On the exploitation of patent protection

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March 1993

MERIT Research Memorandum 93-006

Second draft version

ABSTRACT

Address models from the product differentiation literature are used to describe the competition between a firm which holds a patent and a competitor who invents around that patent. Two dimensions of patent protection are distinguished: patent breadth and height. Breadth gives the extent of protection against imitations, height against improvements. Several models of horizontal, vertical and combined differentiation are explored, and the major conclusion is that a competitor has various opportunities to invent around a patent profitably, although he might be restricted in his choice of imitation and improvement levels. Optimal inventing-around strategies are determined.

* This research was sponsored by the Foundation for the Promotion of Research in Economic Sciences, which is part of the Netherlands Organization for Scientific Research (NWO). I thank Patrick Van Cayseele for valuable comments.

1. Introduction

The central question in this paper is: How can a patentholder exploit his patent if the protection is imperfect and competitors can invent around the patent? In most economic literature related to patents, it is assumed that protection is perfect and inventions are completely appropriated. In the literature on patent races (Reinganum (1989)), for example, the usual assumption is that the winner of the race gets a patent and takes all the benefits from the innovation. Another example is the seminal work of Nordhaus (1969) on patent lifetime, where perfect exclusivity is assumed during the life of the patent. Empirical research (Mansfield et al. (1981), Pakes (1986), Levin et al. (1987), Griliches (1990)), however, has shown that patent protection is not perfect and does not provide pure monopoly power. Patents merely enlarge imitation costs, or alternatively stated, restrict the possibilities for competitors to invent around. Jurists are far more aware of the fact: "To the extent that intellectual property is capable of generating market power, it offers its owner (and his associates) the opportunity to reduce output and raise prices. What it does not bring about is the condition in which the monopolist behaves as though he were the only competitor on the market. Yet the more naive arguments in favour of one or other exclusive right often imply that this alone will be the effect of according the right sought." (Cornish (1989), p.18). If one takes a closer look at a typical patent procedure, the imperfectness of patent protection and the various opportunities for competitors to invent around become clear.

A typical patent granting procedure[1] starts with an application which must contain a specification of the invention. This specification is made up of two parts: a description of the invention, possibly accompanied by drawings, and the claims which indicate where protection is looked for. If an application succesfully passes through the fases of examination and opposition, a patent is granted, possibly after respecification. The protection which the patent provides is partly determined by the specification and partly by the patent office and the court. The exact protection, which was asked for, is written down in the claims. But the court does not have to take these claims literally (in a "fencepost" system it does); it may interpret the claims in a broader and wider sense (a "signpost" system). A similar invention that is slightly different from what is written down in the patent specification can also be judged to fall within the protection. Besides the formulation and interpretation of the claims, the novelty requirements which are used by the patent office in the examination fase, also define the extent of protection of a granted patent. The protection is weak if a current patent can easily be overcome by a small improvement. If the novelty requirements are stronger, patents provide more protection.

[1] I loosely follow the patent granting procedure of the European Patent Office, as described in its publication "How to get a European patent", October 1990. Most patent offices use similar procedures.

From this short description of the patent procedure (see Cornish (1989) for a more extensive overview), it becomes clear that the protection which a patent provides is not perfect. There are several opportunities for competitors to circumvent the patent. They can for example produce an imitation which does not fall within the interpreted claims, or generate an improvement which fulfills the novelty requirements. Given these opportunities, what is the value of the patent for the patentholder? How much profits can he extract from it? And his competitors?

The literature on product differentiation, and more precisely the address branch in it, provides useful tools to examine scenarios with imperfect patent protection[2]. Incremental innovations, in the form of imitations or improvements, can be thought of as being differentiations of a certain basic innovation. The competition which occurs between the basic innovator and incremental innovators, or between incremental innovators themselves, can be described with the use of address models from the product differentiation literature. In this paper, I explore the possibilities for the use of these differentiation models in the context of imperfect patent protection. The analysis is naturally focused on product innovations.

Very briefly, some conclusions are the following. A pure height model shows that weak novelty requirements do not affect the natural equilibrium in product improvements, intermediate requirements make the profits of the patentholder increase and those of the improver decrease and strong novelty requirements provide the patentholder pure monopoly power. In the pure breadth model used, the profits of the patentholder increase in broader protection but stay always smaller than those of the imitator. Combined models of breadth and height finally indicate when a competitor who wants to invent around a patent can best choose a pure improvement strategy or a strategy of imitation with some improvement.

The most important dimensions of patent protection are described in section 2. In section 3, I indicate how these dimensions can be translated into formal language. The shape of the patent protection is described there. The next question is how the patentholder can exploit this protection. The profit opportunities for a patentholder are first studied without competition, in section 4. Later on, in section 5, the patentholder faces competition from firms which invent around the patented product. I conclude in section 6.

[2] See Eaton and Lipsey (1989) for a survey of the product differentiation literature in general. For the merits of using address models of product differentiation, see Archibald, Eaton and Lipsey (1986).

2. Definitions

Patent protection can be described in terms of various dimensions. The *length* of a patent is the best known dimension (Nordhaus (1969)). Patents are temporary by law and thus only provide protection for a limited time, in general 20 years. Another difference of patents compared to other property rights is the strong exclusivity, which not only means that others have no right to copy the patented innovation exactly but are also not allowed to use independently generated similar inventions[3]. Although stronger than usual, the exclusivity is not perfect. Most patent laws and procedures leave open opportunities for others to invent around, as was pointed out in the Introduction. If one accepts the notion that innovations are not perfectly appropriable through patents, three dimensions of protection can be distinguished: breadth, height and width[4]. These dimensions describe the degree of exclusivity of a patent.

The *breadth* of a patent defines how similar *imitations* of a patented invention are allowed to be. Imitations can be seen as varieties which generate the same gross surplus as the imitated product. A way to look at patent breadth is as if it defines how much varieties of a patented product are protected. Take the example a new tennis racket (see Klemperer (1990), p.115). Suppose that a new fibre makes it possible to design an oversized tennis racket of, say, 105 square-inch. The patent breadth protection on the racket may then run from 80 to 130 square-inch. Since not all consumers may prefer the same variety, it can be said that patent breadth *defines a protected region on the horizontal product spectrum.* Gilbert and Shapiro (1990) define patent breadth less explicitly as the ability of the patent owner to raise the price of his product. Klemperer (1990) shows that the conclusions are richer if an explicit model of horizontal differentiation is used.

Closely related to breadth is the dimension of *height* (Scotchmer and Green (1990), Van Dijk (1992)). Patent height indicates how new or how much improved a product must be in order not to infringe a current patent. The stringency of the novelty requirements used by patent examiners mainly determines the height of protection. The dimension of height shows up most clearly if inventions are related. In this paper I will therefore focus on inventions which take the form of improvements of existing products. Height defines the protection that a patent provides against *improvements*. Take again the example of a new tennis racket. Patent height indicates how much improved a 105 square-inch racket must be (for example by the

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[3] Copyright, on the contrary, only protects against exact copying by others.

[4] Patent "scope" is also used by several authors (for example Klemperer (1990) and Merges and Nelson (1992). I interpret patent scope as a *general* indication for the extent of patent protection. A more precise indication distinguishes the dimensions breadth, height and width.

use of a new, stiffer fibre) in order not to infringe the current racket patent. Since it is reasonable to expect that all consumers prefer an improvement to a product that was improved, patent height can be thought of as *defining a protected region on the vertical product spectrum.*

Finally, there is the dimension of patent *width* (Matutes, Regibeau and Rockett (1991)). This dimension is not included in the following analysis, but I will give a definition anyway. An invention may contain an idea which can be applied in various products. Sticking to the tennis racket example, the new material which the original racket or the improvement was made of, could also be used in for example fishing rods or squash rackets. The (limited) number of applications which is reserved for the patentholder is determined by the patent width. If a patent would protect all applications of an invention, it would come close to protecting the idea embodied in the invention. Protecting ideas is generally considered as being too strong. Of course, the total number of possible applications varies among inventions. One would expect that the more basic an invention is, the more applications are possible. It seems therefore appropriate to define patent width as a relative dimension, a proportion a the the total applications. Notice the difference between breadth and width: patent breadth is concerned with the protection on one product spectrum, whereas patent width defines the number of protected product spectra.

3. Cuboid protection

The patent dimensions breadth and height can be defined in a more formal way with the use of simple models of product differentiation. Suppose, all varieties of a patented product can be represented by an address l on the horizontal product spectrum, which extends from 0 to 1. By definition the address of the patentholder is at 0. The ordening in the interval [0, 1] is such that varieties which are located farther away from 0 are less similar imitations[5]. The patent breadth b (≥ 0) protects the range [0, b), where no competitors are allowed. I assume that the border of what is judged to be infringement is precise and known by the patentholder and competitors[6]. The claims and description in the patent file and their interpretation by the patent office and the courts define this protected region.

Improvements of a basic invention can be represented by an address v on the vertical product spectrum, which extends from 0 to V. The address 0 represents the basic invention and V the final improvement possible. Let v be the address, or the improvement level, of the patentholder, which might be the basic invention, $v = 0$, or any improvement $0 < v \leq V$. The height h provides protection in the interval of improvements $[0, v + h]$. Competitors are not allowed here, except of course if they have a license permission. The lower bound of this protected interval is 0 because an improvement must always be larger. The patented improvement is namely the state of the art used by patent examiners as a standard measure. If the improvement v is patented, the next improvement must be larger than $v + h$, in other words it must fulfill the minimum novelty requirements. If not, the patent on innovation v is infringed.

The patent lifetime t is the number of periods during which the patent is valid. Because the focus in this paper is on profits and not on welfare in general, the length of patent protection is less interesting. Calculating the total patentholder's profit is simply a matter of discounting the instantaneous profit, which is determined by breadth and height, over the duration t of the patent. Under certain assumptions, the same holds for the width dimension. If the applications are on independent markets and the reserved applications are present at the start of the patent without extra cost, it is simply a matter of multiplying the profit per application

[5] The patented product can also be assumed to be at the middle of the linear spectrum, or at any point on a circular spectrum, with protection on the "left" and on the "right". Because of the symmetry at both sides, the analysis would not be very different from the one presented here.

[6] Waterson (1990) examines a patent system where it is not clear beforehand whether the patentholder or the possible infringer wins in court. This uncertainty may affect the patenting decision.

with the number of reserved applications[7]. The total patent protection per reserved application is a function of breadth b, height h and duration t. This protection is valid for a new or improved product with the address 0 on the horizontal product spectrum and the address v on the vertical product spectrum. The protection can be represented as a cuboid, like in figure 1, which holds for each reserved application.

[7] Relaxing these rather strong assumptions, the non-address branch in product differentiation (e.g. Dixit and Stiglitz (1977)) might be useful to examine patent width.

4. Patent exploitation by a pure monopolist

What are the profit opportunities for a patentholder who enjoys this cuboid protection? As mentioned above, the focus will be on the breadth and height dimensions and therefore the length of patent protection is assumed to be infinite, although this is a stronger assumption than strictly required. First, I will consider cases where the patentholder does not have to cope with competition, that is where the protection completely covers the interval [0, 1], for $b > 1$, and the height interval [0, V], for $h > V - v$. This will provide some insights in the profit opportunities of the patentholder under perfect protection, as is often assumed in models of patent races. I will examine the separate effects of patent height and breadth before the combined effect.

4.1 Perfect height protection

Since all consumers prefer a larger improvement to a smaller one, a model of vertical differentiation can be used to examine the height dimension. The innovation of the patentholder has an improvement level v, which gives an indication of the gross surplus provided by the innovation. Consumers are heterogeneous in the sense that they evaluate the decision buying v or not buying in different ways, for example because their incomes differ. An (indirect) utility function that catches the idea of vertical differentiation is (see for example Shaked and Sutton (1982) and Tirole (1989)):

$$U = \begin{cases} mv - p \\ 0 \end{cases} \qquad (1)$$

where m is the improvement preference intensity parameter of an individual consumer, which is uniformly distributed with density 1 on the interval [0, 1]. The price of v is p. A consumer buys only if his net utility is non-negative and buys then one unit. This will be the case in each of the models to follow. The demand function which corresponds with utility function (1) and perfect protection against competition through improvement ($h > V - v$) is:

$$x = 1 - p/v \qquad (2)$$

For convenience, I set, here and in the rest of the paper, the marginal production cost equal to 0. The profit function is $\pi = p(1 - p/v)$. The optimal price is $p^* = v/2$ yielding a profit of $\pi^* = v/4$. This is what can be gained with an improvement v, which is perfectly protected against further improvements.

4.2 Perfect breadth protection

The address *l* of the patented product is 0 on the horizontal product spectrum [0, 1]. The

ordening in this interval is such that towards 1, the addresses represent imitations which are less similar to the patented product at 0. At 0 an imitation is an exact duplication and at 1 there is an imitation which is vaguely similar. The gross surplus which these imitations provide, however, is identical to the original innovation. Horizontal differentiation is caught in the following (indirect) utility function:

$$U = \begin{cases} v - p - td \\ 0 \end{cases} \tag{3}$$

where d is the Euclidean distance from the consumer l^* to the patentholder at $l = 0$: $d = |l^* - l|$. Each consumer has a most preferred product variety, which is denoted by l^*. I take a continuum of consumers with a uniform distribution of l^* on [0, 1], with density 1. Since the patentholder is given by $l = 0$ here, the distance d is also uniformly distributed on [0, 1], with density 1. The parameter t is analogous to the transport cost in the Hotelling (1929) model; here it is a utility penalty associated with consuming a less preferred variety. The demand for the product of the patentholder at 0 under perfect protection ($b > 1$) is given by:

$$x = \begin{cases} 0 & \text{if } p \geq v \\ (v - p)/t & \text{if } v \geq p \geq v - t \\ 1 & \text{if } v - t \geq p \geq 0 \end{cases} \tag{4}$$

Based on the second part of the demand function, the profit function is $\pi = p(v - p)/t$, which is maximized for $p^* = v/2$, yielding optimal profit $\pi^* = v^2/4t$. The optimal price p^* must belong to the relevant price interval [v - t, v]. The upper limit, v, is always fulfilled. With respect to the lower limit, p^* is larger than $v - t$ for $v < 2t$. Demand is nonnegative for $v \geq t$. Based on the third part of the demand function, the profit function is $\pi = p$ which increases in p in the relevant price range [0, v - t]. The optimal price for this range is therefore the highest possible $p^* = v - t$, yielding an optimal profit of $\pi^* = v - t$. Compare the optimal profits in both price regimes. For $v < 2t$, the price strategy $p^* = v/2$ yields always larger profits in the second regime than the alternative price strategy $p = v - t$. In the third regime $p^* = v - t$ yields more than $p = v/2$.

Summarizing, the optimal price is $p^* = v/2$ for $t \leq v \leq 2t$, that is for inventions which are small relative to consumer unit travel cost t. Charging this optimal price for such an innovation yields optimal profit $\pi^* = v^2/4t$. For relatively large inventions ($v \geq 2t$), the optimal price is $p^* = v - t$, yielding an optimal profit of $\pi^* = v - t$. The total market [0, 1] is served then. Without any competitors in [0, 1], π^* is what can be gained with an invention at 0, which is perfectly protected against competition through imitation.

4.3. Combining perfect height and breadth protection

Since an innovation is characterized by both its degree of imitation and degree of improvement, I need a utility function which includes *both vertical and horizontal differentiation.* The following utility function is a combination of functions (1) and (3) (see Neven and Thisse (1989) for a similar utility function, and De Palma et al. (1985) and Economides (1986) for less similar functions with two distinguishing characteristics):

$$U = \begin{cases} mv - p - td \\ 0 \end{cases} \tag{5}$$

If all consumers would have an m equal to 1, (5) would be a pure horizontal differentiation model. And if t would be equal to 0 or if all consumers' addresses would be at the producer's, (5) would be a pure vertical differentiation model. Here, each individual consumer has two characteristics. First, as in (1), his improvement preference intensity parameter $m \in [0, 1]$ and second, as in (3), the distance between his most preferred variety and the one that is actually offered, $d \in [0, 1]$.

The derivation of the demand function associated with the patentholder's product at 0 on the horizontal product spectrum and at v on the vertical product spectrum is less simple now because there are two distributions involved. The distributions of m and d are assumed to be independent; the location of a consumer on [0, 1] does not say anything about the intensity m in which he values improvements. The patentholder does not know the combination of characteristics of each consumer. The demand for the patentholder is then given by the shaded area in figure 2[8] and consists of those consumers with characteristics m and d who have a non-negative net utility: $mv - td \geq p$.

In order to simplify notations further on, define $\mu \equiv mv$, so that μ is uniformly distributed on [0, v] with density $1/v$; and define $\delta \equiv td$, so that δ is uniformly distributed on [0, t] with density $1/t$. The joint density of μ and δ is $1/vt$. For $p \leq v \leq p + t$ (that is in the case that even consumers who appreciate improvement most do not buy if their travel costs td are high), the demand is given by:

$$\int_p^v \int_0^{\mu - p} 1/vt \, d\delta \, d\mu = (v - p)^2/2vt$$

For $v \geq p + t$, the demand is given by:

[8] See Papoulis (1984) chapter 6, for an extensive treatment of joint statistics and functions with two random variables.

$$\int_{p}^{p+t}\int_{0}^{\mu-p} 1/vt \, d\delta \, d\mu + t(v - p - t)/vt = 1 - p/v - t/2v$$

Resuming, the demand function based on the utility function which combines horizontal and vertical differentiation is:

$$x = \begin{cases} 0 & \text{if } p \geq v \\ (v - p)^2/2vt & \text{if } v \geq p \geq v - t \\ 1 - p/v - t/2v & \text{if } v - t \geq p \geq 0 \end{cases} \tag{6}$$

This demand function is continuous. The second part ($v \geq p \geq v - t$) is strictly convex, the third part is linear ($v - t \geq p \geq 0$) in p. Demand will never be 1 because price can not be negative. Those consumers who have a low improvement preference intensity parameter combined with large distance do not even buy at zero price. The profit function $\pi(p) = x(p)p$ is also continuous. The optimal price is $p^* = v/3$ for $t \leq v \leq 3t/2$, that is for inventions which have relatively small improvement levels. The optimal profit is $\pi^* = (2v^2)/(27t)$. For relatively large inventions ($v \geq 3t/2$), the optimal price is $p^* = (2v - t)/4$, yielding $\pi^* = (2v - t)^2/16v$. This can be checked in the following way. Based on the second part of the demand function, the optimal price (for $p \leq v$) is the one which maximizes $\pi = (p(v - p)^2)/2vt$, namely $p^* = v/3$. For $v < 3t/2$, p^* belongs to the relevant range $[v - t, v]$. Demand stays nonnegative here for $v \geq t$. The optimal profit with $p^* = v/3$ is $\pi^* = (2v^2)/(27t)$. Based on the third part, the profit function is $\pi = p(1 - p/v - t)/2v$. This profit function is maximized for $p^* = (2v - t)/4$ which yields $\pi^* = (2v - t)^2/16v$. Now compare both optimal profit levels, $\pi^* = (2v^2)/(27t)$ for the second part and $\pi^* = (2v - t)^2/16v$ for the third part. It turns out that for $v < 3t/2$, the optimal profit level for the second part $\pi^* = (2v^2)/(27t)$ is indeed always larger, and for $v \geq 3t/2$ the optimal profit $\pi^* = (2v - t)^2/16v$ is larger in the third part.

What can we learn from this combined model? A patentholder who enjoys perfect protection does not have to cope with competition, but his profit opportunities are restricted by the conditions on the market with respect to demand. If, as is assumed, the patentholder supplies only one product, that is one variety and one improvement level of the product, some consumers will not buy. The reason is that the lost utility associated with buying a less preferred variety is too high, so that there remains no positive net surplus, or that the level of improvement is not sufficiently appreciated, or a combination of both. This can be seen in both cases of small and large improvements, by looking at the expressions for the optimal profits. Both profits increase, be it with different speed, if the travel costs of consumers are lower. Abstracting from any development cost for improvements, the same holds for the improvement level v.

5. Inventing around by competitors

In the previous section, the profit opportunities for a patentholder with the innovation (0, v) were investigated under perfect, that is very broad ($b > 1$) and very high ($h > V - v$), patent protection. Scenarios where the patent does not cover the complete intervals are considered now. Competitors can only locate on the unprotected region of the horizontal spectrum [b, 1]. With respect to the improvement level, competitors are only allowed in the interval [v + h, V]. I will first examine the pure effects of height and breadth, and then the combined effects.

5.1. Patent height

Consider now a scenario where two firms are (potentially) at the market, namely the patentholder and one improver. The model used is a game where three stages of competition are included: the first stage where firms decide whether to enter the market or not, the second stage where, based on the decisions in the first stage, both firms choose their improvement level, and the final stage where price strategies are formulated, given the improvement choices of the previous stage. As usual, the order of the stages is determined by the decreasing degree of flexibility of the decisions. Only pure strategies will be considered. The solution concept is the subgame perfect equilibrium. The game will be solved by backwards induction.

The following utility function is taken:

$$U = \begin{cases} v^F + mv_i - p_i & \quad i = 1,2 \\ 0 & \end{cases} \tag{7}$$

Now there are two improvement levels: v_1 of the patentholder and v_2 of a competitor, where $v_1 < v_2$ (this division of roles is exogenous but seems natural). Each consumer enjoys an autonomous gross surplus v^F, which is taken such that the market is completely served in duopoly: $v^F \geq p_1/v_1$. Define m' as the consumer who is indifferent between v_1 at p_1 and v_2 at p_2. This consumer is given by $m' = (p_2 - p_1)/(v_2 - v_1)$. The demand function for the patentholder then is:

$$x_1 = (p_2 - p_1)/(v_2 - v_1) \tag{8}$$

The demand function for the improver is $x_2 = 1 - x_1$. Still assuming zero marginal product cost, the gross profit functions look like:

$$\pi_1(p_1, p_2; v_1, v_2) = p_1(p_2 - p_1)/(v_2 - v_1) \quad (9.a)$$

$$\pi_2(p_2, p_1; v_1, v_2) = p_2(1 - (p_2 - p_1)/(v_2 - v_1)) \quad (9.b)$$

As a solution concept for the stage of price choices of 1 and 2, I use the non-cooperative Nash equilibrium. A pair of prices (p_1^*, p_2^*) is an equilibrium if $\pi_1(p_1^*, p_2^*; v_1, v_2) \geq \pi_1(p_1, p_2^*; v_1, v_2)$ and $\pi_2(p_2^*, p_1^*; v_1, v_2) \geq \pi_2(p_2, p_1^*; v_1, v_2)$, for all $p_1, p_2 \geq 0$. Using the gross profit functions (9.a) and (9.b), the following Nash equilibrium in prices occurs:

$$p_1^*(p_2^*) = (v_2 - v_1)/3; \qquad p_2^*(p_1^*) = (2(v_2 - v_1))/3 \quad (10)$$

The associated gross profits in equilibrium are:

$$\pi_1(p_1^*, p_2^*) = (v_2 - v_1)/9; \qquad \pi_2(p_2^*, p_1^*) = (4(v_2 - v_1))/9 \quad (11)$$

The Nash equilibrium always exists[9]. Firm 2, the improver, is able to undercut the patentholder because by charging $p_2 < p_1$ he captures the whole market. The undercutting price strategy $p_2 = p_1^* - \varepsilon$, however, is always dominated by the price strategy p_2^* in the Nash equilibrium. Notice that the improver serves the upper segment of the market [1/3, 1], where consumers who appreciate improvement most are located, while the original innovator serves the lower segment [0, 1/3]. The larger improvement level enables the improver to charge a higher price and still have larger demand. This makes his gross profit (four times) larger than that of the original innovator.

The novelty requirements which are used by patent examiners concern the distance between both improvement levels: $v_2 - v_1 \geq h$. Notice that from the revenue side there is a natural tendency for an improver to improve as much as possible in order to create distance and relax price competition. According to Shaked and Sutton (1982), this is due to the vertical differentiation character of competition in product improvements. The gross profits of both the patentholder and the improver increase in distance between improvements in price equilibrium. The novelty requirements can become restrictive for the improver if Research and Development costs of improvements are incorporated. Suppose that increasing R&D

[9] It is assumed here that the patentholder and the improver choose prices simultaneously. A first-mover advantage for the patentholder might be the possibility to choose the price before the improver does. A Stackelberg price equilibrium is then appropriate with the patentholder being the Stackelberg price leader and the improver being the follower. The prices in Stackelberg equilibrium are $p_1^* = (v_2 - v_1)/2$ and $p_2^* = 3(v_2 - v_1)/4$, with corresponding profits of $\pi_1^* = (v_2 - v_1)/8$ and $\pi_2^* = 9(v_2 - v_1)/16$. Notice that these Stackelberg prices and profits of firm 1 and 2 are both higher than the ones in Nash equilibrium.

leads to a higher gross surplus being generated by the product innovation, and suppose that this is decreasingly so. A simple innovation cost function which catches the idea of exhausting improvement opportunities is:

$$c(v) = \alpha v^2 \qquad \alpha > 0 \tag{12}$$

The net profit functions then are $\pi_1(v_1, v_2) = (v_2 - v_1)/9 - \alpha v_1^2$ and $\pi_2(v_1, v_2) = (4(v_2 - v_1))/9 - \alpha v_2^2$. In this case the optimal improvements are independent of each other. A pair of improvements (v_1^*, v_2^*) is an equilibrium if $\pi_1(v_1^*, v_2^*) \geq \pi_1(v_1, v_2^*)$ and $\pi_2(v_1^*, v_2^*) \geq \pi_2(v_1^*, v_2^*)$, for all $v_1, v_2 \geq 0$. The Nash equilibrium in improvements is:

$$v_1^*(v_2^*) = 0; \qquad v_2^*(v_1^*) = 2/9\alpha \tag{13}$$

The net profit of the improver ($\pi_2^* = 4/(81\alpha)$) is (two times) larger than the net profit of the patentholder ($\pi_1^* = 2/(81\alpha)$)[10]. Because of the exhausting improvement opportunities, the relative net profit advantage of the improver is smaller than the relative advantage in gross profit. Three categories of effects of patent height can be distinguished. If the patent height is relatively low ($h \leq 2/9\alpha$), it does not affect the natural choice of the improver. For intermediate heights ($2/9\alpha < h \leq 4/9\alpha$), the improver is restricted and the best he can do is to deviate minimally from his optimal improvement and choose $v_2 = h$. Since he must deviate from his optimal choice v_2^*, his profits decrease in h. The profits of the patentholder increase in h because the distance in improvements becomes larger. For high patent protection ($h > 4/9\alpha$), the improver does not enter because his profits would then be negative. The patentholder becomes a pure monopolist. The categories of height effects shift upwards if improvements are less costly to generate (that is a smaller α in expression (12)). Height is then later restrictive. The effects of patent height on the competition in product improvements indicate how novelty requirements can be used as an instrument of technology policy. If a government wants to stimulate basic research, she can provide more profit to the patentholder, who generates the product innovation, by setting stronger novelty requirements. If she wants to stimulate applied research and development for improvements of existing products, a patent policy of weak novelty requirements is the right instrument. Some minimum incentive for basic research always exists because, even if novelty requirements are negligible weak, the improver freely chooses for some novelty and consequently the patentholder has positive profits.

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[10] Based on the Stackelberg price equilibrium (see footnote 9), the improvement choices are $v_1^* = 0$ and $v_2^* = 9/(32\alpha)$, yielding profits of $\pi_1^* = 9/(256\alpha)$ and $\pi_2^* = 81/(1024\alpha)$. Since the improvement choices are constant, a Stackelberg equilibrium in improvements does not differ from a Nash equilibrium in improvements.

5.2. Patent breadth

Before examining the effects of patent breadth on the competition through imitation, I will give some backgrounds of the model of horizontal differentiation. Hotelling (1929) came to the conclusion that two firms in a linear city will locate both at the center, that is differentiate minimally, if they compete for consumers who have linear travel cost. His conclusion was later qualified by D'Aspremont, Gabszewicz and Thisse (1979), who showed that the price equilibrium at which his statement of minimal differentiation was based, only exists when both firms are located far enough from each other. If located too closely to the other, a firm can gain by undercutting the price and capture the whole hinterland of its opponent. This holds for both firms. As a result, a price equilibrium does not exist when firms are located closely to each other. Several solutions have been proposed over time for this problem. I will shortly expose some of them here. Eaton and Lipsey (1978) use a 'no-mill price undercutting' assumption to rule out the inconsistent expectation of the price undercutting firm. D'Aspremont, Gabszewicz and Thisse (1979) specify quadratic transport costs. The price equilibrium then exists and the principle of minimal turns into one of maximal differentiation. The economic justification of quadratic travel costs, however, is not clear. One can make an argument that travelling involves a fixed cost which may partially offset the marginally increasing variable travel costs, for example because a consumer has to invest a fixed amount in travel equipment, or faces a constant utility penalty if he can not buy his most preferred variety, independent of distance. Another solution is formulated by Salop (1979). He describes spatial competition on a circle where a firm has to undercut a competitive price equal to the marginal cost in order to capture the hinterland of a neighbour. The undercutting price strategy is under these conditions never profitable. Economides (1984) starts from the original Hotelling model and includes a third alternative for consumers, besides the products of both firms, which creates a positive reservation price. A firm has to take into account this third alternative when it tries to undercut its opponent. The result is that the range of existence of the price equilibrium widens.

In the analysis to follow the transport costs are taken linear in distance for at least two reasons. Firstly, as already remarked, marginal increasing transport costs are, though technically handy, economically unappealing. Secondly, the analysis of patent breadth would yield less interesting results if the imitator would always want to differentiate maximally, since patent breadth then never would be restrictive. With linear transport cost, there is no tendency towards maximum differentiation. In fact, the imitator will, by assumption, be located at the border of protection at b. He will, in other words, always have an imitation that is marginally allowed for by court. If imitation would be more costly farther away from the patented product, the tendency for minimal differentiation would be even stronger and b would be the free choice of an imitator. Only the stages of entry and price decisions are included.

(a) One imitator at b

The patentholder is located at 0, the imitator is located at b (< 1). Imitation is costless. The market is assumed to be served completely, which is the case if the gross surplus of the product is larger than the highest delivery price of the patentholder or the imitator: v > max (p_1 + t, p_2 + bt). A consumer buys from firm 1 if v - p_1 - $td_1 \geq$ v - p_2 - td_2. The consumer who is indifferent between buying from the patentholder and the imitator is located at L* = (p_2 - p_1 + tb)/2t. The demand function for the patentholder is:

$$x_1 = \begin{cases} 0 & \text{if } p_1 \geq p_2 + tb \\ (p_2 - p_1 + tb)/2t & \text{if } p_2 + tb \geq p_1 \geq p_2 - tb \\ 1 & \text{if } p_1 \leq p_2 - tb \end{cases} \qquad (14)$$

The demand function for the imitator is $x_2 = 1 - x_1$, by assumption of full market coverage. Firm 1 can undercut firm 2 by charging a price of $p_1 = p_2 - tb - \varepsilon$, where ε is small and positive. At this undercutting price, the consumers in the hinterland of firm 2, given by 1 - b, then buy all from the patentholder, making his total demand 1. At a slightly higher price of $p_1 = p_2 - tb$, the total demand of the patentholder is b because the consumers in the hinterland [b, 1] do not buy from him but buy from the imitator. The demand function of the patentholder is thus discontinuous which makes the profit function also discontinuous. D'Aspremont, Gabszewicz and Thisse (1979) have shown that the discontinuity, or more precisely the non quasi-concavity, is a serious problem in the original Hotelling (1929) location model because a Nash equilibrium in prices does not exist when undercutting is profitable. In the underlying model, price undercutting is only possible for the patentholder. There is no hinterland to capture for the imitator because the patentholder is by definition located at the border 0. The problem of profitable undercutting also rises in this application to patent breadth and as will be shown, a Nash equilibrium in prices only exists if b is sufficiently large.

Based on the second part of the demand function, the price reaction functions of the patentholder and the imitator are $p_1^*(p_2^*) = (p_2 + bt)/2$ and $p_2^*(p_1) = (p_1 + 2t - bt)/2$. If it exists, the prices in Nash equilibrium are:

$$p_1^*(p_2^*) = (t(b + 2))/3; \qquad p_2^*(p_1^*) = (t(4 - b))/3 \qquad (15)$$

The associated profits in Nash equilibrium are:

$$\pi_1(p_1^*, p_2^*) = (t(b + 2)^2)/18; \qquad \pi_2(p_2^*, p_1^*) = (t(4 - b)^2)/18 \qquad (16)$$

Notice that the assumption of the patentholder being located at b is justified in Nash

equilibrium since the imitator indeed wants minimal differentation ($d\pi_2^N/db < 0$, for $b \in [0, 1]$). The closest location for the imitator allowed for by the patent office and by court is b. Now I will first look when these equilibrium prices are in the relevant price interval $p_1^* \in [p_2^* - tb, p_2^* + tb]$. After that I will check when an undercutting price strategy is profitable for the patentholder. It can easily be shown that the equilibrium prices are in the relevant interval for $b \geq 2/5$. This is the first restriction. Now consider the undercutting price strategy of firm 1. The undercutting price is $p_1 = p_2 - tb - \varepsilon$, resulting in a demand of 1 and yielding a profit of $\pi_1 = p_2 - tb - \varepsilon$. It must hold that this undercutting profit is not profitable: $\pi_1(p_1^*, p_2^*) \geq p_2^* - tb - \varepsilon$. So the condition becomes:

$$(t(b + 2)^2)/18 \geq (t(4 - b))/3 - tb - \varepsilon \tag{17}$$

which holds for $b \geq 6\sqrt{6} - 14$ (≈ 0.7). If the patent breadth is smaller than 0.7, then no Nash equilibrium in pure price strategies exists[11]. Conclusions on patent breadth policy only hold for the range $b \in [0.7, 1]$. The market shares of the patentholder and the imitator are respectively $[0, (b + 2)/6]$ and $[(4 - b)/6, 1]$. The imitator has larger profits. Increasing patent breadth levels out market shares, prices and profits. The trade off in patent breadth policy is between providing sufficient innovation incentive on the one hand, and stimulating, through imitation, both competition, which lowers the deadweight losses of patent monopolies, and diffusion, which enlargens consumer surplus, on the other hand[12].

In the remaining of the paper, three scenarios will be examined where the problem of price undercutting by the patentholder is less restrictive. The first (section 5.2 (b)) is fragmentation in the free range. As a result of the free competition in the range [b, 1], the prices will be equal to the marginal cost and undercutting this competitive price can not be profitable. Besides, if the patentholder undercuts the imitator at the border, there will be another

[11] For cases where the patentholder can first choose his price, knowing the expected reaction of the imitator, the Stackelberg equilibrium prices are $p_1^* = t(b + 2)/2$ and $p_2^* = t(6 - b)/4$, yielding profits of $\pi_1^* = t(b + 2)^2/16$ and $\pi_2^* = t(6 - b)^2/32$. The non-undercutting condition is more restrictive in the Stackelberg scenario: only for $b \geq 2\sqrt{41} - 12$ (≈ 0.81) the Stackelberg equilibrium in prices exists.

[12] The exact trade off is relatively easy to determine here. The welfare losses can be split up in two parts. Firstly, there are pure travel costs which occur when both firms charge competitive prices equal to the marginal cost. This welfare loss is here $WL_1 = tb^2/4 + t(1 - b)^2/2$. The second type of welfare loss occurs because both firms charge above-marginal-cost prices. This "price-induced-net-travel cost" is $WL_2 = (p_2 - p_1)^2/4t$. After substituting the prices in Nash equilibrium, the total welfare loss is given by: $WL_T = t(31b^2 - 44b + 22)/36$. If a social planner is only concerned with minimizing the total static welfare loss WL_T without providing a minimum profit for a patentholder, it optimally sets a breadth of $b^* = 22/31$. If the social planner aims at providing a minimum profit level for an inventor at minimum static welfare loss, it optimally sets the patent breadth $b^* = 11/14$, which is the one that minimizes the ratio WL_T/π_1^*. For these optimal patent breadths, the Nash price equilibrium exists.

imitator waiting. The second scenario (section 5.3 (a)) where the price undercutting problem is softened is the introduction of different improvement levels: $v_1 < v_2$. The patentholder has to overcome the difference in improvement before his price undercutting strategy becomes effective. If consumers furthermore evaluate improvements differently, as in the third scenario (section 5.3 (b)), the problem of discontinuous demand and profit functions disappears.

(b) Fragmentation in the free range

The interval $[0, b\rangle$ is protected. Suppose the free range [b, 1] is completely filled up with competitors, for example because imitation is costless. These competitors charge the competitive price equal to the marginal cost, which is in this case $p = 0$. Again I take utility function (3). The consumers who are located in the free region face no travel costs because there are competitors at each location. The patentholder has to compete with the imitator at the border b for the consumers in the interval [0, b). I assume that all consumers buy ($v \geq tb$). The demand function for the patentholder is:

$$x = \begin{cases} 0 & \text{if } p \geq tb \\ (tb - p)/2t & \text{if } 0 \leq p \leq tb \end{cases} \tag{18}$$

The optimal price, based on the second part of the demand function, is $p^* = tb/2$, which is always in the relevant price interval. The optimal profit is $\pi^* = b^2t/8$. The profit of the patentholder is an indication for the effectiveness of patent policy in providing an innovation incentive. Compared to the presence of only one imitator, patent breadth provides less profit with fragmentation in the free range. The reason is that the patentholder must compete now with an imitator who charges a competitive price equal to the marginal cost. The trade off for a social planner is the following: discouraging imitations in the free range enlargens the innovation incentive but lowers the consumer surplus in the free range because consumers then have larger travel costs.

5.3. Inventing aside and above the patent

A duopoly is studied now where a competitor is in the market with a product that is both an imitation and an improvement of the patenholder's product. Like in section 5.1 the stages where firms make entry decisions and choose improvement levels are included and take place before the price competition. Such an analysis serves at least two goals. So far, the patent office only requires a minimum level of improvement or an imitation with a minimum distance away from the patent. A product which is both an improvement *and* an imitation gives the opportunity to study the optimal circumventing strategy of a competitor: aside or above the patented product, or maybe aside and above. It gives furthermore the opportunity to investigate the combined effects of patent policy instruments of breadth and height.

(a) Homogeneous consumers (m = 1) on the Horizontal Product Spectrum

The improvement levels of the patentholder and the imitator differ: $v_1 < v_2$. Consumers are taken homogeneous in the sense that they evaluate the difference in improvement level all in the same way. In the next section consumers are taken heterogeneous in this sense. Consumers only differ in their most preferred varieties. All consumers are assumed to buy (which is the case for $v_2 > \max(p_2 + tb, p_2 + t(1 - b))$). A consumer buys from firm 1 if $v_1 - p_1 - td_1 \geq v_2 - p_2 - td_2$. The indifferent consumer is $L^* = (p_2 - p_1 - (v_2 - v_1) + tb)/2t$. The demand function for the patentholder is:

$$x_1 = \begin{cases} 0 & \text{if } p_1 \geq p_2 + tb - (v_2 - v_1) \\ (p_2 - p_1 + tb)/2t & \text{if } p_2 + tb - (v_2 - v_1) \geq p_1 \geq p_2 - tb - (v_2 - v_1) \\ 1 & \text{if } p_2 - tb - (v_2 - v_1) \geq p_1 \geq 0 \end{cases} \quad (19)$$

The demand function for the imitator is the complement: $x_2 = 1 - x_1$. Notice that the price strategy of undercutting the imitator is not possible if the difference in improvement levels is too large. If $(v_2 - v_1) \geq p_2 - tb$, then the patentholder can never reach the consumers in [b, 1].

The price reaction functions, based on the second part of the demand function are $p_1^*(p_2) = (p_2 + bt - (v_2 - v_1))/2$ and $p_2^*(p_1) = (p_1 + 2t - bt + v_2 - v_1)/2$. If it exists, the price Nash equilibrium is:

$$p_1^*(p_2^*) = (t(2 + b) - (v_2 - v_1))/3; \qquad p_2^*(p_1^*) = (t(4 - b) + v_2 - v_1)/3 \quad (20)$$

with the associated equilibrium gross profits:

$$\pi_1(p_1^*, p_2^*) = (t(b + 2) - (v_2 - v_1))^2/18t; \qquad \pi_2(p_2^*, p_1^*) = (t(4 - b) + v_2 - v_1)^2/18t \quad (21)$$

Notice that the equilibrium profit of the patentholder increases in patent breadth and decreases in distance $v_2 - v_1$. For the competitor, who is imitator and improver at the same time, the reverse holds; his profit decreases in breadth and increases in distance between improvement levels. Compared to the pure imitation scenario 5.2 (a), with profits given by (16), the gross profit of the patentholder decreases because of the difference in improvement level while the gross profit of the imitator/improver increases because of this.

As remarked, the third part of the demand function disappears if $p_2 \leq (v_2 - v_1) + tb$. For the equilibrium price p_2^*, this condition becomes: $b \geq 1 - (v_2 - v_1)/2t$. For these b, undercutting is not possible and the Nash equilibrium in prices always exists. For $\max(2/5 - (v_2 - v_1)/5t, (v_2 - v_1)/t - 2, 0) \leq b < 1 - (v_2 - v_1)/2t$, the Nash equilibrium is consistent with the price

interval on which it is based ($p_2 + tb - (v_2 - v_1) \geq p_1 \geq p_2 - tb - (v_2 - v_1)$). The non-undercutting condition for these b ($\geq \max(2/5 - (v_2 - v_1)/5t, (v_2 - v_1)/t - 2, 0)$) is:

$$(t(2 + b) - (v_2 - v_1))^2/18t \geq p_2^* - \varepsilon - tb - (v_2 - v_1) \quad (22.a)$$

which can be written as:

$$b \geq (v_2 - v_1 - 14t)/t + (6\sqrt{(6t - (v_2 - v_1))})/\sqrt{t} \quad (22.b)$$

Suppose that the patent breadth fulfills these conditions and that the Nash equilibrium in prices exists. Now look at the stage before the price competition, when both firms choose their improvement levels. The net profit functions contain the gross profit functions (21), which are at their equilibrium values after price competition, and the R&D cost function (12):

$$\pi_1(v_1, v_2) = (t(2 + b) - (v_2 - v_1))^2/18t - \alpha v_1^2; \quad (23.a)$$

$$\pi_2(v_2, v_1) = (t(4 - b) + v_2 - v_1)^2/18t - \alpha v_2^2 \quad (23.b)$$

This yields the improvement reaction functions $v_1^*(v_2) = (t(b + 2) - v_2)/(18\alpha t - 1)$ and $v_2^*(v_1) = (t(4 - b) - v_1)/(18\alpha t - 1)$. Notice that $dv_1^*/dv_2, dv_2^*/dv_1 < 0$. If the competitor chooses a larger improvement level, the patentholder can best choose a smaller improvement, and vice versa. We know from the price equilibrium (20) that the patentholder's gross profits decrease in difference between improvement levels and those of the competitor increase. The Nash equilibrium in improvement choices is[13]:

$$v_1^* = (3\alpha t(b + 2) - 1)/(6\alpha(9\alpha t - 1)); \quad v_2^* = (3\alpha t(4 - b) - 1)/(6\alpha(9\alpha t - 1)) \quad (24)$$

Both improvement levels are assumed to be positive, which is the case for ($3\alpha t(b + 2) > 1$). The distance between both improvement levels in Nash equilibrium is:

$$v_2^* - v_1^* = (t(1 - b))/(9\alpha t - 1) \quad (25)$$

[13] The patentholder can have a first-mover advantage and choose an improvement level before the competitor. This first-mover advantage does not necessarily show up in the price stage. The Stackelberg equilibrium in improvements, based on the Nash equilibrium prices in (20), is given by $v_1^*(v_2^*(v_1)) = 6t(3\alpha t(b + 2) - 1)/z$ and $v_2^*(v_1^*) = (t(18\alpha t(4 - b) + b - 10)/z$, where $z \equiv 324\alpha^2 t^2 - 54\alpha t + 1$. The distance in improvements is only positive for $b > 4(9\alpha t - 1)/(36\alpha t - 1)$. Along the same lines, the Stackelberg equilibrium in improvements which is based on a Stackelberg price equilibrium can be determined.

Expression (25) has important implications for patent policy. Broader patent protection makes the "natural" distance in improvements, $v_2^* - v_1^*$, smaller. If a competitor is forced to choose his imitation farther away from the patentholder, he is not prepared to invest as much in R&D for product improvement in order to create vertical distance. The profit of the patentholder directly increases in patent breadth, as can be seen in (21). This direct effect can be enforced by the indirect effect of breadth on the improvement choices. The patentholder faces less competition if the other chooses a smaller improvement (check (21)). Because the distance in improvements shortens, broadening protection has an indirect, positive, effect on the profit of the patentholder. A competitor who keeps sufficient distance with his imitation does not have to fulfill the novelty requirements in order not to infringe the current patent. The novelty requirements can only become restrictive if the improver wants a patent himself. Since broader patents make his profits decrease, there may be a critical patent breadth which makes the strategy of imitation too costly. The competitor can then best choose a duplication at $l = 0$ and focus fully on product improvement. The profit associated with this strategy may depend then on patent height. If patent height is indeed restrictive for the improver, his profits decrease in height. The model used here, however, is not appropriate to examine this trade-off for the improver. The homogeneity of the consumers with respect to improvements makes that all consumers buy from the improver if the improver undercuts with $p_2 = v_2 - v_1 + p_1 - \varepsilon$ the price of the patentholder, corrected for the improvement difference.

After substitution of the equilibrium improvements (24) in the net profit function, it turns out that the equilibrium profit of the competitor is always larger than the one of the patentholder. More precisely, the difference in equilibrium profits is:

$$\pi_2(v_2^*, v_1^*) - \pi_1(v_1^*, v_2^*) = (t(1 - b)(18\alpha t - 1))/(3(9\alpha t - 1)) \qquad (26)$$

This result, which also occured in the pure breadth and height scenarios, suggests that R&D competition can better be described with the use of so called *waiting games* instead of *patent races* as is usually done. In a waiting game for two players, it is better to be second than to be first. Dasgupta (1986) pointed out that the spill-overs from the R&D output of the first firm to the second firm can be a reason for the profit of the second to be larger. In the model described, it is even better to be second in absence spill-overs in R&D for improvements from the patentholder to the improver. This conclusion must of course be qualified in light of the assumptions of the model. It may be weakened by the division of roles imposed in the competition in improvements where the patentholder always has the smaller improvement, by the specification of costly imitation or by the introduction of a time lag for inventing around, during which the patentholder is a monopolist. But it may be enforced when R&D costs for the original product innovation are incorporated.

(b) Heterogeneous consumers

Consumers are characterized now also by the parameter m which is identical for both products and which is uniformly distributed on [0, 1] with density 1. The indirect utility function (5) which combines imitation and improvement is taken again, but now with a constant v^F which is sufficiently large to cover the market completely in duopoly. The demand function of the patentholder is made up of consumers for whom holds $m(v_2 - v_1) - t(d_2 - d_1) \leq p_2 - p_1$. Define $\mu' \equiv m(v_2 - v_1)$ which is distributed uniformly on $[0, (v_2 - v_1)]$ with density $1/(v_2 - v_1)$. The distance from a consumer to the patentholder at 0 is given by d_1 and to the competitor at b, d_2. What I need now is the distribution of $(d_2 - d_1)$. I first focus on the range [0, b] where $d_1 \in [0, b]$ and $d_2 \in [0, b]$ with density 1. In the range [0, b], it holds for all consumers that $d_2 + d_1 = b$. Therefore, $d_2 - d_1 = 2d_2 - b$, so that $(d_2 - d_1) \in [-b, b]$ with density 1/2. In order to simplify notations, define $\delta' \equiv t(d_2 - d_1)$ which is distributed on [-tb, tb] with density 1/2t. The joint distribution of μ' and δ', which are independent, has density $1/2t(v_2 - v_1)$. The division of the market is given by the line: $\mu' - \delta' = p_2 - p_1$ (see figure 3). The consumers who buy from the competitor are located above and to the left of this line. Those who buy from the patentholder are located below and to the right of this line.

Two types of demand functions can be distinguished, depending on the relative size of the product spectra: horizontal dominance and vertical dominance. The demand functions are horizontally dominated if the horizontal spectrum [-tb, tb] is larger than the vertical product spectrum $[0, v_2 - v_1]$. Vertical dominance is present if $v_2 - v_1 > 2tb$. I will work out the case of vertical dominance first. I refer to Appendix A for the overall demand function. The demand function of the patentholder and the competitor are continuous in prices. Since discontinuity of the demand function was the major problem in the original Hotelling model and the model inspired by it in section 5.2 (a), the combination of horizontal and vertical differentiation turns out to be a useful extension. The reason why the discontinuity of demand and profit functions disappears is the following: if the patentholder undercuts the price of his opponent and attracts the consumer at b, he does not attract the complete hinterland of his opponent. There is no mass point at b. It depends on the improvement preference parameter m of a consumer in [b, 1] whether he is attracted or not. However, now there might be a problem at another level. Although the profit functions are continuous, they are not continuous in first derivatives (in price). The price reaction functions are therefore also not continuous. The complexity of determining the Nash equilibrium (equilibria) in prices here, with six-part profit functions, relevant price intervals and discontinuous price reaction functions, makes the research strategy of focusing on one part of the demand and profit function an attractive one. In fact, I will use the (simpliest) third part:

$$x_1 = (b(p_2 - p_1)/(v_2 - v_1) \tag{27}$$

for $p_2 - tb \geq p_1 \geq p_2 - tb - b(v_2 - v_1)$. Demand function (27) is the base for analysis and I will point out later when the use of this part is indeed justified. The gross profit functions here are given by:

$$\pi_1(p_1, p_2; v_1, v_2) = (p_1 b(p_2 - p_1)/(v_2 - v_1); \qquad \pi_2(p_2, p_1; v_2, v_1) = p_2(1 - b(p_2 - p_1)/(v_2 - v_1)) \quad (28)$$

The corresponding price reaction functions are $p_1^*(p_2) = p_2/c$ and $p_2^*(p_1) = (bp_1 + v_2 - v_1)/(2b)$, which generate the price Nash equilibrium:

$$p_1^*(p_2^*) = ((v_2 - v_1))/(3b); \qquad p_2^*(p_1^*) = (2(v_2 - v_1))/(3b) \quad (29)$$

Turning into the stage of improvement competition, the (constant) optimal improvement choices which are based on the net profits, are:

$$v_1^* = 0; \qquad v_2^* = 2/(9\alpha b) \quad (30)$$

Given these equilibrium improvement levels v_1^* and v_2^*, and the equilibrium prices p_1^* and p_2^*, the condition that the prices belong to the relevant interval of the demand function on which the analysis is based (part (iii) of (3) in Appendix A) is: $b \leq (\sqrt{(36\alpha t + 1)} - 1)/(18\alpha t)$. A small development cost parameter α and a small unit travel cost make the relevant range of b larger (for $\alpha = 0.5$ and $t = 0.5$, the range is $b \in [0, 0.48]$. The associated net profits in the Nash equilibrium in improvements are:

$$\pi_1(v_1^*, v_2^*) = 2/(81\alpha b^2); \qquad \pi_2(v_2^*, v_1^*) = 4/(81\alpha b^2) \quad (31)$$

Notice that both profits decrease in patent breadth b. That part of the demand function was chosen where the patentholder only serves consumers in [0, b]. The competitor partly serves the market segment [0, b], but he also serves the complete market segment [b, 1], where he faces no competition from the patentholder. Therefore if b increases, the segment where the competitor has market power shrinks, his optimal price decreases and consequently his profit decreases. The patentholder then faces a lower price of his competitor and the best he can do is lower his price ($p_1^*(p_2) = p_2/2$).

What innovation strategy can a competitor best choose if a product located at 0 on the horizontal and vertical spectrum is protected with patent breadth b and patent height h? He can either choose to focus fully on improvement or generate a sufficiently distant imitation

with some improvement level[14]. Consider first the *improvement strategy*. If he chooses a duplication of the patented product at the horizontal spectrum, he has to fulfill the novelty requirements in order not to infringe the patent. Recall from section 5.1 that a firm which only differentiates through improvement and not through imitation, has a profit of $\pi_2(v_2^*, v_1^*) = 4/(81\alpha)$ with the improvements $v_2^* = 2/9\alpha$ and $v_1^* = 0$ at the market. This improvement choice is allowed for if patent height is low: $h \leq 2/9\alpha$. The improver can best choose $v_2 = h$ if the patent height is intermediate ($2/9\alpha < h \leq 4/9\alpha$). His profit then is $\pi_2(h, 0) = 4h/9 - \alpha h^2$. For high patent protection ($h > 4/9\alpha$), he can not make positive profit with an pure improvement strategy. The other possible, *imitation strategy* of a competitor is to choose an imitation at b which is sufficiently different according to patent breadth rules. His profit then is $\pi_2(v_2^*, v_1^*) = 4/(81\alpha b^2)$. The pure improvement strategy is always dominated, for any combination of breadth and height. To see this take $b = 1$. The imitation strategy then yields $4/(81\alpha)$, which is equal to the unrestricted profit with the pure improvement strategy. For all $b < 1$, the imitation strategy is better. The conclusion here is that a competitor who wants to invent around the patent chooses to imitate sufficiently aside the patent and improve somewhat above the patent. If the improvement level of the circumventer happens to fulfill the novelty requirements, he can get a patent himself. Otherwise he is just tolerated by patent office and court.

Now consider the case of horizontal dominance ($2tb > v_2 - v_1$). The complete demand function of the patentholder (and his competitor) is written out in Appendix B. The middle part (iii), which is linear and easiest to work with, is taken here:

$$x_1 = (2(b + p_2 - p_1) - (v_2 - v_1))/4t \tag{32}$$

for $p_1 \in [p_2 - tb, p_2 + tb - (v_2 - v_1)]$. The price reaction functions based in this part of the demand functions are $p_1^* = (2bt + 2p_2 - (v_2 - v_1))/6$ and $p_2^* = (4t - 2bt + 2p_1 + v_2 - v_1)/4$. They yield the following the Nash equilibrium in prices:

$$p_1^*(p_2^*) = (2t(b + 2) - (v_2 - v_1))/6; \qquad p_2^*(p_1^*) = (2t(4 - b) + v_2 - v_1)/6 \tag{33}$$

The associated gross profits are:

$$\pi_1^* = (2t(b + 2) - (v_2 - v_1))^2/72t; \qquad \pi_2^* = (2t(4 - b) + v_2 - v_1)^2/72t \tag{34}$$

[14] Another possible strategy is to wait till the patent is expired and the restrictions imposed by breadth and height withdraw. See Gallini (1992) on this strategy. Here it becomes clear that the assumption of infinite patents is probably too strong. What in fact is assumed is a patent lifetime which is sufficiently long to exclude the "waiting-for-the-expiration date" strategy.

Based on the net profit functions, the improvement reaction functions are $v_1^* = (2t(b + 2) - v_2)/(72\alpha t - 1)$ and $v_2^* = (2t(4 - b) - v_1)/(72\alpha t - 1)$, yielding Nash equilibrium improvements[15]:

$$v_1^*(v_2^*) = (12\alpha t(b + 2) - 1)/(12\alpha(36\alpha t - 1)) \tag{35}$$

$$v_2^*(v_1^*) = (12\alpha t(4 - b) - 1)/(12\alpha(36\alpha t - 1))$$

The part of the demand function on which the analysis is based is consistent its relevant price interval if $p_1^* \geq p_2^* - tb$ *(a)* and $p_1^* \leq p_2^* + tb - (v_2^* - v_1^*)$ *(b)*. Given p_1^*, p_2^*, v_1^* and v_2^* from (33) and (35), the lower border *(a)* becomes $b \geq (24\alpha t)/(60\alpha t - 1)$ and the upper border *(b)* becomes $b \geq -2(12\alpha t - 1)/(12\alpha t + 1)$. Since $b \geq 0$, the upper border *(b)* is always fulfilled. The lower border is fulfilled for reasonable values of the parameters α and t[16]. The natural distance in improvements depends on patent breadth:

$$v_2^* - v_1^* = (2t(1 - b))/(36\alpha t - 1) \tag{36}$$

As before (check (25) and (30)) patent breadth has a negative effect on the natural distance in improvements. Forcing to keep greater distance on the horizontal spectrum results in smaller distance on the vertical product spectrum. The profit levels in the Nash equilibrium are:

$$\pi_1^* = (72\alpha t - 1)(12\alpha t(b + 2) - 1)^2/(144\alpha(36\alpha t - 1)^2) \tag{37}$$

$$\pi_2^* = (72\alpha t - 1)(12\alpha t(4 - b) - 1)^2/(144\alpha(36\alpha t - 1)^2)$$

Comparing the innovation strategies of the non-patentholder, an improvement or an imitation strategy, yields interesting results. The improvement strategy, that is choosing a duplication on the horizontal spectrum and aiming purely at improvement, yields profit of $\pi_2^{*IMP} = 4/(81\alpha)$. The profit associated with the imitation strategy is given by (37), π_2^{*IMI}. If the patent height is not restrictive in case of the improvement strategy, then the improvement strategy is chosen ($\pi_2^{*IMP} > \pi_2^{*IMI}$) if:

[15] A Stackelberg equilbrium in improvements, whether based on a Nash or Stackelberg price equilibrium, provides qualitatively similar conclusions.

[16] For example:

α	t	b^L
0.25	0.25	0.29
0.1	0.50	0.13
0.1	0.75	0.49

$$b > (3(48\alpha t - 1)\sqrt{(72\alpha t - 1)} - 8(36\alpha t - 1)/(36\alpha t\sqrt{(72\alpha t - 1)}) \quad (38)$$

If (38) does not hold, the competitor can best choose an imitation strategy. If the patent height is restrictive for the improvement strategy ($2/9\alpha < h \leq 4/9\alpha$), the improvement strategy is less appealing because the improver has to deviate from his optimal improvement v_2^*. The improvement strategy stays better for:

$$b > (4(36\alpha t - 1)\sqrt{(\alpha h(4 - 9\alpha h)/(72\alpha t - 1))} + 48\alpha t - 1)/(12\alpha t) \quad (39)$$

Patent height can become so restrictive ($h > 4/(9\alpha)$) that the improvement strategy can never yield positive profits. A competitor then always invents around with an imitation strategy.

6. Conclusions

Patent exploitation by the patentholder is dependent on the standards, breadth and height, which are set by the patent office and the court. These standards affect the profit opportunities of the patentholder because they determine the strategy space for the competition which surrounds the patent. The standards therefore also affect the profit opportunities of competitors. Various simple models of differentiation, horizontal, vertical and combinations, are used to describe the profit opportunities of the patentholder and competitors. The major conclusion is that a patentholder has to deal with two possible restrictions when he exploits his patent. Firstly, in the case of perfect protection it is the variety and the quality of consumers which may restrict him. Profit maximization may imply that not the whole market is covered. The second restriction is present when the protection provided by the patent is not perfect and competitors can circumvent by inventing around the patent.

The analysis raises a number of other questions. What are, for example, the consequences for the competition in R&D as described in most patent race models. According to Mortensen (1982), there is a better allocation of R&D if the winner does not take all, but compensates the loser(s). Patent dimensions might be appropriate to create such a situation, where the winner of a patent race gets such a level of protection that the loser has enough opportunities to make up for his R&D expenditures. Furthermore, the purpose of a patent is to provide an incentive for a firm to do R&D. The incentive is basically given by providing a minimum profit level which can be gained with an innovation. Various mixtures of patent dimensions are possible to provide this minimum profit. The problem then is to find a mixture which causes the least static welfare loss. One part of this problem, the effects of patent dimensions on profit, is discussed in this paper. The other part, the effects on welfare, remain to be studied. Another research question concerns diffusion. The true welfare gains occur in the process of diffusion through the economy, with the innovations preferably being competitively supplied. Since patent dimensions also affect the opportunities for competitors, they may have a decisive influence on the speed and amplitude of diffusion.

APPENDIX

(A) Vertical Dominance ($v_2 - v_1 > 2tb$)

The part of the demand for the patentholder (and for the competitor) on the interval [0, b] is made up of different parts:

$$
x_1 = \begin{cases} 0 & \text{if } p_1 \geq p_2 + tb \\ (bt - p_1 + p_2)^2/(4t(v_2 - v_1)) & \text{if } p_2 + tb \geq p_1 \geq p_2 - tb \\ (b(p_2 - p_1)/(v_2 - v_1) & \text{if } p_2 - tb \geq p_1 \geq p_2 + tb - (v_2 - v_1) \\ b - (bt + p_1 - p_2 + v_2 - v_1)^2/(4t(v_2 - v_1)) & \text{if } p_2 + tb - (v_2 - v_1) \geq p_1 \geq p_2 - tb - (v_2 - v_1) \\ b & \text{if } p_2 - tb - (v_2 - v_1) \geq p_1 \geq 0 \end{cases} \quad (1)
$$

On the interval [b, 1], the difference in travel distance ($d_2 - d_1$) is constant and equal to $-tb$. The market division in the segment [b, 1] is $\mu' = p_2 - p_1 + tb$. In this segment the demand function for the patentholder is:

$$
x_1 = \begin{cases} 0 & \text{if } p_1 \geq p_2 - tb - b(v_2 - v_1) \\ (p_2 - p_1 - tb)/(v_2 - v_1) - b & \text{if } p_2 - tb - b(v_2 - v_1) \geq p_1 \geq p_2 - tb - (v_2 - v_1) \\ 1 - b & \text{if } p_2 - tb - (v_2 - v_1) \geq p_1 \geq 0 \end{cases} \quad (2)
$$

Depending on the patent breadth, the demand in the segment [b, 1] starts in the third or in the fourth part of the total demand function. Consider the case where is starts in the third part (for $v_2 - v_1 \geq 2tb/(1 - b)$). The total demand over the whole market [0, 1] is:

$$
x_1 = \begin{cases} \text{(i)} & 0 \\ \text{(ii)} & (bt - p_1 + p_2)^2/(4t(v_2 - v_1)) \\ \text{(iii)} & (b(p_2 - p_1)/(v_2 - v_1) \\ \text{(iv)} & (b(p_2 - p_1)/(v_2 - v_1) + (p_2 - p_1 - tb)/(v_2 - v_1) - b \\ \text{(v)} & (p_2 - p_1 - tb)/(v_2 - v_1) - (bt + p_1 - p_2 + v_2 - v_1)^2/(4t(v_2 - v_1)) \\ \text{(vi)} & 1 \end{cases} \quad (3)
$$

Resp. if $p_1 \geq p_2 + tb$ (i); if $p_2 + tb \geq p_1 \geq p_2 - tb$ (ii); if $p_2 - tb \geq p_1 \geq p_2 - tb - b(v_2 - v_1)$ (iii); if $p_2 - tb - b(v_2 - v_1) \geq p_1 \geq p_2 + tb - (v_2 - v_1)$ (iv); if $p_2 + tb - (v_2 - v_1) \geq p_1 \geq p_2 - tb - (v_2 - v_1)$ (v); and if $p_2 - tb - (v_2 - v_1) \geq p_1 \geq 0$ (vi).

(B) Horizontal Dominance ($v_2 - v_1 < 2tb$)
The division of the market segment [0, b] is:

$$x_1 = \begin{cases} \text{(i)} & 0 & \text{if } p_1 \geq p_2 + tb \\ \text{(ii)} & (bt - p_1 + p_2)^2/(4t(v_2 - v_1)) & \text{if } p_2 + tb \geq p_1 \geq p_2 + tb - (v_2 - v_1) \\ \text{(iii)} & (2bt + 2p_2 - 2p_1 - (v_2 - v_1))/(4t) & \text{if } p_2 + tb - (v_2 - v_1) \geq p_1 \geq p_2 - tb \\ \text{(iv)} & b - (bt + p_1 - p_2 + v_2 - v_1)^2/(4t(v_2 - v_1)) & \text{if } p_2 - tb \geq p_1 \geq p_2 - tb - (v_2 - v_1) \\ \text{(v)} & b & \text{if } p_2 - tb - (v_2 - v_1) \geq p_1 \geq 0 \end{cases} \tag{4}$$

On the interval [b, 1], the difference in travel distance ($d_2 - d_1$) is constant and equal to $-tb$. The market division in the segment [b, 1] is $\mu' = p_2 - p_1 + tb$. In this segment the demand function for the patentholder is:

$$x_1 = \begin{cases} 0 & \text{if } p_1 \geq p_2 - tb - b(v_2 - v_1) \\ (p_2 - p_1 - tb)/(v_2 - v_1) - b & \text{if } p_2 - tb - b(v_2 - v_1) \geq p_1 \geq p_2 - tb - (v_2 - v_1) \\ 1 - b & \text{if } p_2 - tb - (v_2 - v_1) \geq p_1 \geq 0 \end{cases} \tag{5}$$

The demand on the segment [b, 1] must be added completely to part 4.(iv). The total demand on [0, 1] is:

$$x_1 = \begin{cases} \text{(i)} & 0 \\ \text{(ii)} & (bt - p_1 + p_2)^2/(4t(v_2 - v_1)) \\ \text{(iii)} & (2bt + 2p_2 - 2p_1 - (v_2 - v_1))/(4t) \\ \text{(iv)} & b - (bt + p_1 - p_2 + v_2 - v_1)^2/(4t(v_2 - v_1)) \\ \text{(v)} & (p_2 - p_1 - tb)/(v_2 - v_1) - (bt + p_1 - p_2 + v_2 - v_1)^2/(4t(v_2 - v_1)) \\ \text{(vi)} & 1 \end{cases} \tag{6}$$

Resp. if $p_1 \geq p_2 + tb$ (i); if $p_2 + tb \geq p_1 \geq p_2 + tb - (v_2 - v_1)$ (ii); if $p_2 + tb - (v_2 - v_1) \geq p_1 \geq p_2 - tb$ (iii); if $p_2 - tb \geq p_1 \geq p_2 - tb - b(v_2 - v_1)$ (iv); if $p_2 - tb - b(v_2 - v_1) \geq p_1 \geq p_2 - tb - (v_2 - v_1)$ (v); and if $p_2 - tb - (v_2 - v_1) \geq p_1 \geq 0$ (vi).

References

Archibald, G.C., B.C. Eaton and R.G. Lipsey. 1986. "Address Models of Value Theory." In J.E. Stiglitz and G.F. Mathewson (eds.), *New Developments in the Analysis of Market Structure.* London: MacMillan.

d'Aspremont, C., J. Jaskold-Gabszewicz, J. and J.-F. Thisse, 1979, On Hotelling's stability in competition, *Econometrica* 47: 1145-1150.

Cornish, W.R., 1989, *Intellectual property: patents, copyright, trade marks and allied rights* (Sweet & Maxwell, London).

Dasgupta, P., 1988, Patents, priority and imitation or, the economics of races and waiting games, *Economic Journal* 98: 66-80.

Dijk, van T.W.P., 1992, Patent height and the competition in product improvements, MERIT Research Memorandum 92-019.

Dixit, A.K. and J.E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67: 297-308.

Eaton, B.C. and R.G. Lipsey. 1978. "Freedom of Entry and the Existence of Pure Profit." *Economic Journal* 88: 455-469.

Eaton, B.C. and R.G. Lipsey, 1989, Product differentiation, in: R. Schmalensee and R.D. Willig, eds., *Handbook of industrial organization* (Elsevier, Amsterdam).

Economides, N., 1984, The principle of minimum differentiation revisited, *European Economic Review,* 24: 345-368.

Economides, N., 1986, Nash equilibrium in duopoly with products defined by two characteristics, *RAND Journal of Economics* 17: 431-439.

Gallini, N.T. 1992. "Patent Policy and Costly Imitation." *RAND Journal of Economics* 23: 52-63.

Gilbert, R. and C. Shapiro, 1990, Optimal patent length and breadth, *RAND Journal of Economics* 21, 106-112.

Griliches, Z. 1990. "Patent Statistics as Economic Indicators: A Survey." *Journal of Economic Literature* XXVIII: 1661-1707.

Hotelling, H., 1929, Stability in competition, *Economic Journal* 39: 41-57.

Klemperer, P., 1990, How broad should the scope of patent protection be?, *RAND Journal of Economics* 21, 113-130.

Levin, R.C., A.K. Klevorick, R.R. Nelson, and S.G. Winter. 1987. "Appropriating the Returns from Industrial Research and Development." *Brookings Papers on Economic Activity* 3: 783-820.

Mansfield, E., M. Schwartz, and S. Wagner, 1981, Imitation costs and patents: an empirical

study, *Economic Journal* 91, 907-918.

Matutes, C., P. Regibeau, and K. Rockett, 1991, Optimal patent design and diffusion of innovations, Mimeo, Northwestern University.

Merges, R.P. and R.R. Nelson, 1992, Market structure and technical advance: the role of patent scope decisions, in: T.M. Jorde and D.J. Teece, eds., *Antitrust, innovation, and competitiveness*, (Oxford University Press, Oxford).

Mortensen, D.T., 1982, Property rights and efficiency in mating, racing and related games, *American Economic Review* 72, 968-979.

Neven, D. and J.F. Thisse, 1989, Choix des produits: concurrence en qualité et en variété, *Annales d'Économie et de Statistique* 15/16, 85-112.

Nordhaus, W.D., 1969, *Invention, growth, and welfare, a theoretical treatment of technological change* (MIT Press, Cambridge MA).

Pakes, A. 1986. "Patents as Options: Some Estimates of the Value of Holding European Patent Stocks." *Econometrica* 54: 755-784.

de Palma, A., V. Ginsburgh, Y.Y. Papageorgiou and J.-F. Thisse. 1985. "The Principle of Minimum Differentiation holds under Sufficient Heterogeneity." *Econometrica* 53: 767-781.

Papoulis, A., 1984, *Probability, random variables, and stochastic processes*, (McGraw-Hill, New York).

Reinganum, J.F. 1989, The timing of innovation: research, development, and diffusion." In R. Schmalensee and R.D. Willig (eds), *Handbook of Industrial Organization*, Amsterdam: Elsevier.

Salop, S., 1979, Monopolistic competition with outside goods, *Bell Journal of Economics* 10: 141-156.

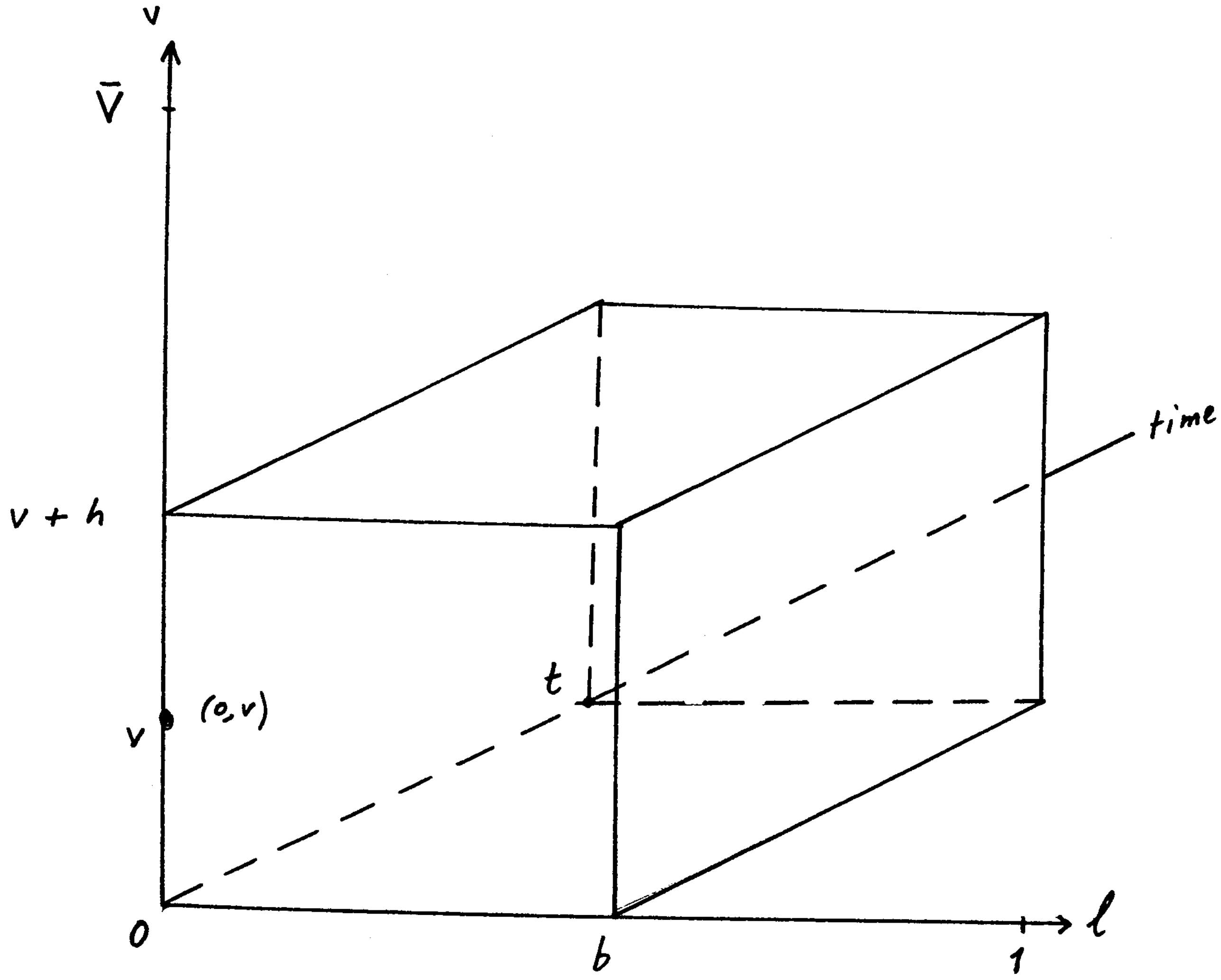
Scotchmer, S. and J. Green, 1990, Novelty and disclosure in patent law, *RAND Journal of Economics* 21, 131-146.

Shaked, A. and J. Sutton, 1982, Relaxing price competition through product differentiation, *Review of Economic Studies* 49, 3-13.

Tirole, J. 1989, The theory of industrial organization, (MIT Press, Cambridge MA).

Waterson, M. 1990. "The Economics of Product Patents." *American Economic Review* 80: 860-869.

FIGURE 1: CUBOID PROTECTION



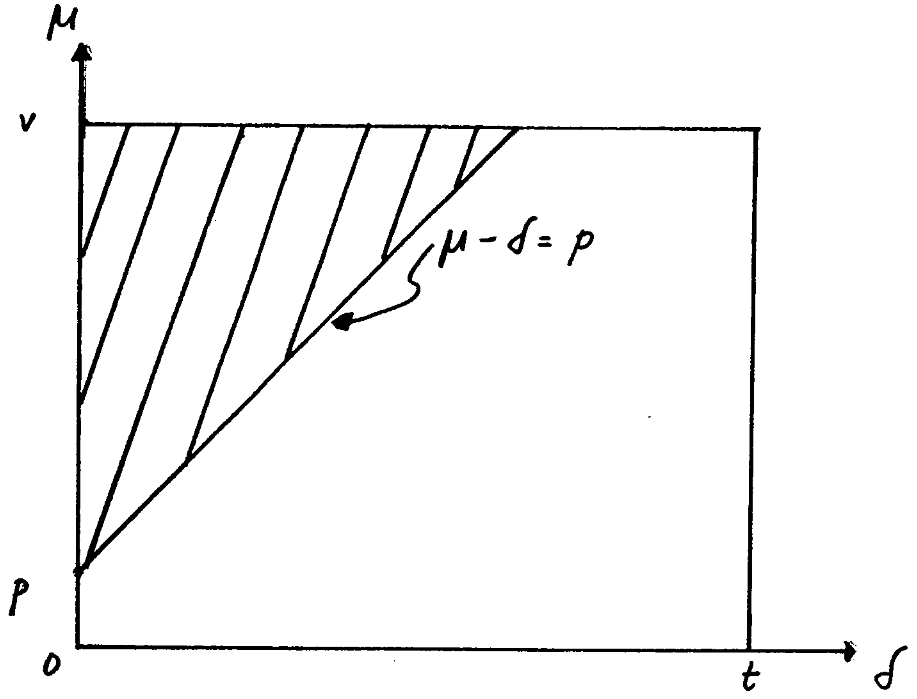


FIGURE 2. DEMAND FOR THE PATENTHOLDER

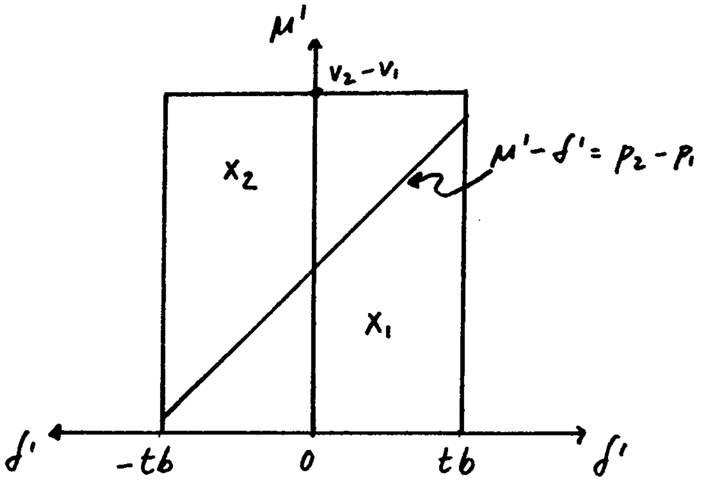


FIGURE 3. VERTICAL DOMINANCE

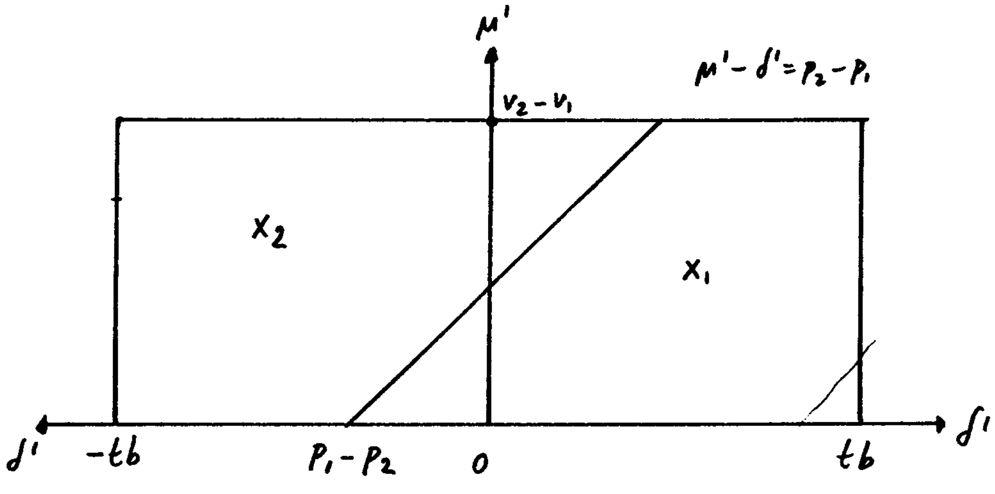


FIGURE 4. HORIZONTAL DOMINANCE