=L|I_t)}{\Pr(v=H|I_t,V=H)\Pr(V=H|I_t)}}.$$

Since the private signal v does not depend on the history and is determined only by the market prospects of the technology, V (see Table with signal probabilities), and taking into account the Remark above this expression can be rewritten as

$$P^t_{hH} = \frac{1}{1 + A_t \frac{1-p}{p}}$$

Similarly to the case of the hH we can write down posteriors for other types of entrepreneurs

$$P^t_{hH} = \frac{1}{1 + A_t \frac{1-p}{p}}, \qquad P^t_{lH} = \frac{1}{1 + B_t \frac{1-p}{p}},$$
$$P^t_{hL} = \frac{1}{1 + A_t \frac{p}{1-p}}, \qquad P^t_{lH} = \frac{1}{1 + B_t \frac{p}{1-p}}.$$

We can also write down posteriors for venture capitalists' beliefs

$$Q^t_{Hh} = \frac{1}{1 + C_t \frac{1-q}{q}} \qquad Q^t_{Lh} = \frac{1}{1 + D_t \frac{1-q}{q}},$$
$$Q^t_{Hl} = \frac{1}{1 + C_t \frac{q}{1-q}} \qquad Q^t_{Ll} = \frac{1}{1 + D_t \frac{q}{1-q}}.$$

Equilibria in likelihood ratios Let us consider an example. Equilibrium ($ADAD$,$AAAD$) requires

$$P_{hH} \geq \frac{q}{1+q} \geq P_{hL}, \qquad Q_{Hh} \geq Q_{Hl} > \frac{1}{2},$$
$$P_{lH} \geq \frac{1-q}{2-q} \geq P_{lL}, \qquad Q_{Ll} \geq \frac{1}{2} \geq Q_{Ll}.$$

Applying the expressions for posterior beliefs for hH and hL via likelihood ratio A_t the first of the inequalities can be rewritten as

$$\frac{1}{1 + A_t \frac{1-p}{p}} \geq \frac{1}{1 + \frac{1}{q}} \geq \frac{1}{1 + A_t \frac{p}{1-p}}$$

or

$$A_t \frac{1-p}{p} \leq \frac{1}{q} \leq A_t \frac{p}{1-p} \Leftrightarrow \frac{1-p}{p} \leq q A_t \leq \frac{p}{1-p}.$$

In the same way we can obtain the conditions for B_t, C_t, and D_t.

Table 10 sums up the conditions for the equilibria in 1-period game in terms of the likelyhood ratios.

3.2.2 Beliefs updating

After one-period has been played the result of the game becomes public knowledge. Now we turn to how this information can be integrated into agents' beliefs.

Let the state of the technology be high, $E = h$. Suppose, that at time t the result of the negotiations is *Result* $\in\{$*Proceed, Not Proceed*$\}$. Consider an entrepreneur, who is to update his belief that $V = H$, P_h^t, given that his prior belief is

$$P_h^t \equiv \Pr(V = H|\text{history by time } t, E = h) = \frac{1}{1 + A_t},$$

According to Bayes' formula P_h^{t+1} (conditional on the *Result*), is

$$\Pr(V = H|Result) = \frac{\Pr(Result|V = H, E = h)P_h^t}{\Pr(Result|V = H, E = h)P_h^t + \Pr(Result|V = L, E = h)(1 - P_h^t)},$$

which can be rewritten (under assumption that $\Pr(Result|V = H)P_h^t \neq 0$)as

$$P_h^{t+1} = \frac{1}{1 + \frac{\Pr(Result|V=L,E=h)}{\Pr(Result|V=H,E=h)} \frac{1-P_h^t}{P_h^t}},$$

or

$$\frac{1}{1 + A_{t+1}} = \frac{1}{1 + \frac{\Pr(Result|V=L,E=h)}{\Pr(Result|V=H,E=h)} \cdot A_t}.$$

Finally, the formula for beliefs updating (in terms of the likelyhood ratios) has the form

$$A_{t+1} = A_t \cdot \frac{\Pr(Result|V = L, E = h)}{\Pr(Result|V = H, E = h)}. \tag{3}$$

Similarly, we can write down the rules for updating probabilities P_l^t, Q_H^t, and Q_L^t, in terms of likelyhood ratios B_t, C_t, and D_t respectively.

$$B_{t+1} = B_t \cdot \frac{\Pr(Result|V = L, E = l)}{\Pr(Result|V = H, E = l)}, \tag{4}$$

$$C_{t+1} = C_t \cdot \frac{\Pr(Result|V = H, E = l)}{\Pr(Result|V = H, E = h)}, \tag{5}$$

$$D_{t+1} = D_t \cdot \frac{\Pr(Result|V = L, E = l)}{\Pr(Result|V = L, E = h)}. \tag{6}$$

Example

Suppose that an entrepreneur knows that at time t equilibrium ($ADAD$, $ADAA$) has been played, and the outcome at time t is *Proceed*. How the entrepreneur should update his beliefs?

E=h, Result=Proceed The outcome is *Proceed* may have happen only if both parties had played *Agree*.

Suppose that $V = H$. Then both entrepreneur and venture capitalist at time t must have received positive signals, $v = H$ and $e = h$, respectively. Therefore,

$$\Pr(Proceed|V = H, E = h) = \Pr(v = H, e = h|E = h, V = H) =$$
$$\Pr(v = H|V = H)\Pr(e = h|E = h) = p \cdot q.$$

Suppose that $V = L$. Once again, the entrepreneur must have received positive signal, $v = H$. However, in contrast with $V = H$, in this case the venture capitalist chooses *Agree* regardless to his private signal.

$$\Pr(Proceed|V = L, E = h) = \Pr(v = H, e = any|E = h, V = L) =$$
$$\Pr(v = H|V = L) = 1 - p.$$

Now we are ready to apply (3). The updating rule is

$$A_{t+1} = A_t \cdot \frac{1-p}{p \cdot q}$$

E=l, Result=Proceed What would be different in the analysis above if $E = l$ instead of $E = h$? Note, that regardless of value of E (h or l) the entrepreneur's decision rule (at t) stays the same, he follows his signal (his strategy is $ADAD$). The value of E does not affect the venture capitalist either, since he does not know what is the true E anyway. The only thing that is going to change is the probabilities of the signals about state of the technology, e. Practically, it means that in the previous formula we should substitute q for $(1 - q)$. Then the beliefs will be updated according to

$$B_{t+1} = B_t \cdot \frac{1-p}{p \cdot (1-q)}$$

Now let them play the same equilibrium ($ADAD$, $ADAA$), but suppose that the outcome at time t happend to be *Not Proceed*. How should entrepreneurs update their beliefs?

E=h, Result=Not Proceed

$$\Pr(NotProceed|V = H, E = h) = 1 - \Pr(Proceed|V = H, E = h) =$$
$$1 - p \cdot q,$$
$$\Pr(NotProceed|V = H, E = l) = 1 - \Pr(Proceed|V = H, E = l) = p,$$
$$A_{t+1} = A_t \cdot \frac{p}{1 - p \cdot q}$$

E=l, Result=Not Proceed Substituting q for $1 - q$ in the formula above we get

$$B_{t+1} = B_t \cdot \frac{p}{1 - p \cdot (1 - q)}$$

In the same way, using the formulas (3)-(6) we can get updating rules for entrepreneurs' and venture capitalists' beliefs for other equilibria. Those rules are listed in Table 11.

3.2.3 General set up

As in BHW we start with prior probabilities (P_E^0 and Q_V^0) of 1/2, or in terms of the likelihood ratios $A_0 = B_0 = C_0 = D_0 = 1$. This corresponds to the equilibrium ($ADAD$,$ADAD$), which we can call 'follow-your-signal' equilibrium, since the players choose *Agree* when they receive a positive signal, and *Decline* if the signal is negative.

Once the signals are received they choose their actions according to their strategies. If both players *Agree*, the result will be positive, otherwise it will be negative. The result of the negotiations will be used by the followers to update their beliefs according to the belief update rules listed in Table 11. Updated beliefs will be used to find the equilirium/equilibria to be played by the next pair etc.

If there are several equilibria that may take place for a given set of beliefs we assume that the equilibrium to be played is determined randomly (each equilibrium has the same chances to be played) and which equilibrium is played will be known to public.

Informational cascades

BHW define an informational cascade as the situation where individual actions does not depend on the private signals. Once it happens the actions of individuals do not reveal any new information to the followers, hence the followers will find themselves in the same situation as their predecessors, therefore, they should ignore their private signals as well. As an informational cascade starts, an observer will see a sequience of uniform outcomes either positive (UP cascade) or negative (DOWN cascade).

What would be an informational cascade in our two-sided setting?

In the terms of the strategies in one-period game ignorance of a private signal means that one uses $**AA$ or $**DD$, if the state of the world is *low* on his side, and $AA**$ or $DD**$, if it is *high* (that is one-sided cascade in BHW model). However, that might not be enough to obtain a sequience of uniform outcomes (which is the primary interest in these models).

There is a difference between the emergence of UP and DOWN cascades in our two-sided model. The difference arises due to the asymmetry in how *Agree* and *Decline* actions are translated into outcomes.

DOWN cascade Once one of the sides starts to reject the deals regardless of their signals, the outcomes will be negative. Indeed, while the opposite side might still change their beliefs, the side that is rejecting the deals has nothing to learn from the results of the negotiations: the outcomes would be negative whatever are the private signals and the actions chosen by the other side. As a result, all followers on this side will find themselves in the same situation (with the same beliefs) as their predecessors, and should reject deals regardless to their signals. Therefore, we can say that one-sided DOWN cascade ($**DD$ or $DD**$) would lead to two-sided DOWN cascade (a never ending series of negative outcomes). Since we know that outside of the regions defined by conditions in Table 9 only equilibria with one of the sides playing $DDDD$ exist, once the beliefs leave those regions we can say that two-sided DOWN cascade emerges.

UP cascade The things are diferent for UP cascades. There are two reasons for that. First, though one of the sides may stick to *Agree* actions, the outcomes might still be negative, if the other side is not in an UP cascade. It would mean that the other side is still learning, and unlike in the case of DOWN cascades this learning may change the story.

For example, consider the case of $E = h$, $V = H$ and assume that ($AAAA$, $ADAD$) equilibrium is played. Although entrepreneurs are in an UP cascade, venture capitalists are not. Beliefs of entrepreneurs do not change, while venture capitalists' beliefs that $E = h$ strengthen when the outcome is positive or weaken when they see a failure of negotiations (see update rules 11). Therefore, an unlucky sequence of negative signals to venture capitalists' might result in that venture capitalists will be locked in a negative cascade, hence to two-sided DOWN cascade.

Second, we have the striking possibility that even if both sides are in a positive one-sided cascade, it may still not be enough for a two-sided UP cascade. Consider an example where $E = l$, $V = L$ and ($ADAA$, $ADAA$) is played. Both sides are playing the strategies which assign them to play *Agree* regardless of their signals. Still this is not an UP cascade. Players on each of the sides hope that the state of the world on the other side is *high*. In this case the $ADAA$ decision rule implies that the other side follow its signals. Hence negative outcomes are expected from time to time, but what agents see is a long lasting series of positive outcomes. It must lead them to the suspicion that the other side is playing not AD but AA. This would imply that the true state is *low*, and consequently they might end up in a DOWN cascade instead of an UP cascade. One can also see this from the updating rules for ($ADAA$, $ADAA$) equilibrium.

We can also remark that the latter example demonstrates what we call gradual 'revelation'. One can note that ($ADAA$, $ADAA$) is a separating equilibrium, where the behaviour of the player depends on the state of the world on his side, and with the time one can infer what is the state of the world on the other side, not from the behaviour of rivals from his own side as in BHW, but from the behaviour of the other side. Revelation does not always have to be gradual, e.g. in equilibrium ($AAAA$,$AAAD$) a negative outcome would immediately lead entrepreneurs to the conclusion that the prospective market for their technology is weak ($V = L$).

Thus for a two-sided UP cascade we have to require from the equilibrium strategy profile that both sides and all types of the players on each side choose to *Agree* regardless of their signals. The only equilibrium that results in an infinite sequence of positive outcomes is ($AAAA$, $AAAA$), which therefore we call an UP cascade in our two-sided POO model.

One remark on the our implementation of the simulations. For some p and q the region of equilibrium ($AAAA$, $AAAA$) may overlap with one of equilibrium ($ADAA$, $ADAA$). In this overlap we choose the equilibrium to play randomly. But since ($AAAA$, $AAAA$) does not change beliefs we will assume that ($ADAA$, $ADAA$) is always played, and UP cascade happens only if there is no equilibrium (excluding 'status quo' equilibria) other than ($AAAA$, $AAAA$).

4 Simulations

As do BHW, we can also compare information regimes with different degrees of limitation on information available to the public. Under *full information*, where E and V are known to the public, every agent chooses correct actions. In the *previous-signals-observable* (PSO) regime agents do not know the state of the opposite side, but can observe the signals their predecessor received. As a result, public information becomes more and more precise, and soon the system will converge to the optimal outcome. In the *previous-actions-observable* (POA) regime, where signals are kept private, but agents can observe actions of their predecessors, informational cascades appear, and there is always non-zero probability that the system may end up in the inferior equilibrium. Now the question to be answered is whether further restrictions on public information as POO regime will further decrease social welfare by increasing the probability of a suboptimal outcome?

We have simulated the POO model for p and q ranging from 0.5 to 1 and have estimated the probability of two-sided cascades. For each values of p and q (on a grid with step size of 0.01) we made 10000 runs. In all series the system converges to one of the cascades. The probability of a cascade was estimated as the share of realizations in our simulations that have been locked into that cascade. We use POA model as benchmark, probabilities of cascades in the POA model are given by equations (1) and (2). We found no qualititive differences in the results, neither for a higher number of runs, nor for finer grids.

Results of simulations

Our primary interest is the probability of informational cascades that lead to convergence to suboptimal choice. For this reason, we will constrain ourselves to two cases: ‘negative’ cascades when $E = h$ and $V = H$, which is an ‘unjustified crunch’, where wide-spread negative feeling about the state of the world, resulting from the chain of the unlucky events, inhibits the diffusion of the good technologies; and ‘positive’ cascade when $E = l$ and $V = L$, which is a socially undesirable ‘two-sided bubble’.

Case *E=h, V=H* The difference between the probailities of ‘incorrect’ cascade (DOWN cascade in this case) in POO model estimated from our simulations and one of ‘benchmark’ POA model is presented in Figure 1.

Notice that for most values of p and q (white region in the density plot) the probability to be locked in the incorrect cascade is higher under POO regime than under POA regime (statistically significant). As private precision of the signals (p and q) is increasing, the difference levels out.

Another interesting feature of the Figure 1 is the deep ‘valley’ at small p and q (< 0.66). The depth of the valley is about 20%. In contrast with what has been discussed above, in the valley the probability to be locked in DOWN cascade is lower under POO regime, than under POA regime.

There are also two steps by the sides of the Figure 1. In the depth of those steps, as in the valley, the probability of DOWN cascades is also lower under POO than under POA regime.

Case *E=l, V=L* Figure 2 shows the difference in the probability of UP cascade under POO and under POA regimes.

As in the previous case, the probability of incorrect cascade (UP cascade in this case) in POO model exceeds one of POA model (about 5% at the top of the hill at $p = 0.5$ and $q = 0.5$). Now, it does so for all p and q.

As in the case of $E = h$, $V = H$, the difference in the probabilities of UP cascades for small values of p and q is substantially higher (‘island’ at p, $q < 0.66$) and it is decreasing with rising p and q.

5 Discussion

As we have anticipated, results of our simulations for the POO model go in line with the result of BHW: restrictions on the information available to public reduce social welfare (in terms of the probability of suboptimal outcome), especially when the private information is only an inferior substitute to public.

The other conclusion from our simulations is rather striking: under the POO regime the agents seem to be 'overoptimistic': when the quality of the private signals is low the system is more prone to UP cascades.

To understand why there is 'overoptimism' in our models let us consider 'minimum series': the shortest series of outcome which result in a cascade. In the POA model similarly to BHW, a cascade starts when the two first pairs receive their positive (for UP cascade) or negative (for DOWN cascade) signals independent of p and q. In terms of the equilibria of the one-period game it means that starting from the third pair we move from ($ADAD$,$ADAD$) to ($AAAA$,$AAAA$) or to ($DDDD$,$DDDD$).

What might be different in POO model?

Notice that when ($ADAD$,$ADAD$) is played, an agent observing a positive outcome unambigiously infers that the players must have received positive signals, i.e. a positive outcome is as informative as the private signals. It means that in the POO model as in POA, two positive outcomes should be enough to start an UP cascade.

However, observing a negative outcome one cannot unambigiously conclude which side rejected the deal, i.e. a negative outcome is less informative than corresponding actions (and private signals). Moreover, the amount of information that an agent can extract from a negative outcome depends on the accuracy of private signals on the other side as well as on the state of the world on his side.

Let $E = h$, and consider an entrepreneur who observes a negative outcome. He wants to know V, therefore he is interested which signal the entrepreneur in that pair received. If the quality of the private information on the other side, q is high, then it is less likely that the venture capitalist received a negative (wrong) signal, thus the deal has to be rejected by entrepreneur, who must have got a negative signal. If, on the other hand, q is low, so it is quite probable that venture capitalist got negative signal, then the outcome is not very informative to entrepreneurs. Thus we might expect that low precision of private information makes agents more tolerant to negative outcomes.

Where lies the border between 'high' and 'low' the precision of the signals? To answer the question we should examine the minimum series. Suppose that the first two deals have failed. Should the third pair follow their signals or join the 'herd'? Using the updating rules for $(ADAD, ADAD)$, and the equilibrium conditions, we can find p and q for which the strategy profile $(ADAD, ADAD)$ remains the equilibrium, that is, for which a cascade has not yet started, and conditions for which a cascade will certainly start ($A_2 \geq \frac{p}{1-p}$ and $C_2 \geq \frac{q}{1-q}$). The two conditions are given by:

$$\left(\frac{1-(1-p)q}{1-pq}\right)^2 \geq \frac{p}{1-p}, \qquad \left(\frac{1-p(1-q)}{1-pq}\right)^2 \geq \frac{q}{1-q} \tag{7}$$

(the conditions on B_t and D_t corresponding to $E = l$ and $V = L$ are not binding in this case). Figure 3 represents the inequalities in equation (7) graphically. The first inequality is represented by the light grey line labelled 1; the second by the dark grey line labelled 2. Below line 1 and left of line 2, the strategy $(ADAD, ADAD)$ remains the equilibrium even after two failures. Outside that area, after two failures a DOWN cascade surely starts.

Examining one other sequence helps to understand the landscapes of Figures 1 and 2. Consider that in the first four meetings there are two negative and two positive outcomes, and that currently the equilibrium strategy is $(ADAD, ADAD)$. Applying again the updating rules and the equilibrium conditions permits us to determine the conditions under which the next equilibrium strategy is $(ADAD, AAAA)$. This is the case in the region above the black line (labelled 3) in Figure 3, and here an UP cascade starts on one side. By contrast, for (p, q) lying between lines 1 and 4 in the figure, the next equilibrium in strategies is $(ADAD, ADAA)$ and no cascade has yet begun ($E = h$, $V = H$).

These minimum sereies results connect well with the results of the simulations. The left diagram in Figure 1 combines a contour plot for the case $E = h$, $V = H$ and lines of Figure 3. As one can see, the valley falls exactly in the range of the values for which after two negative outcomes the players are still following their signals. The two steps of Figure 1 are located in the area where two negative and two positive outcomes result in $(ADAA,AAAA)$ or $(AAAA,ADAA)$ equilibria and so on.

As one can see the main feature of Figure 2, the 'island' at small p and q, can also be explained by the minimum series. What is the intuition here?

One can find that after two negative outcomes neither B_2, nor D_2, which are the likelihood ratios of the agents when $E = l$ and $V = L$, are large enough to abandon conditions for 'follow-your-signal' equilibrium ($ADAD$,$ADAD$). Why, then, for p and q lying outside the island does a DOWN cascade start? As we already know, outside the island $A_2 \geq \frac{p}{1-p}$ and $C_2 \geq \frac{q}{1-q}$ and therefore if one of the sides of the market had been in the high state it would have played $DD**$. The only reasonable reply to this strategy would be a rejection of any deal: $DDDD$. As a result, a DOWN cascade emerges. On the contrary, inside the island the agents continue playing ($ADAD$,$ADAD$) and there is still a chance to be locked into an UP cascade.

Thus, no matter what is the true state of the world in the POO model, in the situation where the quality of the private information is low, the agents tend to be 'overoptimistic' as compared with the POA regime. They put relatively more weight on positive outcomes and less weight on negative ones.

The region of low values of p and q is of particular interest if financing new technology is concerned. The degree of 'asymmetry' of information in this case is high, because investors do not know the technology well, nor do entrepreneurs have knowledge of the market. Overoptimistic bias of individuals in an uncertain environment is well-known in the field of cognitive science. There are studies documenting overconfidence of among entrepreneurs [11] and venture capitalists [12]. Not contesting explanations of this phenomenon from the point of view of cognitive phychology, we conjecture that two-sided interaction and information constraints discussed in this paper may contribute to overconfidence of the agents in real economy.

The recent 'dot com' crash rises questions why the market overvalued many 'new economy' companies with immature products which had vague market perspectives. As we have seen, low quality of information (precision of private signals and incompleteness of information in public domain) results in a overoptimistic bias in interpretation of the history: successful deals get more weight than failures, and agents tend to overvalue performance of their counterparts. Overoptimism based on mutual illusions makes the system more vulnerable to two-sided 'high-tech bubbles'.

Summarizing, we conclude with the following. First, dynamics of two-sided cascades in information structure where only the history of outcomes (rather than history of predecessors's actions) are observable is non-trivial and can be characterized by interactions between the two sides of market arising from learning. In comparison with POA model, where actions are public information, in POO model, although with some exceptions, the probability to end up in socially inferior cascade is higher. Second, in the situation where precision of the private signals is low for both of the sides of the market, agents tend to be 'overoptimistic' about the state of the world.

6 Conclusions

We examine a model in which agents on the both sides of a market have different information sets and are subject to information cascades. We assume some restrictions on available information: instead of observing actions of their predecessors as in one sided information cascade models, agents observe only successes or failures of negotiations. The changes in the information structure lead to increasing probability of locking in socially inferior informational cascade. The results support the general conclusion that can be drawn from literature about information cascades: information structure does matter, and the more restrictions on publicly available information are imposed, the higher is the probability that collective behavior will be suboptimal. Another finding of the paper is that in uncertain environment agents tend to be overoptimistic about the state of the world, which fits with results of empirical studies of financing new technologies. Overoptimism based on mutual illusions makes the system vulnerable to two-sided "high-tech" bubbles, and may be one of the reasons behind "dot com" crash.

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Table 5: EP's expected payoffs, if he chooses *Agree*

VC's strategy	***Agree, E=h***		***Agree, E=l***	
	hH	***hL***	***lH***	***lL***
AAAA	$2P_{hH}-1$	$2P_{hL}-1$	$2P_{lH}-1$	$2P_{lL}-1$
AAAD	$(1+q)P_{hH}-q$	$(1+q)P_{hL}-q$	$(2-q)P_{lH}-(1-q)$	$(2-q)P_{lL}-(1-q)$
AADA	$(2-q)P_{hH}-(1-q)$	$(2-q)P_{hL}-(1-q)$	$(1+q)P_{lH}-q$	$(1+q)P_{lL}-q$
AADD	P_{hH}	P_{hL}	P_{lH}	P_{lL}
ADAA	$(1+q)P_{hH}-1$	$(1+q)P_{hL}-1$	$(2-q)P_{lH}-1$	$(2-q)P_{lL}-1$
ADAD	$q(2P_{hH}-1)$	$q(2P_{hL}-1)$	$(1-q)(2P_{lH}-1)$	$(1-q)(2P_{lL}-1)$
ADDA	$P_{hH}-(1-q)$	$P_{hL}-(1-q)$	$P_{lH}-q$	$P_{lL}-q$
ADDD	qP_{hH}	qP_{hL}	$(1-q)P_{lH}$	$(1-q)P_{lL}$
DAAA	$(2-q)P_{hH}-1$	$(2-q)P_{hL}-1$	$(1+q)P_{lH}-1$	$(1+q)P_{lL}-1$
DAAD	$P_{hH}-q$	$P_{hL}-q$	$P_{lH}-(1-q)$	$P_{lL}-(1-q)$
DADA	$(1-q)(2P_{hH}-1)$	$(1-q)(2P_{hL}-1)$	$q(2P_{lH}-1)$	$q(2P_{lL}-1)$
DADD	$(1-q)P_{hH}$	$(1-q)P_{hL}$	qP_{lH}	qP_{lL}
DDAA	$P_{hH}-1$	$P_{hL}-1$	$P_{lH}-1$	$P_{lL}-1$
DDAD	$-q(1-P_{hH})$	$-q(1-P_{hL})$	$-(1-q)(1-P_{lH})$	$-(1-q)(1-P_{lL})$
DDDA	$-(1-q)(1-P_{hH})$	$-(1-q)(1-P_{hL})$	$-q(1-P_{lH})$	$-q(1-P_{lL})$
DDDD	0	0	0	0

Table 6: VC's expected payoffs if he chooses *Agree*

EP's strategy	***Agree, E=h***		***Agree, E=l***	
	Hh	***Hl***	***Lh***	***Ll***
AAAA	$2Q_{Hh}-1$	$2Q_{Hl}-1$	$2Q_{Lh}-1$	$2Q_{Ll}-1$
AAAD	$(1+p)Q_{Hh}-p$	$(1+p)Q_{Hl}-p$	$(2-p)Q_{Lh}-(1-p)$	$(2-p)Q_{Ll}-(1-p)$
AADA	$(2-p)Q_{Hh}-(1-p)$	$(2-p)Q_{Hl}-(1-p)$	$(1+p)Q_{Lh}-p$	$(1+p)Q_{Ll}-p$
AADD	Q_{Hh}	Q_{Hl}	Q_{Lh}	Q_{Ll}
ADAA	$(1+p)Q_{Hh}-1$	$(1+p)Q_{Hl}-1$	$(2-p)Q_{Lh}-1$	$(2-p)Q_{Ll}-1$
ADAD	$p(2Q_{Hh}-1)$	$p(2Q_{Hl}-1)$	$(1-p)(2Q_{Lh}-1)$	$(1-p)(2Q_{Ll}-1)$
ADDA	$Q_{Hh}-(1-p)$	$Q_{Hl}-(1-p)$	$Q_{Lh}-p$	$Q_{Ll}-p$
ADDD	pQ_{Hh}	pQ_{Hl}	$(1-p)Q_{Lh}$	$(1-p)Q_{Ll}$
DAAA	$(2-p)Q_{Hh}-1$	$(2-p)Q_{Hl}-1$	$(1+p)Q_{Lh}-1$	$(1+p)Q_{Ll}-1$
DAAD	$Q_{Hh}-p$	$Q_{Hl}-p$	$Q_{Lh}-(1-p)$	$Q_{Ll}-(1-p)$
DADA	$(1-p)(2Q_{Hh}-1)$	$(1-p)(2Q_{Hl}-1)$	$p(2Q_{Lh}-1)$	$p(2Q_{Ll}-1)$
DADD	$(1-p)Q_{Hh}$	$(1-p)Q_{Hl}$	pQ_{Lh}	pQ_{Ll}
DDAA	$Q_{Hh}-1$	$Q_{Hl}-1$	$Q_{Lh}-1$	$Q_{Ll}-1$
DDAD	$-p(1-Q_{Hh})$	$-p(1-Q_{Hl})$	$-(1-p)(1-Q_{Lh})$	$-(1-p)(1-Q_{Ll})$
DDDA	$-(1-p)(1-Q_{Hh})$	$-(1-p)(1-Q_{Hl})$	$-p(1-Q_{Lh})$	$-p(1-Q_{Ll})$
DDDD	0	0	0	0

Table 7: Conditions for EP to choose *Agree*

VC's strategy	***Agree, E=h***		***Agree, E=l***	
	hH	***hL***	***lH***	***lL***
AAAA	$P_{hH} \geq 1/2$	$P_{hL} > 1/2$	$P_{lH} \geq 1/2$	$P_{lL} > 1/2$
AAAD	$P_{hH} \geq q/(1+q)$	$P_{hL} > q/(1+q)$	$P_{lH} \geq (1-q)/(2-q)$	$P_{lL} > (1-q)/(2-q)$
AADA	$P_{hH} \geq (1-q)/(2-q)$	$P_{hL} > (1-q)/(2-q)$	$P_{lH} \geq q/(1+q)$	$P_{lL} > q/(1+q)$
AADD	*All* P_{hH}	*All* P_{hL}	*All* P_{lH}	*All* P_{lL}
ADAA	$P_{hH} \geq 1/(1+q)$	$P_{hL} > 1/(1+q)$	$P_{lH} \geq 1/(2-q)$	$P_{lL} > 1/(2-q)$
ADAD	$P_{hH} \geq 1/2$	$P_{hL} > 1/2$	$P_{lH} \geq 1/2$	$P_{lL} > 1/2$
ADDA	$P_{hH} \geq 1-q$	$P_{hL} > (1-q)$	$P_{lH} \geq q$	$P_{lL} > q$
ADDD	*All* P_{hH}	*All* P_{hL}	*All* P_{lH}	*All* P_{lL}
DAAA	$P_{hH} \geq 1/(2-q)$	$P_{hL} > 1/(2-q)$	$P_{lH} \geq 1/(1+q)$	$P_{lL} > 1/(1+q)$
DAAD	$P_{hH} \geq q$	$P_{hL} > q$	$P_{lH} \geq (1-q)$	$P_{lL} > (1-q)$
DADA	$P_{hH} \geq 1/2$	$P_{hL} > 1/2$	$P_{lH} \geq 1/2$	$P_{lL} > 1/2$
DADD	*All* P_{hH}	*All* P_{hL}	*All* P_{lH}	*All* P_{lL}
DDAA	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
DDAD	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
DDDA	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
DDDD	*All* P_{hH}	*All* P_{hL}	*All* P_{lH}	*All* P_{lL}

Table 8: Conditions for VC to choose *Agree*

EP's strategy	***Agree, E=h***		***Agree, E=l***	
	Hh	***Hl***	***Lh***	***Ll***
AAAA	$Q_{Hh} \geq 1/2$	$Q_{Hl} > 1/2$	$Q_{Lh} \geq 1/2$	$Q_{Ll} > 1/2$
AAAD	$Q_{Hh} \geq p/(1+p)$	$Q_{Hl} > p/(1+p)$	$Q_{Lh} \geq (1-p)/(2-p)$	$Q_{Ll} > (1-p)/(2-p)$
AADA	$Q_{Hh} \geq (1-p)/(2-p)$	$Q_{Hl} > (1-p)/(2-p)$	$Q_{Lh} \geq p/(1+p)$	$Q_{Ll} > p/(1+p)$
AADD	*All* Q_{Hh}	*All* Q_{Hl}	*All* Q_{Lh}	*All* Q_{Ll}
ADAA	$Q_{Hh} \geq 1/(1+p)$	$Q_{Hl} > 1/(1+p)$	$Q_{Lh} \geq 1/(2-p)$	$Q_{Ll} > 1/(2-p)$
ADAD	$Q_{Hh} \geq 1/2$	$Q_{Hl} > 1/2$	$Q_{Lh} \geq 1/2$	$Q_{Ll} > 1/2$
ADDA	$Q_{Hh} \geq 1-p$	$Q_{Hl} > (1-p)$	$Q_{Lh} \geq p$	$Q_{Ll} > p$
ADDD	*All* Q_{Hh}	*All* Q_{Hl}	*All* Q_{Lh}	*All* P_{Ll}
DAAA	$Q_{Hh} \geq 1/(2-p)$	$Q_{Hl} > 1/(2-p)$	$Q_{Lh} \geq 1/(1+p)$	$Q_{Ll} > 1/(1+p)$
DAAD	$Q_{Hh} \geq p$	$Q_{Hl} > p$	$Q_{Lh} \geq (1-p)$	$Q_{Ll} > (1-p)$
DADA	$Q_{Hh} \geq 1/2$	$Q_{Hl} > 1/2$	$Q_{Lh} \geq 1/2$	$Q_{Ll} > 1/2$
DADD	*All* Q_{Hh}	*All* Q_{Hl}	*All* Q_{Lh}	*All* Q_{Ll}
DDAA	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
DDAD	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
DDDA	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
DDDD	*All* Q_{Hh}	*All* Q_{Hl}	*All* Q_{Lh}	*All* Q_{Ll}

Table 9: Equilibria in 1-period game

Equilibrium	P_{hv}	P_{lv}	Q_{He}	Q_{Le}
(AAAA, AAAA)	$P_{hH} \geq P_{hL} > 1/2$	$P_{lH} \geq P_{lL} > 1/2$	$Q_{Hh} \geq Q_{Hl} > 1/2$	$Q_{Lh} \geq Q_{Ll} > 1/2$
(AAAA, AAAD)	$P_{hH} \geq P_{hL} > q/(1+q)$	$P_{lH} \geq P_{lL} > (1-q)/(2-q)$	$Q_{Hh} \geq Q_{Hl} > 1/2$	$Q_{Lh} \geq 1/2 \geq Q_{Ll}$
(AAAD, AAAA)	$P_{hH} \geq P_{hL} > 1/2$	$P_{lH} \geq 1/2 \geq P_{lL}$	$Q_{Hh} \geq Q_{Hl} > p/(1+p)$	$Q_{Lh} \geq Q_{Ll} > (1-p)/(2-p)$
(AAAA, AADD)	All	All	$Q_{Hh} \geq Q_{Hl} > 1/2$	$1/2 > Q_{Lh} \geq Q_{Ll}$
(AADD, AAAA)	$P_{hH} \geq P_{hL} > 1/2$	$1/2 > P_{lH} \geq P_{lL}$	All	All
(AAAA, ADAA)	$P_{hH} \geq P_{hL} > 1/(1+q)$	$P_{lH} \geq P_{lL} > 1/(2-q)$	$Q_{Hh} \geq 1/2 \geq Q_{Hl}$	$Q_{Lh} \geq Q_{Ll} > 1/2$
(ADAA, AAAA)	$P_{hH} \geq 1/2 \geq P_{hL}$	$P_{lH} \geq P_{lL} > 1/2$	$Q_{Hh} \geq Q_{Hl} > 1/(1+p)$	$Q_{Lh} \geq Q_{Ll} > 1/(2-p)$
(AAAA, ADAD)	$P_{hH} \geq P_{hL} > 1/2$	$P_{lH} \geq P_{lL} > 1/2$	$Q_{Hh} \geq 1/2 \geq Q_{Hl}$	$Q_{Lh} \geq 1/2 \geq Q_{Ll}$
(ADAD, AAAA)	$P_{hH} \geq 1/2 \geq P_{hL}$	$P_{lH} \geq 1/2 \geq P_{lL}$	$Q_{Hh} \geq Q_{Hl} > 1/2$	$Q_{Lh} \geq Q_{Ll} > 1/2$
(AAAA, ADDD)	All	All	$Q_{Hh} \geq 1/2 \geq Q_{Hl}$	$1/2 > Q_{Lh} \geq Q_{Ll}$
(ADDD, AAAA)	$P_{hH} \geq 1/2 \geq P_{hL}$	$1/2 > P_{lH} \geq P_{lL}$	All	All
(AAAD, AAAD)	$P_{hH} \geq P_{hL} > q/(1+q)$	$P_{lH} \geq (1-q)/(2-q) \geq P_{lL}$	$Q_{Hh} \geq Q_{Hl} > p/(1+p)$	$Q_{Lh} \geq (1-p)/(2-p) \geq Q_{Ll}$
(AAAD, ADAA)	$P_{hH} \geq P_{hL} > 1/(1+q)$	$P_{lH} \geq 1/(2-q) \geq P_{lL}$	$Q_{Hh} \geq p/(1+p) \geq Q_{Hl}$	$Q_{Lh} \geq Q_{Ll} > (1-p)/(2-p)$
(ADAA, AAAD)	$P_{hH} \geq q/(1+q) \geq P_{hL}$	$P_{lH} \geq P_{lL} > (1-q)/(2-q)$	$Q_{Hh} \geq Q_{Hl} > 1/(1+p)$	$Q_{Lh} \geq 1/(2-p) \geq Q_{Ll}$
(AAAD, ADAD)	$P_{hH} \geq P_{hL} > 1/2$	$P_{lH} \geq 1/2 \geq P_{lL}$	$Q_{Hh} \geq p/(1+p) \geq Q_{Hl}$	$Q_{Lh} \geq (1-p)/(2-p) \geq Q_{Ll}$
(ADAD, AAAD)	$P_{hH} \geq q/(1+q) \geq P_{hL}$	$P_{lH} \geq (1-q)/(2-q) \geq P_{lL}$	$Q_{Hh} \geq Q_{Hl} > 1/2$	$Q_{Lh} \geq 1/2 \geq Q_{Ll}$
(ADAA, ADAA)	$P_{hH} \geq 1/(1+q) \geq P_{hL}$	$P_{lH} \geq P_{lL} > 1/(2-q)$	$Q_{Hh} \geq 1/(1+p) \geq Q_{Hl}$	$Q_{Lh} \geq Q_{Ll} > 1/(2-p)$
(ADAA, ADAD)	$P_{hH} \geq 1/2 \geq P_{hL}$	$P_{lH} \geq P_{lL} > 1/2$	$Q_{Hh} \geq 1/(1+p) \geq Q_{Hl}$	$Q_{Lh} \geq 1/(2-p) \geq Q_{Ll}$
(ADAD, ADAA)	$P_{hH} \geq 1/(1+q) \geq P_{hL}$	$P_{lH} \geq 1/(2-q) > P_{lL}$	$Q_{Hh} \geq 1/2 \geq Q_{Hl}$	$Q_{Lh} \geq Q_{Ll} > 1/2$
(ADAD, ADAD)	$P_{hH} \geq 1/2 \geq P_{hL}$	$P_{lH} \geq 1/2 \geq P_{lL}$	$Q_{Hh} \geq 1/2 \geq Q_{Hl}$	$Q_{Lh} \geq 1/2 \geq Q_{Ll}$

Table 10: Equilibria in likelihood ratios

Equilibrium	A_t	B_t	C_t	D_t
(AAAA, AAAA)	$A < \frac{1-p}{p}$	$B < \frac{1-p}{p}$	$C < \frac{1-q}{q}$	$D < \frac{1-q}{q}$
(AAAA, AAAD)	$qA < \frac{1-p}{p}$	$(1-q)B < \frac{1-p}{p}$	$C < \frac{1-q}{q}$	$\frac{1-q}{q} \leq D \leq \frac{q}{1-q}$
(AAAD, AAAA)	$A < \frac{1-p}{p}$	$\frac{1-p}{p} \leq B \leq \frac{p}{1-p}$	$pC < \frac{1-q}{q}$	$(1-p)D < \frac{1-q}{q}$
(AAAA, AADD)	All	All	$C < \frac{1-q}{q}$	$D > \frac{q}{1-q}$
(AADD, AAAA)	$A < \frac{1-p}{p}$	$B > \frac{p}{1-p}$	All	All
(AAAA, ADAA)	$\frac{A}{q} < \frac{1-p}{p}$	$\frac{B}{1-q} < \frac{1-p}{p}$	$\frac{1-q}{q} \leq C \leq \frac{q}{1-q}$	$D < \frac{1-q}{q}$
(ADAA, AAAA)	$\frac{1-p}{p} \leq A \leq \frac{p}{1-p}$	$B < \frac{1-p}{p}$	$\frac{C}{p} < \frac{1-q}{q}$	$\frac{D}{1-p} < \frac{1-q}{q}$
(AAAA, ADAD)	$A < \frac{1-p}{p}$	$B < \frac{1-p}{p}$	$\frac{1-q}{q} \leq C \leq \frac{q}{1-q}$	$\frac{1-q}{q} \leq D \leq \frac{q}{1-q}$
(ADAD, AAAA)	$\frac{1-p}{p} \leq A \leq \frac{p}{1-p}$	$\frac{1-p}{p} \leq B \leq \frac{p}{1-p}$	$C < \frac{1-q}{q}$	$D < \frac{1-q}{q}$
(AAAA, ADDD)	All	All	$\frac{1-q}{q} \leq C \leq \frac{q}{1-q}$	$D > \frac{q}{1-q}$
(ADDD, AAAA)	$\frac{1-p}{p} \leq A \leq \frac{p}{1-p}$	$B > \frac{p}{1-p}$	All	All
(AAAD, AAAD)	$qA < \frac{1-p}{p}$	$\frac{1-p}{p} \leq (1-q)B \leq \frac{p}{1-p}$	$pC < \frac{1-q}{q}$	$\frac{1-q}{q} \leq (1-p)D \leq \frac{q}{1-q}$
(AAAD, ADAA)	$\frac{A}{q} < \frac{1-p}{p}$	$\frac{1-p}{p} \leq \frac{B}{1-q} \leq \frac{p}{1-p}$	$\frac{1-q}{q} \leq pC \leq \frac{q}{1-q}$	$(1-p)D < \frac{1-q}{q}$
(ADAA, AAAD)	$\frac{1-p}{p} \leq qA \leq \frac{p}{1-p}$	$(1-q)B < \frac{1-p}{p}$	$\frac{C}{p} < \frac{1-q}{q}$	$\frac{1-q}{q} \leq \frac{D}{1-p} \leq \frac{q}{1-q}$
(AAAD, ADAD)	$A < \frac{1-p}{p}$	$\frac{1-p}{p} \leq B \leq \frac{p}{1-p}$	$\frac{1-q}{q} \leq pC \leq \frac{q}{1-q}$	$\frac{1-q}{q} \leq (1-p)D \leq \frac{q}{1-q}$
(ADAD, AAAD)	$\frac{1-p}{p} \leq qA \leq \frac{p}{1-p}$	$\frac{1-p}{p} \leq (1-q)B \leq \frac{p}{1-p}$	$C < \frac{1-q}{q}$	$\frac{1-q}{q} \leq D \leq \frac{q}{1-q}$
(ADAA, ADAA)	$\frac{1-p}{p} \leq \frac{A}{q} \leq \frac{p}{1-p}$	$\frac{B}{1-q} < \frac{1-p}{p}$	$\frac{1-q}{q} \leq \frac{C}{p} \leq \frac{q}{1-q}$	$\frac{D}{1-p} < \frac{1-q}{q}$
(ADAA, ADAD)	$\frac{1-p}{p} \leq A \leq \frac{p}{1-p}$	$B < \frac{1-p}{p}$	$\frac{1-q}{q} \leq \frac{C}{p} \leq \frac{q}{1-q}$	$\frac{1-q}{q} \leq \frac{D}{1-p} \leq \frac{q}{1-q}$
(ADAD, ADAA)	$\frac{1-p}{p} \leq \frac{A}{q} \leq \frac{p}{1-p}$	$\frac{1-p}{p} \leq \frac{B}{1-q} \leq \frac{p}{1-p}$	$\frac{1-q}{q} \leq C \leq \frac{q}{1-q}$	$D < \frac{1-q}{q}$
(ADAD, ADAD)	$\frac{1-p}{p} \leq A \leq \frac{p}{1-p}$	$\frac{1-p}{p} \leq B \leq \frac{p}{1-p}$	$\frac{1-q}{q} \leq C \leq \frac{q}{1-q}$	$\frac{1-q}{q} \leq D \leq \frac{q}{1-q}$

Table 11: Beliefs updating rule. *P:Result=Proceed, N:Result=Not Proceed*

Equilibrium		A_{t+1}	B_{t+1}	C_{t+1}	D_{t+1}
(AAAA, AAAA)	***P***	A_t	B_t	C_t	D_t
	N	-	-	-	-
(AAAA, AAAD)	***P***	$A_t q$	$B_t(1-q)$	C_t	$D_t\frac{1-q}{q}$
	N	∞	∞	-	$D_t\frac{q}{1-q}$
(AAAD, AAAA)	***P***	A_t	$B_t\frac{1-p}{p}$	$C_t p$	$D_t(1-p)$
	N	-	$B_t\frac{p}{1-p}$	∞	∞
(AAAA, AADD)	***P***	0	0	C_t	-
	N	∞	∞	-	D_t
(AADD, AAAA)	***P***	A_t	-	0	0
	N	-	B_t	∞	∞
(AAAA, ADAA)	***P***	$A_t\frac{1}{q}$	$B_t\frac{1}{1-q}$	$C_t\frac{1-q}{q}$	D_t
	N	0	0	$C_t\frac{q}{1-q}$	-
(ADAA, AAAA)	***P***	$A_t\frac{1-p}{p}$	B_t	$C_t\frac{1}{p}$	$D_t\frac{1}{1-p}$
	N	$A_t\frac{p}{1-p}$	-	0	0
(AAAA, ADAD)	***P***	A_t	B_t	$C_t\frac{1-q}{q}$	$D_t\frac{1-q}{q}$
	N	A_t	B_t	$C_t\frac{q}{1-q}$	$D_t\frac{q}{1-q}$
(ADAD, AAAA)	***P***	$A_t\frac{1-p}{p}$	$B_t\frac{1-p}{p}$	C_t	D_t
	N	$A_t\frac{p}{1-p}$	$B_t\frac{p}{1-p}$	C_t	D_t
(AAAA, ADDD)	***P***	0	0	$C_t\frac{1-q}{q}$	-
	N	$A_t\frac{1}{1-q}$	$B_t\frac{1}{q}$	$C_t\frac{q}{1-q}$	D_t
(ADDD, AAAA)	***P***	$A_t\frac{1-p}{p}$	-	0	0
	N	$A_t\frac{p}{1-p}$	B_t	$C_t\frac{1}{1-p}$	$D_t\frac{1}{p}$
(AAAD, AAAD)	***P***	$A_t q$	$B_t\frac{(1-p)(1-q)}{p}$	$C_t p$	$D_t\frac{(1-p)(1-q)}{q}$
	N	∞	$B_t\frac{1-(1-p)(1-q)}{1-p}$	∞	$D_t\frac{1-(1-p)(1-q)}{1-q}$
(AAAD, ADAA)	***P***	$A_t\frac{1}{q}$	$B_t\frac{1-p}{p(1-q)}$	$C_t\frac{p(1-q)}{q}$	$D_t(1-p)$
	N	0	$B_t\frac{p}{1-p(1-q)}$	$C_t\frac{1-p(1-q)}{1-q}$	∞
(ADAA, AAAD)	***P***	$A_t\frac{(1-p)q}{p}$	$B_t(1-q)$	$C_t\frac{1}{p}$	$D_t\frac{1-q}{(1-p)q}$
	N	$A_t\frac{1-(1-p)q}{1-p}$	∞	0	$D_t\frac{q}{1-(1-p)q}$
(AAAD, ADAD)	***P***	A_t	$B_t\frac{1-p}{p}$	$C_t\frac{p(1-q)}{q}$	$D_t\frac{(1-p)(1-q)}{q}$
	N	A_t	$B_t\frac{1-(1-p)(1-q)}{1-p(1-q)}$	$C_t\frac{1-p(1-q)}{1-q}$	$D_t\frac{1-(1-p)(1-q)}{1-q}$

Equilibrium		A_{t+1}	B_{t+1}	C_{t+1}	D_{t+1}
(ADAD, AAAD)	$\boldsymbol{P}$	$A_t\frac{(1-p)q}{p}$	$B_t\frac{(1-p)(1-q)}{p}$	C_t	$D_t\frac{1-q}{q}$
	$\boldsymbol{N}$	$A_t\frac{1-(1-p)q}{1-p}$	$B_t\frac{1-(1-p)(1-q)}{1-p}$	C_t	$D_t\frac{1-(1-p)(1-q)}{1-(1-p)q}$
(ADAA, ADAA)	$\boldsymbol{P}$	$A_t\frac{1-p}{pq}$	$B_t\frac{1}{1-q}$	$C_t\frac{1-q}{pq}$	$D_t\frac{1}{1-p}$
	$\boldsymbol{N}$	$A_t\frac{p}{1-pq}$	0	$C_t\frac{q}{1-pq}$	0
(ADAA, ADAD)	$\boldsymbol{P}$	$A_t\frac{1-p}{p}$	B_t	$C_t\frac{1-q}{pq}$	$D_t\frac{1-q}{(1-p)q}$
	$\boldsymbol{N}$	$A_t\frac{1-(1-p)q}{1-pq}$	B_t	$C_t\frac{q}{1-pq}$	$D_t\frac{q}{1-(1-p)q}$
(ADAD, ADAA)	$\boldsymbol{P}$	$A_t\frac{1-p}{pq}$	$B_t\frac{1-p}{p(1-q)}$	$C_t\frac{1-q}{q}$	D_t
	$\boldsymbol{N}$	$A_t\frac{p}{1-pq}$	$B_t\frac{p}{1-p(1-q)}$	$C_t\frac{1-p(1-q)}{1-pq}$	D_t
(ADAD, ADAD)	$\boldsymbol{P}$	$A_t\frac{1-p}{p}$	$B_t\frac{1-p}{p}$	$C_t\frac{1-q}{q}$	$D_t\frac{1-q}{q}$
	$\boldsymbol{N}$	$A_t\frac{1-(1-p)q}{1-pq}$	$B_t\frac{1-(1-p)q}{1-pq}$	$C_t\frac{1-p(1-q)}{1-pq}$	$D_t\frac{1-p(1-q)}{1-pq}$

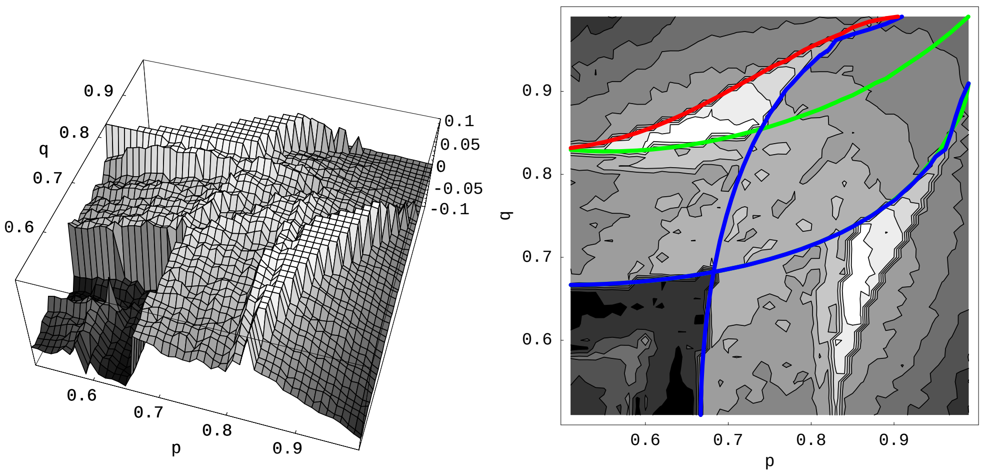

Figure 1: The difference between the probabilities of DOWN cascade in POO and POA models, $E = h$, $V = H$. Left: 3D plot. Right: contour plot.

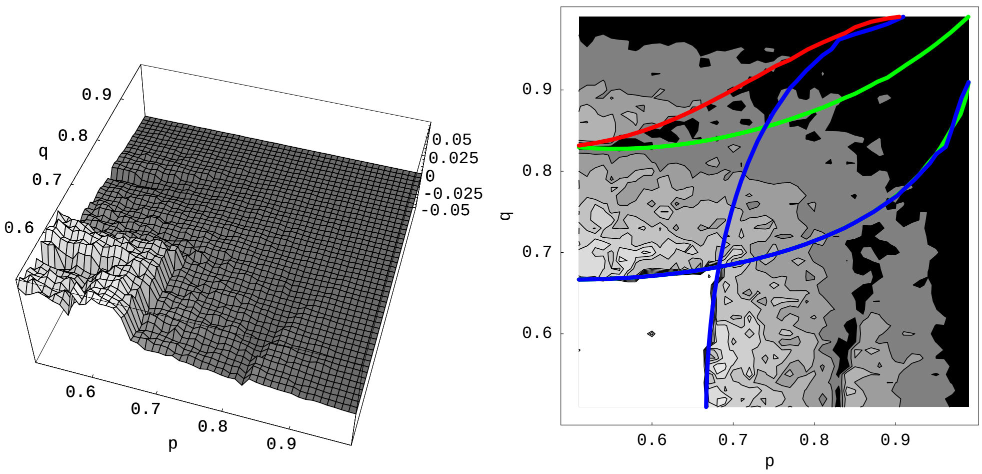

Figure 2: The difference between the probabilities of UP cascade in POO and POA models, $E = l$, $V = L$. Left: 3D plot. Right: contour plot.

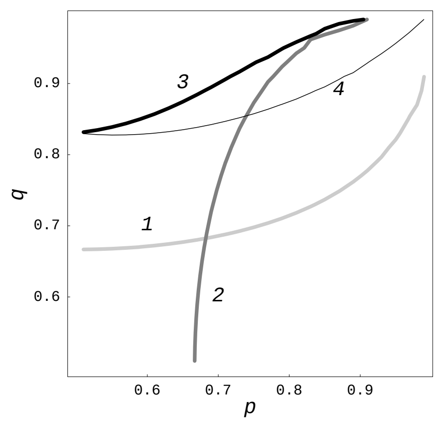

Figure 3: Conditions for minimum series.