On Trickling Chimneys and

Other Unemployment Misery

by

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1 Introduction

One of the most pressing problems of today is the unequal distribution of the burden of unemployment across different skill groups. Especially low-skilled workers are more often unemployed than other skills. For example, Figure 1 below shows the unemployment shares for three different skill categories in the Netherlands and Germany[1]. In the Netherlands, the low-skilled workers are in a unfortunate possition in comparison with the other two skill categories, whereas in Germany, the shares in total unemployment are fairly high for both the low- and medium-skilled people. Next to the fact that the unemployment share of low-skilled workers is very high, they are also unemployed for longer periods of time.

The unevenness of this distribution would seem to indicate that having different skills determines to some extent one's employment/unemployment opportunities. Nonetheless, policy prescriptions meant to alleviate the problems associated with the uneven distribution of unemployment are primarily based on (conceptual) models which effectively ignore skill-differences between workers and which therefore do not directly address the issues involved.

Figure 1

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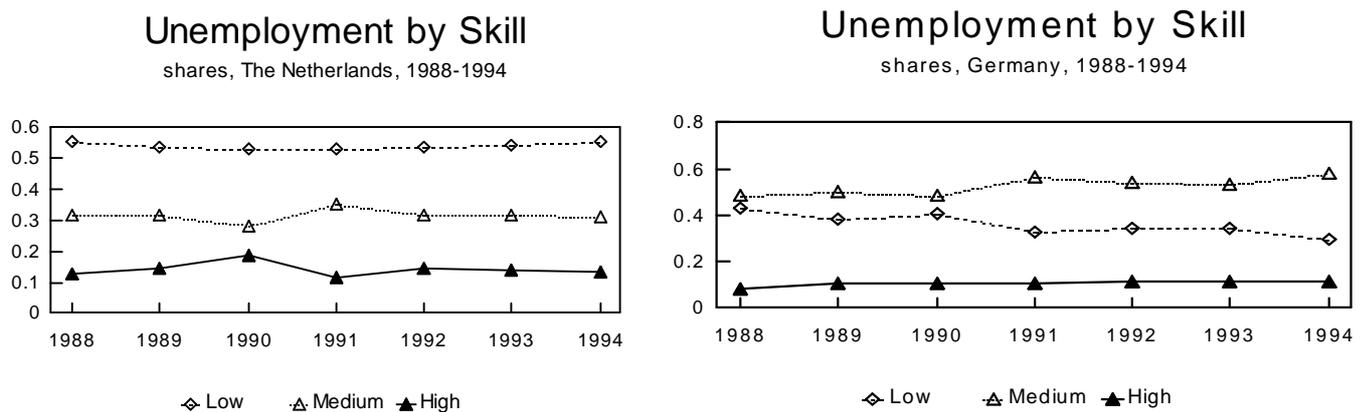
[1] We have used Eurostat's Labour Force Survey data for employment and unemployment. The data are available by country (European Union), by sector of industry, occupation (ISCO 68/88), age and highest level of education attained (corresponding to the ISCED educational classification). The data have been provided only for the period 1988-1994, because Eurostat did not perform an educational desaggregation of (un-)employment data for earlier years. In 1992, two major changes occurred in the classification of the data. First, the classification of sectors of industry changed from NACE-CLIO to NACE-Revision 1. Second, the classification of occupations changed from ISCO 68 to ISCO 88. In the data available, skills refer to the highest level of education attained by workers. We distinguish between a high (H), medium (M) and low (L) level of acquired education. The high educational level corresponds with high vocational and university education. The low educational level corresponds to primary school and low secondary school (up to age 16). Medium education is everything in between.

Policy recommendations involving wage-flexibility or reductions in costs of hiring and firing, for example, seem to be based on the notion that essentially skills are not the distinctive elements in defining the employment opportunities of an individual. Instead, individual 'skill-services' being over-priced or being too costly to hire and/or fire, are often regarded to be the 'real cause' of disparities in unemployment rates by skill. In this paper we will argue that this neglects the fact that people with high skills have intrinsically more employment opportunities than people with low skills. This is because low-skilled people may find it (too) hard to fill high-level jobs. High-skilled people on the other hand may be expected to be able to perform, in principle at least, in both low-level jobs and high-level jobs. In addition, it may well be possible that high-skilled workers are more efficient than low-skilled workers on low-level jobs.

Given these asymmetries between low-skilled and high-skilled workers' employment opportunities, it follows directly that the number of low-level jobs is an upper limit to low-skilled employment, while the number of high-skilled jobs is a lower limit to high-skilled employment. Hence, in the absence of supply bottle-necks, high-skilled employment can not fall below the number of high-level jobs, whereas low-skilled employment may even fall to zero, when high-skilled workers take over low-level jobs. While, therefore, the creation of a high-level job necessarily favours high-skilled employment, effective employment prospects of the low-skilled are not necessarily improved by the creation of low-level jobs. Instead, a combination of measures may need to be taken to overcome the influence of the 'non one-to-one' correspondence between low skills and low-level jobs on low-skilled unemployment as opposed to the one-to-one correspondence between high skills and high-level jobs.

The allocation of high-skilled workers to low-level jobs drives low-skilled workers from the only jobs they can take, so unequivocally reducing employment opportunities for low-skilled workers, ceteris paribus. While these negative employment effects could in principle be mitigated by the creation of still more low-level jobs, it is not at all that clear that this 'trickle down effect'[2] will succeed in furthering employment of the low-skilled. For, the effectiveness of the 'trickle down effect' depends essentially on the infeasibility or undesirability of the allocation of high-skilled workers to low-level jobs: only in the case of a shortage of supply of high-skilled workers or in the case of relatively high costs per efficiency unit of labour generated by high-skilled workers on low-level jobs, low-skilled workers will indeed be hired for low-level jobs.

An additional asymmetry between different skill types of labour lies in the possibility that low-skilled workers may be low-skilled because it is difficult for them to learn enough to become high-skilled. With firms having become 'leaner and meaner' over the last decade, this 'learning-inability' may provide incentives for entrepreneurs to hire workers who are flexible in the sense that they could easily learn different types of tasks instead of hiring people who

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[2] See Drèze and Malinvaud, 1993, 'Growth and Employment, the Scope of European Initiative', Mimeo, CORE/College de France, Louvain-la-Neuve/Paris.

can only perform a limited number of tasks. These learning asymmetries between low-skilled and high-skilled workers may also be part of the explanation of the uneven distribution of unemployment across skills.

The above suggests that there are a number of ways (and combinations thereof) to increase the employment prospects for low-skilled workers. The first one is by increasing the number of low-level jobs, the second by decreasing the relative labour costs of low-skilled workers thus increasing the willingness of entrepreneurs to allocate low- skilled workers to low-level jobs instead of high-skilled workers, and the third by increasing the number of jobs which only high-skilled workers can take. A fourth option would be to uplift the skill-level of low-skilled workers to some extent by means of (re-) training programmes.

The effectiveness of the first possibility depends in part on the availability of high-skilled workers for low-level jobs, as suggested above. The effectiveness of the second measure is limited to the extent that such a change in relative wage costs at first tends to increase the unemployment of high-skilled workers while decreasing the unemployment of low-skilled workers. Only when wage-costs per unit of output would fall, a net expansion of jobs may be expected to occur and therefore a positive net effect on overall employment. Otherwise the bias in unemployment will just shift against high-skilled workers. The third measure structurally decreases the excess supply of high- skilled workers for low-level jobs, and hence increases employment prospects for the low-skilled, almost as a beneficial side-effect. The fourth measure, in as far as it would be feasible, would be highly desirable for it would structurally diminish the constraints put on employment opportunities by the existence of substitution asymmetries between skills.

In this paper we will describe a model which combines most of the notions regarding asymmetries between low-skilled workers and high-skilled workers mentioned above. We will look at the influence of asymmetries in substitution / allocation possibilities, but also of asymmetries in learning capabilities between skills. To this end we specify a model of labour demand defined in terms of jobs, which given the supply of labour distinguished in accordance with skill-levels, requires matches to be made between the skills supplied and the jobs under consideration. The model is highly neo-classical in nature except for the asymmetries mentioned above, and the vintage character of the capital stock, so as to assure a reasonable comparability with the working of models used by the EC or the OECD, for instance. The vintage character of the capital stock is important in practice, because of the embodiment of the job-composition of employment in the capital stock. Hence, changes in employment prospects as they are provided by changes in the job-composition of employment, will generally depend on the rate of investment. This is because investment in new machinery and equipment only provides the means to (marginally) change the job-composition of employment.[3]

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[3] Obviously, from a macro-perspective, changes in the sectoral composition of output may change the job-composition of employment too, but in this paper we disregard this.

The set-up of the paper is as follows. In section 2 we describe the demand for labour in the face of asymmetric substitution/allocation possibilities between different skills on low-level jobs. Section 3 describes the integration of heterogeneous labour in a Bischoff-type vintage model, while section 4 is devoted to a number of simulation experiments. Section 5 contains some concluding remarks.

2 Jobs, Skills and Asymmetries in Substitution Possibilities

2.1 Introduction

High-skilled people may in general be expected to perform better in low-skilled jobs than low-skilled people in high-skilled jobs. Hence, in terms of the allocation of skills to jobs there must be an intrinsic (more or less technical) bias against allocating low-skilled people to high-skilled jobs. Nonetheless, some kind of substitution between higher skills and lower skills should be possible, since empirical investigations in this area all point to a substitution set-up where high-skilled people are complements of capital, and low-skilled people in turn can be substituted for this high-skilled people/capital complex.[4]

In our set-up, a job is a set of tasks which need to be performed, and which require the people engaged in that job to have a certain minimum skill-level in order to be able to perform the tasks in question. These specific skill requirements define the level of the job. Moreover, a low-skilled job requires only low skill-levels in order to be able to execute the tasks belonging to a specific job. And so, everybody who has the minimum required level of skills available can actually be hired for that job. Therefore, generally speaking, high-skilled people can be hired for more levels of jobs than low-skilled people. This provides an asymmetry in employment opportunities for high-skilled and low-skilled people, which, given a certain lack of compensating asymmetries in wage formation, might lead to a bias in employment opportunities in favour of high-skilled people rather than low-skilled people.

2.2 Ex Ante Labour Demand

Let us assume that the design of a production process entails the definition of certain packages of tasks for lower- and higher-skilled people. Let us furthermore assume that technical constraints on combining these tasks are such that a decrease of the number of high-skilled jobs must be compensated by an increase in the number of low-skilled jobs and vice

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[4] Many writers have investigated the possibility of direct substitution between skill-categories. A survey for the US is provided by Hamermesh and Grant (1979), while Hebbink (1990) and Broer and Jansen (1989) provide some results for the Netherlands. Kugler c.s. (1990) do the same for Germany. Mincer (1989) provides additional results for the US. The general conclusions which emerge from these studies are *first* that capital and high-skilled labour are complements, while *secondly* low-skilled labour and the capital/high-skilled labour complex are substitutes.

versa.[5] Furthermore, given the layout of the production process in terms of the number of high-skilled and low-skilled jobs required to generate one efficiency unit of labour, the actual allocation of people to those jobs is a separate issue. Because of the asymmetries described above, we know that low-skilled people can not efficiently perform in high-skilled jobs, whereas high-skilled people can perform efficiently in both low-skilled and high-skilled jobs. This means that the number of low-skilled jobs is an upper-limit to the number of low-skilled people which may actually be employed, while the number of high-skilled jobs is a lower limit to the number of high-skilled people which may actually find employment.

Assuming that the efficiency of a high-skilled person relative to a low-skilled person in a low-skilled job is the constant number e', it follows that the framework sketched above can be pictured as in Figure 2.
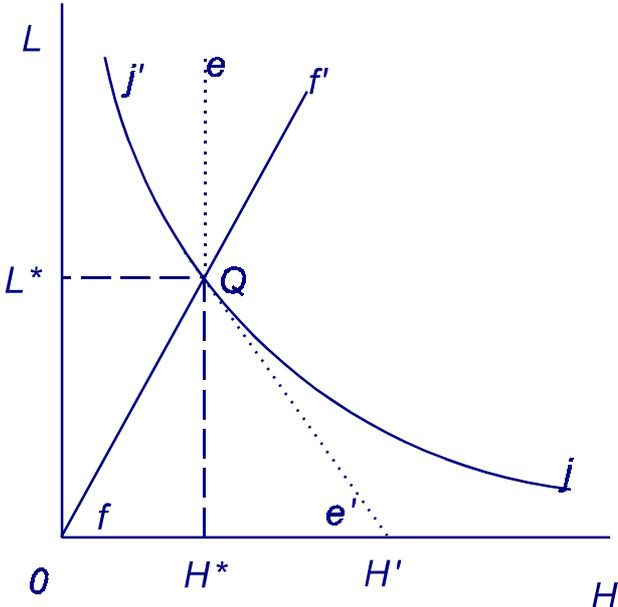

Figure 2. Substitution Between Jobs and Skills

In this figure, the curve j'j defines the number of low-skilled jobs and high-skilled jobs which are required to generate one efficiency unit of labour, assuming, for the moment, that all low-skilled jobs are filled by low-skilled people (L) and all high-skilled jobs are filled by high-skilled people (H).

Let us assume that point Q represents the optimum choice, where the optimum combination

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[5] These are the 'standard' substitution assumptions in neo-classical production theory. We also assume that the implied 'job aggregator function' or 'job substitution function' obeys the usual restrictions on the signs of the first and second partial derivatives.

of jobs requiring only low skills and jobs requiring high skills is represented by the point (L*,H*). Then the kinked line eQH' represents the combinations of all low-skilled and high-skilled people which can generate the same amount and quality of the labour services which would be provided by exactly L* low-skilled people and exactly H* high-skilled people. The relative efficiency of low-skilled people in high-skilled jobs is equal to zero by assumption, as indicated by the vertical line-segment Qe in Figure 2. Hence, H can not fall below H* without reducing the total number of labour-efficiency units. When H is equal to H', then all low-skilled labour L is replaced by high-skilled labour H and still one job efficiency unit is generated, i.e. H'=H* + L*/e'. Actually, all combinations of L and H on the line segment connecting points H' and Q are combinations of low-skilled and high-skilled people which generate one labour efficiency unit. The tangent of the angle of this line segment with the horizontal axis is equal to -e', i.e. minus the relative efficiency.

It should be noted that in this set-up it is assumed that the production process itself is defined/designed in terms of job combinations H* and L*. The design itself is then implemented in terms of the layout of machinery and equipment. It is assumed that this layout can not be altered afterwards. However, the actual allocation of low- and high-skilled people to low- and high-skilled jobs can still be varied in accordance with equation (1) below.

Given the location of H* and L* on the unit-job isoquant, the location of the line-segment Qe' is determined too. Hence the choice of H* and L* also defines the combinations of high-skilled and low-skilled people which may actually be observed to be employed. If we, for reasons of simplicity, would ignore the possibility of supply constraints and adjustment costs in this particular set-up, then the combinations of low- and high-skilled people which may actually be observed at time t, for given levels of L* and H* which have been chosen at time zero, are given by:

$$L_t = L^* \cdot e^{-\delta t} - e' \cdot (H_t - H^* \cdot e^{-\delta t}) \tag{1}$$

where δ enters the analysis because we assume that the job design is embodied in machinery and equipment which is subject to (exponential) technical decay at rate δ. H' in turn is the maximum number of H people required to generate one labour efficiency unit. It can easily be determined from (1) by setting L=0 :

$$H_t = (L^*/e' + H^*) e^{-\delta t} \tag{2}$$

It follows directly that, for given wages by skill w_H and w_L, entrepreneurs can influence the cost of operating a labour-efficiency unit in two different ways, i.e. *first* by choosing a certain combination (H*,L*) and *secondly* by choosing combinations (H,L) given (H*,L*), subject to equation (1). Then, assuming that entrepreneurs can determine the job-layout of their production process (i.e. (H*,L*)) only when new capacity is installed, it follows that the present value of the operating cost of one labour-efficiency unit installed at time zero declines with the progress of time with the discount rate r and with the rate of technical depreciation δ, which is already included in (1) and (2). Hence, the expected present value of total operating

cost of one labour-efficiency unit which is installed at time zero is given by :

$$T = \int_0^\infty \left(w_{H,t}^e \cdot H_t + w_{L,t}^e \cdot L_t \right) \cdot e^{-r \cdot t} \, dt \tag{3}$$

where $w_{H,t}^e$ and $w_{L,t}^e$ are the expected values of the wage rate for high-skilled and for low-skilled people at time t, respectively.[6] We will approach the problem of the minimisation of (3) subject to (1) in two steps, analogous to the two possibilities sketched above.

2.2.1 The Allocation of Skills in the Absence of Adjustment Costs

In the absence of supply constraints and adjustment costs, and given a job-combination (L^*, H^*), the choices of L_t and H_t can be modelled by solving:

$$\text{Min} \quad F_t = \int_0^\infty \left(w_{H,t}^e \cdot H_t + w_{L,t}^e \cdot L_t \right) \cdot e^{-rt} \tag{4}$$

$$\text{s.t.} \quad L_t = L^* e^{-\delta \cdot t} - e' \cdot (H_t - H^* e^{-\delta \cdot t})$$

Given the nature of the problem, (H=H',L=0) is the solution to (4) when $w_H^e / w_L^e < e'$, while $(L = L^* e^{-\delta \cdot t}, \; H = H^* e^{-\delta \cdot t})$ is the solution when the opposite is the case.[7]

Using the shorthand notation $w = w_H^e / w_L^e$ and $\hat{w} = \hat{w}_H - \hat{w}_L$, for given and constant expected exponential growth rates $\hat{w}_H$ and $\hat{w}_L$, there are now four distinct possibilities to consider, depending on whether $w < e'$ or $w > e'$ and $\hat{w} < 0$ or $\hat{w} > 0$. For instance, when $w < e'$ and $\hat{w} < 0$ (case 1) the optimal solution will always be (H=H',L=0). But when $w < e'$ and $\hat{w} > 0$ (case 2) there will be a moment when the other solution (H=H*,L=L*) will be chosen. A switch between states will also occur in case 3, which starts out with $w > e'$ and $\hat{w} < 0$. Case 4 will always have (H=H*,L=L*) since $w > e'$ and $\hat{w} > 0$. The four cases are depicted in Figure 3.

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[6] Note that due to Hicks-neutral disembodied labour augmenting technical change, the effective discount rate of job-operating costs rises with the value of the rate of disembodied technical progress. However, for now we disregard the influence of technical change.

[7] From now on we drop time-subscripts, unless this would lead to confusion.

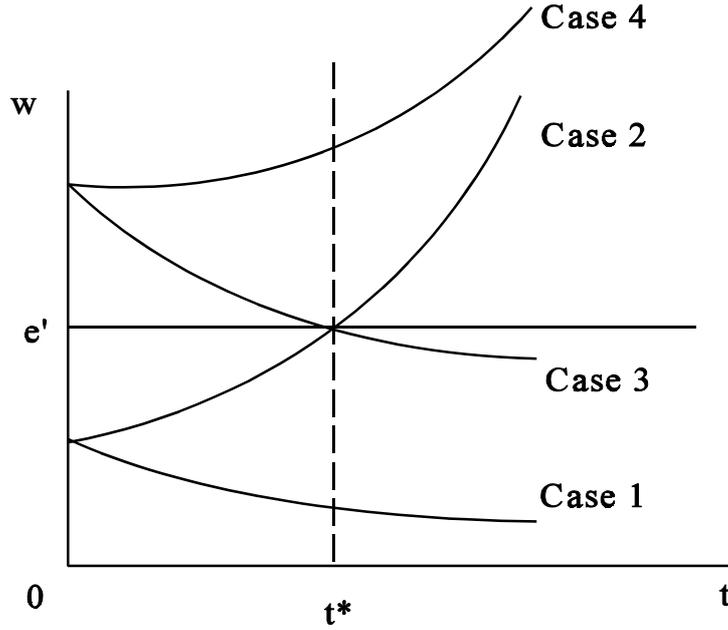Figure 3. Expected Wage Growth and Skill-Switching

In cases 2 and 3 a switch from the one corner solution to the other will take place at time t*.[8] t* itself is easily determined from the condition that :

$$w_{t^*} = \frac{w_{H,t^*}}{w_{L,t^*}} = \frac{w_{H,0}}{w_{L,0}} \cdot e^{(\hat{w}_H - \hat{w}_L) \cdot t^*} = e' \quad \Rightarrow \quad t^* = \frac{\ln\left(e' / \left\{\frac{w_{H,0}}{w_{L,0}}\right\}\right)}{\hat{w}_H - \hat{w}_L} = \frac{\ln(e'/w_0)}{\hat{w}} \qquad (5)$$

Using equation (5), the solution of the original problem over an infinite horizon can be defined as the solution to the problem of minimizing unit operating costs over two consecutive horizons, namely 0-t* and t*-∞.

Choosing Jobs

The solution to the allocation problem defined by the minimisation of (3) subject to (1), is easily obtained by substituting (1) and (5) into (3). The solution of the objective function for case 1 can now be written as :

$$T_1 = \int_0^\infty w_{H,0} \cdot H' \cdot e^{(\hat{w}_H - \rho) \cdot t} dt = -\frac{w_{H,0}}{\hat{w}_H - \rho} \cdot (L^*/e' + H^*) = \alpha_1 \cdot H^* + \beta_1 \cdot L^* \qquad (6)$$

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[8] Although this has not been indicated in Figure 3, t* can of course be different for cases 2 and 3.

where α_1 and β_1 are implicitly defined by (6) and T_1 denotes the value of the objective function given by (3) for case 1 as described in Figure 3. The term ρ is equal to the rate of technical decay plus the discount rate. Note that α_1, $\beta_1 > 0$. Note also that the objective function T1 is linear in L* and H*. Hence, maximisation of T1 by choosing (L*,H*) constrained by the unit job-isoquant ϕ (L*,H*)=1 should lead to a straight forward determination of L*, and H* in terms of e', relative wages, relative growth rates in wages and the parameters of the job-substitution function ϕ (L*,H*). As was the case with T_1, T_2-T_4 can also be written as $T_i = \alpha_i \cdot H^* + \beta_i \cdot L^*$. The resulting values of α and β are summarised in Annex A.

In order to be able to solve the intertemporal cost-minimisation problem, we have had to assume that the sum of the discount rate and the rate of technical decay exceeds the expected rate of growth of wages. These assumptions imply that all the α's and β's are positive, and so the intertemporal cost-minimisation problem has a meaningful solution in all four cases.[9] This solution is implicitly given by the first order condition:

$$\left(\frac{\partial \phi}{\partial H_k^*} \right) \bigg/ \left(\frac{\partial \phi}{\partial L_k^*} \right) = \frac{\alpha_k}{\beta_k} \qquad (7)$$

where L^*_k and H^*_k reflect the optimum values of L* and H* when case k (as depicted in Figure 3) is expected to hold. Note that (7) is totally comparable with the first order conditions of a static cost-minimisation problem, and hence we can conclude that a fall in the ratio α_k/β_k corresponds to a decrease in the slope of the iso-cost lines and hence to a movement down the unit job isoquant (see Figure 2) and hence to a decrease in the L*/H* ratio.

Discussion of the Results

Case 4 may be considered as a bench-mark case in the sense that it generates results which are comparable to the ones which would be obtained for a static cost-minimisation problem. Recall that in case 4 the solution is given by (L=L*,H=H*) and that there is no switching, i.e. L>0 for all t. The only difference between the solution of a static cost-minimisation problem and the problem on hand, is that the (discounted) costs of high-skilled and low-skilled people are cumulated over an infinite horizon, as indicated by the denominators of α_4 and β_4 which reflect the geometric cumulation process.[10]

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[9] See Annex A.

[10] Recall that $1/r = \int_0^\infty e^{-r \cdot t} dt$ for r>0.

It is easy to show from the α/β ratios that case 1 will have the lowest value of the L*/H* ratio, while case 3 will have a larger L*/H* ratio than case 4. Case 2 in turn will have a larger L*/H* ratio than case 1, that is: $(L*/H*)_3 > (L*/H*)_4 > (L*/H*)_1 = e'$ and $(L*/H*)_2 > (L*/H*)_1 = e'$.

Case 1 is an interesting case in the sense that the L*/H* ratio depends on e' only. This is caused by the fact that e' provides a 'technical' limit to the marginal rate of substitution of high-skilled jobs for low-skilled jobs. Indeed, it does not pay to let the marginal rate of job-substitution along the isoquant fall below e', because the corresponding solution to the cost-minimisation problem (L=0,H=H') would in that case result in an upward shift of the iso-cost line again. For, this implies that all job-combinations on the unit job-isoquant to the right of Q would generate higher unit costs than Q itself. Hence, given the substitution possibilities between skills ex post, these ex ante job combinations to the right of Q, have become irrelevant from an economic point of view.

Note that in case 4, L*/H* exceeds the corresponding ratio which is obtained in case 1. This is in line with what one would expect, since in case 1 low-skilled workers are relatively expensive and get ever more expensive, while in case 4 high-skilled workers are more expensive and grow even more expensive over time. Case 3 has an L*/H* ratio which exceeds the corresponding ratio of case 4. This follows from the fact that low-skilled jobs are expected to be filled by high-skilled workers in the future which, although more expensive than low-skilled workers in the beginning, get less and less expensive over time. Hence, the prospective switch from low-skilled workers to high-skilled workers on low-skilled jobs, makes low-skilled jobs less costly on average, for a given development of low-skilled wages. A similar line of reasoning can be followed in the comparison of cases 1 and 2, where high-skilled jobs become relatively more expensive and hence low-skilled jobs (although initially filled with high-skilled people) are substituted for high-skilled jobs.

2.2.2 The Allocation of Skills and Adjustment Costs

We now assume that the allocation of a specific skill to a job gives rise to adjustment costs, which we interpret in terms of learning cost. When a person is allocated to a job, we assume that he needs to spend some time learning all the tricks in the trade. The time spent is a given fraction of 'normal working hours' for a person already trained for the job. We assume furthermore that a trainee does not produce anything while learning. Nonetheless the trainee gets paid for his time. This implies that the learning cost per person is proportional to the wage rate which applies to the type of skill of the person in question. The corresponding factor of proportion depends both on the skill of the person in question and the type of job he is trained for. In addition to this, we assume that each time a job is filled, training costs will have to be incurred. Obviously, people who have been trained already for a certain job do not have to be trained again as long as they remain allocated to the job they have been trained for. Consequently, the experience they have in a certain job brings about an additional asymmetry in the costs of operating a job by either experienced workers or inexperienced workers of the same skill type.

Let the training cost of an allocation of skill i to jobtype j, as a fraction of the relevant wage rate w_i, be given by the symbol $\xi_{i,j}$. Furthermore, let us consider the case where $\hat{w} > 0$. Then, eventually the allocation (L->L*) will be the cost-minimising allocation, because the wage rate of high-skilled people will become infinitely high, while the relative efficiency e' of high-skilled people vis a vis low-skilled people in low-level jobs will remain the same. The question arises whether low-skilled workers (L) should be allocated to a low-level job (L*) from the outset, or high-skilled workers (H) should be allocated to that low-level job first. As in the no-adjustment case, we assume that this depends on the present value of operating the job over an infinite period of time for given expectations of the growth rate of relative wages and for given values of the learning cost parameters. The present value of the cost of 'manning' and operating a low-level job with its final allocation from the outset C_F, is given by:

$$C_F = w_{L,0} \cdot \left(\xi_{L,L^*} + \frac{1}{\rho - \hat{w}_L} \right) \tag{8}$$

Where (8) refers to the costs of manning a low level job with an inexperienced low-skilled worker. For experienced workers initial matching costs can be disregarded. However, for, for a switch in the future, we assume that entrepreneurs will always expect to use inexperienced workers, in order to be on the 'safe side'. Hence, the present value of the cost of 'manning' and operating a low-level job C_S by starting with the allocation (H->L*) first and then switching to the final allocation (L->L*) at time t*, is given by :

$$
\begin{aligned}
C_S &= \xi_{H,L^*} \cdot \frac{w_{H,0}}{e'} + \frac{w_{H,0}}{e'} \int_0^{t^*} e^{(\hat{w}_H - \rho') \cdot t} dt + \xi_{L,L^*} \cdot w_{L,0} e^{(\hat{w}_L - \rho') \cdot t^*} + w_{L,0} \cdot \int_{t^*}^{\infty} e^{(\hat{w}_L - \rho') \cdot t} dt \\
&= \xi_{H,L^*} \cdot \frac{w_{H,0}}{e'} + \frac{w_{H,0}}{e'} \cdot \left(\frac{e^{(\hat{w}_H - \rho') \cdot t^*} - 1}{\hat{w}_H - \rho'} \right) + \xi_{L,L^*} \cdot w_{L,0} \cdot e^{(\hat{w}_L - \rho') \cdot t^*} - w_{L,0} \cdot \left(\frac{e^{(\hat{w}_L - \rho') \cdot t^*}}{\hat{w}_L - \rho'} \right)
\end{aligned}
\tag{9}
$$

where the initial allocation (H->L*) involves an inexperienced high-skilled worker. In equation (9), t* represents the moment of switching. t* itself can be determined by minimisation of (9) w.r.t. t*, giving :

$$t^* = \frac{\ln\left(\left\{ \frac{e'}{w_0} \right\} \cdot \left\{ 1 + (\rho - \hat{w}_L) \cdot \xi_{L,L^*} \right\} \right)}{\hat{w}} \tag{10}$$

t* depends positively on the (effective) rate of discount ρ (which includes the rate of technical

decay).[11] A rise in either of the parameters would decrease the relative importance of future costs in the present value of total cost per low-level job. Hence, the incentive to switch would diminish, thus increasing t*, ceteris paribus. Likewise, a rise in the adjustment cost of the final allocation (L->L*) would tend to postpone the moment of switching, i.e. increase t*, while a rise in the rate of growth of the wage rate of the skill used in the final allocation, would tend to increase the unit cost associated with that allocation, and hence the moment of switching may again be expected to be postponed. This is in line with the fact that $\frac{\partial t^*}{\partial \hat{w}_L} > 0$, for t* > 0.

From equation (10) it is clear that t* is positive, only when $w_0 < e' \cdot (1 + (\rho - \hat{w}_L) \cdot \xi_{L,L^*})$, since $\hat{w} > 0$ by assumption. If t* is positive, then an entrepreneur who had started out with the allocation (H->L*) would be happy to stick with that allocation upto time t*. If $w_0 > e' \cdot (1 + (\rho - \hat{w}_L) \cdot \xi_{L,L^*})$, t* would be negative, and the optimum moment of switching from (H->L*) to (L->L*) would lie in the past, implying that under the circumstances it would have been wise to choose the final allocation (L->L*) from the outset. But even if t*>0, it may be wise to start with the allocation (L->L*) rather than the allocation (H->L*), for differentiation of (8) really assumes that learning costs associated with the allocation (H->L*) have already been sunk. Considering the fact that (L->L*) is the final allocation, the allocation sequence (H->L*) , (L ->L*) involves making matching costs twice rather than just once in the case of starting out with the final allocation (L->L*). However, in the latter case, the present value of the matching cost associated with the allocation (L->L*) increases, ceteris paribus, because the match itself is made at time zero rather than at t*. Moreover, the operating cost of a low-level job during the period upto time t* is now $w_{L,t}$ rather than $w_{H,t} / e'$, which, considering the assumption that $\hat{w} > 0$ from time zero onwards, implies a further cost disadvantage for the allocation (L->L*) vis a vis the allocation (H->L*) during the period upto t*. However, when $\xi_{H,L^*} \cdot w_{H,0}$ is large enough to compensate these cost-disadvantages, (L->L*) should be chosen at time zero instead of the allocation (H->L*). This choice can most easily be made by evaluating and comparing C_F and C_S.

The analysis can be repeated for the case where $\hat{w} < 0$, which implies that ultimately the allocation (H->L*) instead of (L->L*) will be the least costly one. In this case the present value of the cost associated with manning and operating a low-level job L*, by using the allocation (H ->L*) from the outset is given by :

$$C_F = w_{H,0} \cdot \left(\xi_{H,L^*} + \frac{1}{\rho - \hat{w}_H} \right) \tag{11}$$

---

[11] In van Zon and Muysken (1996), switching costs are introduced by means of the notion of a switching threshold Δ. I.e. entrepreneurs switch from (H->L*) to (L->L*) only when w > e'(1+Δ) and $\hat{w} > 0$. Note that Δ can directly be interpreted in terms of matching costs, since the switching threshold approach and the matching costs approach taken here are equivalent when $\Delta = (\rho - \hat{w}_L) \cdot \xi_{L,L}$. Note that this is only an approximate equivalence, however, since matching costs depend on whether one switches to (L->L*) or (H ->L*). Δ, being a constant, is by definition indepedent of the skill involved in the match which is to be realised.

By analogy, the cost C_S of 'manning' and operating a low-level job using the allocation sequence (L->L*),(H->L*), is given by :

$$C_S = \xi_{L,L^*} \cdot w_{L,0} + w_{L,0} \int_0^{t^*} e^{(\hat{w}_L - \rho') \cdot t} dt + \xi_{H,L^*} \cdot \frac{w_{H,0}}{e'} e^{(\hat{w}_H - \rho') \cdot t^*} + \frac{w_{H,0}}{e'} \cdot \int_{t^*}^{\infty} e^{(\hat{w}_H - \rho') \cdot t} dt$$

$$= \xi_{L,L^*} \cdot w_{L,0} + w_{L,0} \cdot \left(\frac{e^{(\hat{w}_L - \rho') \cdot t^*} - 1}{\hat{w}_L - \rho'} \right) + \xi_{H,L^*} \cdot \frac{w_{H,0}}{e'} \cdot e^{(\hat{w}_H - \rho') \cdot t^*} - \frac{w_{H,0}}{e'} \cdot \left(\frac{e^{(\hat{w}_H - \rho') \cdot t^*}}{\hat{w}_H - \rho'} \right)$$

(12)

As before, the optimum switching moment can readily be obtained from (12) by minimising C_S w.r.t. t*, giving:

$$t^* = \frac{\ln\left(\left\{ \frac{e'}{w_0} \right\} \cdot \left\{ 1 + (\rho - \hat{w}_H) \cdot \xi_{H,L^*} \right\}^{-1} \right)}{\hat{w}}$$

(13)

2.3 Linear Programming and the 'Chimney Effect'

2.3.1 Introduction

In order for the chimney effect to work, it is necessary that the allocation of high-skilled workers towards low-level jobs is decreased. In the demand-context of the model defined so far, this constellation of events would require w_H / w_L to rise. The ensuing change in the composition of employment could then be interpreted as a chimney effect. However, this notion would necessarily be false, because demand induced changes in the number of high-level jobs do not automatically and explicitly spill-over into changes in the skill-composition of low-level jobs, as is required in order for the chimney effect to really do its work. When we would introduce supply constraints, however, while taking account of asymmetries in substitution possibilities between high- and low-skilled workers, the chimney effect arises automatically. For, in those circumstances, an increase in the number of high-level jobs would require an increase in the employment of high-skilled workers on those jobs, since these workers are the only ones which are suited for those jobs. The inflow of high-skilled workers into those high-level jobs would in part have to be met by an outflow of high-skilled workers from low-level jobs. The employment-holes this outflow would leave in low-level jobs, creates the employment-opportunities on which the low-skilled workers depend.

In order to add supply constraints to the existing labour demand framework, we can use the technique of Linear Programming, as we have done before in van Zon and Muysken (1992). In this way, we are able to explicitly account for both job demand constraints and skill-supply constraints.

2.3.2 Definition of the LP-Problem

The demand side described so far, is based on the assumption that there are no supply constraints. Consequently, the allocation of skills to jobs referred to first best allocations only. But explicit supply constraints which become binding may force entrepreneurs to use second best allocations instead. In order to be able to cover both first-best and second-best allocations, we assume that entrepreneurs solve a linear programming problem in which they try to meet job requirements as well as skill supply constraints, while minimising the mismatch[12] between skill-levels and job-levels on low-level jobs.

The definition of the LP-Problem is now quite straightforward. Let $E^i_{j,k}$ denote the employment of skill j with experience type i on job k. The index i has two values : experienced and inexperienced with respect to the match in question. The distinction between experienced and inexperienced workers is important because of the fact that the allocation of an inexperienced worker with a certain skill level implies having to bear the training/matching costs associated with that particular combination of skills and jobs, while this is not the case with experienced workers. Consequently, it is important to distinguish between experienced and inexperienced workers at the supply side too. In this respect we simply assume that the experienced workers available for allocation in the present period are those which were allocated to the job under consideration in the previous period, adjusted for a constant exit-rate. Supply from other sources (like the educational system or the unemployment pool, for instance), which is available for employment is inexperienced by assumption.

Given these supply constraints, we can now force the solution of the LP-problem to be as close as possible to the first-best allocation in the following way. First we introduce the notion of the quasi-job-utilisation rate q (with 0<=q<=1). When q=1, the skills allocated to low-level jobs and high-level jobs are exactly sufficient to generate the labour efficiency units in those jobs as implied by the composition and level of labour demand by job. Consequently, the LP-problem should be specified in such a way that q is as close as possible to 1, but not larger than 1, i.e. job-demand constraints should be met as closely as possible. At the same time we want to have these labour efficiency units by job generated in the right proportions at a cost which is as low as possible. However, the costs associated with a certain allocation should also be in line with ex ante behaviour, i.e. an initial allocation should be considered as the first allocation of an allocation sequence, as in the ex ante job-demand problem, rather than being a skill-job allocation which is isolated in time from other allocations to the job under consideration : there is no reason a priori to assume that cost-considerations ex ante and

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[12] Note that the term mismatch used here, usually refers to the a priori notion that the best match is the one where the skill-level is exactly equal to the job-level. From a (learning) cost-minimisation point of view, this is not necessarily the case, however, because of the possibility of 'skill-switching'. In this context therefore, a mismatch refers to a non cost-minimising allocation of skills to jobs, which may be due to the existence of supply constraints, for instance.

ex post as regards the allocation of skills to jobs would be different. Hence, a particular allocation of a skill to a certain job would by necessity be the one which follows from the ex ante cost-considerations, unless supply constraints for the skill under consideration would somehow prove to be binding. Hence, the assignment of skills to jobs at any moment in time can be seen as the implementation of various allocation sequences associated with the different jobs under consideration, within the constraints set by the supply of the various skills involved.

In order to define the objective function of the LP-problem, we now assume a worst-case production scenario, where all jobs are assigned the least-best allocation sequences, for given expectations regarding the growth rates of wages, etcetera. We now define the objective function to be equal to q times the expected present value of the cost of the worst-case scenario when operated at a value of the job-utilisation rate equal to 1, less the costs of the allocation sequences which could be continued/activated within the given supply constraints. Obviously, maximisation of this function would require q to be as close to one as possible, and the expected present value of the costs of the activated/continued allocation sequences to be as small as possible. Hence, cost-reductions vis a vis the worst-case scenario would be maximised for a non-zero value of q.[13] [14]

Let J^*_k denote the optimum job-levels H^*, L^* (where k denotes a job-level), i.e. the number of people with a corresponding skill-level needed to fill the job. Let $P^i_{j,k}$ denote the wage-cost (including the corresponding training cost for inexperienced workers) associated with a non-zero value of first- and/or second best allocation sequences starting with the employment of skill j and experience type i on type k jobs. V denotes the total labour costs associated with the worst case scenario. Then, the LP-problem can be defined as :

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[13] If we would 'value' q with the expected present value of the costs of the first-best allocation sequences, then the objective function would have a maximum value equal to zero, which could also be attained for a zero level of q. Hence, q=0 is both feasible and optimum. But this would imply zero levels for all allocations of skills to jobs, which is not what we are after. Hence, our reformulation of the cost minimisation problem in terms of a maximisation of cost-reductions relative to the worst-case scenario.

[14] Note that because the LP-problem is essentially a static optimisation problem, the intertemporal effects of particular allocations are not taken into account (like, for instance the fact that the inexperienced workers of today are the experienced workers of tomorrow). In the case of intertemporal optimisation under uncertainty regarding the level of demand for output, other interesting aspects of asymmetries in learning costs and substitution possibilities by skill become more relevant. In particular, one can envisage a situation where from an intertemporal cost minimisation point of view under uncertain conditions regarding output growth, we would expect a bias in the demand for skills towards the ones which could be used in as many jobs as possible at intertemporal costs which are as low as possible (including expected matching costs). This provides an intertemporal cost minimisation interpretation to the notion of a 'flexible' worker.

$$\text{MAX} \quad F = q \cdot V - \sum_j \sum_k \sum_i P_{j,k}^i \cdot E_{j,k}^i$$

$$s.t. \quad \sum_i \sum_j e_{j,k} E_{j,k}^i \geq q \cdot J_k^* \qquad \forall \ k$$

$$E_{j,k}^x \leq S_{j,k}^x \qquad \forall \ j,k \qquad \qquad (14)$$

$$\sum_i \sum_k E_{j,k}^x \leq S_j \qquad \forall \ j$$

$$q \leq 1$$

where S_j denotes the total supply of skill j at time t, $S_{j,k}^i$ is the total supply of skill j with experience type i with respect to job k. $e'_{i,j}$ is the relative efficiency of skill-level i on job-level j. Note that $e'_{j,k}=0$ for skill-levels j less than the job-level k, and $e'_{j,j}=1$ for all j, by definition.

The first set of constraints are the job demand constraints. The employment of available skills should generate the required efficiency units of labour in each job. The second set of constraints are the skill supply constraints. They state that the total use of experienced workers can not exceed the available supply. The third set of constraints state that the total use of skills may not exceed total supply (including both experienced and inexperienced workers). The last constraint requires employment to be not larger than needed.
Given the solution to this LP-problem, total employment by skill can now readily be obtained by aggregating over all experience categories and all jobs. The only things which need to be defined are the expected present value of the costs of the relevant allocation sequences per unit of the skill which is the first one in each sequence, i.e. the $P_{j,k}^i$'s.

2.3.3 The Valuation of Alternative Allocation Sequences

In valueing a certain allocation sequence, we assume that entrepreneurs do not expect to face supply constraints in the future, unless they are faced with supply constraints in the present. But even if the latter is the case, they are assumed to act as if these supply constraints would remain binding for a fixed period only, which is equal to Θ years. Non-binding supply constraints are assumed to remain non-binding forever, until proven wrong. Furthermore, future matches/switches, as opposed to present matches, are assumed to involve inexperienced workers in all cases.

Given expectations regarding the growth rate of relative wages by skill, the optimum switching moment t* for each possible allocation sequence starting with a certain skill can be calculated. If t* proves to be positive, then the allocation sequence in question may prove to be first-best. If not, then the allocation sequence must be second best, because switching to the second part of the allocation sequence at time zero (i.e. starting out with the second part of the allocation sequence, as indicated by t*<=0) would lead to a lower expected present value of the costs of the allocation sequence under consideration. Note, however, that this second-best allocation sequence would only be chosen when supply constraints would force entrepreneurs to actually do so. Hence, in this particular case, entrepreneurs expect to stay at least Θ years with this particular skill-job combination, after which they could decide to

return to the unconstrained optimum allocation sequence, if that would prove to be worthwhile, or they could decide to stick with the 'forced' allocation instead. The latter could prove to be the most profitable ex post, since 'returning' to the unconstrained allocation sequence entails the 'sinking' of matching costs in the future. We now assume that the cost to be assigned to the first part of the allocation sequence is equal to the minimum of these two options, i.e. 'sticking' and 'returning'.

When we would evaluate the expected present value of the costs of a certain allocation sequence starting out with an experienced worker, then the corresponding costs of the same allocation sequence starting out with an inexperienced worker can simply be obtained by adding matching costs to the cost of an experienced worker. Hence, all possible matches can be defined knowing the expected present value of allocating an experienced worker to a certain job, as part of a certain allocation sequence. These present value costs are defined in the Table 2.1 below.

Table 2.1 Expected Present Value Costs by Allocation Sequence

	$t^*>0$, FB(H->L*,L->L*) [15]
$\hat{w}>0$	$P_{H,L^*}^{x} = e^{\prime}.(PV(H,0,t^*) + PV(L,t^*,\infty))$
$\hat{w}>0$	$P_{L,L^*}^{x} = MIN(PV(L,0,\infty),PV(L,0,\Theta) + PV(H,\Theta,MAX(\Theta,t^*)) + PV(L,MAX(\Theta,t*),\infty))$
	$t^*>0$, FB(L->L*,H->L*)
$\hat{w}<0$	$P_{H,L^*}^{x} = e^{\prime}.MIN(PV(H,0,\infty),PV(H,0,\Theta) + PV(L,\Theta,MAX(\Theta,t^*)) + PV(H,MAX(\Theta,t*),\infty))$
$\hat{w}<0$	$P_{L,L^*}^{x} = (PV(L,0,t^*) + PV(H,t^*,\infty))$

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[15] FB(AS) denotes a first best allocation sequence AS. When AS consists of two single allocations, a switch between both single allocations is expected to occur at time t*.

	t*<0, FB(L->L*)
$\hat{w}>0$	$P^{x}_{H,L^*} = e'.MIN(PV(H,0,\infty),PV(H,0,\Theta) + PV(L,\Theta,\infty))$
$\hat{w}>0$	$P^{x}_{L,L^*} = PV(L,0,\infty)$
	t*<=0, FB(H->L*)
$\hat{w}<0$	$P^{x}_{H,L^*} = e^{/}.PV(H,0,\infty)$
$\hat{w}<0$	$P^{x}_{L,L^*} = MIN(PV(L,0,\infty),PV(L,0,\Theta)+PV(H,\Theta,\infty))$

PV(S,t0,t1) represents the present value of the cost of operating a low-level job, using skill S during the period t0-t1. If t0>0, then PV includes the expected present value of the matching costs associated with making the match S at time t0. If t0=0, i.e. S starts from the present moment, then matching costs are excluded, since the table above refers to the costs associated with using experienced workers in the first part of the allocation sequence under consideration. PV() is defined as the cost of operating a low-level job, but it needs to be redefined as the costs of an allocation sequence starting with an experienced worker with a certain skill on a low-level job, in order to be of any relevance for the LP-problem which was described above. Hence, the cost of operating a low-level job (i.e. the cost of supplying one efficiency unit of low-level labour on a low-level job) can easily be transformed into the cost of using a certain worker in a certain job, by multiplying PV(S,t0,t1) by $e'_{S,L}$. Note that e' in the table above, really stands for $e'_{H,L}$, while $e'_{L,L}=1$ by definition, and is therefore not shown.

3 A Multi-Level Putty-Clay Production Model with Substitution Asymmetries

3.1 Introduction

As we have discussed above, the job composition of employment can only be altered by investing in new machinery and equipment. We therefore should include the investment decision in our analysis, in order to model the determinants of the skill composition of employment properly. For that purpose we use a putty-clay vintage model based on Bischoff (1971), which by its very nature introduces an asymmetry in time of substitution possibilities between capital and labour. It should be noted that this production model which we will use to run a number of simulation experiments is highly stylized: we have left out every detail which is not directly connected to the representation of production technology as such, since we are at this stage only interested in the principal working of asymmetries in substitution possibilities between different types of labour given the asymmetries in substitution possibilities in time (i.e. differences in substitution possibilities ex ante and ex post) between capital on the one hand and labour on the other. More in particular, we will use a linear

homogeneous production function, where labour and capital are the only factors of production. With regard to technological change, we only consider purely labour augmenting technical change, possibly biased against low-skilled labour.

3.2 Behavioural Assumptions

Apart from the production function features mentioned above, we will assume that producers try to minimise production costs while producing an amount of output which is exogenously given. Since we use a linear homogeneous production function, this implies that producers try to minimise average production costs. As we have explained in section 2, in making their investment decision, producers take into account expectations regarding price developments and the consequences for future (discounted) production costs of the choice of a production technique. However, we assume that producers do not consider the consequences which their current investment plans may have for the formulation and realisation of plans for the not too distant future, which in turn influence the realisation of plans still further in the future, and so on. The solution to such an optimisation problem would have to be obtained by means of a dynamic optimisation procedure which takes into account all future effects of the decisions entrepreneurs have to make today. This goes beyond the illustration purposes which the vintage model is intended to serve.

We also assume that the supply of physical capital is infinitely elastic at the ruling price of capital goods. Furthermore, entrepreneurs share the same expectations regarding the development of prices.

Note that in section 2 we have assumed that labour is a composite factor (or a 'complex' of different types of labour) rather than a purely homogeneous factor. The 'price' of a unit of such a complex then depends on its composition, but, for the purpose of defining the production framework it can be assumed given for the moment.

3.3 The Putty-Clay Vintage Production Model

In the context of finding the cost minimising factor proportions on the newest vintage, it should be noted that factor proportions ex post are assumed fixed. Hence we assume, very much as in the job composition problem described in section 2, that entrepreneurs try to minimise the expected present value of the total costs associated with buying and using a new vintage over an infinite horizon. We furthermore assume that the ex ante production function is a linearly homogeneous CES function and that only embodied labour augmenting technical change occurs. The production structure of the vintage model can then be described by:

$$
\begin{aligned}
1 \quad &= B_N \cdot (e^{\mu T} \cdot v_{T,T})^{-\rho} + B_K \cdot \kappa_{T,T}^{-\rho} \\
\kappa_{T,t} &= \kappa_{T,T} \\
v_{T,t} &= v_{T,T} \\
I_{T,t} &= (1 - \delta)^{t-T} \cdot I_{T,T}
\end{aligned}
\tag{15}
$$

where $\sigma = 1/(1 + \rho)$ is the elasticity of substitution, B_N and B_K are distribution parameters and μ represents the rate of embodied labour augmenting technical change. $v_{T,t}$ and $\kappa_{T,t}$ represent the labour/output ratio and the capital/output ratio at time t of the vintage which was installed at time T, respectively.[16] Minimisation of the expected present value of the total cost per (initial) unit of output on the newest vintage over an infinite horizon then yields the optimim factor proportions on the newest vintage as a function of factor prices.[17] In the absence of disembodied technical change, total capacity output X_t, as well as total capacity labour demand N_t can now be obtained from :

$$X_t = \frac{I_{t,t}}{\kappa_{t,t}} + (1 - \delta).X_{t-1}$$

$$N_t = \frac{v_{t,t}}{\kappa_{t,t}}.I_{t,t} + (1 - \delta).N_{t-1}$$

(16)

The vintage model now works as follows. Output is exogenously determined, and so are wages. Together with prices the latter determine optimum factor proportions on the newest vintage and therefore unit total cost on the newest vintage. Given the volume of capacity output associated with 'old' equipment which has not been completely worn down yet, this determines how 'large' the newest vintage should be. Total capacity labour demand then follows as the capacity labour demand associated with the existing capital stock and capacity labour demand associated with the new vintage.

3.4 Adding Heterogeneous Labour to the Production Function Framework

In section 2 we have postulated that employment is the result of a match between the demand for labour as reflected by the number of jobs on the one hand and the supply of labour in terms of skills on the other. We now assume that capacity labour demand within the context of the vintage model, refers to labour demand in terms of high-level and low-level jobs, which are combined into one job-complex. That is, $v_{T,t}$ in the production function (15) represents the labour/output ratio in terms of job-efficiency units per unit of output: it comprises both low-level and high-level jobs.

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[16] The Bischoff approach disregards economic obsolescence.

[17] The expected present value of the total cost per (initial) unit of output on the newest vintage over an infinite horizon (Λ_T) is equal to :

$$\Lambda_T = \frac{w_T}{\rho + \delta - \hat{w}}.v_{T,T} + q_T.\kappa_{T,T}$$

where q_t represents the price of investment at time t, w_t the wage rate and $\hat{w}$ the growth rate of wages. See van Zon (1990) for a derivation of this result. $v_{T,T}$ and $\kappa_{T,T}$ can now be obtained by minimising Λ_T conditional on the ex ante production function as given in equation (15).

In order to model substitution possibilities between low-level and high-level jobs within the job-complex, we have taken a linear homogeneous job-CES-function, with an elasticity of substitution equal to 0.5.[18] With regard to substitution between capital on the one hand, and jobs on the other, we nested the job-CES-function in another linear homogeneous CES function with elasticity of substitution equal to 0.25.[19]

In the absence of skill supply constraints, the optimum composition of the labour complex in terms of jobs and the allocation of skills to these jobs, defines the expected present value of the total cost associated with using a job efficiency unit over an infinite horizon. Hence, this 'present value' price of a job efficiency unit serves the same function as the 'present value wage rates' used in equations (6) and (7).[20]

3.5 Adding Wage and Price Responses

In standard macro-economic models, wage formation is influenced by labour productivity, inflationary expectations and labour market conditions. Usually an increase in the rate of unemployment decreases the rate of growth of wages, which would in turn increase the demand for labour, ceteris paribus, which would lower unemployment again.

For illustrative purposes we have specified a very simple wage adjustment equation consisting of a constant trend (equal to 2.5 percent in most experiments) and a Phillips effect which is linear in the unemployment rate. Moreover, we have added a uniformly distributed random component to the growth of wages in the range from -2 to 2 per cent. Let $\hat{w}_i$ refer to the value of the growth rate of the wage rate associated with skill i, then:

$$\hat{w}_i = \hat{t}_i - \theta_i . u_i + \hat{\epsilon}_i \tag{17}$$

where $\hat{t}$ refers to the constant trend term, and $\hat{\epsilon}$ refers to the random component in the rate of growth of wages. Assuming that the trend growth of the wage rates of both skills are the

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[18] This function is consistent with the isoquant j'j in Figure 5. The value of the elasticity of substitution is consistent with that found in Hamermesh en Grant (1979) and Hebbink (1990). Note that we do not assume here that high-level jobs are complementary with capital. Rather, the job-complex is a substitute for capital, while low-level and high-level jobs are substitutes within the job-complex. The reason to do so is that substitution reactions between high-skilled and low-skilled workers in response to changes in wage-rates etcetera, should become more pronounced than when high-level jobs would be complementary with capital.

[19] The latter elasticity of substitution is the one found for the Netherlands by Kuipers and van Zon (1982) and Muysken and van Zon (1987), for instance.

[20] The present value price of investment is equal to the price-index of investment (see van Zon (1990) for a description of the assumptions underlying this result), while the present value price of labour is the initial value of the wage rate divided by the sum of the rate of discount and the rate of technical decay less the growth rate of wages (see section 2).

same, it follows from this specification that the ratio of the steady state unemployment rates by skill is defined in terms of the relative strength of the Phillips effect. More in particular, a stable ratio of w_H/w_L for a given supply of labour implies that, on average:

$$\bar{u}_L = \frac{\hat{t}_L - \hat{t}_H}{\Theta_L} + \frac{\Theta_H}{\Theta_L} . \bar{u}_H \tag{18}$$

where $\bar{u}_i$ denotes the steady state rate of unemployment of skill i.[21] Note that this equation implies that for symmetric trend growth of wages and symmetric speeds of adjustment of wage growth to unemployment, the steady state unemployment rates for low-skilled and high-skilled workers are identical. However, if the symmetry would be broken by, for instance, a decreasing low-skilled trend growth of wages for given supply of low- and high-skilled workers, this would lead directly to a disparity in the long term rates of unemployment of low-skilled and high-skilled workers.

With regard to the price of output, we assume that it is equal to unit total cost on the newest vintage. The price of investment is assumed to be equal to the price of output in turn.

4 Simulation Results

4.1 Introduction

In this section we will present the results of some simulation experiments in order to illuminate the working of our model and to emphasise some interactions which might be relevant from a policy point of view. The experiments are:

(1)   an decrease in the trend growth rate of low-skilled wages;
(2)   a lower demand for high-level jobs relative to low-level jobs;
(3)   a higher demand for high-level jobs.

These experiments will be discussed after we have introduced the simulation set-up and presented the base run.

4.2 The Simulation Set-Up

In order to illustrate the working of our model with asymmetric skill substitution, we have defined a very simple general framework with output remaining fixed. This also goes for the supply of the various types of labour. We assume that at zero rates of unemployment,

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[21] Equation (18) follows from (17) and the assumption that w_H/w_L is constant in the steady state.

nominal wage rates would grow with an average rate of 2.5 per cent per year with uniformly distributed random fluctuations of 1 per cent on average. The sensitivity of the growth of wages with respect to the rate of unemployment by skill has been set to 0.4, which is roughly in line with the findings by van Zon et al. (1995) for the Netherlands. The interest rate is assumed to be equal to the nominal rate of discount ρ, which in turn is equal to 10 per cent. Technical decay has been set equal to 5 per cent. The price of output (which is equal to the price of capital goods by assumption) is set equal to marginal production cost. The parameter e' has been set equal to 1.25, which is in line with the wage ratio for high-skilled and low-skilled workers given in Nickel and Bell (1995). Hence, at this particular wage ratio entrepreneurs are assumed to be indifferent between hiring either low-skilled workers or high-skilled workers on low-level jobs, ceteris paribus.

Starting values for the endogenous and exogenous variables were chosen such that the distribution of jobs is 35 per cent in low-level jobs and 65 per cent in high-level jobs (in the sense of not low). This is an exaggeration of the actual distribution, compare for instance the figures in the introduction of this paper, but these values are used for illustrative purposes only.

In order to exclude the influence of the choice of intitial values for the capital stock on the outcomes of the model, we let it run for 225 periods and then apply a shock in accordance with the experiment under consideration. The simulation experiment always ends in period 300, although the shock is ended after 50 periods, i.e. in period 275.

4.3 The Base-Run

In Figures 4.1-4.4 below, we present the outcomes of the base-run. The average rate of unemployment for both types of skill is about 11 percent as can be seen from Figure 4.1, where U denotes the overall percentage rate of unemployment u, and U_L and U_H refer to the percentage rates of unemployment of low-skilled (u_L) and high-skilled workers (u_H), respectively. We see that u is much more stable than u_L and u_H. Moreover, the fluctuations in u_H and u_L are almost perfectly negatively correlated. The reason is that changes in employment are caused primarily by entrepreneurs changing their allocation of skills to jobs, rather than changing the job-composition on the newest vintages.[22]

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[22] Note that in these experiments, changes in the demand for skills due to changes in the vintage composition of the capital stock are necessarily underestimated due to the fact first that output does not grow, and second that in the Bischoff vintage model economic lifetime is infinite, despite changes in unit-wage costs.
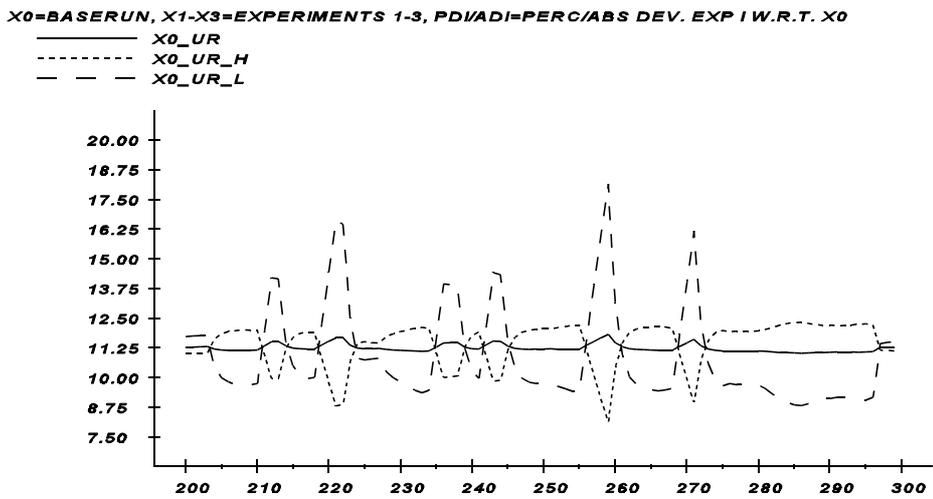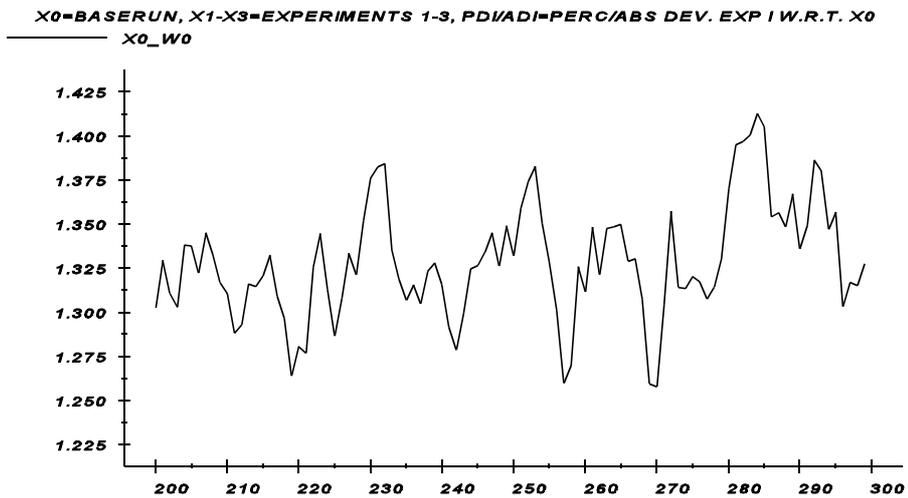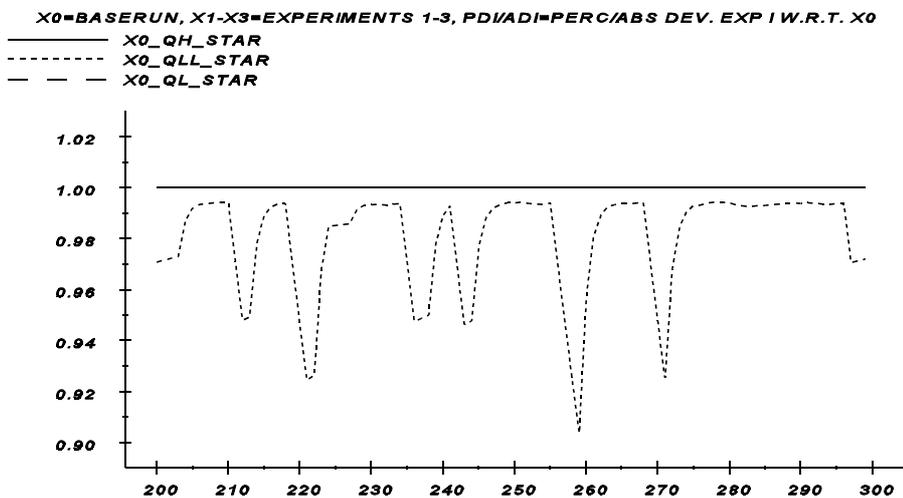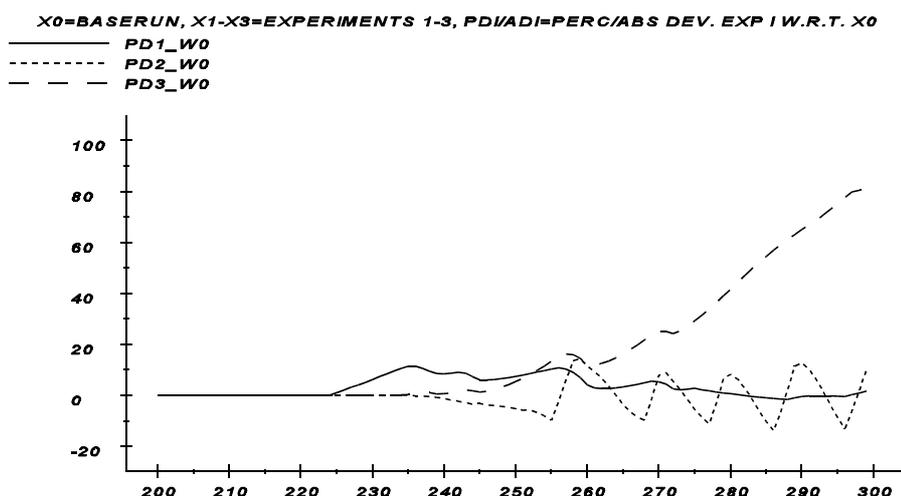
Figure 4.1

Figure 4.2

Figure 4.3

Figure 4.4

X0=BASERUN, X1-X3=EXPERIMENTS 1-3, PD/ADI=PERC/ABS DEV. EXP I W.R.T. X0

———————— PD1_W0
- - - - - - - - PD2_W0
— — — PD3_W0

The ratio of high-skilled wages and low-skilled wages, w_0, is on average equal to 1.4 as is shown in Figure 4.2 - fluctuations in this ratio are caused by random shocks on the one hand and fluctuations in u_H / u_L on the other. Note that there is a positive correlation between fluctuations in w_0 and fluctuations in u_H / u_L, with changes in unemployment rates leading changes in the wage ratio by 1 period by assumption. However, changes in wages, by altering expectations with respect to wage growth also have an effect on employment (and unemployment) which is much smoother that the original changes in relative wages. Seen from this perspective, changes in unemployment rates follow changes in wage rates by a couple of years (expectations are based on realisations over the last three periods by assumption). Figure 4.3 shows the rates of job-utilisation for high- and low-level jobs, QH_STAR and QL_STAR, respectively. These are both equal to one, indicating that there are no quantitative supply constraints in this case. QLL_STAR indicates the share of low-level jobs manned by low-skilled workers. The fact that it is not equal to one, whereas all low-level jobs are fully manned, indicates that in the baserun only a slight fraction of low-level jobs is manned by high-skilled workers.[23] QHL_STAR is not reported here, however.

4.4 A Cut in Low-Skilled Wages

Experiment 1 is concerned with a fall in the trend growth of low-skilled wages by 1 percentage point. In the steady state, this would lead to a fall in the equilibrium rate of unemployment with about 2.5 percentage points (see equation (18)). From Figure 4.4 it can be seen that this leads to a structural fall in the relative wage rate of low-skilled workers during the experimental period and hence to a rise in w_0. From Figure 4.5 one sees that this has the effect of raising the rate of high-skilled unemployment by slightly less than the rate of low-skilled unemployment falls. Note that there are large, negatively correlated swings in both

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[23] This implies, ceteris paribus, that the working of the chimney-effect is by necessity somewhat limited in numerical terms in this example.
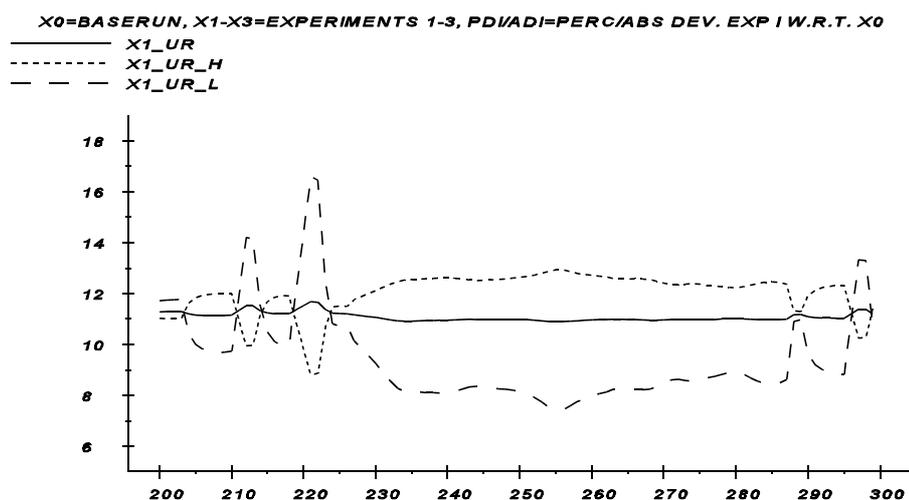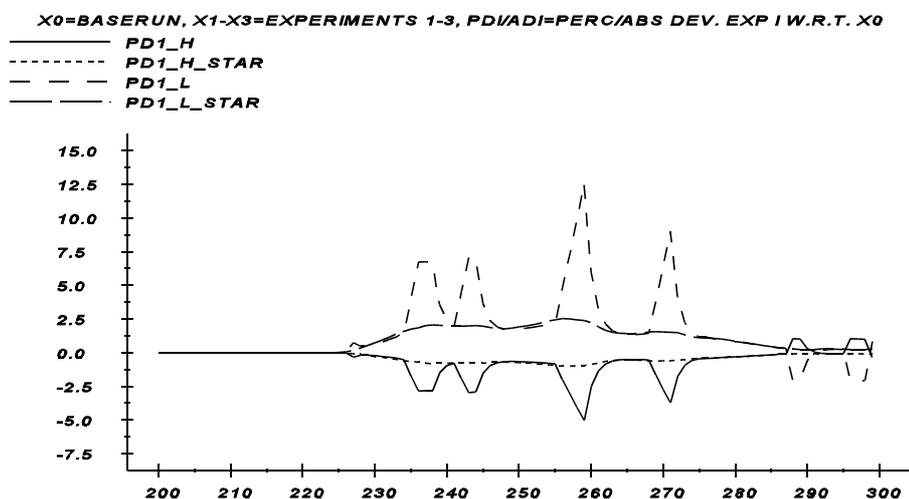
Figure 4.5

X0=BASERUN, X1-X3=EXPERIMENTS 1-3, PDI/ADI=PERC/ABS DEV. EXP I W.R.T. X0
——————— X1_UR
- - - - - - - - X1_UR_H
— — — X1_UR_L

Figure 4.6

X0=BASERUN, X1-X3=EXPERIMENTS 1-3, PDI/ADI=PERC/ABS DEV. EXP I W.R.T. X0
——————— PD1_H
- - - - - - - - PD1_H_STAR
— — — PD1_L
————— · PD1_L_STAR

unemployment rates due to employers switching between skills on low-level jobs, especially before and after the experimental period, when producers do not have a clearcut preference for either low-skilled workers or high-skilled workers on low-level jobs. Note, moreover, that overall unemployment hardly changes. Nonetheless, the relative rise in low-level jobs is about twice as high as the fall in high-level jobs, as is shown in Figure 4.6. From this figure one also sees that the relative changes in the employment of high- and low-skilled workers are much more outspoken than the relative changes in corresponding jobs. Note moreover, that percentage changes in employment are larger (in absolute terms) than the corresponding percentage changes in jobs, for both low- and high-level jobs.

The overall conclusion to be drawn is then that a change in the growth rate of low-skilled wages does change the job composition of employment and employment itself in favour of low-level jobs. However, from a macro-perspective this seems to be very much a zero sum game, i.e. growth of output, c.q. substitution of labour for capital, will be needed to generate net positive results.

4.5 Experiments Concerning the Working of the Chimney Effect

In order to simulate the working of the chimney effect, we have introduced a bias in job augmenting technical change, thus changing the job composition of the demand for labour. In experiment 2 we have assumed a 1 per cent high-level job saving rate of technical change and a 1 per cent low-level job using technical change. Hence, less high-level and more low-level jobs are needed to generate one job efficiency unit, and it is obvious that the job composition of labour demand shifts in favour of low-level jobs. Since the demand for high-skilled workers on account of high-level jobs is now diminished for a given supply of high-skilled workers, relatively more high-skilled workers will be available for low-level jobs. In this case, the creation of low-level jobs does not necessarily have to lead to an increase in low-skilled employment. Rather, the 'excess availability' of high-skilled workers may reduce the employment opportunities for low-skilled workers.

By contrast, experiment 3 assumes only a 1 per cent high-level job using technical change, thus creating the conditions for the chimney effect to occur. Now a given supply of high-skilled labour will be confronted with an ever growing number of high-level jobs, thus reducing the amount of high-skilled labour available for employment on low-level jobs.

The effects on relative wages of both experiments have been depicted in Figure 4.4. The impact on relative wages is quite different in both cases. In experiment 2 w_0 falls initially, because of the shift in demand towards low-level jobs, but after a while the Phillips-effect leads to a stabilisation of the relative wage rate, although w_0 oscillates around this stable value. By contrast, experiment 3 generates a demand for high-skilled labour which cannot be realised due to supply constraints, and consequently high-skilled wages keep on rising.
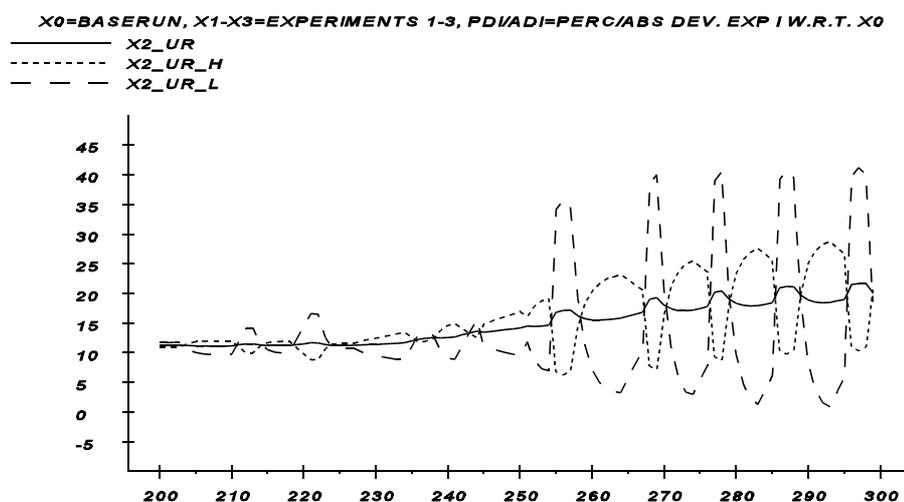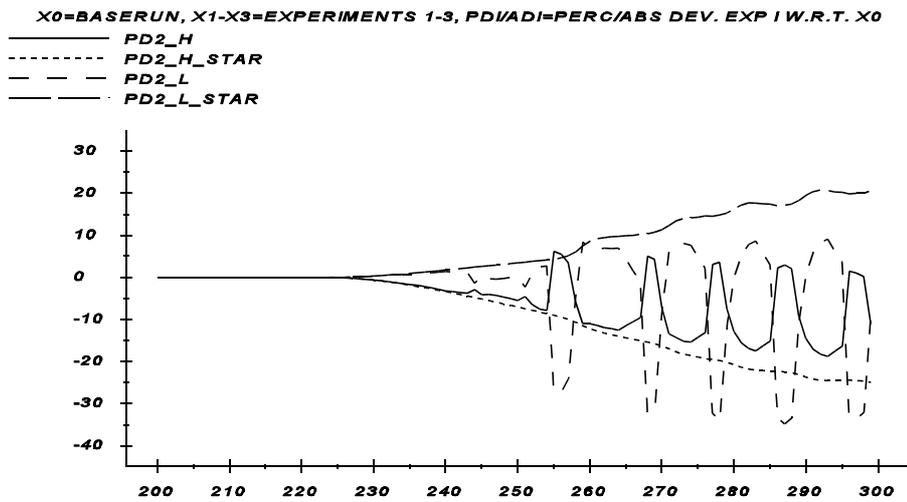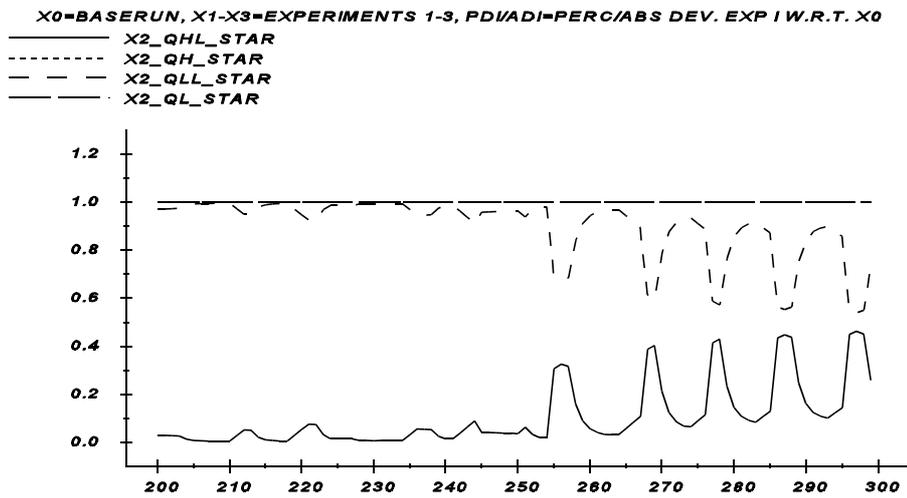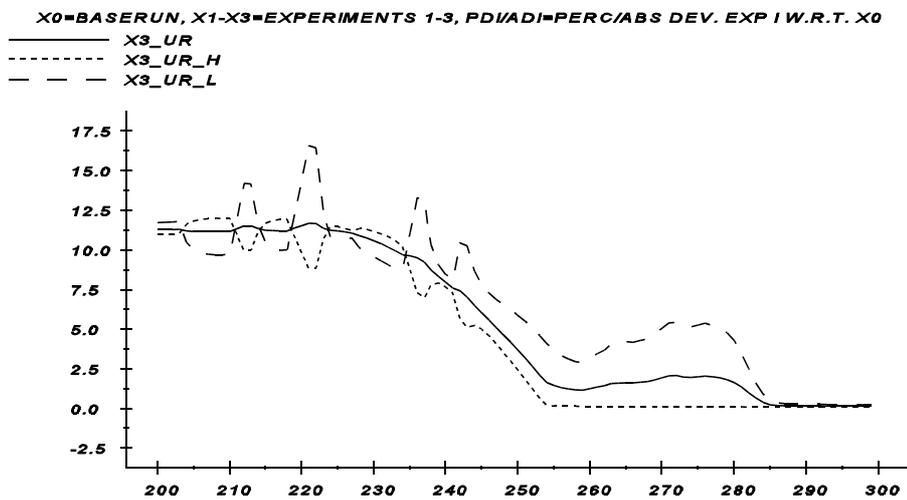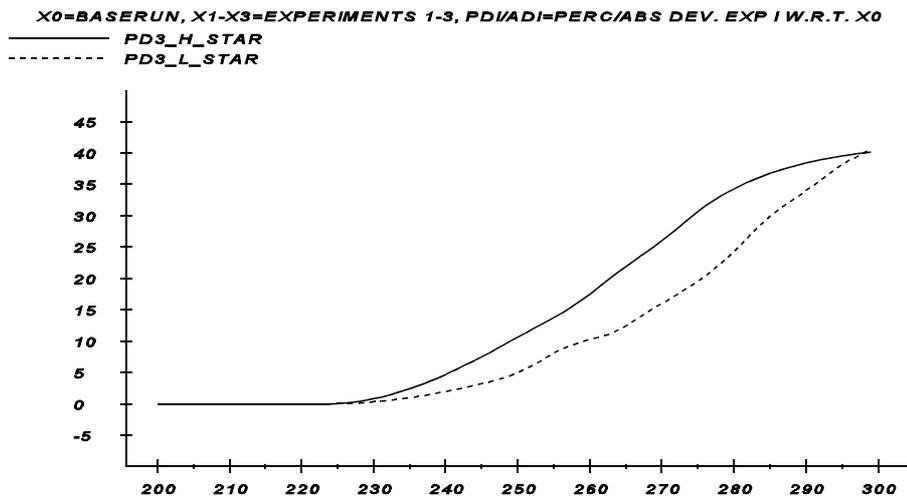
Figure 4.7

Figure 4.8

Figure 4.9

Figure 4.10

Figure 4.11

X0=BASERUN, X1-X3=EXPERIMENTS 1-3, PDI/ADI=PERC/ABS DEV. EXP I W.R.T. X0
——— PD3_H_STAR
- - - - - PD3_L_STAR

Figure 4.12

X0=BASERUN, X1-X3=EXPERIMENTS 1-3, PDI/ADI=PERC/ABS DEV. EXP I W.R.T. X0
——— PD3_H
- - - - - PD3_L

Figure 4.13

X0=BASERUN, X1-X3=EXPERIMENTS 1-3, PDI/ADI=PERC/ABS DEV. EXP I W.R.T. X0
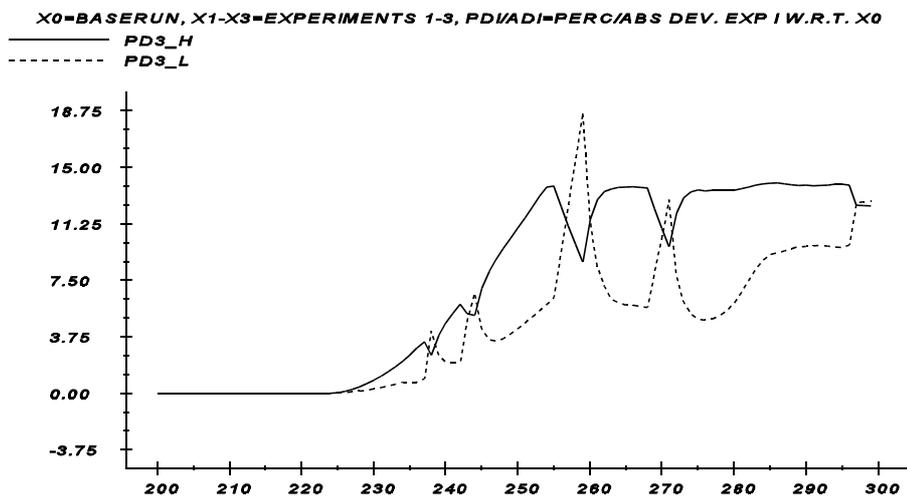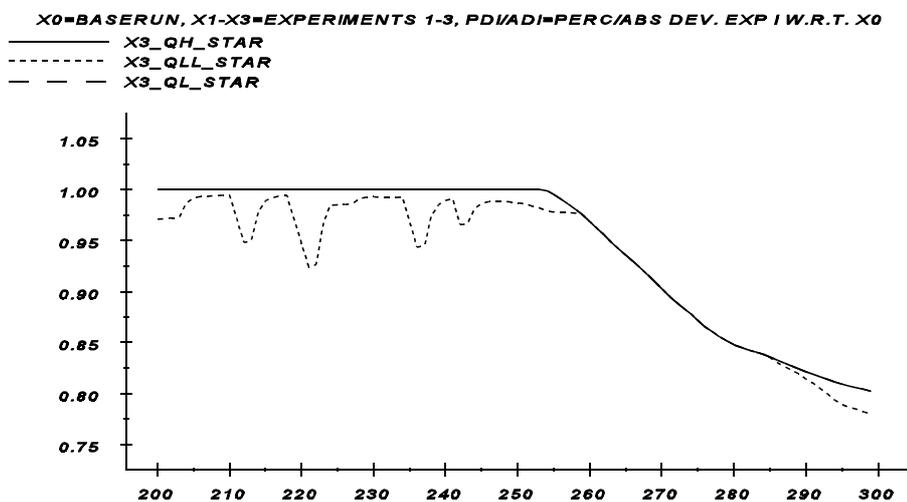——— X3_QH_STAR
- - - - - X3_QLL_STAR
— — — X3_QL_STAR

Figures 4.7-4.13 show the principal differences between both experiments. In experiment 2, relative wages fluctuate more or less around their base-run values. The reason is that the 1 per cent rate of high-job saving technical change increases the availability of high-skilled workers for low-level jobs. Moreover, the initial increase in u_H decreases w_0, but w_0 does not decrease enough to fully counter the increase in u_H. Also u_L starts to rise, which depresses low-skilled wages and hence increases w_0 again. The main effect is that changes in u_L and u_H fluctuate around an upward trend in the overall rate of unemployment, as can be seen from Figure 4.7. The reason is that the additional availability of high-skilled workers makes them replace low-skilled workers on low-level jobs. However, the increase in the number of low-level jobs does still not lead to a one for one rise in employment of low-skilled workers, as can be seen from Figure 4.8, where the percentage rise in low-skilled employment lags behind the percentage rise in low-level jobs. At the same time, the percentage fall in high-skilled employment is far less outspoken than the decrease in high-level jobs, which indicates that at least part of the growth of low-level jobs is absorbed by high-skilled workers rather than low-skilled workers. Moreover, the share of high-skilled workers on low-skilled jobs increases (see Figure 4.9 where QHL_STAR rises over the experimental period, while QLL_STAR falls and QL_STAR and QH_STAR are equal to 1), thus mitigating the effectiveness of the creation of low-level jobs for furthering employment opportunities for low-skilled workers. The bias in technical change is illustrated quite clearly in Figure 4.8 where low-level jobs increase and high-level jobs decrease. Nonetheless, both high-skilled employment and low-skilled employment decrease (with fluctuations around this decreasing trend). Note that due to the existence of unemployment for both types of workers, supply constraints can not be binding, and so, experiment 2 shows that the effectiveness of the creation of low-level jobs in alleviating low-level unemployment is negatively affected by the existence of excess supply of high-skilled workers.

By contrast, experiment 3 shows that the introduction of high-level job using technical change, leading to the creation of additional high-level jobs, may be much more effective in creating employment opportunities for low-skilled workers than the creation of low-level jobs did in the context of experiment 2. In Figure 4.11 we see that the introduction of high-level job using technical change does not only increase the number of high-level jobs, but also the number of low-level jobs. The latter is caused by the phenomenon of substitution of low-level jobs for high-level jobs, which is induced by the increase in high-skilled wages following on the reduction in high-skilled unemployment. Nonetheless, low-skilled employment is only positively affected for part of the experimental period, i.e. from 225-255 as indicated by the fall in QH_STAR below 1 from period 255, as shown in Figure 4.13. However, until the supply of high-skilled workers becomes limitational, low-skilled employment is favoured too, by high-skilled using technical change as indicated in Figure 4.10, where the rate of unemployment for low-skilled workers falls in a slower pace than the rate of unemployment for high-skilled workers does. In period 255, high-skilled unemployment becomes zero, and low-skilled unemployment starts rising again. With minor changes in the allocation of high- and low-skilled workers to low-level jobs after period 255, we see that high-level employment stabilises at a level given by supply, as indicated in Figure 4.12. Hence the increase in demand for high-skilled workers due to the increase in high-level jobs, enlarges employment

opportunities for low-skilled workers too.

The main conclusions of experiment 3 are therefore first that the creation of high-level jobs may favour employment opportunities for low-skilled workers, and secondly that the occurrence of bottlenecks in the supply of high-skilled workers may also diminish employment prospects for low-skilled workers. Both conclusions point to the paradox of needing to monitor the high-skilled side of the labour market, especially in times of low-skilled unemployment.

6 Summary and Conclusion

In this paper we have presented the outlines of a labour allocation model which focuses on asymmetries in substitution possibilities between different types of labour. The general idea underlying this model is that high-skilled people can take over the jobs of low-skilled people but not necessarily the other way around. This implies that the number of low-level jobs is an upper-limit to low-skilled employment, while the number of high-level jobs is a lower limit to high-skilled employment. We assume that a high-skilled person has a non-zero efficiency on a low-level job, whereas a low-skilled person has zero efficiency on a high-level job, which is an exaggeration of the actual immobility of skills between jobs.

We have assumed forward looking behaviour for the firm. Entrepreneurs determine the job composition of employment, as well as the capital intensity of production, based on the expected growth of relative wages. Moreover, in deciding about the job-composition of labour demand, they take account of future changes in the allocation of skills to jobs - when relative wages and relative efficiencies indicate that this would be the most profitable thing to do. We have linked the cost of changing/making an allocation between skills and jobs to the cost of learning. This automatically divides available supply of a certain skill into two groups: the internal pool of people with experience relevant to a certain match, and both internal and external pools of people with no experience in the match under consideration. We solve the allocation problem by means of the definition of a linear programming module, which takes explicitly account of both the existence of asymmetric substitution possibilities between skills on certain jobs, and the available supply of people with and without experience in a certain match between skills and jobs.

A typical solution to the problem of the uneven distribution of unemployment across skills is to rely much on wage flexibility to alleviate this problem. However, one of the results of our model is that asymmetries in substitution possibilities between low-skilled and high-skilled people imply that in order to achieve a positive net employment response by means of a change in the job composition in favour of low-level jobs, quite a fall in low-skilled wages may be required. But even then the largest impact on low-skilled unemployment is to be expected from diminishing the incentives for having high-skilled people replace low-skilled people on low-level jobs.

We also ran two simulation experiments to illustrate the working of the so-called chimney effect. It was shown that the creation of low-level jobs in a situation of excess supply of high-skilled workers may not be an effective way of creating employment opportunities for low-skilled workers. Instead, the creation of high-level jobs which requires reallocations of high-skilled workers from low-level jobs to high-level jobs may be much more effective in generating effective job-openings for low-skilled workers than the direct creation of low-level jobs would be (this is the chimney effect). Moreover, the creation of high-level jobs may lead to the creation of low-level jobs as a 'by-effect', which would essentially increase the effectiveness of the creation of high-level jobs in alleviating low-skilled unemployment problems. These last conclusions point to the paradox of needing to monitor the high-skilled side of the labour market, especially in times of low-skilled unemployment.

Annex A. Solving the Intertemporal Cost Minimisation Problem

For cases 2 and 3, there are two periods to consider, i.e. the pre-switching period and the post-switching period. Case 4 is the case where both L and H skills are used to fill L* and H* jobs. The various objective functions associated with the cases mentioned above are given in equations (A.1)-(A.3).

$$T_2 = \int_0^{t*} w_{H,t}.(L^*/e' + H^*).e^{-(\rho+\delta).t}\,dt + \int_{t*}^{\infty}\left(w_{L,t}.L^* + w_{H,t}.H^*\right).e^{-(\rho+\delta).t}\,dt \qquad (A.1)$$

$$T_3 = \int_0^{t*}\left(w_{L,t}.L* + w_{H,t}.H*\right).e^{-(\rho+\delta).t}\,dt + \int_{t*}^{\infty} w_{H,t}.(L*/e' + H*).e^{-(\rho+\delta).t}\,dt \qquad (A.2)$$

$$T_4 = \int_0^{\infty}\left(w_{L,t}.L* + w_{H,t}.H*\right).e^{-(\rho+\delta).t}\,dt \qquad (A.3)$$

The cost function parameters α and β presented in table A.1, can be derived from for a given moment of switching t*. These are presented in the table below, where z is defined to be equal to $z=(\hat{w}_L - \rho - \delta)/\hat{w}$.

In order for the cost-minimisation problem to have a meaningful solution, it follows that the α's and β's should be positive. This requires the growth rates of high-skilled wages and low-skilled wages to be less than $\rho + \delta$, which we have had to assume in the first place in order to obtain finite objective function values. Hence, all the α's are positive, while it is immediately clear that β_1 and β_2 are positive too. In order for β_3 to be positive, it is necessary that $(e'/w_0)^z < z+1$. Note that in case 3 z>0 and e'/w_0<1, which implies that $(e'/w_0)^z<1<z+1$ for z>0, as required. For case 2 similar results can be obtained. Defining z'=-z, the requirement that $\beta_2>0$ implies that $(w_0/e')^{z'-1}-z'<0$, since z+1<0, by assumption. Because w_0/e'<1 in case 2, this only holds for z'>1, i.e. for z<-1.[24] Hence $\beta_2>0$ when z<-1. Using the definition for z (see above), this requires that $\hat{w}_L - \rho - \delta < \hat{w}_L - \hat{w}_H \ \Rightarrow\ \rho + \delta - \hat{w}_H > 0$, which is true by assumption.

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[24] For z'=1, it follows that $(w_0/e')^0=z'=1$. For 0<z'<1 $(w_0/e')^{z'-1}>1$, while z'<1, by assumption. Hence, $\beta_2>0$ for 0<z'<1.

Table A.1 Cost Function Parameters

Case	α	β
1	$\dfrac{w_{H,0}}{\rho + \delta - \hat{w}_H}$	$\dfrac{w_{L,0}}{\rho + \delta - \hat{w}_L}(w_0/e')\dfrac{z}{1+z}$
2	$\dfrac{w_{H,0}}{\rho + \delta - \hat{w}_H}$	$\dfrac{w_{L,0}}{\rho + \delta - \hat{w}_L}\left((w_0/e')\dfrac{z}{z+1} + (w_0/e')^{-z}\dfrac{1}{1+z}\right)$
3	$\dfrac{w_{H,0}}{\rho + \delta - \hat{w}_H}$	$\dfrac{w_{L,0}}{\rho + \delta - \hat{w}_L}\left(1 - (w_0/e')^{-z}\dfrac{1}{1+z}\right)$
4	$\dfrac{w_{H,0}}{\rho + \delta - \hat{w}_H}$	$\dfrac{w_{L,0}}{\rho + \delta - \hat{w}_L}$

References

Bischoff, C.W., 1971, 'The effect of alternative lag distributions', in G. Fromm (ed.), <u>Tax incentives and capital spending</u>, the Brookings Intitution, Washington.

Broer, D.P., and W.J. Jansen, 1989, 'Employment, Shooling and Productivity Growth', <u>De Economist</u>, no. 137, pp. 425-453.

Draper, N. and T. Manders, 1996, 'Structural Changes in the Demand for Labour', CPB Netherlands Bureau for Economic Policy Analysis, <u>Research Memorandum</u>, nr. 28.

Hamermesh, D.S. and J.H. Grant, 1979, 'Econometric Studies of Labor-Labor Substitution and Their Implications for Policy', <u>Journal of Human Resources</u>, no. 14, pp. 518-542.

Hebbink, G.E., 1991, 'Employment by Level of Education and Production Factor Substitutability', <u>De Economist</u>, no. 139, pp. 379-401.

Kugler, P., U. Muller and G. Sheldon, 1990, 'The Labor Market Effects of New Technologies - an Econometric Study for the Federal Republic of Germany', in Schettkat, R. and M. Wagner (Eds.), <u>Technological Change and Employment Innovation in the German Economy</u>.

Kuipers, S.K. and A.H. van Zon, 1982, 'Output and Employment Growth in the Netherlands in the Postwar Period: A Putty-Clay Approach', <u>de Economist</u>, 130, pp. 38-70.

Lucas, R.E., 1988, ' On the Mechanics of Economic Development', <u>Journal of Monetary Economics</u>, No. 22, pp. 3-42.

<u>MEV: Macro Economische Verkenningen 1997</u>, Centraal Planbureau, 1996, Den Haag.

Mincer, J., 1989, 'Human Capital Responses to Technological Change in the Labor Market', <u>NBER Working Paper</u>, no. 3207.

Muysken, J. and A.H. van Zon, 1987, 'Employment and unemployment in the Netherlands, 1960-1984 : a putty-clay approach', <u>Recherches Economiques de Louvain</u>, Vol. 53, pp. 101-133.

Nickel, S. and B. Bell, 1995, 'The Collapse in Demand for the Unskilled and Unemployment across the OECD', <u>Oxford Review of Economic Policy</u>, Vol. 11, no. 1.

Romer, P., 1990, 'Endogenous Technological Change', <u>Journal of Political Economy,</u> pp. 71-102.

Rijen, P. J. van den, 1996, 'Verdringing geen probleem op de Nederlandse arbeidsmarkt', <u>ESB</u>, nr. 5-6-1996, p. 508-510.

Werkgelegenheidsnota 1996, Ministerie van Sociale Zaken en Werkgelegenheid, Den Haag 1996.

Zon, A.H. van, 1990, Vintage Capital and R&D Based Technological Progress, in Freeman C. and L. Soete (eds.), New Explorations in the Economics of Technological Change.

Zon, A.H. van and J. Muysken, 'Matching Skill Supply and Demand: A Linear Programming Approach', MERIT Research Memorandum, 92-005.

Zon, A.H. van, M. Janssen, H. Meijers, J. van der Meulen, J. Muysken, M. Oude Wansink, 1995, 'Technical Change, Unemployment and Skill Mismatch', Report for DG5 of the European Commission.

Zon, A.H. van, and J. Muysken, 1996, 'MASS: A Model of Asymmetric Skill Substitution', MERIT Research Memorandum, nr. 2/96-016.

Zon, A.H. van, J. Muysken, and H.H.M. Meijers, 1997, 'Asymmetric Skill Substitution, Labour Market Flexibility, and the Allocation of Qualifications', in Heijke, H. and L. Borghans (eds.), Towards a Transparent Labour market for Educational Decisions, Avebury Publishers.