The Continuous Recursive Adjustment Putty-Putty Model: An Outline of Its Main Features

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1 Introduction

In this paper we present an extension of the Quasi-Putty-Putty model as outlined in van Zon (1992). Van Zon (1992) provides a description of a simple hybrid production model which borrows elements of the aggregate production function approach, or APF for short, as well as from the vintage approach, or VAP for short. This paper is about the same subject. The model has changed, however, but not the basic idea that many of the features of a full vintage model can be captured by defining a model which is somewhere in between a VAP and an APF.

As in van Zon (1992), we assume that substitution possibilities before the actual moment of installation of new equipment are different from those after the moment of installation. Moreover, we assume that substitution possibilities ex post are at most equal to those ex ante. The model we are about to present is again of the quasi-putty-putty type, i.e. it is not a full vintage model (quasi), whereas it does exhibit 'putty' characteristics both ex ante and ex post. But contrary to van Zon (1992), the model does not suffer from any of the approximating shortcuts taken there. Instead, we take only one 'tiny' shortcut, which nevertheless generates exact results for a value of the elasticity of substitution of the ex post production functions equal to 0.5. [1]

The model will be called the CRAPP model [2], since it uses a set of recursive adjustment rules which describes the evolution over time of aggregate capital productivity and the aggregate capital/labour ratio in function of the ex post substitution characteristics of the 'old' machinery and of the new machinery just installed.

The advantage of using the CRAPP model instead of a VAP lies in the relative ease by which it can be handled : instead of tracing individual vintages for a considerable length of time, the CRAPP model makes positive use of the fact that it is often not necessary to know all the details about every individual vintage. From a policy point of view, it is usually sufficient to take account

1 We come back to this later.

2 CRAPP stands for Continuous Recursive Adjustment Putty-Putty.

of the impact of differences in elasticities of substitution both before and after the moment of installation on the development over time of aggregate output and employment. This is exactly what the CRAPP model does without drowning in excessive vintage detail and cumbersome numerical exercises. Nonetheless, the CRAPP model is still able to generate responses of employment and investment to changes in factor prices which are present in full putty-clay or clay-clay vintage models (c.f. Malcomson (1975) and van Zon and Muysken (1992)). These responses are lacking in the APF, although they are entirely reasonable on a priori grounds.

The set-up of this paper is as follows. In section 2 we will describe the features of the CRAPP model, and section 3 will be used to sketch the outcomes of some simulation experiments using the model. Section 4 contains a summary and some concluding remarks.

2 The CRAPP Model

We assume that there are two basic factors of production, labour and capital, which can be substituted both ex ante and ex post. We also assume that substitution possibilities are 'smooth' and that there are no costs involved in switching from the one technique to another. Nor are there any costs involved in switching from the one technology to another. A further assumption is that the diffusion of new technologies is an instantaneous process : every single producer invests in the newest technology only. At the same time we assume that capital costs are sunk costs ex post.

Because of the smooth substitution possibilities ex post, it follows that output can be produced using all the technologies which have come into existence from time immemorial. The reason is that it is possible to increase the marginal productivity of labour (and thus decrease variable costs per unit of output) indefinitely for any production function which obeys the Inada conditions. This will become more clear below.

For the ex ante production function as well as the ex post production function we use linear homogeneous CES functions. Denoting the level of output associated with vintage i at time t by $Y_{i,t}$, the amount of labour associated with vintage i at time t by $N_{i,t}$ and the amount of investment associated with vintage i at time t by $I_{i,t}$, we have :

$$Y_{t,t}=\left\{A_t^a\cdot(N_{t,t})^{-\rho_a}+B_t^a\cdot(I_{t,t})^{-\rho_a}\right\}^{-1/\rho_a} \tag{1}$$

where the super-/subscript a denotes the ex ante function. $A^a{}_t$ and $B^a{}_t$ are the CES distribution parameters and $\sigma_a=1/(1+\rho_a)$ is the ex ante elasticity of substitution. Similarly, for the ex post production function we have :

$$Y_{i,t}=\left\{A_i^p\cdot(N_{i,t})^{-\rho_p}+B_i^p\cdot(I_{i,t})^{-\rho_p}\right\}^{-1/\rho_p} \tag{2}$$

where t>i, and where the super-/subscript p denote the ex post parameters. Note that the main difference between (1) and (2) lies in the specification of the distribution parameters A and B. In the ex ante case A depends on time (embodied technical change), whereas in the ex post case A and B depend only on the moment of installation, i.e. there is no disembodied technical change, although the analysis could well be extended to include this case. More in particular, with respect to the ex ante distribution parameters we assume that :

$$A_t^a=A_0\cdot(1+\mu)^{-\rho_a\cdot t} \tag{3}$$

$$B_t^a=B_0$$

Hence, we assume that embodied technical change is of the Harrod-neutral kind. Given the assumptions of sunk capital costs ex post and 'smooth' substitution possibilities ex post and ex ante, it follows that the instantaneous cost minimisation problem which a producer faces can be written as :

$$C_t = w_t \cdot \sum_{i=-\infty}^{t} N_{i,t} + u_t \cdot I_{t,t} \tag{4}$$

$s.t.$

$$X_t = \sum_{i=-\infty}^{t-1} Y_{i,t} + Y_{t,t}$$

$$Y_{i,t} = F^p(N_{i,t}, I_{i,t}) \quad \forall i < t$$

$$Y_{t,t} = F^a(N_{t,t}, I_{t,t})$$

$$I_{i,t} = I_{i,i} \cdot (1-\delta)^{t-i} \quad \forall i \leq t$$

where w_t is the current wage rate, and u_t is the cost of obtaining a unit of capital. Moreover $F^a()$ and $F^p()$ are short hand notations for the ex ante production function and the ex post production function, respectively. X_t is the total amount of output to be produced on both new equipment and old equipment. δ is the technical decay parameter (we assume depreciation by 'radioactive decay').

Minimisation of (4), gives rise to the following first order conditions :

$$\frac{\partial C_t}{\partial N_{i,t}} = w_t - \mu_i \cdot \frac{\partial F^p}{\partial N_{i,t}} = 0 \tag{5}$$

$$\frac{\partial C_t}{\partial N_{t,t}} = w_t - \mu_t \cdot \frac{\partial F^a}{\partial N_{t,t}} = 0$$

$$\frac{\partial C_t}{\partial I_{t,t}} = u_t - \mu_t \cdot \frac{\partial F^a}{\partial I_{t,t}} = 0$$

$$\frac{\partial C_t}{\partial Y_{i,t}} = -\lambda - \mu_i = 0$$

$$\frac{\partial C_t}{\partial Y_{t,t}} = -\lambda - \mu_t = 0$$

where λ and μ_i are Lagrange multipliers and where we have assumed the existing capital stock to be fixed.

Due to the linear homogeneity of $F^a()$, it immediately follows that λ_t is equal to unit total production cost of the new technology, i.e. :

$$\lambda_t = w_t \cdot \nu_{t,t} + u_t \cdot \kappa_{t,t} \tag{6}$$

$$\nu_{i,t} = \frac{N_{i,t}}{Y_{i,t}} \quad \forall i \leq t$$

$$\kappa_{i,t} = \frac{I_{i,t}}{Y_{i,t}} \quad \forall i \leq t$$

From equations (5) and (6) it follows that the optimum allocation of labour (and output) between 'new' equipment and 'old' equipment is determined by the condition that :

$$\lambda_t = \frac{w_t}{\frac{\partial Y_{t,t}}{\partial N_{t,t}}} = \frac{w_t \cdot \Delta N_{i,t}}{\frac{\partial Y_{t,t}}{\partial N_{t,t}} \cdot \Delta N_{i,t}} = w_t \cdot \nu_{t,t} + u_t \cdot \kappa_{t,t} \tag{7}$$

(7) says that the marginal unit of labour allocated to the 'old' capital stock should be able to produce output at a unit variable cost which is equal to the unit total cost of output on new equipment. This is exactly what the Malcomson scrapping condition says with regard to the optimum vintage composition of the capital stock in a situation of cost-minimisation (c.f. Malcomson (1975) and van Zon and Muysken (1992)). It follows furthermore from (5) that for a minimum of (4) the marginal labour productivities of every single existing vintage should be equal to the marginal productivity of labour on the new vintage. Moreover, for the new vintage we should have :

$$\frac{\left(\frac{\partial Y_{t,t}}{\partial N_{t,t}}\right)}{\left(\frac{\partial Y_{t,t}}{\partial I_{t,t}}\right)} = \frac{w_t}{u_t} \tag{8}$$

Using the ex ante function (1), we have :

$$\frac{\partial Y_{t,t}}{\partial N_{t,t}} = A_t^a \cdot \left\{ \frac{Y_{t,t}}{N_{t,t}} \right\}^{\frac{1}{\sigma_p}} \tag{9}$$

$$\frac{\partial Y_{t,t}}{\partial I_{t,t}} = B_t^a \cdot \left\{ \frac{Y_{t,t}}{I_{t,t}} \right\}^{\frac{1}{\sigma_a}}$$

which, combined with (5) and (6), leads to:

$$\nu_{t,t} = \kappa_{t,t} \cdot \left\{ \frac{B_t^a \cdot w_t}{A_t^a \cdot u_t} \right\}^{-\sigma_a} = \kappa_{t,t} \cdot h_t \tag{10}$$

where h_t is implicitly defined by (10). Using (10) and (1), we find :

$$\kappa_{t,t} = \left\{ A_t^a \cdot h_t^{-\rho_a} + B_t^a \right\}^{\frac{1}{\rho_a}} \tag{11}$$

$$\nu_{t,t} = \left\{ A_t^a + B_t^a \cdot h_t^{\rho_a} \right\}^{\frac{1}{\rho_a}}$$

Essentially, equation (11) says that the optimum labour intensity is determined by the wage-rental ratio, as well as the distribution parameters of the CES ex ante function next to the ex ante elasticity of substitution.

In figure 1, the ex ante unit iso-quant has been labelled e.a., while two of the ex post iso-quants have been labelled e.p. The ex ante iso-quant has been drawn as an envelope of all possible ex post iso-quants.[3] In figure 1, we have drawn two different wage rental ratios, which give rise to two different optimum values of the labour intensity of production on new (and old) equipment. Suppose that at time 0 the ruling wage rental ratio

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3 Note that for the type of embodied technical change we have assumed that the entire iso-quant field would shift towards the origin, where the N/Y dimension would 'shrink' faster than the I/Y dimension, ceteris paribus (embodied technological change is assumed to be Harrod neutral).

is such that point A would be chosen. Then, at time 1, the wage rental ratio changes such that on new equipment point B becomes optimum. [4]

With the rise in the relative wage rate, the labour/capital ratio has a tendency to fall. But on old equipment, substitution possibilities between labour and capital are more limited by assumption, and therefore the rise in the relative wage rate invokes only a moderate adjustment of the labour/capital ratio on old equipment. This is depicted by the move from point A to point C along the ex post function, as opposed to the 'move' from point A to point B along the ex ante function, where point C which is the optimum capital labour combination for equipment installed at time 0 at the new price vector. Obviously, when substitution possibilities ex post would be equal to those ex ante, the distinction between old equipment and new equipment vanishes entirely in the absence of embodied technical change.

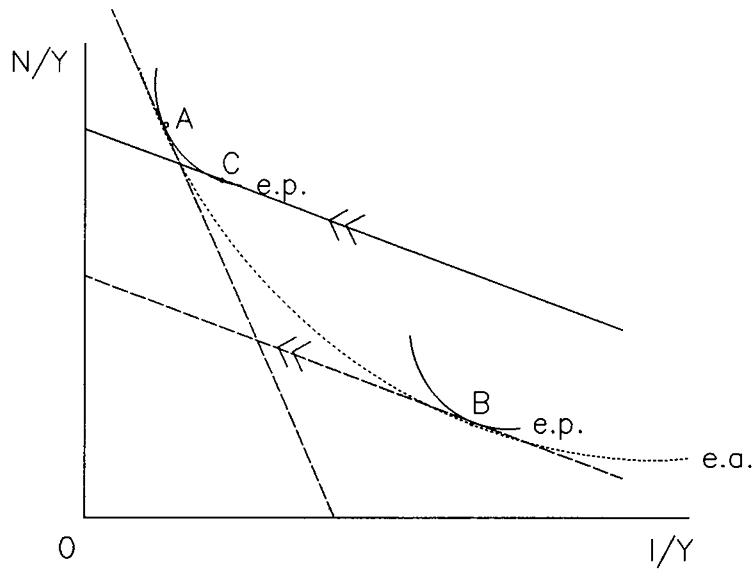

Fig. 1 The ex ante production function envelope

4 Note that for the moment we disregard the possibility of embodied technical change, since, for ease of exposition, we assume the position of the unit ex ante isoquant to be fixed. However, in the full CRAPP model we do have Harrod-neutral embodied technical change.

From figure 1 it is clear that the exact location of an ex post iso-quant in the N/Y, I/Y plane depends, among other things, on the value of the wage/rental ratio. More specifically, the notion of the ex ante iso-quant as an envelope of ex post unit iso-quants can help to derive the values of the distribution parameters $A^p{}_i$ and $B^p{}_i$ in function of the substitution parameters ex ante and ex post, the ex ante distribution parameters and h_t. For, the figure shows that for the ex post production function two conditions should hold in order for the ex post iso-quant in question to be consistent with the ex ante choice set :

1 the optimum technique should be part of the ex ante envelope as well as part of its associated ex post unit iso-quant ;

2 since the envelope has only one technique in common with each ex post unit iso-quant, and since substitution possibilities ex ante and ex post are 'smooth' by assumption, it follows that for the optimum labour/capital ratio the slopes of both the ex ante iso-quant and the ex post iso-quant should be the same.

From requirement 2 it follows that :

$$\frac{d\nu^a_{t,t}}{d\kappa^a_{t,t}} = \frac{d\nu^p_{t,t}}{d\kappa^p_{t,t}} \Rightarrow \tag{12}$$

$$\frac{B^a_t}{A^a_t} \cdot \left\{ \frac{\kappa^a_{t,t}}{\nu^a_{t,t}} \right\}^{-1/\sigma_a} = \frac{B^p_t}{A^p_t} \cdot \left\{ \frac{\kappa^p_{t,t}}{\nu^p_{t,t}} \right\}^{-1/\sigma_p}$$

From requirement 1 it follows moreover that the labour coefficient ex post as well as the capital coefficient ex post should be equal to their ex ante counterparts. Thus the capital/labour ratios ex ante and ex post are identical and equal to $1/h_t$ (c.f. equation (10)). Using this result we can rewrite (12) to obtain :

$$B^p_t = A^p_t \cdot h_t^{\rho_a - \rho_p} \cdot \frac{B^a_t}{A^a_t} \tag{13}$$

Moreover, from requirement 1 and equation (11), we can derive that :

$$A_t^p = A_t^a \cdot \left\{ A_t^a + B_t^a \cdot h_t^{\rho_a} \right\}^{\frac{\rho_p}{\rho_a} - 1} \tag{14}$$

Equations (13) and (14) taken together show how a change in h_t may cause changes in the distribution parameters of the ex post iso-quants. Moreover, it is easily seen that embodied technical change has an influence on the ex post distribution parameters too, since the latter are directly dependent on the ex ante parameters. Note also that, in the case of identical elasticities of substitution ex ante and ex post, the ex post distribution parameters are identical to their ex ante counterparts.

From the first order conditions for a costminimum (c.f. (5)) it follows that all marginal labour productivities should be the same for existing machinery and equipment and for new machinery. Using (9) we therefore have :

$$A_t^a \cdot \{\nu_{t,t}\}^{-1/\sigma_a} = A_i^p \cdot \{\nu_{i,t}\}^{-1/\sigma_p} \Rightarrow \tag{15}$$

$$\nu_{i,t} = \{\nu_{t,t}\}^{\frac{\sigma_p}{\sigma_a}} \cdot (A_t^a)^{-\sigma_p} \cdot (A_i^p)^{\sigma_p} = \psi_t \cdot (A_i^p)^{\sigma_p}$$

where ψ_t is implicitly defined by (15). (15) shows that the optimum value of the labour coefficient on an existing vintage consists of a vintage specific part and a general part. The corresponding value of the capital coefficient can be obtained from (2) and (6), since :

$$\{\kappa_{i,t}\}^{-\rho_p} = \frac{1}{B_i^p} - \frac{A_i^p \cdot \{\nu_{i,t}\}^{-\rho_p}}{B_i^p} \tag{16}$$

Using (15) and (16), we immediately obtain :

$$\zeta_{i,t} = \left\{ \frac{1}{\kappa_{i,t}} \right\}^{\rho_p} = \frac{1}{B_i^p} - (\psi_t)^{-\rho_p} \cdot \frac{(A_i^p)^{\sigma_p}}{B_i^p} \tag{17}$$

Equation (17) provides one of the central equations of the CRAPP model. Note that $\zeta_{i,t}$ is implicitly defined as the capital productivity of vintage i at time t, raised to the power of ρ_p. Let us now define :

$$\bar{\zeta}_t = \sum_{i=-\infty}^{t} \zeta_{i,t} \cdot \frac{I_{i,t}}{\sum_{j=-\infty}^{t} I_{j,t}} = \sum_{i=-\infty}^{t} \zeta_{i,t} \cdot \frac{I_{i,t}}{K_t} = \sum_{i=-\infty}^{t} \zeta_{i,t} \cdot S_{i,t} \tag{18}$$

We see that $\bar{\zeta}_t$ is a weighted average of all individual 'capital productivities' of the separate vintages with the investment shares in the total capital stock (K_t) as weights. Note that when ρ_p is equal to 1, i.e. the ex post elasticity of substitution is equal to 0.5, then (18) provides the 'exact' value of the aggregate capital productivity. When ρ_p is not equal to 1, we will assume that the average capital productivity ($\pi_{k,t}$) can be obtained as:

$$\pi_{k,t} = \{\bar{\zeta}_t\}^{1/\rho_p} \tag{19}$$

Note that (19) implies that for ρ_p not equal to 1 the aggregate productivity of capital is obtained as a 'CES average' of the individual capital productivities, since (18) and (19) imply :

$$\pi_{k,t} = \left\{ \sum_{i=-\infty}^{t} S_{i,t} \cdot \left(\frac{1}{\kappa_{i,t}} \right)^{\rho_p} \right\}^{1/\rho_p} \tag{20}$$

Equation (20) provides the 'tiny' approximation needed to define the CRAPP model. Assuming that (20) is indeed a good approximation of the arithmetical average of capital productivity [5] we can use (17) and (18) to obtain :

$$\bar{\zeta}_t = \sum_{i=-\infty}^{t} \frac{S_{i,t}}{B_i^p} - (\psi_t)^{-\rho_p} \cdot \sum_{i=-\infty}^{t} S_{i,t} \cdot \frac{(A_i^p)^{\sigma_p}}{B_i^p} \tag{21}$$

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5 We will come back to this issue later on in the form of some illustrative simulations.

Equation (21) can be redefined as :

$$\bar{\xi}_t = T_{1,t} - (\psi_t)^{-\rho_p} \cdot T_{2,t} \tag{22}$$

Note that since we have assumed that disembodied technical change is absent, the only way in which $T_{1,t}$ and $T_{2,t}$ depend on time is through $S_{i,t}$ and through the upperlimits of the respective summations. For $T_{1,t}$ we conclude therefore that its value must be equal to $T_{1,t-1}$ except for the fact that the overall weight of already existing vintages in the determination of $T_{1,t}$ must have decreased when gross investment is positive, while on the other hand the relative weights $S_{i,t}/S_{j,t}$ for $i,j<t$ are not changed at all (c.f. (18) and (4)). Thus the result of the transition from t-1 to t implies that the weight of existing machinery (i.e. the machinery installed upto and including time t-1) in the determination of the average value of capital productivity at time t has become $(1-\delta)\cdot K_{t-1}/K_t$, whereas the weight of the new vintages capital productivity in aggregate capital productivity is equal to $I_{t,t}/K_t$. A similar reasoning holds for the change in the value of $T_{2,t}$. We therefore have :

$$T_{1,t} = T_{1,t-1} \cdot \frac{(1-\delta)\cdot K_{t-1}}{K_t} + \left(\frac{1}{B_t^p}\right) \cdot \frac{I_{t,t}}{K_t} \tag{23}$$

$$T_{2,t} = T_{2,t-1} \cdot \frac{(1-\delta)\cdot K_{t-1}}{K_t} + \left(\frac{(A_t^p)^{\sigma_p}}{B_t^p}\right) \cdot \frac{I_{t,t}}{K_t}$$

$$K_t = (1-\delta)\cdot K_{t-1} + I_{t,t}$$

$$\pi_{k,t} = \left\{T_{1,t} - \psi_t^{-\rho_p} \cdot T_{2,t}\right\}^{1/\rho_p}$$

and equation (23) shows that the capital productivity 'book-keeping' of an infinitely large family of vintages can be reduced to a fairly small set of equations. [6] Moreover, equation (23) shows that the value of aggregate capital productivity can

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6 Note that a related approach is described in Eigenraam (1987), although the link between productivity aggregates and the development of the capital stock is less direct there than in the CRAPP model.

be obtained by means of a (time-) recursive adjustment of its composing terms, rather than by explicitly obtaining it from the underlying individual vintages.

With regard to the determination of the aggregate labour/capital ratio, we can use a similar approach. Defining $\theta_{i,t} = \nu_{i,t}/\kappa_{i,t}$, the aggregate labour capital ratio ($\bar{\theta}_t$) can (implicitly) be written as :

$$\begin{aligned}
\bar{\theta}_t^{\rho_p} &= \sum_{i=-\infty}^{t} S_{i,t}\cdot(\theta_{i,t})^{\rho_p} \\
&= \sum_{i=-\infty}^{t} S_{i,t}\cdot(\nu_{i,t})^{\rho_p}\cdot\zeta_{i,t} \\
&= \sum_{i=-\infty}^{t} \frac{S_{i,t}\cdot(\nu_{i,t})^{\rho_p}}{B_i^p} - \sum_{i=-\infty}^{t} \frac{A_i^p}{B_i^p}\cdot S_{i,t} \\
&= \psi_t^{\rho_p}\cdot\sum_{i=-\infty}^{t} S_{i,t}\cdot(A_i^p)^{\frac{\rho_p}{1+\rho_p}} - \sum_{i=-\infty}^{t} \frac{A_i^p}{B_i^p}\cdot S_{i,t} \\
&= \psi_t^{\rho_p}\cdot T_{3,t} - T_{4,t}
\end{aligned} \tag{24}$$

where we have used equation (17). Again, the terms $T_{3,t}$ and $T_{4,t}$ can be obtained by means of continuous recursive adjustment :

$$\begin{aligned}
T_{3,t} &= T_{3,t-1}\cdot\frac{(1-\delta)\cdot K_{t-1}}{K_t} + \frac{A_t^{p\,\frac{\rho_p}{1+\rho_p}}}{B_t^p}\cdot\frac{I_{t,t}}{K_t} \\
T_{4,t} &= T_{4,t-1}\cdot\frac{(1-\delta)\cdot K_{t-1}}{K_t} + \frac{A_t^p}{B_t^p}\cdot\frac{I_{t,t}}{K_t}
\end{aligned} \tag{25}$$

Equations (24) and (25) can be used to obtain total capacity labour demand as :

$$N_t = \bar{\theta}_t\cdot K_t = \left\{\psi_t^{\rho_p}\cdot T_{3,t} - T_{4,t}\right\}^{1/\rho_p}\cdot K_t \tag{26}$$

Of course, the replacement of a full putty-putty vintage model by its CRAPP representation has its price. First of all, for ex post elasticities of substitution not equal to 0.5, the CRAPP model is only an approximation of the full vintage model (although a good one), while secondly the terms $T_{1,t}$ through $T_{4,t}$ are recursively defined, and hence need to be initialised. However, in a growing economy it follows that the term $(1-\delta)\cdot K_{t-1}/K_t=(1-\delta)/(1+g)$ (where g is the rate of growth of the capital stock) is smaller than one, and it is easily seen that the influence of any initial value of the individual terms $T_{1,t}$ through $T_{4,t}$ tends to diminish over time. Moreover, this happens more rapidly when the rate of technical decay is high or when the rate of growth of the economy is high. Nonetheless, for short sample periods, initial values $T_{1,0}..T_{4,0}$ will have to be 'estimated' next to the other parameters of the production structure.

The logic of the model is now as follows. First, the technological characteristics of the $F^a()$ function together with factor prices determine the value of λ_t as well as h_t. This determines the value of the marginal labour productivity which is required to minimise production costs on the newest vintage. This in turn determines the reference value for marginal labour productivity to be used for the allocation of labour to existing equipment. Thus we obtain the level of output on existing machinery for a given value of the stock of existing capital, as well as the associated amount of labour. Then we obtain the amount of output to be produced on the newest equipment as the difference between the total amount of output required, and the amount of output to be produced on the existing equipment. After that, the required amount of investment as well as the associated amount of labour on new equipment can be obtained from the optimum values of the factor productivities on new equipment.

In the following section, we will provide some simulation results using this particular model, without going into the problem of the econometric estimation of the CRAPP model yet. As a reference model, we will also use the full putty-putty vintage model which was put forward in this section, with the proviso that we only do the 'book-keeping' for the last hundred vintages installed. [7] The vintage reference model will further be referred to as 'VRM'.

3 The CRAPP Model : Some Simulations

In this section we present the outcomes of a number of experiments we have conducted with the CRAPP model. First, however, we present the values of the parameters and the initial values of the model variables we have used in order to obtain a base-run. The values are listed in table 3.1 below. In this table G(Q) stands for the proportional rate of growth of Q : $G(Q) = Q_t/Q_{t-1} - 1$. w_0 is the initial value of the wage rate, and the average rate of growth of nominal wages is assumed to be 2.5 percent, while the rate of growth of output is assumed to be 3 percent. X_0 is the initial value of output.

Table 3.1 Parameter Values and Initial Values

Parameter/ Variable	Value	Parameter/ Variable	Value
σ_a	0.667	G(w)	0.025
σ_p	0.25	w_0	1
A_0	0.5	u	1
B0	0.5	X_0	500
μ	0.04	G(X)	0.03
δ	0.1		

Using these parameter values we ran 4 different experiments, the outcomes of which are labelled X0-X3, respectively. The first experiment is called the base-run, and it will be used as a frame of reference some of the other experiments. The second experiment illustrates the exact representation of the VRM by means of the CRAPP model when the elasticity of substitution ex post is equal

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7 Note that the assumption of a 10 percent technical decay per annum implies that we can safely ignore the vintages installed before and upto time t-100.

to 0.5. The third experiment shows the performance of the CRAPP model in response to short term random fluctuations in wage growth. The fourth experiment shows the reaction of the CRAPP model towards a shock in the rate of embodied labour augmenting technological change.

Apart from the series generated during the different experiments, we generally distinguish between two types of series : those generated by the CRAPP model (labelled with post-fixes '_Y' or '_BAR') and those generated by the full putty-putty vintage reference model (VRM) based on the same parameter values and data as the CRAPP model. The latter series are labelled with the post-fixes '_X' or '_AVERAGE'. The series associated with the newest vintage have post-fix '_0'.

In figure 2 below, we show how the volume share of investment in the newest vintage falls from an initial value of 1 (i.e. we start off with a capital stock consisting of one vintage only) to about 12.5 percent of the total capital stock in the long run. We start the simulation at period 100, and the capital stock becomes more diverse when more and more different vintages are installed. At period 145 a slight disturbance can be noticed. This disturbance is caused by the initialisation of the VRM, which lends its investment series to the CRAPP model. Note that, since the investment shares are identical, the calculation of the vintage capital stock over the last 100 vintages generates the same results as the perpetual inventory stock method which is essentially used in the CRAPP model. With a relatively high value of δ of about 10 percent, this is not too surprising.

In figure 3 we have depicted average capital productivity as generated by the VRM model and the CRAPP model. The series labelled '_AVERAGE' is generated by the VRM, and the one labelled '_BAR' is generated by the CRAPP model. One sees that they coincide almost perfectly.

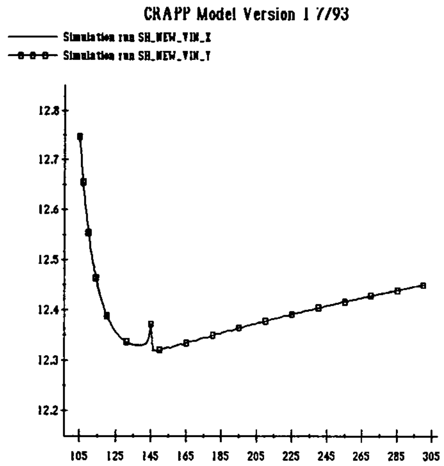

Fig 2. Share Newest Vintage

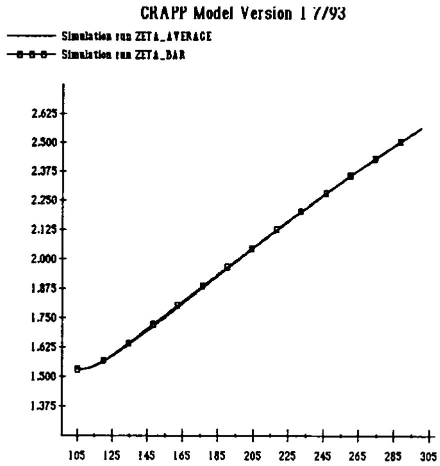

Fig 3. Average capital productivity

This also goes for the series presented in figure 4. Both labour/capital ratios are virtually the same. Note that due to the occurrence of embodied technical change, capital productivity shows a tendency to rise (c.f. figure 5), whereas labour productivity rises even faster (also due to the growth in relative wages of 2.5 percent per annum), since the labour capital ratio falls. With regard to the growth of capital productivity, we notice that it has a tendency to decelerate, where capital productivity growth on the new vintage shows a slight tendency to fall. Average capital productivity growth rises fast at first, but slows down after the installation of about 50 vintages or so, and then starts to decelerate. Note that the 'spike' in aggregate capital productivity growth is more or less missing in the CRAPP case. This does not mean that the CRAPP model is unable to react to 'sudden changes' in the economic environment, as we will see later.

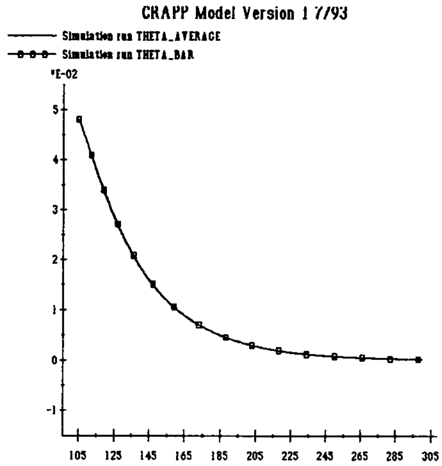

Fig 4. Capital/Labour Ratios

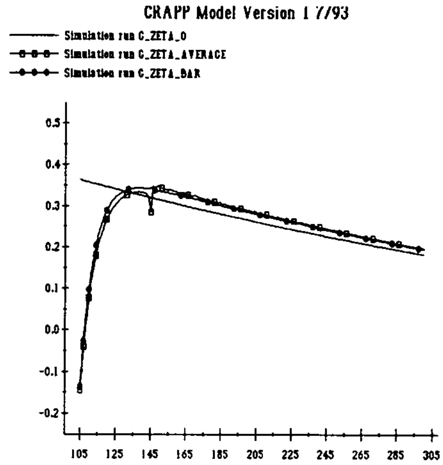

Fig 5. % Growth Capital Productivity

In figure 6, we have depicted the rate of growth of the aggregate labour/ capital ratio. We see <u>first</u> that in the long run the aggregate rate of growth tends to the rate of growth of the labour/capital ratio on the newest vintage, while <u>secondly</u> the VRM and the CRAPP model generate nearly identical results.

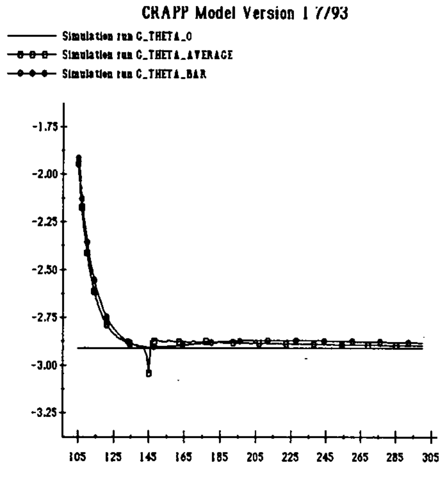

Fig 6. % Growth Labour/Capital Ratio

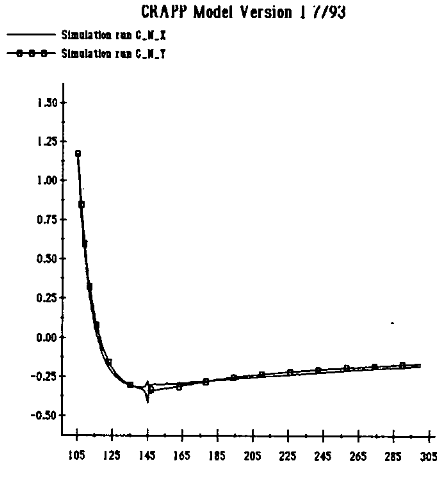

Fig 7. % Growth Capacity Labour Demand

In figures 8-12 we present similar results for the case where, except for a very slight numerical anomaly, the CRAPP model generates exact results. We do notice that for a value of ρ_p equal to 1, the initialisation 'spike' has disappeared in the investment share. Moreover, in figure 9 we have presented the percentage deviation between the CRAPP results with respect to aggregate capital productivity and the aggregate labour/capital ratio on the one hand, and corresponding the VRM results on the other. Noting that the scale is measured in 1/1000 percentage points, we conclude that there is still some sort of a spike in 1965 this time, but a very tiny one indeed, which shows moreover a tendency to disappear.

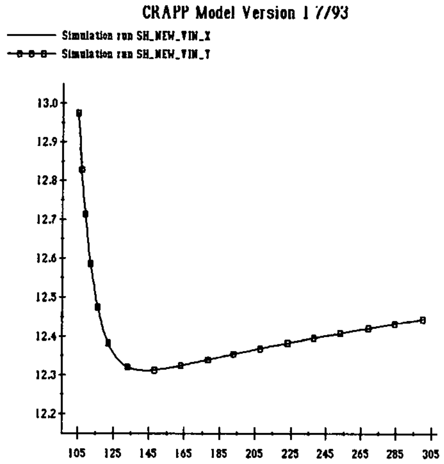

Fig 8. Share Newest Vintage

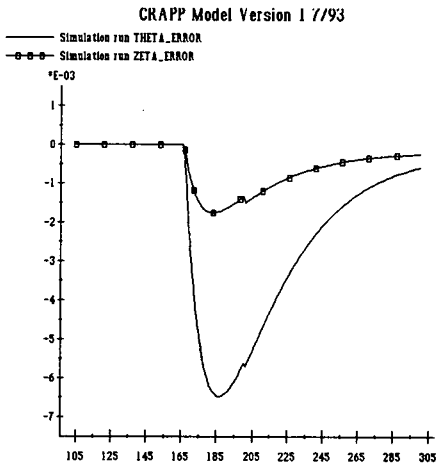

Fig 9. % Deviations Between CRAPP and the Vintage Reference Model

In figure 10 we show the development over time of the labour/capital ratio again, and a comparison between figures 6 and 10 shows that the labour/capital ratios on the newest vintage are identical (as they should be because only the elasticity of substitution ex post has changed), while differences between both models are mainly to be found in the speed of adjustment of the aggregate capital/labour ratio to its asymptotic value as provided by the newest vintage. Note that in figure 11 the latter values coincide completely, while the growth rate of the labour/capital ratio falls much faster to its asymptotic value than is the case with the base-run (labelled by means of the prefix 'X0_'). This is only natural, since the elasticity of substitution ex post has risen from a value of 0.25 to a value of 0.5, which is quite close to the ex ante elasticity of substitution of 0.667. We conclude therefore first that again CRAPP and the VRM generate identical results in this particular case, while secondly asymptotic behaviour is influenced by changes in the elasticity of substitution ex post only to the extent that a higher elasticity of substitution ex post implies a faster approach towards the asymptote.

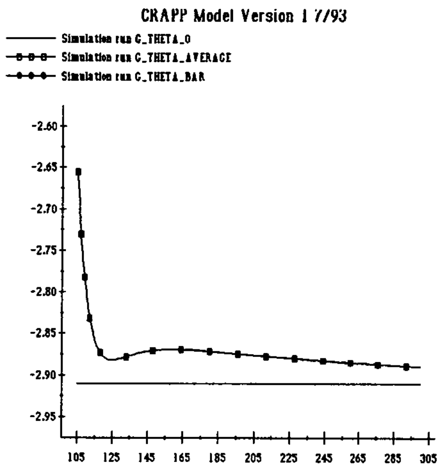

Fig 10. % Growth Labour/Capital Ratio

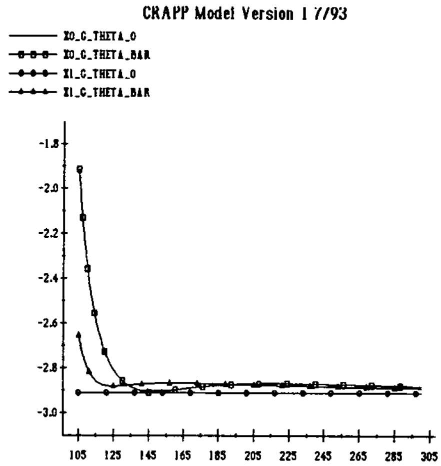

Fig 11. θ X0_ and X1_ Compared

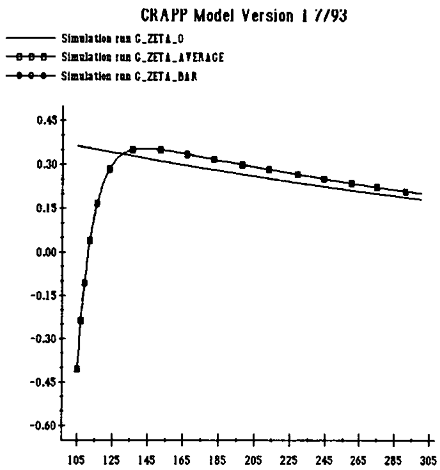

Fig 12. Capital Productivity

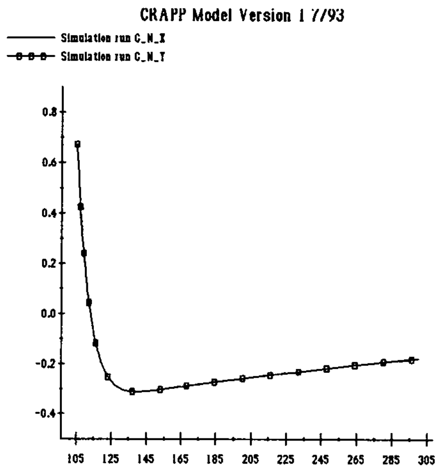

Fig 13. Capacity Labour Demand Growth

With respect to capital productivity (c.f. figure 12) we see that CRAPP and the VRM generate identical results, and the growth in capacity labour demand is the same as well (see figure 13).

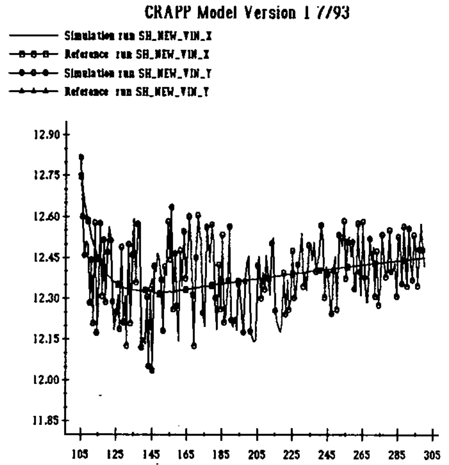

Fig 14. Investment Share Newest Vintage Experiment X2

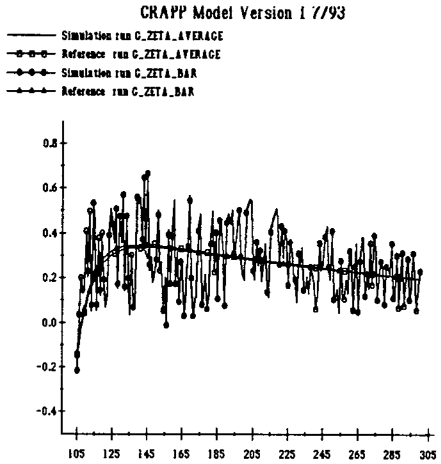

Fig 15. % Growth Capital Productivity

In figures 14-19 we present the results of an experiment in which we generated uniformly distributed random wage growth in between 0% and 5% per annum. Average wage growth is therefore the same as before, and this also goes for the average development over time of the CRAPP model and the corresponding VRM outcomes. [8] In figure 14 we notice that the investment shares coincide as they should, while the base-run development of the investment share of the newest vintage in the total capital stock seems to follow quite closely the average development of the random wage growth experiment. This goes for figures 15-17 too.

8 The reference run mentioned in figures 14-17 is identical to the base-run mentioned earlier.

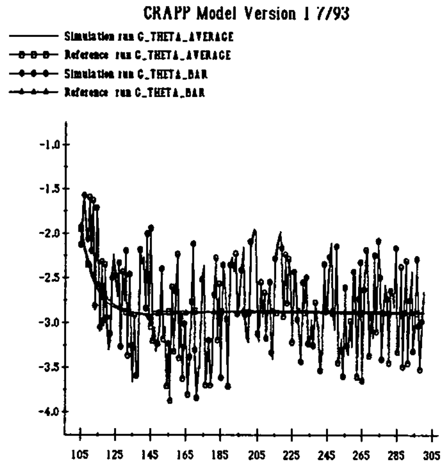

Fig 16. % Growth Labour/Capital Ratio

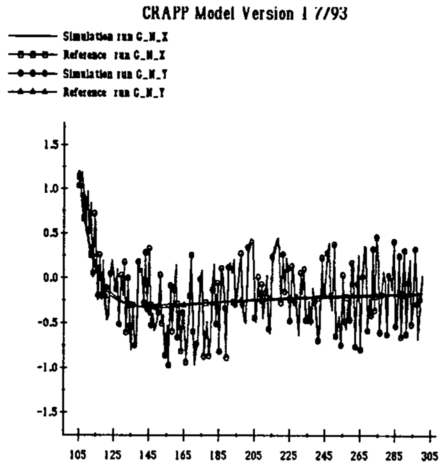

Fig 17. % Growth Capacity Labour Demand

In figure 18, we have plotted the growth of capital productivity on the newest vintage against aggregate capital productivity as generated by the CRAPP model.

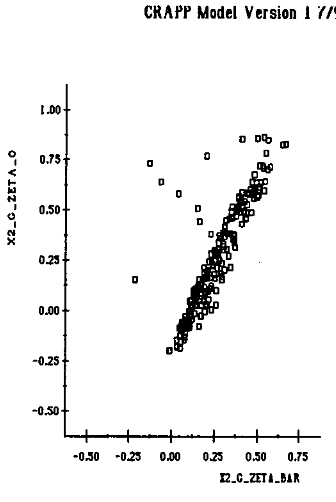

Fig 18. Capital Productivity Growth

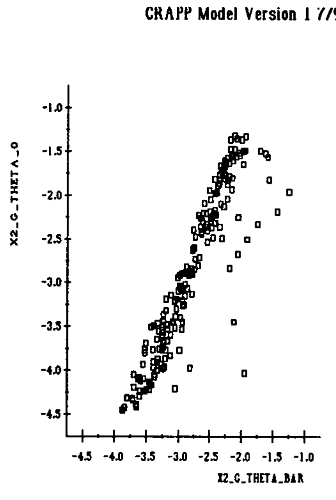

Fig 19. Growth Labour Capital Ratio

From figures 18 and 19 it is clear that the growth rates of the aggregate capital productivity growth and the growth of capital productivity on the newest vintage are positively correlated. Moreover, we see that the angle of the slope is larger than 45 degrees, which means that capital productivity growth and the labour/capital ratio on the newest vintage react faster to changes in wage growth than their aggregate counterparts. Of course, this is a consequence of the existence of limited substitution possibilities ex post.

In the final experiment to be presented, we have halved the rate of embodied technical change in between the periods 200 and 250 and raised it again to its initial value from period 251. We are interested in seeing how the CRAPP model responds to changes in technological circumstances rather than to changes in substitution characteristics, or changes in relative prices. Note that we have assumed non random wage growth at a rate of 2.5 percent a year.

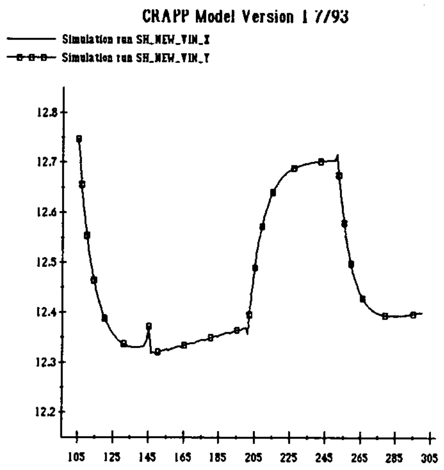

Fig 20. Investment Share Newest Vintage

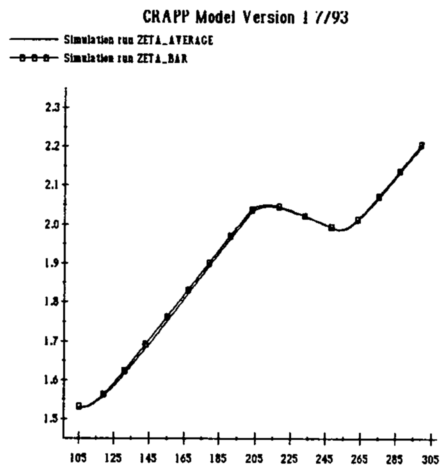

Fig 21. Capital Productivity Growth

Figure 20 shows much the same pattern as in figure 2 except for the experimental period from 200-250. In this year, the investment share suddenly rises in response to a fall in the rate of embodied Harrod neutral technical change. The reason is simply that, relative to the base run, labour has become effectively more expensive in the experimental period, and consequently the capital intensity of production on the newest vintage is increased. The latter increases the amount of investment for a given size of the newest vintage (note that due to the baseness of disembodied technical change, the cost of labour on the older vintages remains the same). The counterpart of figure 20 can be observed in figure

21. Here capital productivity falls during the experimental period, while it takes up its old growth path when the rate of embodied technical change is reset to its original value from period 251.

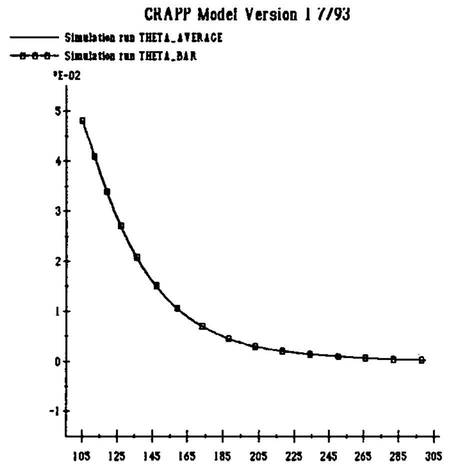

Fig 22. Labour/Capital Ratio

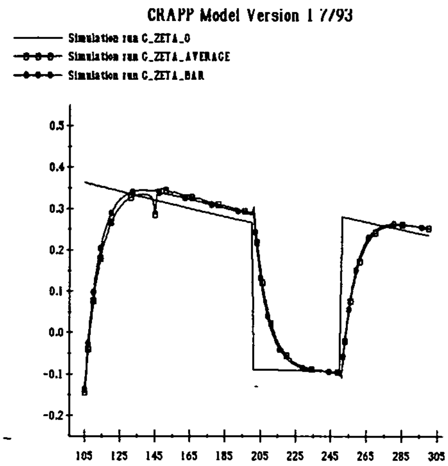

Fig 23. % Growth Capital Productivity

In figure 22 the impact of a change in embodied technical change on the labour/capital ratio is depicted. And although both the CRAPP series and the VRM series are virtually the same, no shock as in figures 20 and 21 can be observed. However, magnification of the shock by reverting to a rate of growth specification shows a different picture.

In figure 23 we see that a fall in the rate of labour augmenting technological change leads to a fall in the value of aggregate capital productivity which asymptotically reaches the fall in capital productivity growth on the newest vintage. Again the CRAPP model and the VRM generate virtually identical results. Note that with regard to the labour/capital ratio, similar things are happening, although the labour/capital ratio rises rather than falls. Moreover, the rise in the (negative) growth rate is about 0.5 percentage points, and given the fact that the experimental period falls well within the flat region of the labour/capital ratio as depicted in figure 22, it is not too surprising that we the impact of the embodied technical change experiment remained hidden at first. Note that due to the fall in marginal labour productivity consequent on the drop in the rate of embodied labour augmenting technical change, marginal productivities on older vintages may fall too, and therefore the aggregate labour/capital falls more slowly than before.

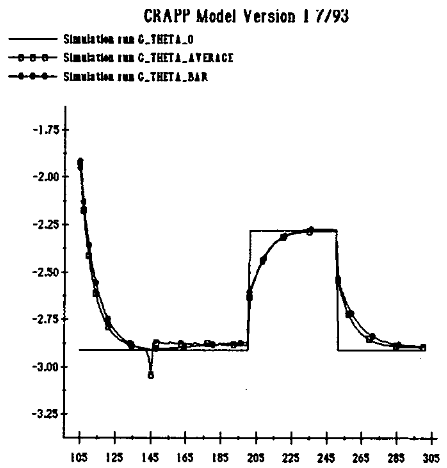

Figure 24. % Growth Labour/Output Ratio

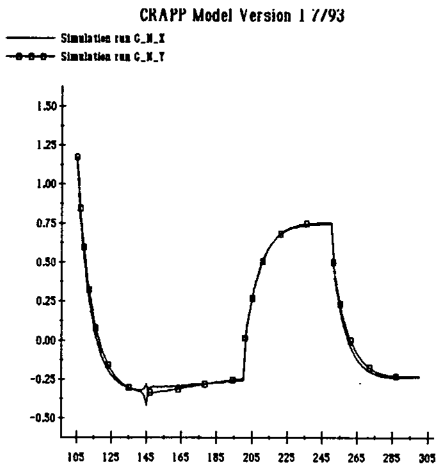

Fig 25. % Growth Capacity Labour Demand

In figure 25 we have depicted the reaction of aggregate capacity labour demand towards a fall in the rate of labour augmenting technological change. We see that the effect is positive, ceteris paribus. Note however, that the level of output is assumed given, and therefore, the cost increasing effect of a drop in the rate of embodied technical change does not feed back onto demand. Note again that the CRAPP model and the VRM model generate virtually identical results.

4 Summary and Conclusion

In this paper we have set out a model which is half way between a full vintage model of production and an aggregate production function framework. We have assumed that producers face limited substitution possibilities between the physical factors of production (i.e. labour and capital) after they have installed the machinery in question, while substitution possibilities are more abundant before the moment of installation. In order to fully specify the model we used a linear homogeneous CES function both in the ex ante case and in the ex post case. We showed that producers minimise their overall production cost by allocating labour to new equipment and to old equipment in such a way that the respective marginal labour productivities in both cases are equal. Given a fixed amount of capital (at least in the very short run) ex post, a required rise in the ex post labour productivity must be matched by a fall in employment per unit of capital and therefore also to a fall of output on old equipment, since both labour and capital have strictly positive marginal productivities, both ex ante and ex post.

We also showed that it is not strictly necessary to engage in extensive vintage 'book-keeping' exercises. Rather a continuous recursive adjustment of the aggregate capital productivity and the aggregate labour/capital ratio in function of the characteristics of the new vintage to be installed, is sufficient to reproduce all the relevant information generated by a full putty-putty vintage model.

We illustrated the working of the CRAPP model in a number of experiments. The general conclusion one can draw from the experiments is that, apart from disturbances caused by initialisation errors in the VRM model, the CRAPP model and the VRM model are indeed almost perfect 'look-alikes' with respect to aggregate behaviour.[9] The experiments have shown that not only asymptotic behaviour of the CRAPP model and the VRM are very much the same , but also short term fluctuations in the VRM environment are captured almost perfectly by the CRAPP model. This goes for fluctations in the growth of relative prices, but also for fluctuations due to technology shocks.

In conclusion we may state then that the CRAPP model is half way between a full vintage model and the aggregate production function approach. It has the flavour of a full putty-putty vintage model, and in practice it works as one. At the same time it consists of a very limited set of equations which use the idea of a time-recursive update of aggregate capital productivity and the aggregate capital/labour ratio in function of the characteristics of new investment. The CRAPP model behaves as if it is a full vintage model, while at the same time it avoids tracing individual

---

9 Note that with respect to marginal behaviour, the equations describing the behaviour regarding the installation of new capital goods are completely identical.

vintages during their (infinite) lifetime. Nonetheless the CRAPP model is able to generate exact results for an elasticity of substitution ex post of 0.5, and nearly exact results for a value of the elasticity of substitution ex post of 25 percent.

Considering its performance during a number of different experiments, we may conclude that the CRAPP model can serve as a comprehensible and manageable alternative to the large computational and 'book-keeping' burden which a standard vintage model approach usually entails.

Appendix A : The CRAPP Model Listing

```
{ July 1993/ MESS Model Source CRAPP Model
  MESS (C) Menhir Software Group
           Ringweg 46
           6271 AK Gulpen
           the Netherlands
}
{
  CRAPP model stands for Continuous Recursive Adjustment Putty
Putty Model.
 QPP model with variable distribution coefficients in CES function
  and recursive updating of average capital productivity old
equipment
  and average labour intensity of old equipment
}

{
  only labour augmenting embodied technical change
}
{
  rrandom is random number generator (uniformly distributed).
 maximum value is equal to 1. Before first draw initialise random
  generator uin such a way that for all simulations the same
sequence
 is generated => define function rrandom in separate Turbo-Pascal
Unit
}

{wagerate = wagerate(-1)*(1+rrandom*gw),}

wagerate = wagerate(-1)*(1+gw),

a_exante = a0*p_(1+mu,-rho_exante*time),

b_exante = b0,

{
  optimum (ex ante) labour capital ratio, depends on user cost
  of capital (ucc) and the wagerate (wagerate))
}
```

```
h = p_((wagerate/ucc)* (b_exante/a_exante), -1/(1+rho_exante)),

{ 0 is youngest vintage }

kappa0 = p_(a_exante * p_(h,-rho_exante) + b_exante, 1/rho_ex-
ante),

nu0 = h * kappa0,

a_expost = a_exante * p_(a_exante + b_exante * p_(h,rho_exante),
                         (rho_expost/rho_exante-1)),

b_expost = a_expost * (b_exante/a_exante) * p_(h,rho_exan-
te-rho_expost),

psi = p_(nu0,(1+rho_exante)/(1+rho_expost))*
         p_(a_exante,-1/(1+rho_expost)),

{
 nux is labour coefficient at t of vintage installed at time t
- x, i.e.
 a vintage of x years old. Because no dismebodied tc => a_expost(i)
has
 remained the same as at time of installation. Note only dis-
tribution
 parameters are vintage specific, NOT SUBSTITUTION parameter
}

dot_(dt=1 to 99 :
 nu. = psi * p_( a_expost(dt), 1/(1+rho_expost))
),

{kappax see nux above}

dot_(dt=1 to 99 :
  kappa. = max(0,p_((1 - a_expost(dt) * p_(nu.,- rho_ex-
post))/b_expost(dt),
                  -1/rho_expost))
),

{xz production capacity vintage z at time t}
```

```
dot_(dt=1 to 99 :
 x. = (p_(1 - delta,abs(dt)) * inv(dt))/kappa.
),

xold = sum_(i=-99 to -1 : ref_(x,i)),

{nz capacity labour demand vintage z at time t}

dot_(dt=1 to 99 :
 n. = nu. * x.
),

nold = sum_(i=-99 to -1 : ref_(n,i)),

y = x,

{determine size newest vintage}

x0 = max(0 , y - xold),

{determine required amount of capital}

inv = x0 * kappa0,

n0 = nu0 * x0,

n_x = nold + n0,

{ vintage capital stock}

k_x = sum_(i=-99 to 0 : P_(1-delta,abs(i))*inv(i)),

inv_x = inv,

{now updating of average capital productivity and average labour
intensity}

inv_y = inv,

k_y = (1 - delta)* k_y(-1) + inv_y,
```

```
t1 = t1(-1)*(1-delta)*k_y(-1)/k_y + (inv_y/k_y)/b_expost,

t2 = t2(-1)*(1-delta)*k_y(-1)/k_y +
      (inv_y/k_y)*p_(a_expost,1/(1+rho_expost))/b_expost,

{ now calculate average value of p_(phi,rho), and calculate
  phi as p_(p_(phi,rho),1/rho) }

zeta_bar = p_(t1 - p_(psi, -rho_expost)* t2, 1/rho_expost),

t3 = t3(-1)*(1-delta)*k_y(-1)/k_y +
      (inv_y/k_y)*p_(a_expost, rho_expost/(1+rho_expost))/b_ex-
post,

t4 = t4(-1)*(1-delta)*k_y(-1)/k_y + (inv_y/k_y)*a_expost/b_ex-
post,

theta_bar = p_(p_(psi, rho_expost) * t3 - t4, 1/rho_expost),

{calculate CRAPP aggregates + growthrates}

n_y = theta_bar * k_y,

x_y = zeta_bar * k_y,

zeta_average = y/k_x,

theta_average  = n_x/k_x,

zeta_error = 100*(zeta_bar/zeta_average-1),

theta_error = 100*(theta_bar/theta_average-1),

g_zeta_bar = 100*(zeta_bar/zeta_bar(-1)-1),

g_zeta_average = 100*(zeta_average/zeta_average(-1)-1),

g_theta_bar = 100*(theta_bar/theta_bar(-1)-1),

g_theta_average = 100*(theta_average/theta_average(-1)-1),
```

```
{ calculate capital productivity growth newest vintage}

g_zeta_0 = (kappa0(-1)/kappa0-1)*100,

{calculate growth labour/capital ratio newest vintage}

g_theta_0 = ((nu0/kappa0)/(nu0(-1)/kappa0(-1))-1)*100,

{calculate share of newest vintage in total capital stock}

sh_new_vin_x = (inv_x/k_x)*100,

sh_new_vin_y = (inv_y/k_y)*100,
```

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