Endogenous technological change by cost-reducing and demand-creating innovations

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1. Introduction

Technological progress is generally accepted as the most important factor contributing to economic growth. But contrary to what standard empirical implementations of technological change suggest, it is the intentional outcome of the behaviour of economic agents rather than an "exogenous" residual. Economic agents "invest" in technological change in order to be able to reap its benefits in the future. Research and Development (R&D) expenditures are an important component of such investments in technological change. The R&D process can therefore be seen as an imperfect approximation of the technology generation process of a firm.

It is possible to divide R&D expenditures into two categories (Scherer, 1982, 1984). First, R&D can be directed to improve the efficiency of the production process with the aim of obtaining cost-saving benefits in the future. This will be called process R&D. Secondly, one can engage in R&D in order to improve the quality of ones products which increases expected demand and experience the benefits of demand creation in the future. This will be called product R&D. It is important to recognise that in both cases the benefits of current R&D are spread over the future.

The main aim of this paper is to determine the optimal level of R&D directed at cost-reducing innovations and demand-creating innovations given initial quality and productivity levels. Other important aims are to determine which factors will favour or temper technological change and whether product and process R&D are substitutes or complements.

Levin and Reiss (1989) developed a static model in which the amount of product and process R&D were endogenously determined. Kamien and Schwartz (1969), Sato and Ramachandran (1974), Sato and Suzawa (1984) considered also the dynamic effects of process R&D but neglected the effects of product R&D. We will develop a model which considers the dynamic effects of process R&D as well as those of product R&D. Because of the intertemporal effects of R&D, we will use optimal control theory to solve these models. The latter usually implies that the models themselves are (over-) simplified representations of economic behaviour/constraints.

In section 2 we discuss the case in which a firm can engage in product R&D in order to achieve a higher expected demand (demand creating R&D). In section three we consider the case in which a firm can engage in process R&D in order to achieve efficiency gains in the production (cost reducing R&D). In the fourth part we discuss the case in which a firm can engage in both demand creating R&D as well as cost-reducing R&D. Section 5 contains a summary and some concluding remarks.

2. Demand Creating R&D

In this part we discuss the case in which an entrepreneur can engage in product R&D in order to increase the quality of his products. With regard to quality we distinguish between two different types: *intrinsic* quality and *perceived* quality. The former can be regarded as a weighted score on a number of relevant aspects of the product in question, while the latter can be thought of as the intrinsic quality of some product relative to the intrinsic quality of other similar/competing products. Instead of specifying the R&D behaviour of competitors in detail, we simply assume that everybody engages in product R&D directed at improving the perceived quality of ones product by increasing its intrinsic quality. We disregard the possible role of marketing here, which might increase the perceived quality of a product without a corresponding rise in its intrinsic quality characteristics.

The main effect of increasing the perceived quality of a product is to raise (expected) demand. We use perceived quality, which is defined as the level of quality experienced by people. If the perceived quality level of a product doesn't change, it will be less appreciated at time t+1 than at time t. Without doing any R&D, the perceived quality level will decline over time for example because other products, which serve as a reference group, are improving. When perceived quality influences expected demand, a firm has to do "defensive" product R&D just to maintain its marketshare. An entrepreneur can therefore influence his expected demand not only by the standard price instrument, but the perceived quality of a product serves as an additional "marketing" instrument. When the perceived quality of a product can be induced by means of advertising the general idea of this approach resembles very much the treatment of advertising by Dorfman and Steiner (1954) .

Hoos (1959), Nerlove and Arrow (1962) and Gould (1970) assumed that current advertising expenditures can be seen as a form of investment in a stock of goodwill which affects the present and future demand for the product. We use a related approach in which current product R&D expenditures influence the quality level not only in the current period but also in the future. A firm which maximises profits must therefore take account of the intertemporal benefits of product R&D which result from current R&D expenditures. The intertemporal effects of product R&D which change the intrinsic as well as the perceived quality could be expected too be more important than the intertemporal effects of advertising which shifts only the perceived quality level.

We assume that the expected demand for a product is negatively influenced by the price of the product and positively by the perceived quality of the product. We postulate the following expected demand function:

$$Y_t^d = X_0 . Q_{p,t}^b . P_t^{-a} \tag{2.1}$$

where Y_t^d is the expected demand at time t, P_t is the price level at time t, X_0 is a positive scale parameter and $Q_{p,t}$ is the perceived quality level at time t. a and b are both positive parameters.

The perceived quality level of a product can be raised by performing product R&D. We assume decreasing marginal quality increases when product R&D expenditures grow. Without doing any product R&D the level of perceived quality will decrease due to the R&D activities of ones competitors. For reasons of simplicity we assume a constant exponential decay of perceived quality.

$$\frac{dQ_p}{dt} = \gamma . R^{\delta}_{d,t} - w . Q_{p,t} \tag{2.2}$$

where $R_{d,t}$ is the amount of product R&D at time t and w is the perceived quality depreciation rate. When w=0 the perceived quality level coincides with the intrinsic quality level. When we interpret Q_p as Goodwill and R_d as advertising expenditures and assume that the costs of adding to goodwill are linear (δ=1) we obtain the dynamic Nerlove/Arrow equation. As Gould (1970) we assume non-linear costs which implies different dynamic behaviour than linear costs.

We assume that a firm will choose its price level and product R&D budget so as to maximize the present value of its expected profits (π), given its initially perceived quality level. We can specify the intertemporal profit maximisation problem of a producer by:

$$\begin{aligned} \underset{R_d,\, P}{Max}\ \pi(0) &= \int_0^{\infty} e^{-rt} \left[(P_t - C_t) . Y_t^d(P_t, Q_{p,t}) - R_{d,t} . q_{d,t}\right] dt \\ s.t.\ & Y_t^d = X_{0,t} . Q^b_{p,t} . P_t^{-a} \qquad with\ \ 0 \le b \le 1 \\ & dQ_p/dt = \gamma . R^{\delta}_{d,t} - w . Q_{p,t} \qquad with\ \ 0 \le \delta \le 1 \\ & Q_{p,t=0} = Q_{p,0} \end{aligned} \tag{2.3}$$

where C_t is the unit production cost at time t[1], r is the discount rate and $q_{d,t}$ is the price of product R&D at time t. a, b and δ are non-negative parameters.

By using the current-value Hamiltonian, we can in effect disregard the discount factor (Chiang 1992). The current value Hamiltonian of this problem is:

$$H^c = (P_t - C_t) X_{0,t} . Q^b_{p,t} . P_t^{-a} - R_{d,t} . q_{d,t} + \lambda_t (\gamma . R^{\delta}_{d,t} - w . Q_{p,t}) \tag{2.4}$$

where P_t and $R_{d,t}$ are control variables. $Q_{p,t}$ is the state variable and λ_t is it's co-state variable. The first two components on the right hand side of (2.4) are simply the profit function at time t which depends on the value of the control variables to be chosen. The third component can be rewritten as $\lambda . dQ_{p,t}/dt$. Where λ can be interpreted as an increase in future profits resulting from an increase in Q_p at time t. This third term represents therefore the amount of additional profits in the future due to an increase in the state variable $Q_{p,t}$. So, the Hamiltonian maximizes the value of current and future profits (see, also Dorfman, 1969).

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[1] In this model we assume that product R&D increases the intrinsic quality of the product without affecting unit costs. A logical extension of this model is that a higher quality level can only be obtained by higher unit costs: $C = C(Q_p)$ where $C' > 0$.

Necessary conditions[2] for a path to be optimal under these assumptions are the following first order conditions[3]:

$$\frac{\delta H_c}{\delta P} = Y^d - (P-C).\frac{a.Y^d}{P} = 0 \Rightarrow P = \frac{-a}{1-a}.C \quad ; \quad a>1 \tag{2.5}$$

$$\frac{\delta H_c}{\delta R_d} = -q_d + \lambda.\gamma.\delta.R_d^{\delta-1} = 0 \tag{2.6}$$

$$\frac{\delta H_c}{\delta Q_p} = -\frac{d\lambda}{dt}+\lambda.r=(P-C).X_0.P^{-a}.b.Q_p^{b-1}-w.\lambda \tag{2.7}$$

$$\frac{\delta H_c}{\delta \lambda} = \frac{dQ_p}{dt}=\gamma.R_d^{\delta}-w.Q_p \tag{2.8}$$

$$TVC : \lim_{t\to\infty} \lambda_t.e^{-rt} = 0 \tag{2.9}$$

The first equation shows that the price will be determined as a mark-up over marginal costs. This is the familiar Amoroso-Robinson relation, which also holds in this case, where we have assumed a linear homogenous production function and instantaneous adjustment of labour and capital, i.e. the standard factors of production. The mark-up depends on the price elasticity of demand (a). Because the price has to be positive the price elasticity of demand has to be larger than one.

The second equation implies that in the optimum situation, a marginal increase in the amount of product R&D, which decreases current profits with its price (q_d) has to be counterbalanced by an increase in future profits; the marginal increase in quality times its marginal value (λ).

In the third equation denotes $d\lambda/dt$ the rate at which the marginal value of a unit of quality is depreciating when $d\lambda/dt<0$ and appreciating when $d\lambda/dt>0$. In the optimum situation the marginal value of quality depreciates at the rate at which quality

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[2]The necessary conditions are also sufficient for global maximization if the Mangarasian Sufficiency Theorem is satisfied. This theorem contains two conditions. First, the profit function and the perceived quality generation function are differentiable and concave in the variables (Q_p, R_d) and secondly, in the optimal solution it is true that $\lambda \geq 0$ for all $t \in [0, \infty>$ if dQ_p/dt is nonlinear in Q_p or R_d (see Chiang 1992, p 215-p 221).

Our model satisfies the first condition, the second condition has to be checked for any optimal solution because dQ_p/dt is nonlinear in R_d.

[3] For reasons of simplicity we have dropped time subscripts.

is contributing to the current profits (this is represented by the first term at the left hand side). The marginal value of capital appreciates at the rate of the discount rate and the perceived quality depreciation factor (respectively the second term on the right hand side and the second term on the left hand side).

The fourth equation is just the dynamic constraint, i.e. the quality generation function. The fifth equation is the transversality condition (TVC). This condition implies that the marginal present value of quality is zero when $t \rightarrow \infty$. The intuition of the latter condition is that the firm maximizes profits in the interval [t=0, t=∞], so the firm has to exhaust all the possible profit possibilities during this interval. This is only the case when at the end of the interval the marginal present value of increasing the quality level is zero.

A firm has to form expectations about the future course of the input prices and the autonomous scale of demand in order to make decisions about its output price level and its product R&D level. Gould (1970) in his related dynamic model of advertisements assumed constant prices. We will assume that prices and the exogenous scale of demand increase at given constant rates:

$$\begin{aligned} q_t &= q_0.e^{l_d t} \\ C_t &= C_0.e^{mt} \\ X_{0,t} &= X_{0,0}.e^{nt} \end{aligned} \qquad \textbf{(2.10)}$$

We assume that the marginal cost (C) is increasing at rate m, the price of product R&D (q_d) at rate l_d and the autonomous scale of demand (X_0) at rate n.

Steady State growth rates

We will differentiate the first order conditions with respect to time to obtain the equilibrium (steady state) rates of growth:

$$\hat{P} = \hat{C} \qquad \textbf{(2.11)}$$

$$\hat{\lambda} + (\delta - 1).\hat{R}_d = \hat{q}_d \qquad \textbf{(2.12)}$$

$$\hat{\lambda} = \hat{C} - (1-b).\hat{Q}_p - a.\hat{P} + \hat{X}_0 \qquad \textbf{(2.13)}$$

$$\hat{Q}_p = \delta.\hat{R}_d \qquad \textbf{(2.14)}$$

where we assume that the growth rates of the system will be constant in the steady state. We have therefore put the changes over time of the growth rates equal to zero. After making a few straightforward substitutions we get the following steady state growth rates:

$$
\begin{aligned}
\hat{R}_d^* &= \frac{\hat{X}_0-\hat{C}.(a-1)-\hat{q}_d}{1-b.\delta} = \frac{n-m(a-1)-l_d}{1-b.\delta} \\
\hat{Q}_p^* &= \delta.\hat{R}^* \\
\hat{Y}^{d*} &= \frac{n-m.(a-b.\delta)-l_d.b.\delta}{1-b.\delta}
\end{aligned}
\tag{2.15}
$$

The steady state growth rates are constant because we assume that the growth rates of X_0, q_d and C are also constant (see equation 2.10). The steady state growth rates of R_d, Q_p and Y^d are positively influenced by the growth rate of the autonomous scale of demand and negatively influenced by the growth rates of the marginal cost and the price of R&D. The elasticity of demand (a) has a negative impact and the quality elasticity of demand (b) and the R&D elasticity of the quality generation process (δ) have a positive influence on the steady state growth rate of R_d and Q_p.

Transversality Condition

The growth rate of product R&D is constant in the steady state, which in combination with equation (2.12) implies that the steady state growth rate of the marginal value of perceived quality is also constant. We can use this information when we put the transversality conditions in growth rates:

$$
TVC \quad \lim_{t\to\infty} \quad \lambda.e^{-rt} = 0 \Rightarrow \hat{\lambda}-r <0 \Rightarrow r > \hat{q}+(1-\delta).\hat{R}_d^* \tag{2.16}
$$

The transversality condition is satisfied if r> $\hat{q}+(1-\delta).\hat{R}_d^*$. The economic rational for this sufficiency condition is simple: the growth rate of the marginal value of perceived quality has to be less than the discount rate, otherwise one would invest in producing perceived quality without bound.

Elimination of Time

The differential equations which describe the dynamics of the systems which we will study are non-linear. This makes these systems analytically less tractable. In this paper we will therefore use a qualitative-graphic (phasediagram) analysis to study its dynamics[4]. The two differential equations which describe the dynamics of the system are $dQ_p/dt=f(Q_p, R_d)$ and $dR_d/dt=g(Q_p, R_d)$ and can be calculated from the first order conditions. Because we assume that input prices and the autonomous part of the scale

[4] For details of the qualitative-graphic analysis of a nonlinear differential-equation, see A.C. Chiang, 1984. Concerning the value of this approach Chiang says: 'The two variable phase diagram,.., is limited in that it can only answer qualitative questions- those concerning the location and dynamic stability of the intertemporal equilibrium(s). But,.., it has the compensating advantages of being able to handle nonlinear systems as comfortable as linear ones and to address problems couched in terms of general functions as readily as those in terms of specific ones' (Chiang, 1984, p 629).

of demand grow with a constant rate, the first order conditions and therefore the differential equations do depend on time. For the application of the phase-diagram technique it is a prerequisite that the variable t (time) does not enter into the differential equations as a separate argument (the system has to be autonomous) because otherwise each point in a phase space can imply different directions of the system over time. When this is the case it is not possible to make qualitative statements about the characteristics of a possible equilibrium (see, Chiang 1984).

In order to apply the phase diagram analysis to the case with exponentially growing input prices, we have to remove the time component from the profit maximization problem by a suitable redefinition of the variables of the system. Equation (2.15) showed that both perceived quality and the product R&D budgets grow with constant rates which are directly linked to the constant rates of growth of the exogenous variables of the system and to the various elasticities of the system. When we deflate Q_p and R_d with their steady state growth rates we will get an autonomous system of differential equations (see, Lucas 1988). To this end, we define the following new variables:

$$Q_p'' = Q_{p,t}.e^{-\left(\frac{\delta.(n-m(a-1)-l_d)}{1-b.\delta}\right).t} = Q_{p,t}.e^{-\delta.\sigma.t} \tag{2.17}$$

$$R_d'' = R_{d,t}.e^{-\left(\frac{n-m(a-1)-l_d}{1-b.\delta}\right).t} = R_{d,t}.e^{-\sigma.t} \tag{2.18}$$

$$P'' = P_t.e^{-mt} \tag{2.19}$$

where $\sigma=(n-m(a-1)-l_d)/(1-b.\delta)$.

We will redefine the profit maximization problem given by equation (2.3) in terms of these three new variables. In order to get a new dynamic constraint in terms of Q_p'', we can differentiate equation (2.17) with respect to time. When we use this result in combination with the old constraint we can solve for dQ_p''/dt:

$$\frac{dQ_p''}{dt} = \gamma.(R_d'')^{\delta} - (w+\delta.\sigma).Q_p'' \tag{2.20}$$

Notice that the depreciation factor has increased with the steady state growth rate of Q_p in comparison with the old constraint (equation 2.2). Using the assumption that all exogenous variables grow with a constant rate (see equation 2.10) and the equations (2.17-2.20), we can rewrite the current value Hamiltonian as:

$$H_c = (P'' - C_0)X_{0,0}.(Q_p'')^{b}.(P'')^{-a} - R_d''.q_{d,0} + \lambda.(\gamma.(R_d'')^{\delta} - (w+\delta.\sigma).Q_p'') \tag{2.21}$$

The associated discount factor of this problem is $\sigma+\hat{q}_d-r$, where $\sigma+\hat{q}_d$ represent the steady state growth rate of the revenues, costs and product R&D budget.

We have now reached the stage where every explicit time dependency has been removed. We can calculate the first order conditions associated with this problem:

$$\frac{\delta H_c}{\delta P''} = 0 \;\Rightarrow\; P'' = \frac{-a}{1-a}.C_0 \quad ; \quad a>1 \tag{2.22}$$

$$\frac{\delta H_c}{\delta R_d''} = -q_{d,0} + \lambda.\gamma.\delta.(R_d'')^{\delta-1} = 0 \tag{2.23}$$

$$\frac{\delta H_c}{\delta Q_p''} = -\frac{d\lambda}{dt} + \lambda.(r-\sigma-\hat{q}_d) = (P''-C_0).X_{0,0}.b.(P'')^{-a}.(Q_p'')^{b-1} - (w+\delta.\sigma).\lambda \tag{2.24}$$

$$\frac{\delta H_c}{\delta \lambda} = \frac{dQ_p''}{dt} = \gamma.(R_d'')^{\delta} - (w+\delta.\sigma).Q_p'' \tag{2.25}$$

$$TVC: \lim_{t\to\infty} \lambda_t.e^{(\sigma+\hat{q}_d-r).t} = 0 \tag{2.26}$$

When we put these first order conditions in growth rates and calculate the steady state growth rates of these redefined variables we find that the steady state growth rates of Q_p'', R_d'' and λ are zero. A zero growth rate for λ in the steady state implies that the transversality condition is satisfied when $r > \sigma+\hat{q}_d$ in the steady state.

When we combine 2.22-2.24 and use that $d\lambda/dt=0$ in the steady state we can derive the following expression for the product R&D/sales ratio:

$$\frac{Q_p''}{P''.Y''} = \frac{b}{a.(r+w-\hat{q}_d-(1-\delta).\sigma)}.\frac{\gamma.\delta.R_d''^{\,\delta-1}}{q_o} \tag{2.27}$$

This equation contains the familiar Dorfman/Steiner and Nerlove/Arrow advertising to sales ratio as a special case. The Nerlove/Arrow ratio can be obtained when there are constant returns to R&D ($\delta=1$), when the exogenous growth rates of all variables are zero ($\sigma = \hat{q}_d = 0$) and when we use R&D expenditures ($q_d=1$). The static Dorfman/Steiner result can be obtained when in addition the discount rate and depreciation factor are zero.

Dynamics

To study the dynamics of the autonomous system we need the two differential equations, $dQ_p''/dt=f(Q_p'',R_d'')$ and $dR_d''/dt=g(Q_p'',R_d'')$, which describe the movements in the (Q_p'', R_d'') space. The first differential equation is equation (2.25), i.e. the dynamic constraint. The second differential equation can be calculated from the other first order conditions. We can solve equation (2.23) for λ, and differentiate this result with respect to time.

Now we substitute $d\lambda/dt$ and λ in equation (2.24) and obtain the differential equation for dR_d''/dt:

$$\frac{q_0.(r+w-\hat{q}_d-(1-\delta).\sigma)}{\gamma.\delta}.(R_d'')^{1-\delta}-\frac{q_{d,0}.(1-\delta).(R_d'')^{-\delta}}{\gamma.\delta}.\frac{dR_d''}{dt}=\frac{C_0X_{0,0}.b}{(a-1).(P'')^a}.(Q_p'')^{b-1}$$

(2.28)

Note that, in this redefined system, dQ_p''/dt and dR_d''/dt depend only on Q_p'' and R_d'' and that the time variable does not enter anymore into these equations as a separate argument.

To construct the phase diagram, we need two demarcation loci, the $dQ_p''/dt=0$ and $dR_d''/dt=0$ loci. Each demarcation locus provides the location for any prospective equilibrium and separates the phase space into two regions, one characterised by $di/dt<0$ and the other by $di/dt>0$, where $i = Q_p''$, R_d''[5]. An important implication of the autonomous character of this system is that these demarcation lines have positions which remain fixed overtime. The two demarcation lines represent the graphs of the two equations of (zero) motion:

$$\frac{dQ_p''}{dt}=0 \quad \Rightarrow \quad R_d''=\left(\frac{w+\delta.\sigma}{\gamma}\right)^{\frac{1}{\delta}}.(Q_p'')^{\frac{1}{\delta}} \qquad \textbf{(2.29)}$$

$$\frac{dR_d''}{dt}=0 \quad \Rightarrow \quad \left(\frac{q_{d,0}.(r+w-\hat{q}_d-(1-\delta).\sigma)}{\gamma.\delta}\right).(R_d'')^{1-\delta}=\left(\frac{C_0X_{0,0}.b.(P'')^{-a}}{a-1}\right).(Q_p'')^{b-1} \qquad \textbf{(2.30)}$$

The perceived quality level and product R&D level have to be positive in equation (2.30), which implies $r+w-\hat{q}_d-(1-\delta).\sigma>0$. This condition is automatically satisfied when the transversality condition is met (see equation (2.26)). The two demarcation lines are drawn in figure 2.1, they constitute a "phase diagram" drawn in the (Q_p'', R_d'') space. The $dR_d''/dt=0$ locus is negatively sloped because $0\leq b\leq 1$, $0\leq\delta\leq 1$ and $a>1$. The locus $dQ/dt=0$ is positively sloped.

The economic interpretation of $dR_d''/dt=0$ can best be seen from equation (2.30). The right hand side of (2.30) can be rewritten as $(P''-C_0).\delta Y^d/dQ_p''$, which is the marginal increase of the profits due to an expansion of Q_p''. The left hand side can be rewritten as $(r+w-\hat{q}_d-(1-\delta).\sigma).q_d)/(\partial(dQ_p''/dt)/\partial R_d'')$, which are the marginal costs of obtaining this extra unit of Q_p''. So, on the $dR_d''/dt=0$ locus, marginal costs of increasing Q_p'' are equal to the associated marginal benefits. The negative slope of this locus can be explained as follows. On the right hand side of equation (2.30) the marginal benefits are declining when Q_p'' increases because $\delta^2Y^d/dQ_p''^2$ is negative ($b<1$). On the $dR_d''/dt=0$ locus marginal benefits are equal to marginal costs, so when marginal benefits are declining, marginal costs have to decline too. The marginal costs of 'producing' Q_p'' are increasing when R_d'' gets larger because $\delta^2(dQ_p''/dt)/\delta R_d''^2$ is negative. So, when the lower

[5]See Chiang, 1984 p 629.

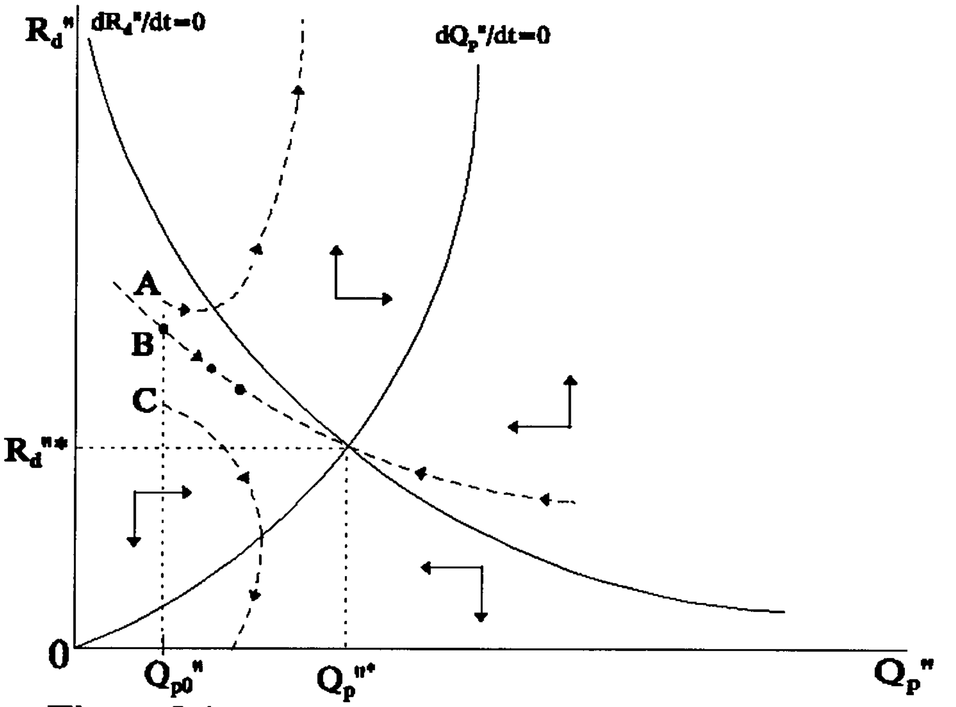

Figure 2.1
The dynamics of perceived quality and R&D

benefits associated with a higher Q_p" level have to be met with lower marginal costs on the dR_d"/dt=0 locus, the level of R_d" has to decline. This implies a negative slope of the dR_d"/dt=0 locus.

The economic interpretation of the dQ_p"/dt=0 locus is simpler. On this locus the depreciation of the perceived quality level is equal to the generation of perceived quality. This locus has a positive slope because a higher perceived quality level implies more depreciation which has to be met with more quality generation which can only be achieved by doing more product R&D.

In order to study the dynamics of the system, we use the phase diagram in figure 2.1. The demarcation locus dQ_p"/dt=0 is upward sloping. Using the dQ_p"/dt differential equation (equation 2.25) to describe the horizontal movements in the phase diagram we find that anywhere below the dQ_p"/dt=0 locus, the perceived quality level Q_p" is decreasing: the amount of product R&D is not enough to maintain the perceived quality level. Similarly, Q_p" is increasing above the dQ_p"/dt=0 locus. The horizontal arrows demonstrate these directions of motion. The second demarcation locus, dR_d"/dt=0 is downward sloping. According to the dR_d"/dt differential equation (equation 2.28) anywhere above the dR_d"/dt=0 locus, product R&D R_d" will increase and below this locus R_d" will decrease. The vertical arrows demonstrate these directions of motion. The equilibrium (steady state) is at the intersection of the two curves. The steady state levels of the perceived quality and product R&D are given by $Q_p"^*$ and $R_d"^*$, respectively. This configuration of arrows surrounding the equilibrium implies a saddle point equilibrium[6].

The configuration of arrows in figure 2.1 leads us to conclude that the equilibrium at ($Q_p''^*$, $R_d''^*$) is a saddle point. This conclusion is taken on a qualitative judgement of the phase diagram. Because the position of the curves are drawn with considerable latitude in the positioning of the curves, we examined the validity of this conclusion by an investigation of the local stability characteristics around the steady state. We applied a linearisation (first order Taylor expansion) of the nonlinear differential-equation system near the steady state and this analysis showed us that the equilibrium is indeed locally saddle point stable[7].

In order to describe the dynamics, assume now that the firm has the initial perceived quality level Q_{p0}'' and that it is furthermore free to choose the amount of product R&D. The figure shows three possible trajectories which start from this initial level Q_{p0}'' and an arbitrarily chosen level of product R&D[8]. Along each of these trajectories the necessarily static and dynamic first order conditions are satisfied. Moreover, the trajectories fulfil the directional requirements imposed by the horizontal and vertical arrows.

Let us first consider what will happen if a firm chooses a level of product R&D which is higher than the product R&D level which is associated with point B, say at A. The assumed location of the saddle path through B and the general directional requirements imply that the level of product R&D declines until one gets an intersection with the $dR_d''/dt = 0$ locus. After this intersection the level of product R&D will increase without bound. All the trajectories which start above point B will therefore end up with an infinitively high level of R_d'' and Q_p''. But when $Q_p'' \rightarrow \infty$ the marginal increase of expected demand is approaching zero. This makes the left hand side of equation 2.24 negative which implies a violation of the transversality condition, as λ, $\hat{\lambda} >> 0$. Because this path violates the transversality condition it cannot have been optimal to start at A.

Consider any trajectory that starts below point B, say at C. The level of the perceived quality increases until the intersection with the $dQ_p''/dt=0$ locus, and decreases afterwards. The level of product R&D is so low that the perceived quality level decreases. The level of product R&D decreases all the time and becomes eventually zero or negative. When R_d'' is zero λ will also be zero and this gives in combination with equation (2.24) that $d\lambda/dt<0$ and λ becomes negative. A negative R_d'' level implies by (2.23) also a negative value for λ. A negative marginal value for perceived quality cannot be optimal because the last R&D expenditures are inefficient and it violates the Mangasarian sufficiency theorem. Therefore, it cannot have been optimal to start from C.

Because on all these trajectories either the transversality condition or the

[6]The characteristics of a saddle point equilibrium are described in Chiang: 'A saddle point is an equilibrium with a double personality-it is stable in some directions, but unstable in others. More accurately, a saddle point has exactly one pair of streamlines-*the stable branches* of the saddle point-that flow directly and consistently toward the equilibrium, and exactly one pair of streamlines-*the unstable branches*-that flow directly and consistently away from it. All the other trajectories head toward the saddle point initially but sooner or later turn away from it'(Chiang, 1984, p 633).

[7]The mathematical description of this analysis is given in Appendix 1. For more details of the procedure of linearization of a non-linear system, see A.C. Chiang, 1984, Section 18.6.

[8]Given the initial quality level Q_{p0} a firm can choose from an infinite number of values of product R&D. All these possibilities are located on the vertical Q_{p0}ABC locus.

Mangarasian sufficiency theorem are violated the unique saddlepath which leads to the steady state equilibrium is the only possible candidate for an optimal solution which is left. In the equilibrium the values for Q_p'' and R_d'' are constant and positive. A positive constant equilibrium product R&D level ($R_d''^*$) implies also a positive constant level for λ (see equation 2.23). This satisfies the transversality condition and the Mangarasian sufficiency theorem. The saddle path is therefore the unique optimal path. For each initial value of the perceived quality level of a firm, this implies a unique optimal product R&D level. In our case with Q_{p0}'' the optimal initial level of product R&D is R_{d0}''. In the long run the perceived quality level converges to the steady state level Q_p^* along the saddle path.

Steady State

We get the steady state values for R_d'' and Q_p'' if we solve equations (2.29) and (2.30):

$$R_d''{}^* = \left[\frac{\gamma^b.\delta.(w+\delta.\sigma)^{1-b}.X_{0,0}.b.(a-1)^{a-1}}{q_{d,0}.(r+w-\hat{q}_d-(1-\delta).\sigma).C_0^{a-1}.a^a} \right]^{\frac{1}{1-b.\delta}} \tag{2.31}$$

$$Q_p''{}^* = \left[\frac{X_{0,0}.b.\delta.\gamma^{\frac{1}{\delta}}.(w+\delta.\sigma)^{\frac{\delta-1}{\delta}}.(a-1)^{a-1}}{q_{d,0}.(r+w-\hat{q}_d-(1-\delta).\sigma).C_0^{a-1}.a^a} \right]^{\frac{\delta}{1-b.\delta}} \tag{2.32}$$

The steady state levels of quality and product R&D and therefore technical change are positively influenced by the autonomous scale of demand ($X_{0,0}$), the productivity of the quality generation process (γ), the elasticity of the quality generation process (δ), the quality elasticity of demand (b). The rate of technical change due to product R&D is negatively influenced by the unit cost of production (C), the price of product R&D (q_d), the discount rate (r) and the elasticity of demand (a). The perceived quality depreciation factor (w) has a negative influence on the steady state quality level. The influence of w on the steady state product R&D level is ambiguous. On the one hand a higher level of R&D is required to maintain a given perceived quality R&D level, while on the other hand less R&D is needed because the associated steady state quality level will be lower.

Notice that the discounted steady state level of product R&D should be just enough to maintain the discounted perceived quality level. It should be stressed that in the equilibrium only the redefined (discounted) variables (R_d'', Q_p'') are constant but that the "original" variables R_d and Q_p have a constant steady state growth rate (see equation 2.15).

We can derive the following simple expression for the product R&D/sales ratio in the steady state:

$$\frac{R_d''^{*} q_{d,0}}{P''^{*}.Y''^{*}} = \frac{\delta.b.(w+\delta.\sigma)}{a.(r+w-\hat{q}_d-(1-\delta).\sigma)} \quad (2.33)$$

The product R&D intensity is larger when technological opportunities are larger (δ), the impact of quality on demand is larger (b), depreciation rate is larger ($w+\delta.\sigma$), price elasticity of demand smaller (a) and the marginal opportunity costs are less $(r+w-\hat{q}_d-(1-\delta).\sigma)$.

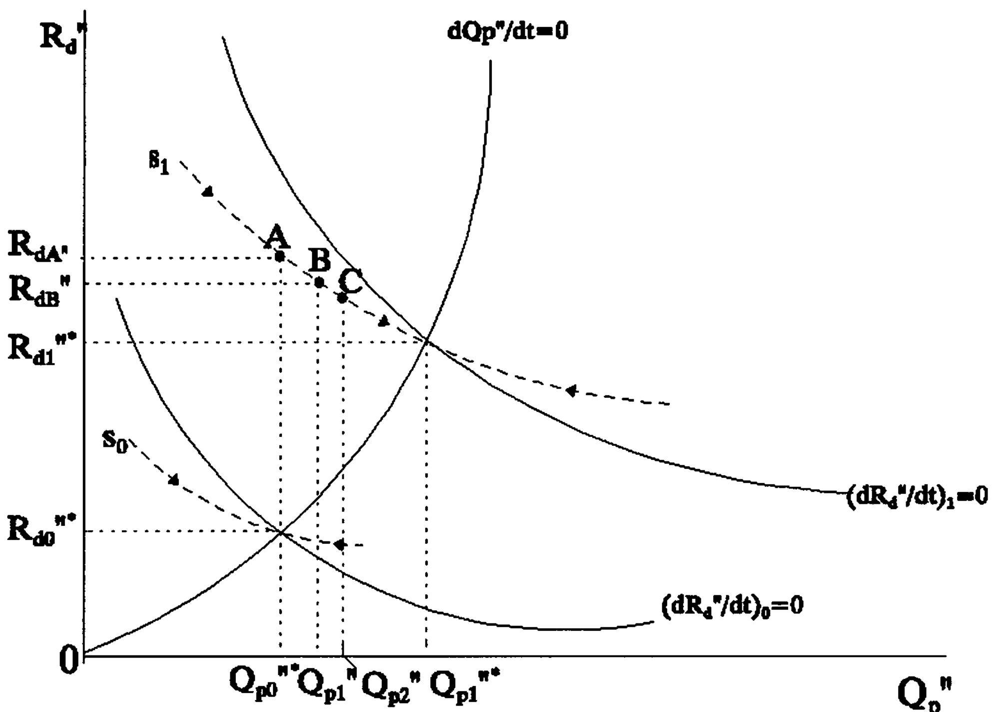

Figure 2.2
Increase of the autonomous scale of demand

Graphical Illustration of Transitional Dynamics

What will happen if the value of an exogenous variable or a parameter changes? First, equations (2.31) and (2.32) show us that the steady state values of R_d'' and Q_p'' will change. Second, the loci in the phase diagram will move. We will illustrate this case with the help of figure 2.2.

We assume the following initial situation: the firm is in the steady state, it has the perceived quality level $Q_{p0}''^{*}$ and the corresponding product R&D level $R_{d0}''^{*}$. The $(dR_d''/dt)_0=0$ and the $dQ_p''/dt=0$ loci of this situation are also drawn in figure 2.2. Their mathematical expressions are given by equation (2.29) and equation (2.30).

Suppose that there is a rise of the autonomous scale of demand($X_{0,0}$). The first locus $dQ_p''/dt=0$ will not be affected by this change. The second locus $dR_d''/dt=0$ will change. This curve will move upwards from $(dR_d''/dt)_0=0$ to $(dR_d''/dt)_1=0$ in figure 2.2. The steady state will change from R''^{*}_{d0} and Q''^{*}_{p0} to R''^{*}_{d1} and Q''^{*}_{p1} . The saddle path which belongs to this new equilibrium is given by the locus with arrows s_1. So an increase in the autonomous demand moves the steady state including the saddle path to the north east. The firm is now no longer in a steady state equilibrium. Given its initial quality level Q''^{*}_{p0} a firm will choose the corresponding optimal level of product R&D on the new saddle path s_1, in point A (R_{dA}''). Its new quality level(Q_{p1}'') will be (approximately):

$$Q''_{p1} = Q''^{*}_{p0} + \gamma.(R''_{d,A})^{\delta} - w.Q''^{*}_{p0} \tag{2.34}$$

With this new perceived quality level Q_{p1}'' the firm will choose the product R&D level in point B (R_{dB}'') on the saddlepath s_1. When we repeat this process we find that the new quality level will be Q_{p2}'' and the corresponding R&D level is given by point C. Note that $R_{dA}''>R_{dB}''>R_{dC}''$ which implies by an increasing quality level that $\Delta Q_{pA}''>\Delta Q_{pB}''>\Delta Q_{pC}''$, which causes the adjustment process to slow down.

With figure 2.3 we will illuminate two characteristics of the adjustment process.

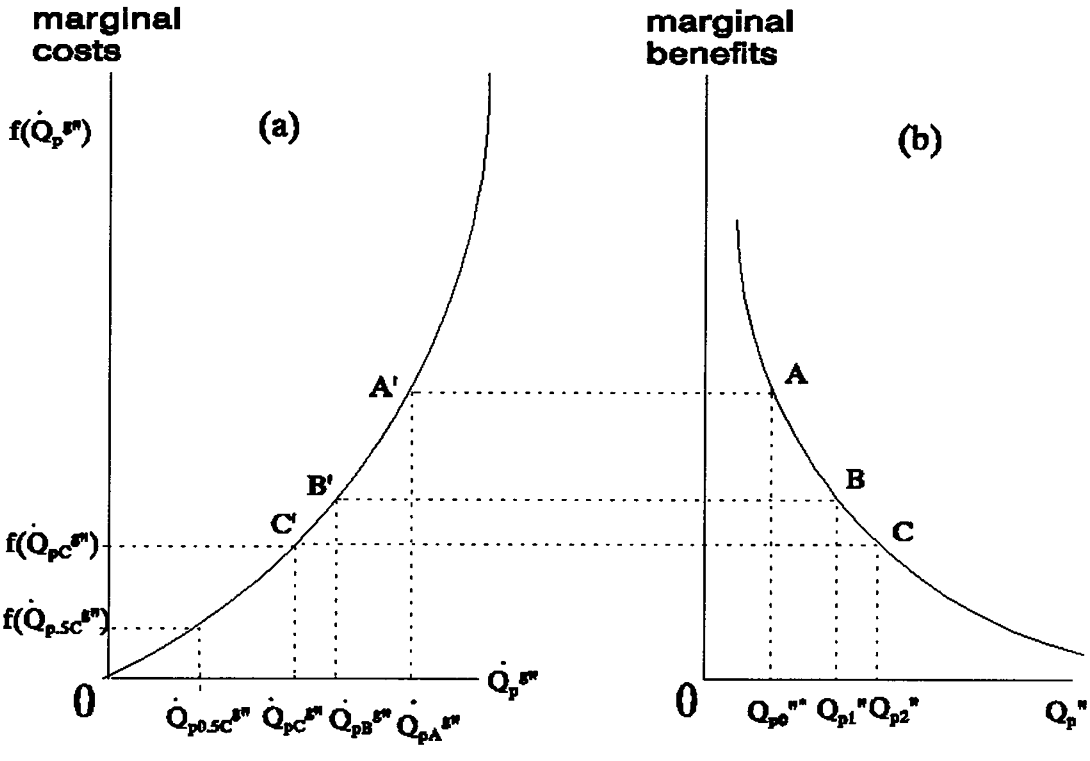

Figure 2.3
marginal costs and marginal benefits

First, a firm doesn't jump immediately to the steady state by choosing a higher R&D level but adjusts his quality level gradually. Secondly, The adjustment process slows down when one gets closer to the steady state.

In the left part of figure 2.3, i.e. figure (a), we have drawn the relation between marginal costs (R&D expenditures: $f(\dot{Q}_p^{g\prime\prime})$) and the gross change in the perceived quality level $dQ_p^{g\prime\prime}/dt$[9]. The quality generation function, i.e. equation 2.20, implies a convex relation between the marginal costs and $dQ_p^{g\prime\prime}/dt$. This convex relation explains the first characteristic because it implies that it is always cheaper to achieve a certain increase in quality in two smaller steps in stead of one big step. We can also demonstrate this with figure 1. The marginal costs of achieving $\dot{Q}_{pC}^{g\prime\prime}$ in one time are $f(\dot{Q}_{pC}^{g\prime\prime})$, which are larger than two times the marginal costs $f(\dot{Q}_{p0.5C}^{g\prime\prime})$ of achieving $\dot{Q}_{p0.5C}^{g\prime\prime}$. Another implication of this process is that the steady state level will only be approached in the long run.

In the right part of figure 2.3, i.e. figure (b), we depicted the relation between the marginal benefits with respect to the perceived quality level and the perceived quality level itself. The marginal benefits (increase in expected demand) are declining when the level of Q_p'' gets higher (see the demand function (equation 2.1)).

On the optimal quality generation plan the marginal costs involved in generating one extra unit of quality must always balance the marginal benefits of the generation of the extra unit of quality. Figure (a) and (b) together can therefore explain the second characteristic[10]. Given the initial quality level $Q_{p0}''^{*}$, the marginal benefits of an increase in the quality level are given by point A in figure (b). Because marginal costs have to be equal to marginal benefits the R&D expenditures are given by point A' in figure (a). The corresponding gross change in the quality level is $\dot{Q}_A^{g\prime\prime}$. $Q_{p0}''^{*}$ and the net change in quality level gives the new quality level Q_{p1}''[11]. We can repeat this process and see that given this new quality level the firm has R&D expenditures in point B' and a change in gross quality of $\dot{Q}_B^{g\prime\prime}$. When we compare $Q_{p0}''^{*}$AA' with Q_{p1}''BB' we can conclude that a higher perceived quality level implies lower R&D expenditures and a smaller gross and therefore implicitly net change in perceived quality. This implies that the adjustment process slows down when the perceived quality level gets closer to the steady state value.

Figure 2.3 is also well suited to illustrate the dynamic behaviour of the system with the Nerlove and Arrow dynamic equation which describes the case with constant returns to R&D. With constant returns to R&D the marginal costs of R&D are constant (independent of dQ"/dt level). We can imagine them by a horizontal line in part (a) of figure 2.3, say through point B". This implies that the steady state quality level is given

[9] The gross change in Q_p'' ($dQ_p^{g\prime\prime}/dt$) is the net change in Q_p'' (dQ_p''/dt) plus the depreciation of perceived quality ($w'.Q_p''$): $\dfrac{dQ_p^{g\prime\prime}}{dt} = \dot{Q}_p^{g\prime\prime} = \dfrac{dQ_p''}{dt} + w'.Q_p''$.

[10] We discuss the same situation as we considered in figure 2.2. We postulate that the increase in the scale of the autonomous demand has already taken place in figure 2.3. The effect by the way of an increase in the scale of the autonomous demand in figure 2.3 is an upward shift of the marginal benefit curve in figure (b).

[11] The net change in the perceived quality level is the gross change in the perceived quality level less the depreciation of perceived quality. The net change in the perceived quality level is positive because the points A and A' in figure 2.3 correspond to point A in figure 2.2, which implies a positive change in the perceived quality level because it is situated to the left of the $dQ_p''/dt=0$ locus.

by $Q_{p,1}$" because marginal costs are equal to marginal benefits in the steady state. Let us first consider what this implies for the transitional dynamics when the initial quality level is lower than this steady state level, say $Q_{p,0}$". The marginal benefits of R&D in this situation are equal to A and the marginal costs are equal to B. The marginal benefits of R&D are greater than the marginal costs and the firm will invest in R&D until marginal benefits are equal to marginal costs, which is the case by $Q_{p,1}$". The firm will jump from $Q_{p,0}$" to $Q_{p,1}$" at t=0. There are no transitional dynamics in this case because the firm jumps immediately to the steady state. Let us now consider what will happen if the initial quality level is higher than the steady state level, say at $Q_{p,2}$. The marginal benefits are equal to C in this situation which are lower than the marginal costs. This implies that there will be no investment in R&D and the perceived quality level decreases in proportion with the depreciation rate until the steady state perceived quality level is met.

We can conclude that changes in exogenous variables or parameters will shift the steady state, but optimal behaviour of firms implies that they will only adjust gradually to this new steady state when marginal return to R&D are decreasing and jumps immediately to the steady state as marginal returns to R&D are constant.

3. Cost Reducing R&D

In this part we discuss the case in which an entrepreneur can engage in process R&D in order to increase the efficiency of his production process. Just like the case with product R&D, the benefits of process R&D are spread over the future. Current process R&D expenditures increase the productivity of the production process in a permanent fashion. A pioneering attempt to deal with this problem is the work of Kamien and Schwarz (1969) which is based on a situation with constant prices. Sato and Ramachandran (1974) discussed this kind of problem in a situation with exponentially growing prices which could be removed from the dynamic optimization problem right from the beginning. Sato and Suzawa (1983) developed a similar model in which a firm can invest in applied and basic R&D with exponentially growing prices. In this section we discuss this kind of problem with cost reducing R&D in the presence of exponentially growing prices and output which, unfortunately, can not be removed from the optimization problem right from the start.

To keep things as simple as possible, we assume a Cobb-Douglas production function with constant returns to scale in which two factors of production, i.e. capital and labour, are combined.

$$Y_t = F(A_t, L_t, K_t) = A_t . K_t^{\alpha} . L_t^{1-\alpha} \tag{3.1}$$

where Y_t is the production at time t, K_t is the capital stock at time t, L_t is the amount of labour at time t and A_t is the total factor productivity index or TFP variable.

In this section we will use the total cost function which is associated with this Cobb-Douglas production function[12]:

$$TC_t = \frac{Y_t}{A_t} . \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} . \left(\frac{v_t}{\alpha}\right)^{\alpha} = Y_t . C_t \tag{3.2}$$

where w_t is the expected wage level at time t, v_t is the expected user cost of capital at time t and C_t is the unit costs at time t. The advantage of this total cost function approach in comparison with a direct production function approach is that the dual cost function approach automatically implies the optimum allocation of labour and capital. In this way we can save two control variables, capital and labour. Another advantage is that there is now a perfect equivalence with the profit-maximization problem of section two.

Producers may engage in cost reducing R&D, which we call process R&D. Process R&D is aimed at increasing the efficiency of the production process, i.e. it is aimed at producing more output with the same amount of inputs[13]. We assume decreasing marginal productivity increases with respect to process R&D expenditures.

[12] Sato and Suzawa (1983) used also this cost function approach.

[13] C.f. Kennedy (1964).

We can specify the following productivity generation function[14]:

$$\frac{dA}{dt} = F(R_{c,t}) \qquad with \quad F' \geq 0 \ \wedge \ F'' \leq 0 \tag{3.3}$$

where $R_{c,t}$ is the amount of process R&D at time t.

We assume that the expected demand is negatively influenced by the price level of the product. We will use the following expected demand curve:

$$Y_t^d = X_0.P^{-a} \tag{3.4}$$

We assume now that a firm will maximize the present value of its expected profits, by choosing the optimal price and process R&D level, given its initial productivity level. We can specify the intertemporal profit maximisation problem as follows[15]:

$$\begin{aligned}
\underset{P,\, R_d}{Max}\ \pi(0) &= \int_0^{\infty} e^{-rt} \left[P_t.Y_t^d(P_t) - TC(A_t, Y_t^d, w_t, v_t) - R_{c,t}.q_{c,t}\right] dt \\
s.t.\quad & Y_t^d = X_{0,t}.P_t^{-a} \\
& TC_t = \frac{1}{A}.Y_t^d.w_t^{1-\alpha}.v_t^{\alpha}.(1-\alpha)^{\alpha-1}.\alpha^{-\alpha} \\
& dA/dt = \eta.R_{c,t}^{\theta} \qquad with \ \ \theta = \pm 0.1 \\
& A_{t=0} = A_0
\end{aligned} \tag{3.5}$$

where $q_{c,t}$ is the expected process R&D price at time t.
We assume that all exogenous variables grow with a constant growth rate:

$$\begin{aligned}
q_{c,t} &= q_{c,0}.e^{l_{c}t} \\
w_t &= w_0.e^{ht} \\
v_t &= v_0.e^{it} \\
X_{0,t} &= X_{0,0}.e^{nt}
\end{aligned} \tag{3.6}$$

We assume that the exogenous scale of demand (X_0) is increasing at rate n, the price of process R&D (q_c) at rate l_c, the wage rate (w) at rate h and the user cost of capital (v) at rate i.

[14] Note that Sato and Ramachandran (1974) and Sato and Suzawa (1983) use a different dynamic constraint. Their specification is: $\hat{A} = F(R_{c,t})$. This specification implies that a constant level of process R&D is enough to obtain a constant growth rate of the productivity level. Similar specifications in the context of human capital accumulation can be found in Lucas (1988) and Romer (1990).

[15] Griliches (1991) and Mohnen (1992) reviewed the empirical literature about own (process) R&D and interindustry spillovers and concluded that the elasticity of output with respect to own (process) R&D was ±0.1.

The current value Hamiltonian of this problem is:

$$H^c = X_{0,t}.P_t^{1-a} - \frac{1}{A_t}.X_{0,t}.P_t^{-a}.w_t^{1-\alpha}.v_t^{\alpha}.Z_1 - R_{c,t}.q_{c,t} + \mu.\eta.R_{c,t}^{\theta} \tag{3.7}$$

where $Z_1 = (1-\alpha)^{\alpha-1}.\alpha^{-\alpha}$. The first order condition with respect to the price level is:

$$P = \left(\frac{-a}{1-a}\right).\frac{w^{1-\alpha}.v^{\alpha}.Z_1}{A} = \frac{-a}{1-a}.C_t \tag{3.8}$$

Notice that this equation shows again that the price will be determined by a mark-up over unit costs which depends on the price elasticity of demand. The difference with section two is that these unit costs are not given for an entrepreneur but can be influenced by doing process R&D which influences total factor productivity. Differentiation of all the first order conditions with respect to time gives rise to the following equilibrium (steady state) rates of growth:

$$\begin{aligned} \hat{R}_c^* &= \frac{\hat{X}_0-(a-1).\alpha.\hat{v}-(a-1).(1-\alpha).\hat{w}-\hat{q}_c}{1+\theta-a.\theta} \\ \hat{A}^* &= \theta.\hat{R}_c^* \\ \hat{P}^* &= \frac{-\theta.\hat{X}_0+\alpha.\hat{v}+\theta.\hat{q}_c+(1-\alpha).\hat{w}}{1+\theta-a.\theta} \end{aligned} \tag{3.9}$$

We assume a positive steady state growth rate for R_c and A. Just like in the case with demand creating innovations, the growth rate of the exogenous scale of demand has a positive influence on the steady state growth rates of R_c and A the growth rates of the prices of labour, capital and process R&D have a negative influence on the steady state growth rates of R_c and A. The process R&D elasticity of the productivity generation process (θ) has a positive influence on the steady state growth rates of R_c and A.

To determine the influence of the negatively sloped demand curve on the steady state growth rates of R_c and A we will put the price elasticity of demand to zero (a=0). When a=0 we have an exogenous level of production and the firm minimizes costs to meet this exogenous production level. When we put a=0 in equation (3.9) it follows immediately that the growth rates of the prices of labour and capital have a positive influence on the steady state growth rates of A and R_c. This is the case because higher growth rates of the prices of labour and capital imply that the potential benefits of process R&D per unit of output increase. The level of output is given, and therefore the total amount of potential benefits increases which justifies a higher growth rate of process R&D. When we have an elastic demand curve (a>1) we have the opposite result, a higher growth rate of input prices decreases the steady state growth rates of A and R_c. The reason for this result is that the level of output is not exogenous but endogenously determined. In this situation (a>1) higher input prices mean not only that the potential benefits per unit of output of engaging in process R&D are higher, but also that the output level will decrease because the price level will increase. In a situation with an elastic demand curve (a>1) the level of output decreases more than proportionally, which implies that the total potential benefits for process R&D are smaller. Smaller potential benefits imply a lower steady state growth rate of R_c and A.

When one puts the price elasticity of demand equal to zero one gets another striking result; the influence of the R&D elasticity of the productivity generation process (θ) becomes negative on the steady state growth rate of process R&D and stays positive on the steady state growth rate of A. The process R&D elasticity of the productivity generation process (θ) has a positive influence on the steady state growth rate of A. The reason for this is that the costs of achieving a certain productivity increase declines when θ rises. But θ has a negative influence on the steady state growth rate of R_c. In this case there are two opposite effects. First, as we have seen, productivity production costs decline which results in a higher growth rate of the productivity level which implies a higher growth rate of process R&D. Second, the amount of process R&D to achieve a certain level of productivity growth is less. In this particular situation the second effect dominates the first effect. With a negatively sloped demand curve the influence of θ is positive on the growth rate of R_c. The reason for this is that next to the two opposite influences of the situation when a=0, we have a third effect. A higher θ means a higher productivity growth which lowers unit costs. In the case of an elastic demand curve this actually implies a more than proportionally higher output level. The total effect in this case is a higher growth of process R&D.

Time elimination method

As in section 2 we first redefine the variables of the system. Because we found constant steady state growth rates for A, R_c and P we normalize the productivity level, the process R&D level and the price level with their corresponding constant steady state growth rates. We thus obtain the following new variables:

$$\begin{aligned} R_c'' &= R_{c,t}.e^{-\left(\frac{n-(a-1).\alpha.i-(a-1).(1-\alpha).h-l_c}{1+\theta-a.\theta}\right).t} = R_{c,t}.e^{-\sigma''.t} \\ A'' &= A_t.e^{-\theta.\sigma''.t} \\ P'' &= P_t.e^{-\left(\frac{-\theta.n+\alpha.i+(1-\alpha).h+\theta.l_c}{1+\theta-a.\theta}\right).t} \end{aligned} \tag{3.10}$$

where $\sigma''=(n-(a-1).\alpha.i-(a-1).(1-\alpha).h-l_c)/(1+\theta)$.

Again we obtain the new dynamic constraint in terms of A" by differentiating the second equation of (3.10) with respect to time. When we use this result in combination with the old constraint (equation 3.3) we can solve for dA"/dt and obtain:

$$\frac{dA''}{dt} = \eta.(R_c'')^{\theta} - \theta.\sigma''.A'' \tag{3.11}$$

The profit maximization problem (equation 3.5) can be rewritten in terms of these new variables.

The current value Hamiltonian of this redefined problem is:

$$H^c = X_{0,0}.(P_t'')^{1-a} - \frac{X_{0,0}}{A''}.(P_t'')^{-a}.w_0^{1-\alpha}.v_0^{\alpha}.Z_1 - R_c''.q_{c,0} + \mu.(\eta.(R_c'')^{\theta} - \theta.\sigma''.A'') \tag{3.12}$$

The discount rate is $\sigma'' + \hat{q}_c - r$.

Using the first order conditions, we are able to acquire the two differential equations, dA"/dt=f(A", R_c") and dR_c"/dt=f(A", R_c"), which we need to study the dynamics. The dA"/dt locus is equation (3.11) and the dR_c"/dt locus can be obtained from the other first order conditions:

$$\frac{q_{c,0}.(r-\hat{q}_c-(1-\theta).\sigma'')}{\eta.\theta}.(R_c'')^{1-\theta} - \frac{q_{c,0}.(1-\theta).(R_c'')^{-\theta}}{\eta.\theta}.\frac{dR_c''}{dt} = X_{0,0}.\left(\frac{1-\alpha}{w_0}\right)^{(a-1).(1-\alpha)}.\left(\frac{\alpha}{v_0}\right)^{(a-1).\alpha}.\left(\frac{a-1}{a}\right)^{a}.(A'')^{a-2} \tag{3.13}$$

Now we can calculate the steady state levels of A" and R_c" by putting these two differential equations to zero:

$$\frac{dA''}{dt} = 0 \quad \Rightarrow \quad R_c'' = \left(\frac{\theta.\sigma''}{\eta}\right)^{\frac{1}{\theta}}.(A'')^{\frac{1}{\theta}} \tag{3.14}$$

$$\frac{dR_c''}{dt} = 0 \quad \Rightarrow \quad \frac{q_{c,0}.(r-\hat{q}_c-(1-\theta).\sigma'')}{\eta.\theta}.(R_c'')^{1-\theta} = X_{0,0}.\left(\frac{1-\alpha}{w_0}\right)^{(a-1).(1-\alpha)}.\left(\frac{\alpha}{v_0}\right)^{(a-1).\alpha}.\left(\frac{a-1}{a}\right)^{a}.(A'')^{a-2} \tag{3.15}$$

To get an economically meaningful situation we require that $r-\hat{q}_c-(1-\theta).\sigma'' > 0$, which is automatically satisfied when the transversality condition is met[16].

Steady State

When we solve equations (3.14) and (3.15) for A" and R_c" we get the following steady

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[16] The transversality condition is satisfied when $r > \hat{q}_c + \sigma''$.

state values:

$$R_c''^{*}=\left[\frac{X_{0,0}.(a-1)^{a}.(1-\alpha)^{(a-1)(1-\alpha)}.\alpha^{(a-1)\alpha}.\eta^{a-1}.(\sigma'')^{2-a}.\theta^{3-a}}{q_{c,0}.w_0^{(a-1)(1-\alpha)}.v_0^{(a-1).\alpha}.(r-\hat{q}_c-(1-\theta).\sigma'').a^{a})}\right]^{\frac{1}{1+\theta-a.\theta}} \tag{3.16}$$

$$A''^{*}=\left[\frac{X_{0,0}.(a-1)^{a}.(1-\alpha)^{(a-1)(1-\alpha)}.\alpha^{(a-1)\alpha}.\eta^{\frac{1}{\theta}}.(\sigma'')^{\frac{\theta-1}{\theta}}.\theta^{\frac{2\theta-1}{\theta}}}{q_{c,0}.w_0^{(a-1)(1-\alpha)}.v_0^{(a-1).\alpha}.(r-\hat{q}_c-(1-\theta).\sigma'').a^{a})}\right]^{\frac{\theta}{1+\theta-a.\theta}} \tag{3.17}$$

First, just like in the case with the steady state growth rates the influence of the prices of labour and capital is negative when demand is elastic (a>1) and negative when demand is totally inelastic (a=0).

Secondly, in the situation with an elastic demand curve, the productivity of the productivity generation process (η) has a positive influence on $R_c''^{*}$ and a negative influence when the demand is totally inelastic. There are three influences which cause this result. First, the costs associated with a certain productivity growth are declining, which results in a higher steady state productivity level. Secondly, the amount of process R&D needed to achieve a certain productivity growth is less. These two effects together result in the case with an totally inelastic demand curve in a lower process R&D level per unit of output. Third, the introduction of the elastic demand curve implies that the higher steady state productivity level implies lower unit costs. This in turn results in a lower steady state price level. With an elastic demand this increases output more than proportionally. The overall results of these three forces is a positive influence of η on $R_c^{*''}$.

Similarly to the case with only demand creating innovations we can derive a simple equation for the R&D intensity in the steady state:

$$\frac{R_c''^{*}q_{d,0}}{p''^{*}.Y''^{*}}=\frac{\theta.(a-1).(\theta.\sigma)}{a.(r-\hat{q}_c-(1-\theta).\sigma)} \tag{3.18}$$

The process R&D intensity is larger when technological opportunities are larger (θ), depreciation rate is larger ($\theta.\sigma$), price elasticity of demand smaller (a) and the marginal opportunity costs are less $(r-\hat{q}_c-(1-\theta).\sigma)$.

Dynamics

The two demarcation lines which divide the (A'', R_c'') space into four principally different areas are given by equations (3.14) and (3.15). The first demarcation line, which is given by equation (3.14), has the same general form as in section two. The only difference is the value of the steady state growth rate. The shape of the second demarcation line, as given by equation (3.15), is totally dependent on the value of the price elasticity of demand. To illuminate these statements we can simplify these two equations by putting all constant variables and parameters in one expression. Equations (3.14) and (3.15) become respectively:

$$R_c^{a''} = Z_2.(A'')^{\frac{1}{\theta}} \tag{3.19}$$

$$R_c^{b''} = Z_3.(A'')^{\frac{a-2}{1-\theta}} \tag{3.20}$$

As before, note that the first first order condition implies that a>1 and that we assume $\theta=\pm 0.1$. In order to obtain both loci we take the first and second derivative with respect to A":

$$\frac{\delta R_c^{a''}}{\delta A''} = \frac{Z_2}{\theta}.(A'')^{\frac{1-\theta}{\theta}} \quad , \quad \frac{\delta^2 R_c^{a''}}{\delta A''^2} = \frac{1-\theta}{\theta^2}.Z_2.(A'')^{\frac{1-2\theta}{\theta}} \tag{3.21}$$

$$\frac{\delta R_c^{b''}}{\delta A''} = \left(\frac{a-2}{1-\theta}\right).Z_3.(A'')^{\frac{a-3+\theta}{1-\theta}} \quad , \quad \frac{\delta^2 R_c^{b''}}{\delta A''^2} = \frac{(a-2)(a-3+\theta)}{(1-\theta)^2}.Z_3.(A'')^{\frac{a-4+2\theta}{1-\theta}}$$

(3.22)

The first and second derivative of $R_c^{a''}$ are positive. This means that the slope of the dA"/dt=0 locus is increasing at an ever increasing rate, the locus is convex. When we look at the first and second derivative of $R_c^{b''}$ with respect to A", we can identify four different situations. We will describe the dynamics in these four situations.

A. Low Price Elasticity of Demand: $1<a\leq 2$

In this situation has the first derivative of $R_c^{b''}$ a negative value and the second derivative a positive value. This is the situation we know from section two. The dR_c"/dt=0 has a negative slope which get's less negative as A" get larger. The phase diagram we can draw is familiar to us and is given in figure 2.1. It has the same dynamic characteristics. The steady state level is the only viable path in the long run and can only be reached by means of the unique saddlepath. The optimal decisions of a firm regarding its choice about the amount of process R&D is given by this saddle path, when the initial productivity level is not equal to the steady state level.

When a=2, the dR_c"/dt=0 locus is independent of A". This demarcation line becomes a horizontal line at R_c"=Z_3. We have drawn this situation in figure 3.1.1. If we draw some trajectories which fulfil the directional requirements implied by the arrows, we see again a saddlepoint configuration. The saddle path is in this situation the dR_c"/dt=0 locus. We assume that the firm has an initial productivity level of A_0". The firm will do a constant level of discounted process R&D which is equal to the constant discounted steady state level of process R&D. This implies that the change in productivity level is the same. Because the initial discounted process R&D level of the firm is lower than the constant discounted steady state process R&D level and the change in productivity level is the same the growth rate of the discounted productivity level of this firm will be larger than the constant steady state growth rate. Its new discounted productivity level in the next period (A_1'') is therefore located to the right of A_0''. Given A_1'' the firm will choose again the same discounted process R&D level which results in a higher growth rate than the constant steady state growth rate. But the difference with the constant steady state growth rate is less, because its initially discounted productivity level is higher. This results in a slower adjustment speed.

When we compare the adjustment speed between the situation with $1<a<2$ and the situation in which $a=2$, we can notice that the adjustment speed is slower in the last situation. The reason for this is that the positive saddle path of the first situation implies a faster adjustment than the horizontal saddle path in the second situation. We can conclude that a higher price elasticity of demand decreases the adjustment speed.

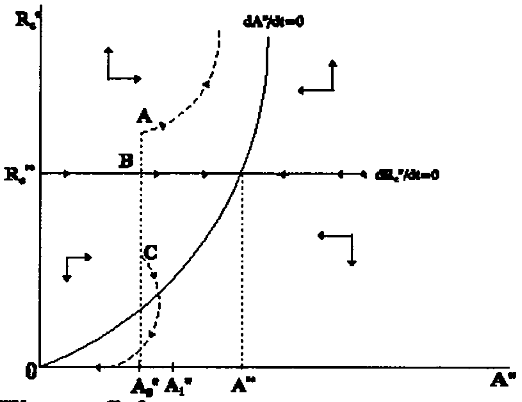

Figure 3.1
price elasticity of demand:a=2

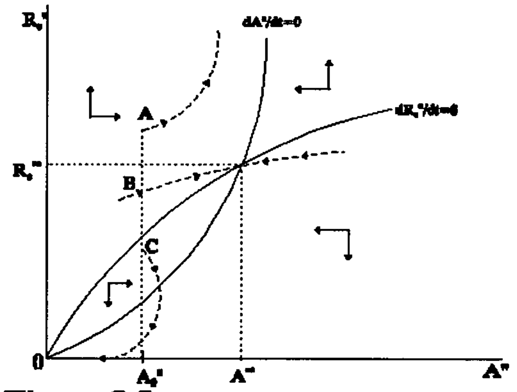

Figure 3.2
moderate price elasticity of demand

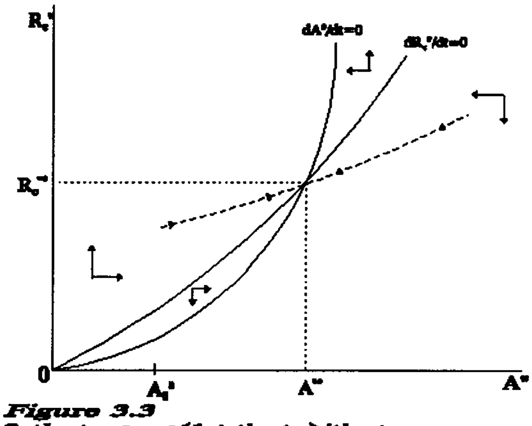

Figure 3.3
3-theta<a<(1+theta)/theta

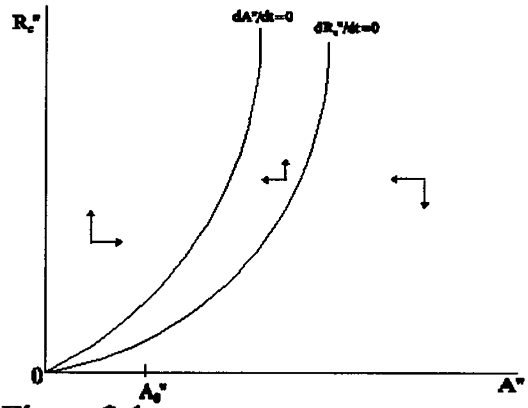

Figure 3.4
a=(1+theta)/theta

B_1. Moderate Price Elasticity of Demand: $2<a<3-\theta$

We will illustrate the dynamics of this situation with figure 3.2. The dA"/dt=0 is again convex. The first derivative of the dR$_c$"/dt=0 locus ($R_c^{b\prime\prime}$) is positive which differs from all the previous situations. The second derivative of $R_c^{b\prime\prime}$ is negative. The dR$_c$"/dt=0 has a decreasing positive slope, is concave (see figure 3.2). The economic interpretation of the positive slope of the dR$_c$"/dt=0 locus can be seen from equation (3.15). We noticed in section 2 that on the dR$_c$"/dt =0 locus the marginal costs are equal to the marginal benefits of increasing the state variable, which is in this case the productivity level. The left hand side of equation (3.15) shows that nothing important has changed on this cost side. The right hand side of equation (3.15), i.e. the marginal benefits, has changed a lot. When a>2 an increase in the productivity level has a positive influence on the marginal benefits instead of the negative influence when a<2. When prices are constant, a higher productivity level implies declining marginal benefits in terms of cost savings. The marginal cost savings, δTC/δA", per unit of output are declining as A" increases. But with an elastic demand curve a higher productivity level implies also a lower price, which results in a more than proportionally increase in output. The total effect is that the benefits increase as the productivity level increases. As marginal benefits as well as marginal costs increase as the productivity level gets higher we get a positive slope of the dR"/dt=0 locus. We have drawn the two demarcation loci in figure 3.2. The convex nature of dA"/dt=0 and the concave nature of dR$_c$"/dt=0 imply a positive point of intersection. This intersection point is again the steady state level in which the redefined variables (A", R$_c$") are constant and $R_{c,t}$ and A_t grow at a constant rate. The dynamics imply again a saddlepoint stable equilibrium. The difference with all the previous saddlepaths is that this saddlepath has a positive slope.

When A_0"<A"*, a positively sloped adjustment path implies that the discounted process R&D level which a firm chooses is lower than the discounted steady state level of process R&D. The productivity growth rate is still higher than the steady state growth rate of such a firm because its lower initial productivity level. The adjustment speed implied by a positive saddlepath is slower than the adjustment path implied by a negative or horizontal saddle path.

B_2. Moderate Price Elasticity of Demand: $3-\theta\leq a<(1+\theta)/\theta$, $\theta=\pm0.1$

The first and second derivative of $R_c^{b\prime\prime}$ are positive. This implies that the amount of process R&D rises at an ever increasing rate as the productivity level gets higher. The dR$_c$"/dt=0 locus is just like the dA"/dt=0 locus convex. But as long as the price elasticity of demand is not to high, a<(1+θ)/θ, we get a positive point of intersection, in which the dR$_c$"/dt=0 locus cuts the dA"/dt=0 from above. Two demarcation lines which fulfil these conditions are drawn in figure 3.3. The directional requirements are again given by the horizontal and vertical arrows. The implied dynamics of these arrows is very similar to situation B_1.

C. High Elastic Demand: $a\geq(1+\theta)/\theta$

When a=(1+θ)/θ both curves are again convex. In this special situation there is no intersection point. We have drawn this situation in figure 3.4. The trajectories, which are drawn in accordance with the directional requirements of the arrows, show us that this system is unstable. The only point which lies on both demarcation lines is (0,0). This point is no economically viable point, because the productivity level is zero in this point. When $a>(1+\theta)/\theta$, we get a positive intersection point, in which the dA"/dt=0 locus cuts the dR_c"/dt=0 from above. If we draw a phase diagram it is not that easy to say what the characteristics of the equilibrium are. We postpone our graphical analysis of this situation after we have checked the local stability of the steady state.

In appendix 2 we checked our qualitative conclusions about the characteristics of the equilibria in the four situations. We examined the local stability around the steady state in the four situations with a linearisation of the non-linear differential system around the steady state. This analysis confirmed that the equilibria in situations A, B_1 and B_2 were locally saddle point stable and showed that situation C is characterised by an unstable equilibrium. The latter can be an unstable node of an unstable focus[17]. When the price elasticity of demand (a) is not too large the steady state is an unstable node. This situation is depicted in figure 3.5. The steady state is an unstable focus, as par example a is very large. This situation is drawn in figure 3.6.

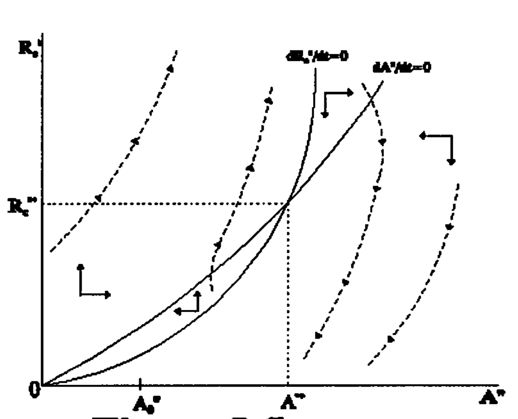

Figure 3.5
unstable node

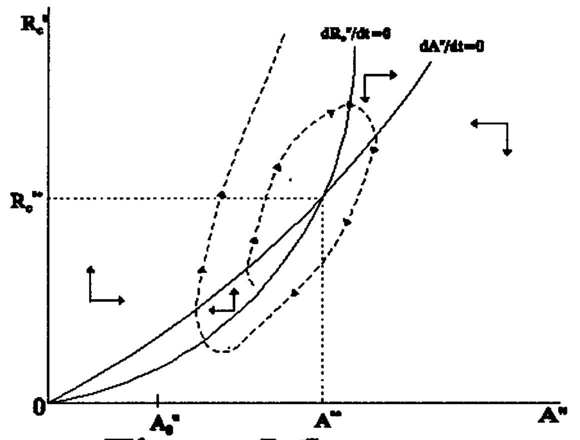

Figure 3.6
unstable focus

[17]A node is an equilibrium such that all the trajectories associated with it either flow noncyclically toward it (*stable node*) or flow noncyclically away from it (*unstable node*). A focus is an equilibrium which is characterised by whirling trajectories, all of which either flow cyclically toward it (stable focus), or flow cyclically away from it (unstable focus) (Chiang, 1984, p633-634).

$$
\begin{aligned}
\hat{R}_c^* &= \frac{\hat{X}_0-(a-1).\alpha.\hat{v}-(a-1).(1-\alpha).\hat{w}-b.\delta.\hat{q}_d-(1-b.\delta).\hat{q}_c}{1-(a-1).\theta-\delta.b}\\
\hat{R}_d^* &= \frac{\hat{X}_0-(a-1).\alpha.\hat{v}-(a-1).(1-\alpha).\hat{w}-(a-1)\theta.\hat{q}_c-(1-(a-1)\theta).\hat{q}_d}{1-(a-1).\theta-\delta.b}\\
\hat{A}^* &= \theta.\hat{R}_c^*\\
\hat{Q}_p^* &= \delta.\hat{R}_d^*\\
\hat{P}^* &= \frac{-\theta.\hat{X}_0+(1-b.\delta).(\alpha.\hat{v}+(1-\alpha).\hat{w})+\theta.b.\delta.\hat{q}_c+\theta.(1-b.\delta).\hat{q}_d}{1-(a-1).\theta-\delta.b}
\end{aligned}
\qquad \textbf{(4.3)}
$$

The directions of the influences of the growth rates of the exogenous scale of demand and the various input prices on the steady growth rates of R_c, R_d, A, Q_p and P are the same as in previous sections. But the strength of the influences of these growth rates is larger than in the case where one can only engage in either product R&D or process R&D. One can see this in the denominator of the steady state growth equations. The (a-1).θ term characterises the *cost-reducing effect* in the presence of a negatively sloped demand curve (section 3). The b.δ term characterises the *demand creating effect* (section 2). These cost-reducing and demand-creation effects diminish both the denominator and increase therefore the strength of the influence of the various exogenous growth rates.

A distinction with the previous sections is also the appearance of the growth rate of process R&D as well as the growth rate of product R&D. Note that the influence of both prices of R&D are negative on both R&D steady state growth rates. This implies that process and product R&D are complements rather than substitutes. Notice furthermore that $\hat{R}_c^*+\hat{q}_c=\hat{R}_d^*+\hat{q}_d$, which implies that the growth rate of the process and product R&D budget are equal. The steady state growth rate of process (product) R&D is higher than that of product (process) R&D if the exogenous growth rate of the product (process) R&D price level is higher than that of process (product) R&D. When $\hat{q}_c=\hat{q}_d$ the steady state growth rates of process and product R&D are equal because of constant elasticities everywhere.

The steady state growth rates of the situations in which a firm can only engage in one particular type of R&D are special cases of this situation in which a firm can do both kinds of R&D. When there are no technological opportunities to perform process R&D, θ=0, the $\hat{R}_d^*$, $\hat{Q}_p^*$ become identical to the steady state growth rates of section 2 in which we treated only product R&D (see equation 2.15[18]). When there are no technological opportunities to perform product R&D, i.e. δ=0, the steady state growth rates of R_c and A become similar to the steady state growth rates of section 3 (see equation 3.9).

Time elimination method

First we redefine the endogenous variables, $Q_{p,t}$, A_t, R_{dt}, R_{ct} in the same manner as in

---

[18] Note that $\hat{C} = \alpha.\hat{v}+(1-\alpha).\hat{w}$.

4. Demand Creating and Cost Reducing R&D

In this part we will integrate sections 2 and 3. Just like in Levin and Reiss (1989) a firm can perform product R&D to enhance its perceived product quality level as well as process R&D to increase the efficiency of its production process. Our approach differs from Levin and Reiss in that we treat this problem in a dynamic setting whereas Levin and Reiss work in a static setting. In this section we allow R&D prices and technological opportunities to differ between product and process R&D. We will examine the effects of for example these exogenous variables and parameters on the amount and composition of R&D. Moreover, we will try to answer the question whether product and process R&D are complements or substitutes.

By integrating the profit maximisation problems of sections 2 and 3, the profit maximization problem for a producer becomes:

$$\max_{P,R_c,R_d} \pi(0) = \int_0^\infty e^{-rt}\left[P_t.Y_t^d(P_t,Q_{p,t})-TC(A_t,Y_t^d,w_t,v_t)-R_{c,t}.q_{c,t}-R_{d,t}.q_{d,t}\right]dt$$

$$\begin{aligned}
\text{s.t. } & Y_t^d=X_{0,t}.Q_{p,t}^b.P_t^{-a} && \text{with} \quad 0\le b\le 1\\
& TC_t = \frac{1}{A}.Y_t^d.w_t^{1-\alpha}.v_t^{\alpha}.(1-\alpha)^{\alpha-1}.\alpha^{-\alpha} && \text{with} \quad 0\le \alpha\le 1\\
& dA/dt=\eta.R_{c,t}^{\theta} && \text{with} \quad 0\le \theta\le 1\\
& dQ_p/dt=\gamma.R_{d,t}^{\delta}-w.Q_{p,t} && \text{with} \quad 0\le \delta\le 1\\
& A_{t=0} = A_0 \;,\; Q_{p,t=0} = Q_{p,0}
\end{aligned} \tag{4.1}$$

Notice the difference between w_t (with a time subscript) and w: the first variable is the price of labour at time t and the second is the perceived quality depreciation factor. Just as in other sections, we assume that prices and the exogenous scale of demand have a constant growth rate.

The current value Hamiltonian associated with this problem is:

$$\begin{aligned}
H^c &= X_{0,t}.Q_{p,t}^b.P_t^{1-a}-\frac{1}{A}X_{0,t}.Q_{p,t}^b.P_t^{-a}.w_t^{1-\alpha}.v_t^{\alpha}.Z_1-R_{c,t}.q_{c,t}-R_{d,t}.q_{d,t}\\
&\quad + \mu.\eta.R_{c,t}^{\theta} + \lambda.(\gamma.R_{d,t}^{\delta}-w.Q_{p,t})
\end{aligned} \tag{4.2}$$

where $Z_1 = (1-\alpha)^{\alpha-1}.\alpha^{-\alpha}$. The technological opportunities of process R&D are given by the values of η and θ, and the technological opportunities of product R&D are given by the values of δ and γ. The higher the values of these parameters the higher are the technological opportunities.

Steady State Growth Rates

When we assume that the growth rates are constant in the steady state we can derive the following values of these steady state rates of growth:

the previous sections; we normalize them using their corresponding steady state growth rates. The two dynamic constraints in terms of the redefined variables become:

$$\frac{dA''}{dt} = \eta.(R_c'')^{\theta} - \sigma_a.A'' \tag{4.4}$$

where $\sigma_a = \theta.(n-(a-1).\alpha.i-(a-1).(1-\alpha).h-b.\delta.l_d-(1-b.\delta).l_c)/(1-(a-1)\theta-b.\delta)$.

$$\frac{dQ_p''}{dt} = \gamma.(R_d'')^{\delta} - (w+\sigma_q).Q_p'' \tag{4.5}$$

where $\sigma_q = \delta.(n-(a-1).\alpha.i-(a-1).(1-\alpha).h-(1-(a-1)\theta).l_d-(a-1).\theta.l_c)/(1-(a-1)\theta-b.\delta)$.

The current value Hamiltonian of the redefined problem is:

$$\begin{aligned} H^c = {} & X_{0,0}.(P'')^{1-a}.(Q_p'')^{b} - \frac{X_{0,0}}{A''}.(P'')^{-a}.(Q_p'')^{b}.w_0^{1-\alpha}.v_0^{\alpha}.Z_1 - R_c''.q_{c,0} - R_d''.q_{d,0} \\ & + \lambda.(\gamma.(R_d'')^{\delta} - (w+\sigma_q).Q_p'') + \mu.(\eta.(R_c'')^{\theta} - \sigma_a.A'') \end{aligned} \tag{4.6}$$

The associated discount rate is σ_1, where $\sigma_1 = (n-(a-1).\alpha.i-(a-1).(1-\alpha).h-b.\delta.l_d-(1-a).\theta.l_c)/(1-(a-1)\theta-b.\delta)$[19]. σ_1 is again the steady state growth rate of revenues, costs and both R&D budgets.

The four differential equations which characterise this system can again be calculated from the first order conditions:

$$\frac{dR_d''}{dt} = \phi_1.(R_d'')^{\delta}.\left(\phi_2.(R_d'')^{1-\delta} - \phi_3.(Q_p'')^{b-1}.(A'')^{a-1}\right) \tag{4.7}$$

$$\frac{dR_c''}{dt} = \phi_4.(R_c'')^{\theta}.\left(\phi_5.(R_c'')^{1-\theta} - \phi_6.(Q_p'')^{b}.(A'')^{a-2}\right) \tag{4.8}$$

where $\phi_1 = \gamma.\delta/(q_{d,0}.(1-\delta))$, $\phi_2 = (r+w-\sigma_1+\sigma_q).q_{d,0}/(\gamma.\delta)$, $\phi_3 = X_{0,0}.b.w_0^{(1-a)(1-\alpha)}.v_0^{(1-a).\alpha}.Z_1^{1-a}.a^{-a}.(a-1)^{a-1}$, $\phi_4 = \eta.\theta/(q_{c,0}.(1-\theta))$, $\phi_5 = (r-\sigma_1+\sigma_a).q_{c,0}/(\eta.\theta)$, $\phi_6 = X_{0,0}.w_0^{(1-a)(1-\alpha)}.v_0^{(1-a).\alpha}.Z_1^{1-a}.a^{-a}.(a-1)^{-a}$.

The dA"/dt and dQ_p/dt differential equations are given by respectively equation (4.4) and (4.5).

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[19]The similarity with the discount factors, after the application of the time elimination method, in other sections is apparent when one notes that $\sigma_1 = \hat{q}_c + \hat{R}_c'' = \hat{q}_d + \hat{R}_d''$.

Steady State

When we put dA"/dt, dQ$_p$"/dt, dR$_c$"/dt and dR$_d$"/dt equal to zero in the four differential equations and solve for A", Q$_p$", R$_d$" and R$_c$" we get the following steady state values:

$$R_c''^{\,*} = \left[\frac{X_{0,0}.\gamma^{b}.(b.\delta.Z_3)^{b.\delta}.(\sigma_a)^{2-a-b.\delta}.(w+\sigma_q)^{-b(1-\delta)}.\eta^{(a-1)}.\theta.(Z_2.\theta)^{1-b.\delta}}{w_0^{(a-1)(1-\alpha)}.v_0^{(a-1).\alpha}.(q_{c,0}.(r-\sigma_1+\sigma_a))^{1-b.\delta}.(q_{d,0}.(r-\sigma_1+w+\sigma_q))^{b.\delta}}\right]^{\frac{1}{1-(a-1).\theta-b.\delta}}$$

$$A''^{\,*} = \left[\frac{X_{0,0}.\gamma^{b}.(\delta.b.Z_3)^{b.\delta}.\sigma_a^{-(1-\theta)(1-b.\delta)/\theta}.(w+\sigma_q)^{-b(1-\delta)}.\eta^{(1-b.\delta)/\theta}.(Z_2.\theta)^{1-b.\delta}}{w_0^{(a-1)(1-\alpha)}.v_0^{(a-1).\alpha}.(q_{c,0}.(r-\sigma_1+\sigma_a))^{1-b.\delta}.(q_{d,0}.(r-\sigma_1+w+\sigma_q))^{b.\delta}}\right]^{\frac{\theta}{1-(a-1).\theta-b.\delta}} \tag{4.9}$$

$$R_d''^{\,*} = \left[\frac{X_{0,0}.\gamma^{b}.(b.\delta.Z_3)^{1+\theta-a.\theta}.\sigma_a^{-(1-\theta)(a-1)}.(w+\sigma_q)^{1+\theta-a.\theta-b}.\eta^{a-1}.(Z_2.\theta)^{\theta(a-1)}}{w_0^{(a-1)(1-\alpha)}.v_0^{(a-1).\alpha}.(q_{c,0}.(r-\sigma_1+\sigma_a))^{\theta.(a-1)}.(q_{d,0}.(r-\sigma_1+w+\sigma_q))^{1+\theta-a.\theta}}\right]^{\frac{1}{1-(a-1).\theta-b.\delta}}$$

$$Q_p''^{\,*} = \left[\frac{X_{0,0}.\gamma^{1+\theta-a.\theta/\delta}.(b.\delta.Z_3)^{1+\theta-a.\theta}.(w+\sigma_q)^{-(1-\delta)(1+\theta-a.\theta)/\delta}.\eta^{a-1}.(Z_2.\theta)^{\theta(a-1)}}{w_0^{(a-1)(1-\alpha)}.v_0^{(a-1).\alpha}.(q_{c,0}.(r-\sigma_1+\sigma_a))^{\theta.(a-1)}.(q_{d,0}.(r-\sigma_1+w+\sigma_q))^{1+\theta-a.\theta}.\sigma_a^{(1-\theta)(a-1)}}\right]^{\frac{\delta}{1-(a-1).\theta-b.\delta}}$$

The denominator of the general exponent of all the steady state values contains again the *cost-reducing* ((a-1).θ) and *demand-creation* (b.δ) effects. In comparison with the analyses, in which we treated these effects separately, the value of the exponent is increased. The result is that the effects of exogenous variables and parameters are more pronounced.

In general the direction of the influences of the various parameters and exogenous variables, on the steady state levels, is the same as in the cases in which product and process R&D are treated separately. The exogenous scale of demand has a positive influence on the steady state level, while the various costs variables have a negative influence on the steady state level. The productivity parameter and R&D elasticity of the productivity (perceived) quality generation process have again a positive influence on the steady state productivity (perceived quality) level, but also on the steady state perceived quality (productivity) level. Factors which favour the productivity (quality) level have also a positive influence on the quality (productivity) level. Product and process R&D are again *complements* in stead of substitutes. The reason for this is quite intuitive. For example, a higher perceived quality level increases the demand for the firms product. Cost-savings per unit of output stay the same but a higher output level implies an increase in the total costsavings (benefits) of doing process R&D. Factors which favour perceived quality favour therefore also productivity. Factors which increase the productivity level on the other hand, reduce cost per unit of output which result in a lower price level. This lower price level implies a higher marginal increase in demand as Q_p" increases. This justifies a higher level of product R&D. Therefore factors which favour the productivity level favour also the perceived quality level.

The R&D intensities of product and process R&D in the steady state are very similar to these intensities in section two and three (respectively equation (2.33) and equation (3.18)). The only difference is that the steady state growth rates of A and Q_p have changed.

Dynamics

The dynamics of the system in which demand-creating and cost-reducing innovations are possible is characterised by four non-linear differential equations. To draw the dynamics of dQ_p", A", R_d" and R_c" we need a four-dimensional space. Because this is not possible we will clarify the dynamics in two two-dimensional phasediagrams to get a qualitative impression of the dynamic behaviour of the system.

First, we will describe the dynamics of process R&D and productivity in the (A", R_c") space. The two demarcation lines, dA"/dt=0 and dR_c"/dt=0, can be calculated from equations (4.4) and (4.8):

$$\frac{dA''}{dt} = 0 \;\Rightarrow\; R_c'' = \left(\frac{\sigma_a}{\eta}\right)^{\frac{1}{\theta}} . (A'')^{\frac{1}{\theta}} \tag{4.10}$$

$$\frac{dR_c''}{dt} = 0 \;\Rightarrow\; R_c'' = \left(\frac{\phi_6}{\phi_5}\right)^{\frac{1}{1-\theta}} . (Q_p'')^{\frac{b}{1-\theta}} . (A'')^{\frac{a-2}{1-\theta}} \tag{4.11}$$

This system has only changed in comparison with the "pure" process R&D case of section 3 by the occurrence of the perceived quality level in equation (4.11). Imagine first that the perceived quality level is constant. Then we are back in the situation of section 3 and the dynamics can be described by the phasediagrams (3.1-3.6). The system is saddle point stable as long as the price elasticity of demand is $1<a<(1+\theta)/\theta$ and unstable as $a\geq(\theta+1)/\theta$. If the value of Q_p changes we get a corresponding shift in the dR_c"/dt =0 locus. This locus moves upwards (downwards) as the perceived quality level is higher (lower). These movements have no influence on the stability characteristics. The dR_c"/dt=0 locus shifts because on this locus the marginal costs of doing process R&D are equal to the marginal benefits of doing product R&D. A higher (lower) perceived quality level increases (decreases) the benefits while cost stay the same, which results in an upward (downward) shift of the dR_c"/dt=0 locus.

Second, we will describe the dynamics of product R&D and perceived quality in the (Q_p", R_d") space. Equation (4.5) and (4.7) give us the two demarcation lines, dQ_p"/dt=0 and dR_d"/dt=0 which divide this area in four parts:

$$\frac{dQ_p''}{dt} = 0 \;\Rightarrow\; R_d'' = \left(\frac{w+\sigma_q}{\gamma}\right)^{\frac{1}{\delta}} . (Q_p'')^{\frac{1}{\delta}} \tag{4.12}$$

$$\frac{dR_d''}{dt} = 0 \quad \Rightarrow \quad R_d'' = \left(\frac{\phi_3}{\phi_2}\right)^{\frac{1}{1-\delta}} .(Q_p'')^{\frac{b-1}{1-\theta}} .(A'')^{\frac{a-1}{1-\delta}} \tag{4.13}$$

This system is very similar to the system which treats the case in which one could do only product R&D. The principal difference is the occurrence of the endogenous productivity level in the $dR_d''/dt=0$ demarcation line. If this productivity level is constant we are back in the situation of section 2 and the system is saddle point stable. A higher (lower) productivity level shifts the $dR_d''/dt=0$ locus upwards (downwards) which doesn't influence the stability properties.

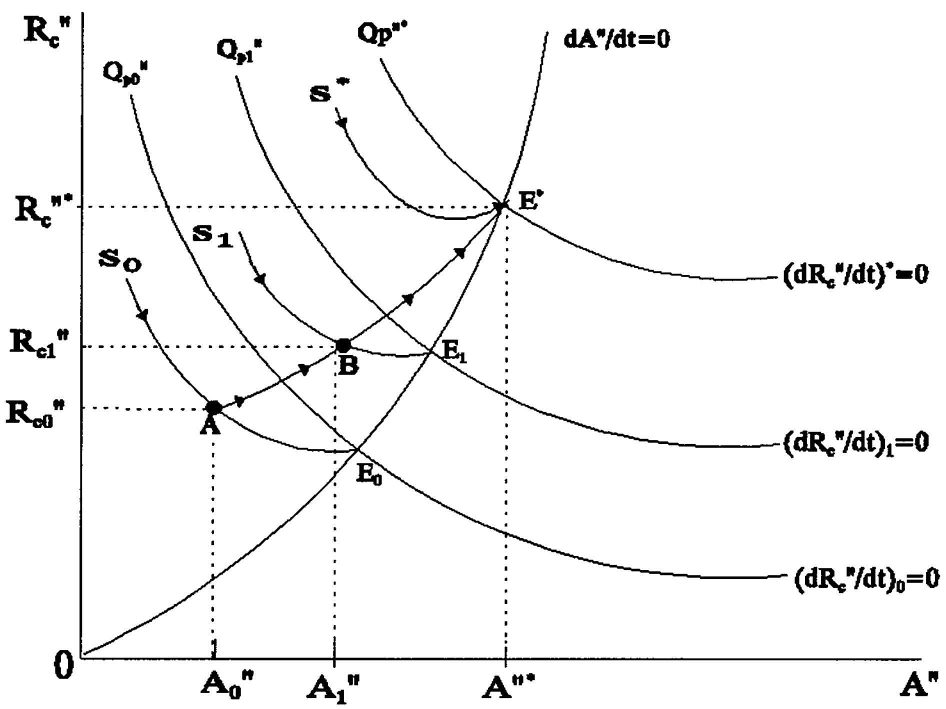

Figure 4.1
productivity generation process

Now we will integrate these two parts and examine the dynamics as A" and Q_p" both change. We assume that the initially perceived quality level and productivity level are smaller than the steady state values; $Q_{p,0}''<Q_p''^*$ and $A_0''<A''^*$. Furthermore we assume that $1<a<(1+\theta)/\theta$ which implies that the first system is saddle point stable.

In phasediagram 4.1 we have drawn the demarcation loci of the first system. The $dA''/dt=0$ locus is independent of the perceived quality level and the $dR_c''/dt=0$ locus does depend on the perceived quality level. On time t=0 the given initially perceived quality level is equal to Q_{p0}'' for the firm. The $(dRc''/dt)_0=0$ locus depends on this level. We get a steady state which is located in point E_0 and an associated saddlepath s_0. Given its initial productivity level (A_0''), the firm will choose the process R&D level of $R_{c,0}''$ on this saddle path.

In phasediagram 4.2 we have drawn the demarcation loci of the second system. Only the $dR_d''/dt=0$ locus is dependent on the productivity level. The $(dR_c''/dt)_0$ locus is drawn dependent on the initially given productivity level A_0''. The steady state is located in point E_0' and the accompanying saddle path is s_0'. A firm with an initially perceived quality level of $Q_{p,0}''$ will choose the product R&D level of $R_{d,0}''$.

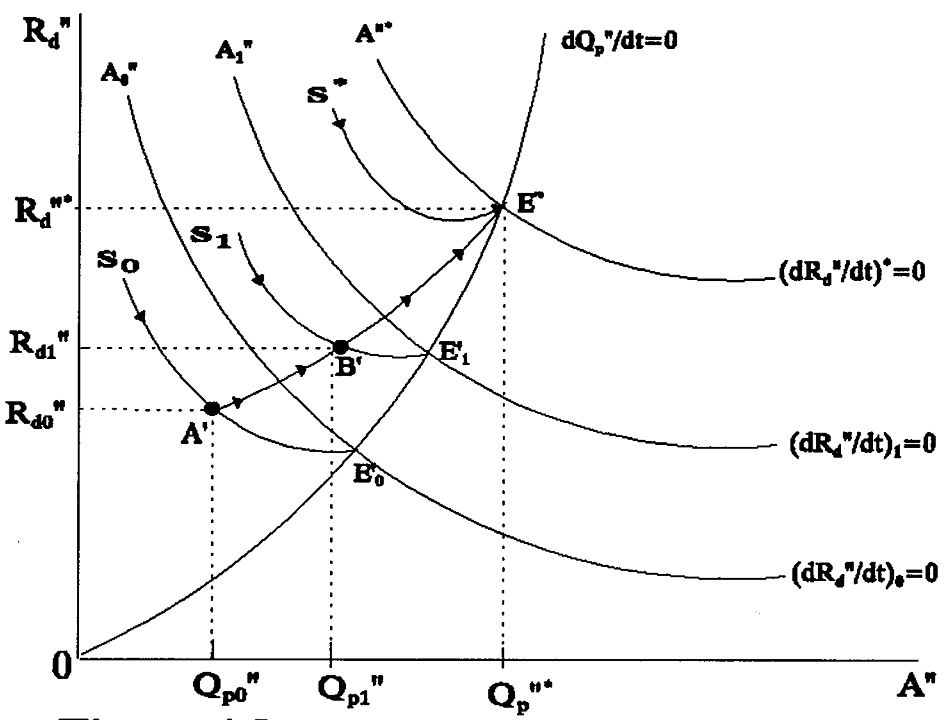

Figure 4.2
perceived quality generation process

The firm has chosen its product and process R&D level at time t=0. Given these values we can calculate its new productivity level and perceived quality level (approximately):

$$
\begin{aligned}
A_1'' &= A_0'' + \eta.(R_{c,0}'')^{\theta} - \sigma_a.A_0'' \\
Q_{p,1}'' &= Q_{p,0} + \gamma.(R_{d,0}'')^{\delta} - (w+\sigma_q).Q_{p,0}''
\end{aligned}
\qquad (4.14)
$$

The new productivity level (A_1'') and the new perceived quality level ($Q_{p,1}''$) have both increased. The higher perceived quality level ($Q_{p,1}''$) shifts the $dR_c''/dt=0$ locus upwards to $(dR_c''/dt)_1=0$ in figure 4.1. The steady state equilibrium shifts from E_0 to E_1. A higher perceived quality level implies a higher steady state productivity level.The higher productivity level (A_1'') shifts the $dR_d''/dt=0$ locus upwards to $(dR_d''/dt)_1=0$ in figure 4.2. The steady state equilibrium shifts from E_0' to E_1'. A higher initial productivity level results in a higher steady state perceived quality level. This analysis shows again that process and product R&D are complements in stead of substitutes.
Given the new productivity level (A_1'') the firm will choose the amount of process R&D, $R_{c,1}''$ which is located on the new saddlepath s_1 in figure 4.1 Given its new perceived quality level ($Q_{p,1}''$) the firm will choose the level of product R&D, $R_{d,1}''$ on the new saddle path s_1'in figure 4.2. With this new product and process R&D levels we can calculate the new productivity and perceived quality levels, $Q_{p,2}''$ and A_2''. The process

repeats again and continues until the steady state values ($Q_p''^*$ and A''^*) are approached. When the productivity level and perceived quality level of the firm are equal to the steady state values, the firm will choose the process and product R&D levels which are just enough to maintain the current perceived quality and productivity levels. The $dR_c''/dt=0$ and $dR_d''/dt=0$ will not shift anymore and the firm will choose $R_d''^*$ and $R_c''^*$ from now on. We have reached the steady state ($Q_p''^*$, A''^*) in which the discounted values of Q_p and A are constant and $Q_{p,t}$ and A_t grow at a constant rate. When the initial productivity and perceived quality level are not equal to these steady state values we get a long-run adjustment path for the amount of process R&D and product R&D which is given by respectively ABE* in figure 4.1 and A'B'E'* in figure 4.2.

We examined the validity of the saddle point steady state property of this four dimensional differential equation system again by a linearisation of the nonlinear differential-equation system around the steady state. Feichtinger and Hartl (1986) describe the conditions for saddle point stability of an optimal control system with two state and co-state variables[20]. The system is saddle point stable when two characteristic roots are real and negative and the other two characteristic roots are real and positive. In this case has the system a saddlepoint plane. When we know the two initial conditions A_0'' and $Q_{p.0}''$, the begin point of the unique dynamic path in this saddlepoint plane is exactly determined. We checked in appendix 3 the conditions of Feichtinger and Hartl for a saddle point plane and concluded that this system is saddle point stable when the value of the price elasticity of demand is not too large.

[20]For details on the stability of a non-linear differential system with more than one state variable see Feichtinger and Hartl, 1986, p. 122-154.

5. Conclusion:

In this paper we used optimal control theory to determine the optimal amount of product and process R&D which a firm can choose given its perceived quality and productivity level. An advantage of this method is that it can deal with the intertemporal benefits which characterise the technology generation process. In combination with the time elimination method we can deal easily with prices which have an arbitrary constant growth rate.

Our analyses showed that the various systems are characterised by saddle point stability. There exists a unique saddle path which "prescribes" the optimal choices of the amount of product and/or process R&D. Only when the price elasticity of demand becomes large and the firm can do process R&D in order to improve its productivity level, the system becomes unstable.

Technological progress (innovations) is generally favoured by factors which increase demand or by factors which increase the efficiency of the perceived quality or productivity generation process. Factors which temper technological progress are the wage rate and the user cost of capital if the price elasticity of demand is relatively high. These factors favour technological progress when demand is totally price inelastic. The price of R&D tempers technological progress independently of the price elasticity of demand.

Product and process R&D turn out to be complements rather than substitutes in our analysis because they reinforce each other, while the total R&D budget can be freely chosen.

The first elaboration of this analysis is to include the effect of perceived quality on unit costs. A higher quality level of a product can only be obtained by increasing unit production costs. A profit maximizing firm who considers to engage in product R&D to raise its quality level must therefore not only take into account an increase in expected demand but also an increases in unit production costs.

Another deficiency of this analysis is that it doesn't take into account spill-over effects. When spill-over effects are introduced a firm can do own R&D and/or use results of R&D done by other firms. It would be very interesting to investigate whether the introduction of spill-over effects in these models would favour or temper technological progress.

Appendix 1: Checking the Saddle Point of section 2 by Characteristic Roots

The configuration of arrows in figure 2.1 leads us to conclude that the equilibrium at ($Q_p''^*$, $R_d''^*$) is a saddle point. These positions are taken on a qualitative judgement of the phase diagram. Because the position of the curves are drawn with considerable latitude in the positioning of the curves, we can examine the validity of this conclusion by examining the local stability characteristics around the steady state. These characteristics can be examined by a linearisation (first order Taylor expansion) of the nonlinear differential-equation system near the steady state[21].

To keep this analysis as straight forward as possible, we simplify the two differential equations (2.25) and (2.28) somewhat:

$$\dot{Q}_p'' = \frac{dQ_p''}{dt} = \gamma.(R_d'')^{\delta} - w'.Q_p''$$

where $w'=w+\delta.\sigma$

$$\dot{R}_d'' = \frac{dR_d''}{dt} = \phi_1.(R_d'')^{\delta}.(\phi_2.(R_d'')^{1-\delta} - \phi_3.(Q_p'')^{b-1})$$

where $\phi_1=\gamma.\delta/(q_{d,0}.(1-\delta))$, $\phi_2=(r+w-\hat{q}_d-(1-\delta).\sigma).q_{d,0}/(\gamma.\delta)$ and $\phi_3=C_0.X_{0,0}.b.(P'')^{-a}/(a-1)$.

We first construct the Jacobian matrix and evaluate it at the steady-state point ($Q_p^{*''}$, $R_d^{*''}$).

$$J_{(Q_p''^*, R_d''^*)} = \begin{bmatrix} \frac{\delta \dot{Q}_p''}{\delta Q_p''} & \frac{\delta \dot{Q}_p''}{\delta R_d''} \\ \frac{\delta \dot{R}_d''}{\delta Q_p''} & \frac{\delta \dot{R}_d''}{\delta R_d''} \end{bmatrix}_{(Q_p''^*, R_d''^*)}$$

The four partial derivatives evaluated at ($Q_p''^*$, $R_d''^*$) turn out to be:

[21] For more details of the procedure of linearization of a non-linear system, see A.C. Chiang, 1984, Section 18.6.

$$\left.\frac{\delta \dot{Q}_p''}{\delta Q_p''}\right|_{Q_p''^*, R_d''^*} = -w' \qquad <0$$

$$\left.\frac{\delta \dot{Q}_p''}{\delta R_d''}\right|_{Q_p''^*, R_d''^*} = \gamma.\delta.(R_d''^*)^{\delta-1} \qquad >0$$

$$\left.\frac{\delta \dot{R}_d''}{\delta Q_p''}\right|_{Q_p''^*, R_d''^*} = -\phi_1.(R_d''^*)^{\delta}.\phi_3.(b-1).(Q_p''^*)^{b-2} \qquad >0 \qquad \because\ 0<b<1$$

$$\left.\frac{\delta \dot{R}_d''}{\delta R_d''}\right|_{Q_p''^*, R_d''^*} = \phi_1.\delta.(R_d''^*)^{\delta-1}.\left[\phi_2.(R_d''^*)^{1-\delta}-\phi_3.(Q_p''^*)^{b-1}\right]+\phi_1.(R_d''^*)^{\delta}.\phi_2.(1-\delta).(R_d''^*)^{-\delta} \qquad >0$$

Note that the term between the brackets in the last equation is equal to zero in the steady state. The Jacobian matrix evaluated at ($Q_p''^*$, $R_d''^*$) is:

$$J_{(Q_p''^*, R_d''^*)} = \begin{bmatrix} <0 & >0 \\ >0 & >0 \end{bmatrix}$$

To check the dynamic stability of the equilibrium, we have to know the signs of the two characteristic roots, r_1 and r_2. A saddle point stable equilibrium implies that the two roots have opposite signs. Important information in this context can be derived from the determinant of the Jacobian matrix.

$$r_1.r_2 = \left|J_{Q_p''^*, R_d''^*}\right| = (-).(+)-(+).(+) = <0$$

The determinant of the Jacobian matrix is negative, which implies that the two characteristic roots have opposite signs. This immediately enables us to conclude that the equilibrium ($Q_p''^*$, $R_d''^*$) is locally a saddle point.

Appendix 2: Checking the Saddle Points of section 3 by Characteristic Roots

The configuration of arrows in the various situations leads us to conclude that when $1<a<(1+\theta)/\theta$ we have a saddle point equilibrium and when $a>(1+\theta)/\theta$ the stability characteristics are not clear. These conclusions are based on a qualitative judgement of the phasediagram. Because the position of the curves are drawn with considerable latitude in the positioning of the curves, we can examine the validity of these conclusions by examining the local stability characteristics around the steady state. We will again examine this using a linearisation of the nonlinear differential-equation system (Chiang, 1984).

We first simplify the two differential equations (3.11) and (3.13):

$$\dot{A} = \frac{dA''}{dt} = \eta.(R_c'')^{\theta} - \theta.\sigma''.A''$$

$$\dot{R}_c'' = \frac{dR_c''}{dt} = \phi_1.(R_c'')^{\theta}.\left(\phi_2.(R_c'')^{1-\theta} - \phi_3.(A'')^{a-2}\right)$$

where $\phi_1=\eta.\theta/(q_{c,0}.(1-\theta))$, $\phi_2=(r-\hat{q}_c-(1-\theta).\sigma'').q_{c,0}/(\eta.\theta)$ and $\phi_3=X_{0,0}.(w_0/(1-\alpha))^{(1-a)(1-\alpha)}.(v_0/\alpha)^{\alpha.(1-a)}.((a-1)/a)^a$.

The Jacobian matrix evaluated at the steady-state point (E) is

$$J_E = \begin{bmatrix} -\theta.\sigma'' & \eta.\theta.(R_c''^{*})^{\theta-1} \\ -(a-2).\phi_1.\phi_3.(R_c''^{*})^{\theta}.(A^{*})^{a-3} & \phi_1.\phi_2.(1-\theta) \end{bmatrix}$$

The determinant of the Jacobian matrix becomes:

$$r_1.r_2 = \left| J_{A''^{*},R_c''^{*}} \right| = -\theta.\sigma''.\phi_1.\phi_2.(1-\theta) + (a-2).\eta.\theta.\phi_1.\phi_3.2.(R_c''^{*})^{2\theta-1}.(A'')^{a-3}$$

The value of the determinant of the Jacobian matrix is dependent on the value of the price elasticity of demand. When $1<a\leq 2$ we can immediately conclude that the system is *saddlepoint stable* because the value of the determinant is negative. This verifies our conclusion in situation A. To examine the local stability around the steady state when $a>2$ we have to compute the sign of the determinant of the Jacobian matrix. We can simplify the determinant to[22]:

$$r_1.r_2 = \theta.\sigma''.\phi_1.\phi_2.(a.\theta-1-\theta)$$

The determinant of the Jacobian matrix is negative as $a<(1+\theta)/\theta$ and positive as $a>(1+\theta)/\theta$. With a negative determinant the only possibility is again that the steady

[22] We have used the fact that in the steady state the following two equalities should hold: $\phi_2.(R_c'')^{1-\theta} = \phi_3.(A'')^{a-2}$ and $\eta.(R_c'')\wedge\theta = \theta.\sigma''.A''$.

state is locally saddle point stable. This confirms our conclusion in situation B_1 and B_2 that the steady state is a *saddle point* if $a<(1+\theta)/\theta$. When the determinant is positive we are not able to take directly inference about the local stability of the system[23]. To be able to make this inference we have to calculate the trace of the Jacobian matrix ($tr\ J_E$)[24]. The trace of this Jacobian matrix is:

$$r_1 + r_2 = trJ_E = -\theta.\sigma'' + \phi_1.\phi_2.(1-\theta) = r - \hat{q} - \sigma'' > 0$$

where $r > \hat{q} + \sigma''$ if the transversality condition is satisfied. Relevant for the ability to make inference is the positive sign of the trace of the Jacobian. A positive determinant in combination with a positive trace implies an *unstable equilibrium*, which clarifies our discussion in situation C. If we are dealing with an unstable node or with an unstable focus is dependent on the values of the parameters and exogenous growth rates[25]. Or more explicitly is this dependent on:

$$(trJ_E)^2 \geq 4\left| J_E \right| \quad , \quad (r-\hat{q}-\sigma'')^2 \geq 4.\frac{(r-\hat{q}-(1-\theta).\sigma'').\theta.\sigma''}{1-\theta}.(a.\theta-\theta-1)$$

If this condition holds, for example when a is slightly larger than $(\theta+1)/\theta$, we have an unstable node. When this condition is not satisfied we have an unstable focus.

---

[23]See, A. C. Chiang, 1984, p.643, table 18.1.

[24]The trace of J_E, symbolised by tr J_E, is the sum of the principal-diagonal elements of the Jacobian. There is a simple relation between the trace of J_E and the characteristic roots: $r_1+r_2=tr\ J_E$ (Chiang, 1984, p 641).

[25]A node is an equilibrium such that all the trajectories associated with it either flow noncyclically toward it (*stable node*) or flow noncyclically away from it (*unstable node*). A focus is an equilibrium which is characterised by whirling trajectories, all of which either flow cyclically toward it (stable focus), or flow cyclically away from it (unstable focus) (Chiang, 1984, p633-634).

Appendix 3: Checking the Saddle Point of section 4 by Characteristic Roots

Feichtinger and Hartl state that the general condition for a saddle point plane without loops is characterised by the following conditions[26]:

$$1)\ \det J > 0 \qquad 2)\ K < 0 \qquad 3)\ 0 < \det J \leq K^2/4$$

First, we have to compute the determinant of the Jacobian matrix evaluated at the steady-state point (E):

$$J_E = \begin{bmatrix} -(w+\sigma_q) & 0 & \gamma.\delta.R_d''^{\,*\delta-1} & 0 \\ 0 & -\sigma_a & 0 & \eta.\theta.R_c''^{\,*\theta-1} \\ -\phi_1.\phi_3.R_d''^{\,*\delta}.Q_p''^{\,*\underline{b}-2}A^{*\underline{a}-1} & -(a-1)\phi_1.\phi_3.R_d''^{\,*\theta}.Q_p''^{\,*\underline{b}-1}A''^{\,*\underline{a}-2} & \phi_1.\phi_2.(1-\delta) & 0 \\ -b.\phi_4.\phi_6.R_c''^{\,*\theta}.Q_p''^{\,*\underline{b}}A''^{\,*\underline{a}-1} & -(a-2).\phi_4.\phi_6.R_c''^{\,\theta}.Q_p^{*\underline{b}}A''^{\,*\underline{a}-3} & 0 & (1-\theta).\phi_4.\phi_5 \end{bmatrix}$$

The determinant of the Jacobian matrix can be simplified to:

$$|J| = B_1.(1-a.\theta+\theta)-B_2$$

where B_1 and B_2 are dependent on the parameters and exogenous variables of the system and the steady state values of A", Q_p", R_c" and R_d". It is difficult to give an explicit value for the value of the price elasticity of demand which makes this determinant zero. When we solve this determinant numerically we find that when the value of the price elasticity of demand increases the determinant is first positive, gets zero and eventually becomes negative. The first condition, a positive determinant of the Jacobian matrix is satisfied when the price elasticity of demand is not too high.

Second, we have to calculate the value of the following matrix K:

$$K = \begin{bmatrix} \dfrac{\delta\dot{Q}_p''}{\delta Q_p''} & \dfrac{\delta\dot{Q}_p''}{\delta R_d''} \\ \dfrac{\delta\dot{R}_d''}{\delta Q_p''} & \dfrac{\delta\dot{R}_d''}{\delta R_d''} \end{bmatrix} + \begin{bmatrix} \dfrac{\delta\dot{A}''}{\delta A''} & \dfrac{\delta\dot{A}''}{\delta R_c''} \\ \dfrac{\delta\dot{R}_c''}{\delta A''} & \dfrac{\delta\dot{R}_c''}{\delta R_c''} \end{bmatrix} + 2.\begin{bmatrix} \dfrac{\delta\dot{Q}_p''}{\delta A''} & \dfrac{\delta\dot{Q}_p''}{\delta R_c''} \\ \dfrac{\delta\dot{R}_d''}{\delta A''} & \dfrac{\delta\dot{R}_d''}{\delta R_c''} \end{bmatrix}$$

[26]For details on the stability of a non-linear differential system with more than one state variable see Feichtinger and Hartl, 1986, p. 122-154.

From the first matrix, on the right hand side of equation (4.18), we calculate the determinant of this "demand creating sub-system" which turns out to be negative. From the second matrix, on the right hand side, we calculate the value of the determinant of this "cost-reducing sub-system" which has a negative value as long as $a < (\theta+1)/\theta$. The determinant of the third matrix, which describes cross influences is zero. Overall we can conclude that the second condition, a negative value for K, is met for a very large range of the price elasticity of demand (a).

We examined the third condition, $0 < \det J \leq K^2/4$, again numerically. We found that for all values which satisfy the first condition the second and third condition are also satisfied. We can conclude that this system is saddle point stable when the value of the price elasticity of demand is not too large.

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