Essays on Economies with Heterogeneous Interacting Consumers

Zakaria Babutsidze

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Essays on Economies with Heterogenous Interacting Consumers

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To Elené

STELLINGEN

Essays on Economies with Heterogenous Interacting Consumers

Zakaria Babutsidze

1. Information diffusion through fixed social networks naturally generates clustering in demand.

2. The producer of the better product does not necessarily have higher incentives to advertise it.

3. Deliberate R&D efforts of optimistic, myopic firms generates fat tailed firm size distributions in a wide range of industries.

4. Global consumer interaction results in higher innovation incentives in many industries. However, these incentives are non-uniformly distributed across firms of different sizes.

5. Many economists feel comfortable working with models where the representative of the pride of lions is a rabbit.

6. There are two ways of doing research. One is to identify an important research question and then use whatever is in your head in order to come up with the model to answer it. The other is to use whatever is in your head to come up with the model and then think about an important question to answer with it. You have much greater chance of being published using the former approach, but the latter is definitely more fun.

7. *"If we knew what it was we were doing, it would not be called research, would it?"*

   Albert Einstein

8. *"When an elderly and distinguished scientist tells you that something is possible, he is most probably right. When he tells you that something is impossible, he is most probably wrong!"*

   Arthur Clarke

The Fight*

"Walk this way!"
Igor
Mel Brooks' "Young Frankenstein" (1974)

He had never met it.

He had never even seen its face in print or on holograph.

Nobody had.

That made it so much harder to fight for it. For the empty, meaningless, faceless creature. The beast. He was the one that had to shape it, give the meaning to it. He had to shape it and name it. He had to look for its parts in fields. Many vast, ragged, unwelcoming fields. And he had to find all parts... He understood it was okay if it would limp, or have different colored eyes, but it should have been functional, it had to help somebody. At the very least – him.

He was not alone, though. He had never been. For all this time, that became recoded in history as "The Fight," while known universe took its chance to expand for five more light years in radius, he felt the support of his master. Romeo of the Charlie clan from the black dwarf part of the common galaxy. Great Jedi master that used much different weapons than lightsabers for accomplishing his goals. It was due to Romeo that he adopted similar style of weaponry that seemed much too fruity for many. He consciously and unconcsiously took much from this powerful source and yet always remained hungry, wanting more. He always filled astonished by its quality and endlessness. Romeo was the one pushing him towards different fields and making sure he always did a thorough job in searching for the parts. For the functional parts of the beast.

There were other supporters, many other fighters for the greater causes than this one. One, the most distinguished among all was Mike Victor from the supernova part of the common galaxy. Mike has always been eager to help. Always prompt

*The acknowledgments for the inspiration for "The Fight" go to I. Asimov, F. P. Herbert Jr., O. S. Card, F. G. MacIntyre, A. C. Clarke and R. A. Heinlein.

and responsive. Always there to discuss different filling and shaping tactics. Always warning about the dangers of waking several fields simultaneously.[1] And more, naming of who would only confuse the reader. One can be said with high confidence: if you are reading this piece, there is a great chance you were one of those.

There were yet others who aided him in auxiliary, yet valuable, ways during the Fight. Alfa November, who constantly provided parts of other beasts for him to feed on, who generously donated much of his own powers for the cause of shaping the beast. Mike-Juliet Hotel, who took care of this power contribution to run smoothly. Hotel Papa, who ensured that he got the exposure necessary for the success. Echo Bravo, who graciously took care of his arrangements with the authorities at Hilbert Hotel, where he had to be accommodated for the duration of the Fight. Whiskey Charlie (not to be confused with the Charlie clan that Romeo was coming from) and Mike Romeo that made sure he was never hungry.

And there were two more. Two most significant helpers in his life. Sweet November, who had been there from the start. And his little Echo who had been around only the last half of the Fight. They were the ones giving the necessary (and sufficient) strength.

With the help of all of the people mentioned and implied here, the Fight resulted into the limping beast you are about to probe.

Zulu Bravo
May 2-22, 2010
College Park MD

"Wait! Where are you going?... I was going to make espresso!"
The Blindman
Mel Brooks' "Young Frankenstein" (1974)

[1] By then this was already possible, And even schoolchildren did not think of Tibbles not being able to be dead and alive at the same time, or Pixel not being able to walk through the walls.

- The problem [of the modern economic theory] seems to be embodied in what is an essential feature of a centuries-long tradition in economics, that of treating individuals as acting independently of each other.

Alan Kirman (Economic Journal 1989, 99:137)

Contents

Introduction

Economics is an exciting discipline. It studies broad range of phenomena that can be only loosely connected to each-other. This, in my opinion, is reflected in the multiplicity of definitions of the discipline. Perhaps the oldest definition of economics as a separate discipline is due to Adam Smith. In his "Wealth of Nations" Smith writes that

> *"[e]conomics is an enquiry into the nature and causes of wealth of nations."*
>
> Smith (1776/1974)

It is obvious that what Smith had in mind was the understanding of differences in living standards across countries/nations. For Smith, economics was defined by wealth, or welfare.

Since Smith economics has been evolving and reshaping. Therefore, new definitions have been suggested. Arguably the most famous definition is the one offered by Alfred Marshall. A passage from his "Principles of Economics" reads:

> *"Economics is a study of mankind in the ordinary business of life; it examines the part of individual and social actions which is most closely connected with the attainment and with use of material requisites of well being."*
>
> Marshall (1890)

The big leap from Smith to Marshall is the appearance of "individual and social actions" in the definition of economics. With this definition economics clearly falls under social sciences. Apparently, economists are ought to study (the part of) social actions.

A more concise definition has been offered by another prominent economist Lionel Robbins. He suggested, that

> *"[e]conomics [was] a science that studie[d] human behaviour as a relationship between given ends and scarce means which ha[d] alternative uses."*
>
> Robbins (1932)

Robins' words add another important key-word – "scarcity" (of resources) – to the definition, but they maintain Marshall's spirit: economics studies human behaviour!

Human behaviour can be analyzed on many different levels and in many different incarnations. If we avail to Smith's definition, the major subjects of economics are humans as actors who derive welfare. Wealth is good as long as humans can consume it. Thus, major subjects of economics are consumers. There are two more types of subjects of study of modern economics: firms and governments. Both of these types of enterprises are run by humans and, thus, their behaviour is a derivative of human behaviour. Actions of firms and governments only augment consumers' actions in creating welfare.

However, a typical firm, and even more a typical government, is usually much larger in size than a typical consumer. Therefore, their actions have stronger repercussions than those of a single consumer. This is perhaps what led the discipline to divert its course away from the classical definitions and to put much effort into the analysis of firms and governments instead of consumers. Another reason for deviating from the original path could be the fact that the objectives of firms and governments are clearly defined, in contrast, objectives of consumers can be somewhat obscure.

Behaviour of consumers has been analyzed from many different prospectives. Various disciplines have used various consumer behaviour models at different levels of analysis. Hansen (1972) offers an insightful classification of these models. According to him, the models can be divided in four groups. The first one collects psychological models, which deal with the consumers at the individual level. The second unites social-psychological models, which deal with the individual and her environment. The third – sociological models, which deal with the segments of society. The last one comprises anthropological models, which deal with complete societies. Economics, being a social science, is somewhere close to anthropological group. What we should aim at is understanding the behaviour of the consumer population as the unified system. This is crucial for deriving a useful policy advise.

Thus, of a central importance is an aggregate consumer behaviour. However, the society consists of heterogenous, interacting consumers. And, as Hansen (1972) points out, the further you go in modeling from psychological towards anthropological models the more important the aggregation issues become. To put it in other words, it becomes harder to derive the aggregate behaviour from the behaviour of single agents. But, in order to gain insights into the aggregate behaviour it is important to understand how consumers behave.

The accent of the present thesis is on two inherent features of consumer populations. These are the heterogeneity of consumers and the interaction among them. I argue that these two features, that have been long downplayed by the discipline, are in fact important for understanding, and therefore predicting, the behaviour of demand, which is the ultimate constraint on the production process.

Agent heterogeneity has been recognized as the major problem for deriving aggregate implications. An early solution to this problem was found in models with the representative agent. Since then these models have become the major workhorse of macroeconomics. It has become a common practice to use these models without deep considerations about the degree and the structure of agent heterogeneity. Interaction is meaningful only in case of population that is heterogenous in some respect. Therefore, excessive usage of representative agent framework induced the removal of interaction from research agendas for decades.

However, during the last decade things have been changing. The criticism of the modern state of economics has been rising. Various types of crises, which helped to demonstrate the poor accuracy of policy recommendations, have contributed to this process. Dissatisfaction with the current economics is so high that the year 2009 saw the establishment of the Institute for New Economic Thinking, that brought together many heavy-weight academicians and analysts in order to plot the new path for the development of research into economics. As noted during the inaugural conference of the institute on April 8 2010 at King's College, one of the three major issues the new establishment will puts its energy in finding alternatives to is the usage of representative agent framework in economics. The other two are the usage of rational expectations and efficient market hypotheses. It was also the last decade that saw the establishment of the Society for Economic Science with Heterogeneous Interacting Agents. The society that explicitly promotes the research into the implications of agent heterogeneity and interaction.

Although research into heterogeneous interacting systems gets encouraged and accumulates, it develops asymmetrically (just like the whole economics, as argued above): a bulk of it looks at "agents" as being firms. Differing practices and competences of firms, firm collaboration for R&D purposes, exchange or sharing of technical information are few of popular topics among the researchers in the field. Meanwhile research on consumer heterogeneity and interaction stays in minority. There still are numerous important questions on different topics currently not answered at a satisfactory level. For example, on the topic of consumer choice and demand dynamics, how strong is the offsetting force of interaction on consumer heterogeneity? Does everyone converge to the same decision, or is there a heterogeneity in the long run? Do answers to these questions depend on the nature of interaction, or on the structure of the interaction network? Does knowing consumer interaction network help in identifying the demand structure and location?

As interaction plays an important role in the information diffusion it is also important from the advertising prospective. In this respect it is important to know how the intensity/amount of optimal advertising depends on the characteristics of social network? Does the consumer interaction imply the modification of optimal advertising strategies? How can firms save on advertising costs by using viral marketing? What types of consumers have to be initially seeded with information so that the rate of information diffusion is maximized? This is important also for the innovation strategies: firms are concerned whether the diffusion of information about their products will be wide enough for it to cover the whole target consumer base, and whether the adoption rate of their product will be high enough for them to recover R&D costs in a desired timeframe. As consumer heterogeneity and interaction affect firm behaviour, they also become important from the policy maker's prospective: affecting consumer interaction processes might become an important tool for industrial policies. For example, facilitation of consumer interaction by organizing special fora might be the cheapest way to stimulate higher R&D expenditures and healthier competition.

The present thesis contributes to answering few of these questions. But before going into its original contribution, in the remaining of introduction I survey the existing literature on consumer behaviour, heterogeneity and interaction. I raise the issues of aggregation of this behaviour, from which most of the relevant economic work suffers, and analyze two modern solutions to it. I also outline the rest of the thesis, which mainly consists of applications of agent-based modeling techniques to consumer behaviour in different contexts.

Consumer choice process

As consumers, we make decisions every day: what to eat, what to wear, what to read and so on. These are types of decisions that generate incentives for firms to produce and create welfare. These are central decisions for study of economics if you synthesize definitions presented above. Therefore, we should understand and be able to model them well. Several related disciplines have gone much deeper into the study of consumers than economics. Two that stand out are marketing and psychology. First I present findings from these disciplines and later I try to understand how economics has been using these findings for achieving its goals.

Everyone agrees that consumers use information in order to make decisions. Understanding how consumers collect and use this information is of central importance. There are two views on how consumers use information. One argues that consumers use information directly, like standard economic tool of utility maximization would suggest. The other view suggests that consumers use some kind of algorithm for deducing a specific action plan from different pieces of information. Extensive evidence from marketing and psychology supports the latter. For example, in marketing Bettman (1971) directly identifies heuristics and further analyses the question whether they are stable or change over time, Leong (1993) finds that in the process of certain types of products (e.g. less durable) heuristics are extensively used. On the other hand, in psychology Chaiken (1980) finds that if the involvement rate of the consumer in the choice process is lower than a certain threshold she is highly likely to use the heuristics instead of trying to solve any kind of optimization problem. Shirai and Meyer (1997) find that heuristics are used by more experienced consumers as well as by novices, but that complexity of heuristics reduces with the increase of the consumer experience. In short, the long tradition of consumer research in these fields resulted in conclusion that consumers extensively use heuristics in the choice process. As a result, collected information affects consumer action plans only through the filter of heuristics. Heuristics are simplified rules for handling the available information.

Thus, our decisions are based on behavioural rules. Consequently, it is important to understand whether these rules are stable over time (i.e. stored) or whether they are constructed on the spot as the consumer faces the problem. Bettman and Zins (1977) contrast these two views about the heuristics used in the choice process and try to identify which of them is correct by using think aloud protocols with the large sample of grocery shoppers. The conclusion they arrive to is that constructive heuristics are usually used when consumers have little experience or/and when the choice is difficult. Stored heuristics are used when consumers are already experts or the choice problem is trivial. This is intuitive, as in the first two cases (unexperienced consumers and difficult choice) consumers lack high quality ("internal") information about a product. In the latter two cases "internal" information is either present, or not required.

A more recent study by Dahr et al. (2000) is concerned with the level of sophistication of heuristics used in consumer durable purchases. This study finds that people use quite complicated heuristics, which are continuously updated and modified, thus, these heuristics incorporate systematic information processing. Shirai and Meyer (1997) analyze the dynamics of the sophistication level of heuristics along the consumer expertise on the example of a mountain bike and conclude that heuristic rules get simpler as consumers acquire more experience. Similar results are presented by Coupey (1994) who finds that simple heuristics are used only by experienced consumers. So, it seems that people use complicated constructive heuristics, which get simplified and turn into stored rules as consumers acquire experience.

There is an alternative view on consumer choice process which synthesizes the two views described earlier. This view acknowledges the existence of heuristics, but argues that they might be replaced by more thorough decision process in certain cases. The "functional perspective" formulated by Chaiken (1980) decomposes purchase concerns into two: reliability of the product and price. It claims that simple heuristics will only be used if price concerns are predominant (usually with cheap, nondurable goods). If reliability concerns are predominant people will engage in more systematic processing of information.

Why do people use heuristics? The main justification can be found in psychology (e.g. Chaiken, 1980). Cognitive psychology claims that people are not able to process all the available information systematically. The costs of information processing become so high that people are forced to search for simple methods to handle the information. An alternative explanation is McGuire's (1969) principle of "lazy organism." Here the message recipient tries to utilize the information about the information source in order to evaluate the reliability of information itself, rather than systematically processing the content of information received. For example, people usually easily accept the piece of information that is coming from their friends to be true, compared to the information coming from advertising. In this case, agents appeal on the intention of the source of information to influence a receiver (Hansen, 1972). To cut the long story short, the use of heuristics is the saving of energy and is extensively done by consumers where appropriate.

Consumers in all the essays in this thesis (except essay 4) use some sort of heuristics. In the second essay product valuations are updated and purchase decisions are made based on the information accumulated by consumers. In the third essay decisions are made based on the level of consumption skills of each product. These two essays do not use conventional utility maximization toolbox. Here I rather model deeper constructs – valuations. Although essay 5 employes the utility maximization, the process is augmented by behavioural rules in order to describe the dynamics of demand.

Consumer heterogeneity

We have established that consumers are not using information about available options directly, but rather through behavioural rules. Then it becomes important whether these rules are similar across agents, which would simplify the modeling task, or they are different from each-other.

Early marketing academicians argued that decisions made by consumers were heavily influenced by their perception of the environment and the understanding of separate events. For example, Bauer (1960) claimed that what mattered in consumer choice was the "perceived risk" of purchase. Later, Holbrook and Hirschman (1982) emphasized the role of the emotional state of the consumer when making the decision. In psychology, Bartlet (1932) claimed the individuality of the decisions much before that. He argued that memory retrieval was based on the individual's understanding of the event. More recently, Freimuth (1992) has emphasized the role of the fit of one's perception about the world with the reality.

These considerations about individuality are nicely summed up in the discussion about the uniqueness of the consumer's environment (e.g. Simonson and Tversky, 1992; Payne et al., 1992). This approach emphasizes the role of consumer's individual understanding of the surrounding environment, and argues that decisions are contingent upon these individual perceptions (e.g. Payne, 1982; Moorthy et al., 1997).

Larrick (1993) gives a useful distinction between two general groups of existing theories of consumer behaviour. The author distinguishes between the approach that gives universal explanations to consumer behaviour and the one that gives individual-difference explanations. Cardinal utility theory (Bernoulli, 1954) and prospect theory (Kahneman and Tversky, 1979) belong to the first group. These are psychological theories that explain the behaviour with general/universal behavioural laws. These laws apply to all humans to the same extent, thus the theories do not account for individual differences. The second group of theories does take into account individual differences. In this group the major theory is the expected utility theory (Friedman and Savage, 1948). This theory is based on the individual differences in feelings about the risk associated with an action. Larrick (1993) classifies two more theories in this group. They are Atkinson's (1957) theory of differences in motivation and Lopes' (1987) two-factor model of risk preference.

There is a crucial difference in the usage of these two groups of theories by economists for purposes of describing the behaviour of the group of consumers. The second group requires much greater detail in modeling that easily makes models non-tractable. Besides, modeling different behavioural rules for different consumers can be used only for a very narrow purpose of describing one particular population, but no other. Theories in the first group demonstrate that there are certain behavioural rules that are universal across the population, that consumers behave similarly. However, this does not mean that consumers are homogenous. As emphasized earlier, even if two consumers live in the same environment, they perceive it differently. Therefore, their observed behaviour will be different even though they use the same universal behavioural rules. All the essays collected in this thesis belong to the first class of models. Consumers are homogenous in respect to their behavioural rules.

Even if the behavioural rules are similar, consumers can still be heterogenous in (at least) three aspects. These are information, consumption skill, and taste profiles. Not only these profiles can be different across consumers, they can also change over time. New bits of information can become available to a whole population or to a part of consumers. Consumer skills can be accumulated through experience or learned through interaction with friends. Even tastes can change due to external influences (e.g. fads). Examples of applications of changes in all three of these profiles are presented in this thesis: changes in information profile in essay 2, in skill profile in essay 3 and in taste profile in essay 5.

Changes in information, skill or taste profiles can be due to two types of sources: internal sources or external sources. Internal sources are usually modeled through experience. These are well-known mechanisms of learning curves (Ebbinghaus, 1885/1964) or learning by doing (Arrow, 1962a). External sources are usually modeled as broadcasting, advertising, or face-to-face interactions among consumers.

Interaction

In general we can distinguish between two types of interactions: global and local. Interaction is global when forces generated by consumers affect every consumer. Interaction is local when different forces affect different consumers. This distinction is best demonstrated on the example of information. If information about the product, say an impression of a consumer, is somehow broadcasted to all the consumers, the interaction is global. If this information is communicated only to a part of the society, the interaction is local. Local interactions must have temporal stability of links. In other words, the same consumers should share information with the same subset of population over time. These linkages are usually determined by the geographical location or social status.

Overwhelming evidence (including much of the studies surveyed hereafter) suggests that consumers interact intensively. Besides peer-to-peer consumer interactions there are other sources providing valuable information to consumers (e.g. mass media). Next to interaction there are internal sources of information acquisition. Thus, the question arrises: which are the sources that consumers utilize the most?

The literature in psychology (e.g. Fazio and Zanna, 1978) and marketing (e.g. Smith and Swinyard, 1983) suggests that weights put on the information obtained through the immediate experience are higher than those of information obtained through any type of external source (Muthukrishnan, 1995). This is not surprising as internal information is usually perceived as being of higher quality. An important question remains: which of the external sources are most valuable for consumers?

Marketing academicians have been interested in this question since 1960s. Bennett and Mandell (1969) and Duncan and Olshavsky (1982) report the evidence on the ranking of external information sources by the intensity of their usage. They consider consumer reports and dealer visits to be the most widely used source of information. Next come experts' and friends' opinions (Beatty and Smith, 1987). Advertising and mass media score considerably lower on these scales. These rankings are by frequency of usage and do not tell much about which sources do consumers really trust. Psychology offers more relevant results. Myers and Robertson (1972) claim that consumers judge about information sources according to the source's intention to influence the information receiver. On this scale personal communication among consumers scores the highest (Hansen, 1972). These studies arrive to such a conclusion after carefully analyzing the opinion leadership process in small groups of people. More recent studies provide more support to earlier findings of consumers putting high weights on information received through social interaction (e.g. Gershoff and Johar, 2006).

So, interpersonal relations among consumers (communication with friends) is a very important information source. But how do consumers choose whom to receive the information from among all of their friends? Simon (1958) and Cyert and March (1963) suggest that consumers follow the least effort rule. This means that consumers are minimizing the overall information search effort given the trade-off between search costs and the quality of information received. More specifically, consumers are asking friends who are the most knowledgeable about a certain good and who will give them a piece of information that needs least complementation from other information sources. This is done so that they can terminate their search effort right away. More recent consumer behaviour models (e.g. Gershoff et al., 2001) suggest that, for the sake of search effort minimization it is optimal to acquire information from a friend that can rank all the alternatives and pass this ranking to the inquirer. The first approach I utilize in essay three, the second one – in essay two.

We have seen that people receive information from their friends, but what type of information do they receive? Is it detailed information about different characteristics of a good? Or, is it a general evaluation of a good? This obviously depends on what exactly people remember about the goods they consume. The early work in psychology on this topic (e.g. Johnson and Russo, 1978) has presented evidence that people remember a general impression about the goods more easily. Even if immediately after consumption people remember exact features of the product, as time goes by they tend to forget the characteristics of the good and the information distills to general impression about the good. Biehal and Chakravarti (1983) arrive to the similar conclusion by examining a behaviour of large sample of consumers of pocket calculators in experimental setting. The group of consumers who was allowed to choose one out of four brands and consume on the first stage of experiment as an alternative to learn about the separate characteristics of different brands memorized only a general impression about the product they have consumed. Then, it seems straight forward that most of the information received by the agents through the interpersonal communication is in form of a general evaluations of the goods that their friends have consumed.

Later, cognitive psychologists working on memory retrieval found another explanation to the phenomenon. Wyer and Srull (1989) have presented the model of impression formation. In this model, a consumer gets information about the product piece by piece. Every piece of information is transfered into a special 'bin' in the memory of an agent upon its arrival. And the information retrieval from this memory bin works in a way that the later you have put a piece of information in the bin the easier it is to recall it. The general impression about a certain product is formed based on all the information available in the bin and is used as a separate piece of information. Of course, it is stored on the top of the bin and, thus, it is easiest to retrieve. According to this theory, it is more likely that consumers will share general impressions about the goods. Park and Wyer (1993) have empirically examined the theory by studying the information remembering about the TV sets by a large sample of agents. They split the sample in two subsamples assigning slightly different tasks to each of two. They have concluded that general impressions are not always formed in minds of people. They are only formed if there is a purpose to do so. But when they are formed, indeed they are the easiest pieces of information to retrieve and communicate.

Thus, there is an extensive evidence of local interaction in consumer behaviour. The second and the third essays in the thesis use these findings for modeling local interactions among consumers. The second essay models local information transfer process where consumers are passing their general impressions about products to their neighbours. The third essay models the local interaction process for skill sharing. In this case not general impressions (information), but rather very specific pieces of knowledge are passed from consumer to consumer. In both cases, consumers also have internal information/knowledge sources in line to the findings reported above.

In both of the essays the social network structure is assumed to be regular and periodic. In particular, I assume that consumers are located on equidistant locations on the circumference of a circle and that each of them interacts with the same number of immediate neighbours from her left and right. The reason for assuming periodicity (a circle) is that this setup provides analytical convenience as we do not have to deal with boundary conditions of the model. The reason for assuming regularity (the same number of social contacts) is that what I am interested is the effects of interaction as such. Introducing non-regular architecture of social network brings the additional agent heterogeneity and increases the burden of model analysis. It also helps in presenting and understanding of the micro results of the models. Changing these two assumptions are likely to change the results of the model quantitatively, but I believe they are not crucial for the qualitative results. The reason is that the major parameter that I discuss in these essays is the intensity of interaction, which has the same, straightforward meaning in any type of the network. Of course, if the social network is not regular or periodic consumer roles in information transmission process are heterogenous (different agents will have different degrees of centrality). This factor is expected to moderate the effects of model parameters. However, heterogeneity in agent centrality will not change the fact that high interaction intensity implies faster information diffusion.

In today's age of information society it is not hard to imagine that consumer interaction becomes global. Every piece of information can be easily shared through internet, blogs and social networking sites. However, global interactions can have many more incarnations. Most acts of consumption are pretty conspicuous (Veblen, 1899). And information about the most popular items in consumption baskets spread easily. This contributes to the rise of "fad" behaviour: popular products attract more buyers (e.g. Young, 1993; Bernheim, 1994). Kim and Chung (1997) empirically find the positive effect of popularity. Raj (1985) finds popularity of the products is further reinforced by the larger share of loyal buyers.

Network effects can be viewed as a type of global interaction. In this case, for one reason or another, the number of market share of products contributes towards its desirability (Mayer and Sinai, 2003). This contribution can be either positive (e.g. Rysman, 2004) or negative (e.g. Mechoulan, 2007).

Essays 1 and 5 in this thesis model global interactions among consumers. Essay 1 describes the process when agents in the whole population are randomly matched for learning purposes. In this case, interaction is global as consumption skills do not go through any temporarily stable pass. In essay 5 I model industries with positive network effects. Here popularity of a product generates positive feedbacks and allows products to fight for being the standard in the industry.

Aggregation issue and solutions

As we have seen consumers are heterogenous and they interact through various channels outside the marketplace. Even if they have similar behavioural rules they live in different environments, or at least they perceive environment differently. Due to the immense importance of the aggregate demand for major economic processes (e.g. innovation, growth) aggregation of consumer behaviour becomes of central importance.

For a long period the main instrument for deriving the aggregate behaviour has been a construct named "the representative agent." The representative agent is an adaptation of the Alfred Marshall's (1890) idea of the representative firm. The idea behind the representative agent is that the aggregate behaviour of collection of agents can be described by the behaviour of one agent. Then, aggregate demand can be easily derived by analyzing the economy from the prospective of the representative agent. This mathematical tool has dominated the discipline for decades.

However, the representative agent had its critics. Geweke (1985) and Schlee (2001) present examples when the representative agent predicts the aggregate behaviour of a collection of agents incorrectly. Kirman (1992) reviews many similar examples and argues strongly against the use of representative agents. Besides higher accuracy of results in case of heterogenous agent models, it turns out that these models are superior in explaining various empirically observable phenomena in economics (e.g. Hansen and Singelton, 1983; Cooper and Haltiwanger, 1990).

Next to the criticism demonstrating obvious flaws in the methodology based on the representative agent, there is another big push for the move towards models with heterogenous agents for solving aggregation issues. This push is coming from the desire to incorporate non-market interactions into economic models. As I have pointed out earlier, consumers interact with each other outside the market place and this interaction is bound to have an effect on aggregate outcomes. Models with the representative agent are not suited for analysis of interactions.

The are two major approaches to the analysis of social interactions. These are game theoretic and complex systems approaches. There are many game theoretic setups in non-market environments. A part of these games explicitly models social interactions between agents. The most prominent example of this type of games are coordination games. These are the games with multiple pure strategy Nash equilibria that are Pareto rankable. Examples of such games include "stag hunt" and the "battle of sexes" (Cooper, 1998). The major problem is equilibrium selection. In simple forms of coordination games rationally playing agents choose Pareto inferior outcome (Gibbons, 1992). Prospects of overcoming the coordination failure with additional social interaction has recently been examined experimentally (Manski, 2000). Other studies have explored the possibility of achieving non-equilibrium socially efficient outcomes (Fehr and Gächter, 2000). In these researches social interaction seems to ease the achievement of better equilibrium as it simplifies the coordination necessary to punish free riders. Other examples of modeling social interactions from game theoretic prospective include minority games (Arthur, 1994) and household behaviour games (McElroy, 1990).

Social networks represent the most prominent application of complex systems approach to the analysis of interactions. Modern day graphical toolbox for the analysis of social networks was first developed in sociology (Coleman, 1964; Holland and Leinhardt, 1970). Recently the methodology is becoming popular in business and management (e.g. Reingen et al., 1984; Brass et al., 1998). This methodology has great potential for empirical analysis (Burt, 2001), but deriving exact implications for different phenomena using social networks as the basis for agent interaction is somewhat complicated. The reason is that, like in any other complex system, small perturbations in the topology of networks lead to large differences in outcomes. However, with modern computational capabilities the applications of complex systems theories to economic problems becomes viable. Computational tools like agent-based modeling can contribute great deal to the understanding of important economic processes (Fagiolo et al., 2007).

Besides social networks there is one other application of complex systems widely used to model social interactions. This is network effects (Katz and Shapiro, 1994). Network effects are used to model global interactions.[2] The importance of network effects for social behaviour is best demonstrated by the models of informational cascades (Bikhchandani et al., 1992). In these models of sequential decision-making bandwagon (or herd) behaviour can emerge as an outcome (Banerjee, 1992).

Game theoretic and complex systems approaches to social interactions are usually used in combination in economics. For example, a great deal of effort has gone into analyzing network formation through game theoretic setups. This line of research has been concerned with the formation of social (Bala and Goyal, 2000; Ehrhardt et al., 2007), as well as collaborative (Goyal and Moraga-Gonzalez, 2001) and buyer-seller networks (Wang and Watts, 2006). In addition, large games where agents are matched not randomly but through networks have also been developed. Here networks on which games are played can be exogenously given (Schelling, 1969; Epstein and Axtel, 1996) or endogenously formed (Fagiolo, 2005; Fagiolo and Valente, 2005).

Essays in this thesis use complex systems tools for modeling social interactions. Essays 2 and 3 use social networks as channels of communication among consumers, while essay 5 uses network effects to model social influence on individual decisions.

Structure of the thesis

The thesis is organized in two parts. The first part collects three contributions to the modeling of consumer behaviour, while the second one unites two contribu-

[2] Although there are exceptions when interactions modeled through network effects are not entirely global (e.g. Akerlof 1997; Cowan et al., 2004).

tions to the modeling of firm behaviour in environments with heterogenous and interacting consumers.

The first essay proposes a simple setup for the study of consumer behaviour. Consumer choices depend on consumption skill levels with respect to individual products. If the agent has good consumer skills for a product she can utilize it better, thus can derive higher level of utility from consuming it. Consumer skills are dynamic. They increase through experience as well as through interaction with other, better skilled consumers. I examine whether the representative agent can describe the evolution of the average skill level in this simple economy. I conduct two exercises that analyze two forces of skill augmentation (experience and interaction) separately. It turns out, that the representative agent fails in both cases. Consumer behaviour in this essay is similar to their behaviour in the rest of the thesis. Therefore, the failure of the representative agent in this instance motivates the usage of heterogenous agent models in later essays.

In the second essay I present the discrete choice model of consumption. In this case consumers are making decisions based on the information they have about each of the alternatives. Similar to the first essay, there are two sources of information: consumption and communication. Communication takes the form of local interaction: agents talk to their friends and in the process convey their general impressions about available products. The network describing social linkages is not disconnected. I analyze whether this kind of decentralized consumer choice and communication can result in a stable distribution of behaviour over the social network. The result is that for large number of initial conditions clustering in economic behaviour emerges as an equilibrium outcome. I also examine the out-of-equilibrium behaviour of the model which turns out to be accurately predictable given the equilibrium results.

The third essay goes back to the choice process based on consumer skills, while maintaining the local nature of consumer interaction. In this case consumers share consumption skills. Besides interaction, accumulation of skills through experience is in place. Products in this setup have two characteristics: quality and the level of user-friendliness. I consider producers that are able to influence consumption decisions at the onset of the industry through their efforts in advertising. I analyze the producer incentives to advertise products on a duopolistic market. I find that the relation between the quality of the product and returns to advertising is not monotonic as suggested by earlier studies. Rather, returns have and inverted U shape, given the characteristics of the competing product.

In the fourth essay I temporarily abandon the interaction among consumers. I describe the consumer side relatively schematically. Consumer behaviour is modeled through well-known toolbox of utility maximization. Of crucial importance in this setup is the consumer taste heterogeneity. The main concern turns on the producer innovation incentives. Given the behaviour of consumers, firms engage in an uncertain process of research and development with the aim to create a new product that would appeal to a certain group of consumers. The industry is organized through submarkets of products over which consumer tastes are defined. Starting from firms with equal prospects, I analyze the development of different industries. I find that fat-tailed firm size distributions emerge as equilibrium outcomes for a large variety of industries. The thickness of the tail depends on only one industry-level parameter of the model.

The fifth essay extends the model from the fourth to include consumer interaction. In this case interaction is modeled as a network effect and takes a global form. Consumers adapt their tastes in order to take into account the popularity of the submarket on which the product are traded. The aim is to understand how innovation incentives change due to consumer interaction. I show that the economies with the same potential support higher levels of innovation in case of consumer interaction. This is true not only in equilibrium but also during out-of-equilibrium dynamics, although in this case innovation frequency is increased only for certain types of industries. I also analyze the distribution of these additional innovation incentives across different firms. It turns out that depending on the characteristics of industry, innovation incentives might increase either for larger or for smaller firms.

Part I

Contributions to Modeling of Consumer Behaviour

Essay 1

On the Performance of the Representative Agent During Out-of-Equilibrium Dynamics*

Abstract. This essay contributes to the discussion about the representative agent's ability to characterize the collection of agents. I perform two exercises in context of a fairly general multi-product multi-agent environment and show that in order the representative agent to be able to describe the out-of-equilibrium dynamics of the society additional assumptions are required. It is established that this feature is not specific to the economies with interacting agents.

1.1 Introduction

Whether the behaviour of one individual can accurately describe the behaviour of the collection of agents is the topic of decades-long debate in economics. Some people believe the approximation is useful, even if not very accurate, while others think approximation is fundamentally misleading. This central player, on which the bulk of the discipline is built, is called the "representative agent."

This essay aims to contribute to the debate by demonstrating that the representative agent can well describe the economic system only in special cases. Most of the previous effort has gone into the analysis of the representative agent's powers in equilibrium (Kirman, 1989; 1992). Here I am concerned with her ability to describe the out-of-equilibrium behaviour of dynamic economy. This essay, to some

*I am grateful to Robin Cowan and Bulat Sanditov for their insightful comments on earlier versions of this essay.

extent, motivates the remaining of the thesis methodologically by demonstrating that representative agent models cannot be used for studying the economies that are under discussion in the remaining of the thesis. Thus the use of heterogenous models is justified.

To analyse the problem I set up a very simple behavioural model and study its behaviour out-of-equilibrium. The important feature of the model is that agents are learning while the model is out of equilibrium. Learning is the force that drives the economy to a time-invariant state, that I use as the definition of equilibrium. The equilibrium of this model is not particularly interesting: it is characterized by agent homogeneity. Thus we can predict, without any mathematical analysis, that in this equilibrium state the representative agent will be powerful. This is done intensionally so that we can analyse the out-of-equilibrium performance of the representative that is powerful in equilibrium.

In my simple model agents are consumers, and they do only one thing: upgrade their consumption skills. I consider two variants. In the first, consumers are learning from own consumption history. This is the case where consumers are heterogenous but do not interact with each other. In the second variant, consumers are interacting with each other and sharing their skills. Here heterogenous consumers interact with each other. I demonstrate that in both cases one needs certain strong functional requirements in order the representative agent to be able to describe the dynamics of the society out-of-equilibrium.

The remainder of the essay is organized as follows. Section 1.2 reviews the literature on the debate about the performance of the representative agent. Section 1.3 presents the setup of the economy for the exercises. Section 1.4 outlines the results. Section 1.5 concludes.

1.2 The Representative Agent and Its Problems

The predecessor of the modern representative agent is, without a doubt, Marshall's (1890) "representative firm." Marshall used this notion in order to speculate about the industry-level supply curve. He recognized that firms in the industry might have differing costs. This created problems as it brought a confusion into which of those costs determined the unique selling price in the industry. Marshall invented the notion of the representative firm in order to resolve this problem. By his definition it was a "fairly" successful firm that was managed with "normal" ability. Hence, Marshall seems not to have had in mind the representative firm to be some kind of a statistical construct.

Marshall's notion of the representative firm was met with the fierce criticism from colleagues (e.g. Robbins, 1928) and it was ultimately abandoned. However, the idea kept on living and found a revival as the central player of the modern neoclassical macroeconomics. But today's version is somewhat different from Marshall's original idea. Today the representative agent is a statistical construct. It is an "average" agent of the economy. Although I was not able to pin down an exact definition of the modern representative agent anywhere in the literature, after the analysis of numerous uses of the concept I feel confident to state that the representative agent is the construct that describes the "average" values of agent-specific variable distributions.

The definition of the representative agent in the previous paragraph seems fairly clear. However, there are two problems with it. Firstly, it is not obvious what does the "average" exactly mean? There are two different answers to this question in the literature: the simple average of the distribution (mean) (e.g. Schlee, 2001) or the average weighted with some other characteristic[1] (e.g. Constantinides and Duffie, 1996).[2] Although having two definitions of the average does not create a conceptual problem (after all you use the one most relevant in your case) it does create the problem of the comparability of results.

Even if one ignores the problem of *which average*, a more important problem remains. This problem is: the average of *which variable* is the representative agent supposed to describe? This is an acute problem especially in models where an agent has more than one function. Geweke (1985) constructs a model where firms are playing three roles: they produce, demand production factors and supply products. He demonstrates that in this economy there will be three different representative agents: the average producer, the average supplier and the firm placing average demand on production factors. In all three cases representatives are different from each other. Modelers working with the representative agent somehow choose one of the definitions (the choice criterion is rarely presented), which again creates the problem of compatibility.

Besides problems with the definition of the representative agent, the decades-long literature has brought up three major topics of discussion. Firstly, whether the representative agent can be constructed at all? In this respect the fundamental contribution is due Rubinstein (1974) who provided fairly general sufficient conditions under which a representative agent can be constructed. However, as Kirman (1992) argues there can be cases where these kind of "representatives" will not accurately represent the society. In particular, one can construct a situation in which this kind of representative agent prefers the first option over the second, while *every* member of the society prefers the second over the first. Then we have an issue of whether the representative agents that can be constructed are useful.

Secondly, imagine we construct the agent that closely describes the behaviour of the average of some variable. It might well be that behavioural rules of this agent are different from the behavioural rules of the society that it represents. Caballero (1992) points out that the representative agent framework has "blured the distinction between statements that are valid at the individual level and those that apply to the aggregate." Schlee (2001) calls this the problem of "normative representativeness" of the representative agent. By his definition, representative

[1] For example income in case of consumers or size in case of firms.

[2] In principle there is a third one, which is the usage of the sum instead of the average (e.g. Gollier and Zeckhauser, 2005), but this approach is equivalent to the usage of the simple average.

agent is a "normative representative" of the society if, and only if, his behavioural rules are not different from those of agents comprising the society.

Therefore, we can distinguish two essential features of the representative agent. One is "normativeness" in Schlee (2001) sense, the other is "functionality," which means that the construct is able to describe the averages of important variables. Then, we can have two types of representative agents: "normative" and "functional." These two constructs need not coincide. Maliar and Maliar (2003) present a recent example of the economy where the functional representative agent is not the normative representative of the society.

Researchers feel comfortable working with models using functional (but not normative) representative (e.g. Gollier and Zeckhauser, 2005), as the ultimate goal of describing average behaviour is accomplished. However, there is a fundamental problem with this approach which makes it redundant. If every agent in the economy has one behavioural rule and the representative agent (although functional) has a different behavioural rule (is a non-normative representative) – this rule has to be constructed. In order to construct the behavioural mechanism of the representative agent one has to solve the model without the representative agent. If the solution without the representative agent is obtained there is no use in further reformulation of the problem as the problem with the representative agent. Thus, I believe that the only useful representative agent has to be "functional" as well as "normative" representative of the society.

The third, relatively minor, problem with the representative agent framework is that when using it one can not address distributional issues (Stoker, 1986). As the representative agent is a single individual, distributions of the characteristics over the population can not be observed or analysed. More recently Caselli and Ventura (2000) proposed a methodology for constructing the representative agent that will be able to describe the distribution of characteristics in the society. However, again, the agent that they use is not a normative representative of the society. I think these kinds of framework are useful, however they have to be called something like single-agent (rather than representative agent) economies.

As noted above "functional" and "normative" representatives might be different from each-other. Of course there are exceptional cases when these two constructs do coincide. Two such examples are presented in Huffman (1986) and Salyer (1988). Then, it becomes important to understand generally when this is the case. If one demonstrates that the problem at hand is in this category, there will be no further objections for using representative agent methodology.[3]

All the works referred to above are concerned with the ability of the represen-

[3] However, there is a substantial body of literature demonstrating that certain empirically validated features of the economy cannot be obtained with representative agent frameworks. Examples of these kind of work are Grossman and Shiller (1981) who find that the rate of risk aversion implied by the representative agent model are implausibly high, and Mehra and Prescott (1985) who obtain implausibly high risk premium. Several studies contrasting representative and heterogenous agent models have confidently rejected representative agent models (e.g. Hansen and Singelton, 1983; Cooper and Haltiwanger, 1990).

tative agent to describe the equilibrium properties of the economy. There is no contribution analysing the out-of-equilibrium performance of the representative agent. This seems reasonable as equilibrium is of central importance in modern economics. However, in recent years there has been substantial increase in research into out-of-equilibrium behaviour. In fact in all the remaining essays of this thesis out-of-equilibrium dynamics plays a crucial role. Thus for the choice of the methodology it is important to evaluate the performance of the representative agent in types of environments that will be discussed in the remaining of the thesis.

Closest to the exercise in this essay comes the research by Geweke (1985) who analyses the performance of the representative agent in evaluating the effect of the policy change. This is studying the adjustment process to the new policy, which is similar to the analysis of the out-of-equilibrium dynamics, as the original arrangement is not equilibrium after the policy change. Geweke (1985) constructs three different types of representative agents and demonstrates that not one of them can predict the average effect of the policy change correctly.

What I do in this essay is somewhat similar: I start the economy from the point out-of-equilibrium and study the transition to equilibrium. But I analyse a simpler and more general setup. My agents play a single role in the economy, and they are homogenous at all times with respect to all characteristics except the one under discussion. Therefore, there is no need for several representative agents (which was the case in Geweke, 1985). Agents do one simple task in my setup: they learn. They can learn through own experience, or they can learn through interaction with others. I analyse both of these mechanisms separately: first learning without interaction, next learning through interaction. This way we can detect whether problems with the representative agent are not specific to any of these two environments.

1.3 Framework

The setup of the economy that I use is extremely simple. There is continuum of agents indexed by i conveniently placed on interval $[0;1]$. Think of them as consumers that have to make decision about which product to consume in a multi-product environment. Time is discrete. Number of products is finite. Each consumer has a certain skill level s for each product, such that $s_t(i)$ represents the skill level of the consumer i at time t for the product under discussion. Besides these skill levels consumers are absolutely identical (e.g. no differences in income).

Laws of motion of skill levels are identical for every consumer. The skill levels are positive, non-decreasing and bounded from above. Their dynamics is such that they approach the bound asymptotically. This bound is homogenous across agents. Therefore, if we define the equilibrium as the time invariant state of the economy, all equilibria will be characterized by the homogeneity of agents. Of course, in this case the representative agent will have no problem describing the equilibrium. However, my concern is with the out-of-equilibrium dynamics.

In this environment we can define the representative agent.

Definition 1.1. *The representative agent is an agent that behaves identically to every other agent in the economy and describes the evolution of the average skill level from the initial state to the equilibrium.*

From the definition above it is obvious that we are looking for the representative agent that is "functional" and "normative" at the same time. As argued earlier, construction of any other type of representative agent would not make much sense. If the representative agent, as described in definition 1.1, exists it can be further used for the analysis. If it does not exist, in order to construct another representative agent (that would clearly have different behavioural rules from every consumer in this economy) we have to first solve for the evolution path of the average skill level and only after that create the representative agent that would mimic it. Even if one ignores the possibility of multiple behavioural rules being able to replicate the exact same route of average skill evolution, the use of such representative agent will be pretty limited: analysis of a slightly different problem might call for a different representative of the economy.

Next I analyse two problems of skill evolution in turn and evaluate the performance of the representative agent. More precisely I ask the question whether the representative agent (as defined in definition 1.1) can be constructed in an arbitrary economy.

1.4 Results

1.4.1 Learning by Consuming

Consider the situation where in order to acquire consumption skills agents have to consume products. This construct is similar to the notion of learning by doing (Arrow, 1962a). With time as agents are consuming products they learn and ultimately (as $t \to \infty$) everybody's skill level for every product converges to the maximum. To simplify the presentation assume that there are several products on the market for already long enough so that every agent already has maximum skill levels for all of them. Then the economy is in equilibrium and all the product choices are time invariant.

Now consider a new product entering the market. At time zero consumer skills for the new product will be distributed over the population. This distribution is described by $s_0(i)$. Assume, reasonably, that consumer skills are increasing (at a decreasing rate, although this is not crucial for the results) in number of times the consumer has used the product. Without loss of generality I assume the skill levels are bounded by unity. Then we can write the law of motion of the skill level for the new product for agent i

$$s^n(i) = 1 - (1 - s_0(i))\, e^{-\delta n}, \tag{1.1}$$

where n is how many times an agent i has consumed this particular product and δ is the speed of learning.

From equation (1.1) we can derive the change in skill levels between two consumptions

$$s^{n+1}(i) - s^n(i) = \gamma\left(1 - s^n(i)\right), \tag{1.2}$$

where $\gamma = 1 - e^{-\delta}$.

The skill level dynamics for each of the agents depend on her product choices. In order to derive the expected path for the skills, assume $P(x)$ is the probability that the consumer will choose an entrant product if her skill level is x. $P(\cdot)$ is a continuous, increasing function: the more skills an agent has for the new product, the higher the chance that she will purchase it. Naturally, probabilities for every agent sum to one across all the products at every time period.

To study the skill level development we need the expected law of motion for the skill of every agent. The agent i with skill level $s_t(i)$ consumes the product at time t with probability $P\left(s_t(i)\right)$. Therefore, with the same probability agent i's skills increase by $\gamma(1 - s_t(i))$, but with probability $1 - P\left(s_t(i)\right)$ they remain at the same level. With this logic, we can write down the expected law of motion:

$$s_{t+1}(i) = s_t(i) + \gamma\left(1 - s_t(i)\right) P\left(s_t(i)\right). \tag{1.3}$$

Starting from the initial distribution, using equation (1.3), we can calculate the expected path of the average skill for the new product in the economy.

Notice that the equation (1.3) is agent-specific and if one assumes that the initial distribution is not given by Dirac's delta function, we will have heterogeneity in skill development paths across population. In this context the question arises: do we need to track the skill level of every agent or can we construct a representative agent which can describe the dynamics of the society? According to the definition, the representative agent has to have the (expected) average skill level in the economy if she wants to claim to be the representative.

Thus, we know that my representative agent has the average skill level in the initial skill distribution: $s_0^r = \bar{s}_0 = \int\limits_0^1 s_0(i)di$. Then, her skill level next period should be

$$s_1^r = \bar{s}_0 + \gamma\left(1 - \bar{s}_0\right) P(\bar{s}_0). \tag{1.4}$$

The average skill level in the economy at time one is

$$\bar{s}_1 = \int\limits_0^1 s_1(i)di = \int\limits_0^1 \left(s_0(i) + \gamma\left(1 - s_0(i)\right) P\left(s_0(i)\right)\right) di, \tag{1.5}$$

which can be re-written as

$$\bar{s}_1 = \bar{s}_0 + \gamma \int\limits_0^1 (1 - s_0(i))P(s_0(i))di. \tag{1.6}$$

From equations (1.4) and (1.6) it is obvious that for $s_1^r = \bar{s}_1$ to hold we need

$$(1 - \bar{s}_0)\, P\left(\bar{s}_0\right) = \int\limits_0^1 (1 - s_0(i))P(s_0(i))di. \tag{1.7}$$

Equality (1.7) is the requirement for my representative agent at time zero to be representative at time one. But the representative agent has to be able to describe the average skill level in the economy at every time period. Thus, we need the following general equality to hold

$$(1 - \bar{s}_t)\, P\left(\bar{s}_t\right) = \int\limits_0^1 (1 - s_t(i))P(s_t(i))di, \tag{1.8}$$

$\forall t$.

Now we have all the material in order to be able to answer the question: can the representative agent be constructed in this economy? Normative representative will only be functional if $P(\cdot)$ satisfies the general requirement (1.8) $\forall t$, when law of motion of $s_t(i)$ is described by (1.3). Unfortunately, I was not able to derive the general form of $P(\cdot)$ that would satisfy the requirement. However, there are a few special cases when we can demonstrate that representative can be constructed. Three of these cases are presented here.

Special Cases: There are three straightforward examples that one can construct when the representative consumer will describe the economy precisely. First of them is when γ is zero. This is easy to infer from equation (1.4). However, this case means that there is no learning in the economy, thus no dynamics of skill levels. Consequently the economy is already in equilibrium. And by construction, the representative agent is perfectly able to describe the economy. Another example is when $s_0(i) = s$, $\forall i$. This case implies that there is no heterogeneity across population (initial distribution is Dirac's delta). In this case my best guess of the skill level development is the same for every agent, of course including the representative. Therefore, representative agent is powerful. This case is not interesting as it does not permit any heterogeneity, and consequently, it does not require the representative agent. An example involving heterogeneity and learning with powerful representative is the case when the function $P(\cdot)$ is constant. However, this case completely undermines the model's central assumption that consumer choices are determined by the skill levels for products. Here, although there is skill level dynamics, there is no dynamics in purchases. Thus, the skill levels themselves become irrelevant for describing agent behaviour.

Any other instance, where the will exist the representative agent combining features of “functionality” and “normativeness,” will involve restrictions on the function $P(\cdot)$. Without these restrictions the representative agent at one time period will not be representative of the economy in consecutive periods. Thus, in general, the representative agent that can describe the out-of-equilibrium dynamics of this simple economy can not be constructed in an arbitrary economy (an economy with arbitrary $P(\cdot)$).

1.4.2 Skill Sharing

Now consider the case when there is an interaction among agents. I do not assume anymore that consumers learn by consuming. Rather, I assume that they learn by interaction with each other. In this environment multiplicity of products does not play a role, thus I consider there is only one product in the economy. The probability density of the population skill for this product at time zero is described by $f_0(s)$. The interaction structure is as follows. Every period, every agent (i) randomly picks one other agent (j) from the population. If j has higher skill level than i, i learns from j. As a result her skills increase by $\mu(s(j) - s(i))$, where μ $(\in [0;1])$ is the speed of learning. If j's skill level is lower than that of i's, i cannot learn anything.

There are several things to note in this scheme. The agent who has the highest skill level in initial distribution (denote it with s'), cannot learn anything from anybody in the population. Everyone else's skill level approaches her skill level as $t \to \infty$. Here again, I am interested in out-of-equilibrium dynamics, and whether the representative agent can predict how the average skill level approaches its time-invariant value.

Given the description of the model we can specify the expected law of motion for skill level of an agent i

$$s_{t+1}(i) = s_t(i) + \mu \int_{s_t(i)}^{s'} f_t(s)(s - s_t(i))ds, \tag{1.9}$$

Due to the fact that $f_t(s)$ is the probability density function of skill distribution at time t, the second summand in the right hand side gives the expected increase in the skill level.

Our representative agent at time zero has to have a skill level equal to the average of the society

$$s_0^r = \bar{s}_0 = \int_0^1 s_0(i)di. \tag{1.10}$$

Then, her skill level at time one will be

$$s_1^r = \bar{s}_0 + \mu \int_{\bar{s}(0)}^{s'} f_0(s)(s - \bar{s}_0)ds. \tag{1.11}$$

We can also calculate the average skill level in the economy at time one

$$\bar{s}_1 = \int_0^1 s_1(i)di = \int_0^1 \left(s_0(i) + \mu \int_{s_0(i)}^{s'} f_0(s)\,(s - s_0(i))\,ds \right) di, \tag{1.12}$$

that can be re-written as

$$\bar{s}_1 = \bar{s}_0 + \mu \int_0^1 \int_{s_0(i)}^{s'} f_0(s)\,(s - s_0(i))\,dsdi. \tag{1.13}$$

By looking at equations (1.11) and (1.13) for the equality $s_1^r = \bar{s}_1$ to hold I need

$$\int_{\bar{s}_0}^{s'} f_0(s)\,(s - \bar{s}_0)ds = \int_0^1 \int_{s_0(i)}^{s'} f_0(s)\,(s - s_0(i))\,dsdi. \tag{1.14}$$

Or, in general

$$\int_{\bar{s}_t}^{s'} f_t(s)(s - \bar{s}_t)ds = \int_0^1 \int_{s_t(i)}^{s'} f_t(s)\,(s - s_t(i))\,dsdi, \tag{1.15}$$

$\forall t$.

Now, again, we have all the material to be able to answer whether the proper representative agent can be created in this economy. And, again, the normative representative will only be functional if $f_t(\cdot)$ satisfies the general requirement (1.15) $\forall t$, when law of motion of $s_t(i)$ is described by (1.9). Unfortunately, similarly to the case presented in section 1.4.1 it is not possible to derive the general form of $f_t(\cdot)$ that would satisfy the requirement. However, few special cases can be constructed.

Special Cases: Examples of the cases when the representative agent will be able to describe the economy accurately are when $\mu = 0$ and/or when $s_0(i) = s = s_0^r$, $\forall i$. Both of these cases imply that the economy starts off at equilibrium, thus, the performance of the representative agent out-of-equilibrium cannot be evaluated.

Any other case in which the representative agent of the economy can be constructed would require restrictions on the skill probability density functions $f_t(\cdot)$,

$\forall t$. Thus, again, in general the representative agent that could describe the out-of-equilibrium dynamics of the simple economy cannot be constructed for arbitrary $f_t(\cdot)$.

1.5 Conclusion

In this essay I have discussed two types of behaviour of collection of heterogeneous agents. I have demonstrated that even if the representative agent is powerful in equilibrium she might not be able to describe the out-of-equilibrium dynamics of the society. In particular, I have shown that the representative agent at time t will not be representative at time $t+1$ if she follows the same behavioural rules as every other agent in the economy (i.e. is the "normative representative" of the society). By analysing two setups, one with interaction, another without, I have also established that this phenomenon is not specific to models with interaction.

The definition of the representative agent that I have used combines the "functional representativeness" with the "normative representativeness." The first feature means that the representative agent should be able to describe the development of the average value of the interesting variable(s). The second feature implies that the representative has to follow the same behavioural rules that are followed by every other agent in the economy. Of course, in these simple economies one can construct an agent that will mimic the dynamics of averages of interesting variables in the economy (be only functional representative). But as I have demonstrated, this kind of agent will have different behaviour from every agent in the economy (she will not be a normative representatives). Besides unfairness of the title "representative" in this case, there are severe problems with this kind of central agents.

Firstly, in order to construct them one has to solve the heterogenous agent model completely. For example in my case, I had to obtain the expected path of the average skill level in the economy before thinking of constructing the representative agent that could describe it. Thus, the usefulness of this kind of an agent is questionable. Secondly, even if we have the full solution there can be multiple protocols that can replicate the solution. Because in case of "non-normative" representativeness the behaviour of actual agents is not the restriction for the behaviour of the representative, we are confronted with the choice problem. Of course the choice can be aided by some other criteria, for example there features of the economy that we want to describe with our representative agent. However, in this case again in order to make a selection we have to have the full solution of that part of the model too. Adding dimensions to the model might narrow down the list of representative agents, but it definitely does not make choice easier. And finally, even if we somehow make a choice, we cannot use the chosen representative to analyse any aspect of the economy that has not been included in the choice criteria (thus has not been solved for). We simply cannot be sure that the representative agent is the representative for that part of the model.

Essay 2

Inertia, Interaction and Clustering in Demand*

Abstract. In this essay I present a discrete choice model of consumption that incorporates two empirically validated aspects of consumer behaviour: inertia in consumption and interaction among consumers. I specify the interaction structure as a regular lattice with consumers interacting only with immediate neighbours. I investigate the equilibrium behaviour of the resulting system and show analytically that for a large range of initial conditions clustering in economic behaviour emerges and persists indefinitely. Short-run behaviour of the model is investigated numerically. This exercise indicates that equilibrium properties of the system can predict short-run behaviour of the model quite accurately.

2.1 Introduction

One of the challenges in modelling consumer behaviour lies on the observation that consumption is in many ways a social activity. This has been observed both in the context of bandwagon behaviour or conspicuous consumption (Leibenstein, 1950; Smith, 1776/1974; Veblen, 1899), but also in the context of learning to consume (Witt, 2001). Consumers often face incomplete information both about what is available, and how to get the most out of the goods they consume. In both cases agents rely on friends and neighbours as sources of information. In addition, though, consumers appear to form habits (Guariglia and Rossi, 2002), depending on rules of thumb and past behaviour to guide future choices.

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In this essay I model the dynamics of individual consumer behaviour and analyze its implications for the distribution of the demand for goods over a social space. There are empirical studies of this issue, reporting on the impact of social space on demand (e.g. Birke and Swann, 2006), but those papers tend to explain their results entirely through network externalities. In this essay I use more general constructs and show that the network structure of social interactions can be reflected in demand. Key to the consumer's decision-making, and thus to the dynamics of demand, is the consumer's on-going, or repeated evaluation of her alternatives. In my model valuations are based on two things: the consumer's own consumption history; and the consumer's neighbours' consumption histories. Consumers repeatedly decide which products to buy, and learning by consuming increases the future valuation of a product for a consumer. Consumers also routinely interact with their neighbours and exchange information about products on the market. Based on these two distinct information streams consumers update their valuations for each product and in response (possibly) change their behaviour.

From this starting point I model two aspects of consumer behaviour: inertia in consumption; and local influence of peers through interaction. The model can be interpreted in two ways.[1] One is to say that there is an imperfect informational structure in the economy and consumers are aware of that fact. They try to reduce uncertainty in the decision process by using two sources of information (Jacoby et al., 1994). One is the information they receive through own experience. As consumers have the better understanding of the value of the goods they have already consumed, consuming the same good avoids possible disappointment. The other is the information they receive from their social networks about the available goods. Information gathered from "friends" can similarly reduce the risk of disappointment.

The second interpretation of the two parts of consumption dynamics would be that people form habits for the goods they consume, but that there is also an interdependence in the utilities of nearby consumers. With regard to habit formation, I assume that in the consumption process a consumer forms some special skills for using the product and as a result receives higher utility every time she consumes the same product. Interdependencies arise because people get higher utility if their consumption bundles are similar to those of their neighbours. This is similar to the effect of a "peer group" discussed by Bourdieu (1984) and addressed in a formal model of consumption by Cowan et al. (1997).

I analyze the long-run (equilibrium) dynamics of a population of consumers subject to these two forces and show that spatial clustering in economic behaviour emerges as a stable, long run equilibrium pattern for a large set of initial conditions. Additionally though, analysis of the short-run behaviour indicates that equilibrium properties of present complex system can predict the short-run dynamics of the model quite accurately.

[1] Throughout the paper I use these two interpretations interchangeably.

The remainder of the essay is organized as follows. The first section briefly reviews related literature. The second section presents the model. In the third section I present the analysis of the long- and short-run behaviour of the model. The fourth section presents one particular extension to the model. The last section concludes.

2.2 Literature

Of central interest here is information. Economists have long known that an assumption of perfect information was a strong one, and it has been relaxed in a variety of contexts. Early theoretical relaxations of the perfect information structure were applied to market organization (see Rothschild, 1973 for a survey), credit rationing (e.g. Jaffee and Russell, 1976; Stiglitz and Weiss, 1981) as well as to a general consumer behaviour (e.g. Nelson, 1970). But recently, the consumer's lack of and need for different types of information have been studied more closely. For example, uncertainty about prices is discussed by Galeotti (2004), who examines the welfare implications of search costs when the distribution of prices is unknown. Similarly to the model presented in this essay, Samuelson (2004) models interdependency among consumers. There, consumers observe the actions of relatively successful consumers and use that information to impute which actions are likely to be good for themselves. In that model consumers are differentially successful, and information flow consists only of agents observing each others' actions. By contrast, in the model I develop below, agents are successful in optimizing at each step (given their current information), and have a richer information flow in that they pass to each other opinions about the values of all goods. Another important distinction between these two models is that Samuelson models the decision of "how much" to consume, while I model the decision "what" to consume. My consumers act in complex environment and use information communicated to them in deciding on which product to buy, unlike Samuelson's consumers who are deciding on the consumption budget based on the information available to them.

In any situation in which information is imperfect, information acquisition can be valuable. Research in both marketing and psychology stresses the immense importance of information collection for the consumer decision process (Bettman, 1971), in that it permits consumers to make better (in the sense of utility-increasing) choices. A large empirical literature shows that people tend to collect information through many different sources, such as the media, sellers or other consumers. However, in a seminal work, Hansen (1972) shows that information received from peers through social networks, is the dominant source of knowledge about goods considering both the information's reliability and its ability to affect the receiver. Thus, if one wants to understand the influence of external information on consumer decisions, it seems reasonable to concentrate on information coming from peers, rather than from any other external source. So, while not denying the importance of other sources of more general external information, in this essay I focus on socially localized peer effects.

The view that agents use both internal and external sources of information in making decisions is not new in economics and has been applied to related fields. For example, information cascade models (two canonical papers being Banerjee, 1992 and Bikhchandani et al., 1992) consider a population of agents, sequentially making decisions using both public and private information. The interest there is the conditions under which public information can overwhelm private, and the possibility of that creating a sub-optimal (aggregate) outcome. In a certain sense, the model presented in this essay is also an information cascade model, but it differs from the conventional models in two ways. First, consumers make repeated choices. This allows us to study the effects of the change in internal information driven by the consumption process itself. Second, cascade models agents receive information only about other agents' actions. In my model they receive (somewhat subjective, though higher bandwidth) information not only about the current actions of others, but also about available options not chosen. Thus, information about *any* particular option, even if it is not being taken by any agent, can form a cascade as it flows within the population. Agents use information (which may or may not be cascading) about each of the options to make a choice for one of them.

A second literature that relates closely has to do with habit formation. Habit formation in consumption was discussed early by Duesenberry (1949) and Brown (1952). These approaches are concerned with the formation of the general habit of consuming, meaning that people form habits to consume in general, rather than the habit of consuming some particular good. More recently, habit formation has been rigorously incorporated into consumer decision models by Abel (1990), Constantinides (1990) and others. These models have been extensively used to explain equity premium and risk-free rate puzzles (Constanides, 1990; Otrok et al., 2002) as well as the stylized fact that higher growth rates lead to higher savings rates (Carroll et al., 2000). By contrast to the formation of the general habit of consuming the present essay is concerned with habit formation for isolated products. These are the habits that people develop themselves through the consumption process.

One good, and well-studied example would be eating habits. Smith (2004), for example, drawing on empirical literature from a wide variety of behavioural and hard sciences, shows that people acquire very strong eating habits that persist for a long period. He refers here not to the habit of eating generally, but to habits regarding particular foods. He also shows that people are more likely to consume products that they see other people consuming, which is a basic assumption of my model.

The marketing and psychology literatures referred to above have shown that friends and neighbours are an important source of information. The "externalities in consumption" literature has a similar feature in that externalities are often seen as (spatially or socially) limited in scope. The possibility that interactions can be localized in various dimensions has been raised in other contexts. Scheinkman and Woodford (1994), or Weisbusch and Battiston (2007), for example, examine non-market interactions between consumers and producers; Eshel et al. (1996), and Cowan et al. (1997) look at interactions among consumers. In general, interactions generate feedback loops that affect the decisions of the economic agents. But as noted by Glaeser and Scheinkman (2000) the structure of those interactions can make significant differences both for the sorts of equilibria that emerge and for the dynamics leading to them. In particular, they show that when interactions are local the economy generates richer dynamic possibilities, having multiple equilibria and the possibility of moving from one equilibrium to another.

More contextualized work on interactions shows that they can explain certain interesting phenomena in economics or other social sciences, such as the standardization process (e.g. Arthur, 1989; Cowan, 1991; Eshel et al., 1998), waves in consumption across the population classes (Cowan et al., 2004), or contagious justice (Alexander and Skyrms, 1999).

2.3 The model

The model I develop here can be seen as a repeated discrete choice model in which consumers' evaluations of goods are determined by internal and external information sources.

Consider an economy inhabited by a large, finite number (S) of agents, indexed by s. Each is a single consumer faced with the same fixed, finite set of substitute goods, indexed by n. In each period, each consumer consumes one unit of one good. The consumption choice is based on the consumer's "valuation" of the goods.

The valuation a consumer ascribes to a given good is the maximum price she is willing to pay for it. Using very basic consumer theory, the utility a consumer derives from consuming a good will be the difference between its valuation and price that she pays. I define $v^s_{n;t}$ as the net valuation consumer s ascribes to good n at time period t.

I adopt a standard discrete choice approach (Andersen et al., 1992) and assume that each consumer buys one and only one product each time period. Under this assumption the utility of individual agent can be written as

$$U^s_t = v^s_{n^*,t}, \tag{2.1}$$

where n^* is the good consumed by consumer s in period t. I assume that consumers are unable to deliberately manipulate the choices of their neighbours, and so do not choose "strategically", but rather simply maximize instantaneous utility. Under this setup, utility maximization implies that in each period the consumer chooses $n^{s*}_t = \arg\max(v^s_{n;t})$.

What I seek to model here is the dynamics of product purchases as they respond to changes in the valuations of consumers of the goods available on the market. Following the discussion in the introduction to the essay, I assume that valuation is derived from information of two types: internal and external. So we can write:

$$v^s_{n;t} = f(x^s_{n;t}, y^s_{n;t}), \tag{2.2}$$

where $x^s_{n;t}$ is determined by own consumption history, and $y^s_{n;t}$ by the consumption history of other members of the same social group as consumer s.

Both parts of the valuation are subject to change over time: $x_{n;t}$ is subject to change due to habit formation (which results in inertia in consumption) and $y_{n;t}$ is subject to change due to local interaction (because of information exchange or network externalities). Assume that $f(\cdot)$ is additive, and write the dynamics of v^s_n as[2]

$$\Delta v^s_n = \Delta x^s_n + \Delta y^s_n. \tag{2.3}$$

To model interaction among consumers I assume that every consumer has a fixed social location and a fixed neighbourhood. A neighbourhood is the set ($\mathcal{H}^s$) of other agents with whom an agent (s) interacts directly. In this context, interaction is tantamount to information exchange. Each information exchange consists of two agents revealing to each other their private evaluations of each of the goods. The information revealed is assumed to be "convincing" in the sense that the post-exchange valuations of each of the two agents partially converge. Hence, this exchange process can be expressed simply in terms of the dynamics of beliefs of a single agent, s, following her exchanges with all of her neighbours, i:

$$\Delta y^s_n = \sum_{i \in \mathcal{H}^s} \frac{\mu}{|\mathcal{H}^s|}(v^i_n - v^s_n), \tag{2.4}$$

where $|\mathcal{H}^s|$ is the cardinality of the set $\mathcal{H}^s$ (number of neighbours of agent s), and μ ($\in [0, 1]$) is the intensity of interaction. I assume that all products are substitutes and there are no *ex ante* systematic differences among consumers, so interaction intensity is the same across all the goods and agents.

For concreteness, assume that consumers are located on a one-dimensional, regular, periodic lattice such that the distance between any two agents corresponds to the social distance between them, and the distance between immediate neighbours is constant across all the population. In this case I can define the neighbourhood of an agent ($\mathcal{H}^s$) simply by specifying the number of agents (H^s) with whom this consumer interacts on the left and on the right. Then $|\mathcal{H}^s| = 2H^s$.

If we assume neighbourhood size to be equal across the population, that is $H^s = H \; \forall s$, we can write

[2]From here on I drop the time subscript, but it should be borne in mind that the model is inherently dynamic and time is implicitly present in all the variables used throughout the essay.

$$\Delta y_n^s = \frac{\mu}{2H} \sum_{h=1}^{H} \left[(v_n^{s+h} - v_n^s) + (v_n^{s-h} - v_n^s) \right], \tag{2.5}$$

where s can be interpreted as a "serial number" of an agent, or her address (consequently, $s+1$ and $s-1$ are her immediate neighbours to the right and left respectively).

Re-arranging, (2.5) can be rewritten as

$$\Delta y_n^s = \frac{\mu}{2H} \left[\sum_{h=1}^{H} (v_n^{s+h} + v_n^{s-h}) - 2H v_n^s \right]. \tag{2.6}$$

Valuations are also influenced by habit formation.[3] Habits are formed only for goods that are consumed. Thus,$\Delta\ x_n^s$ is equal to zero for the goods that are not consumed in a given period and is equal to some positive value for the good that has been consumed:

$$\Delta x_n^s = \begin{cases} \zeta & \text{if } \ n = n_t^{s*} \\ 0 & \text{otherwise,} \end{cases} \tag{2.7}$$

where ζ (> 0) is a constant and n_t^{s*} is a product that agent s has purchased in period t.

To summarize the model we can make explicit the sequence of consumers' actions. At the start of each period every agent decides which good to consume. After purchase she consumes it and forms habits for it. At the end of the period each agent meets all of her neighbours and passes to them all the information (that is, her valuations of all goods) that she possesses. Based on the information communicated to them by neighbours all agents adjust their valuations of all goods.

I am interested in whether this kind of behaviour has implications for the social geography of demand; more precisely, whether any specific patterns emerge in the long-run. Essentially I ask whether one can determine anything about the consumption basket of a consumer by looking at the consumption baskets of her neighbours.

2.4 Equilibrium analysis

In this section I analyse the long-run equilibria of the model. It is not possible to solve the model as presented in section 2.3, so in the process of solution I make two modifications. First, I assume that the habit formation process can be well-approximated (at least in the region of interest) by a linear function. Second, I re-write the model as continuous in time and space.

[3] I should once again make clear, that by habit formation I mean individual habit formation for a single product, rather than formation of a general habit to consume.

Linearization. Above, equation (2.7) shows habit formation: a consumer forms habits only for the good he consumes, and the effect on her valuation takes place in discrete jumps. This describes a path dependent process. This is problematic, as analysis of the system at any point in time requires analyst's knowledge of the whole history of the system. However, employing a standard way of modeling expected product choices, allows us to approximate the dynamics of (2.7) with a Markov process (Andersen et al., 1992). I model the choice of the consumers as a conventional discrete choice, where it is based on probabilities: agent s chooses good n with probability $p^s_{n;t}$ at time period t. In this case, the law of motion in equation (2.7) becomes:

$$\Delta x^s_n = \begin{cases} \zeta & \text{with probability } \; p^s_{n;t} \\ 0 & \text{with probability } \; 1 - p^s_{n;t}. \end{cases} \tag{2.8}$$

Further, $p^s_{n;t}$ will be a function of the vector of valuations for the agent s at period t. Thus we can write $p^s_{n;t} = p_n(\mathbf{V}^s_t)$, where $\mathbf{V}^s_t$ is the vector of valuations. Then the expected change in valuation due to habit formation, $x^s_{n;t}$, can be written as:

$$E\left(\Delta x^s_n\right) = \zeta p_n(\mathbf{V}^s_t). \tag{2.9}$$

The choice probability for a product n depends on valuations of all the products. However, it is reasonable to assume that the contribution of changes in valuations of products other than n are of second order significance. This is easy to see if we consider the effects of an increase in the valuation of good n. This will increase its purchase probability byΔ p_n. This will also decrease the purchase probabilities of all the other products, each byΔ p_j. As probabilities are normalized it should be the case that $|\Delta p_n| = \sum\limits_{j \neq n} |\Delta p_j|$. If I have relatively large number of products in the economy, it will in general be true thatΔ $p_n \gg \Delta p_j$, $\forall j \neq n$. Thus a change in the valuation of one good will cause the change in its purchase probability. It will also cause the changes in purchase probabilities of other goods, but the size of each of these changes will be considerably smaller. Therefore I impose a restriction on my probability function: it has to satisfy the following relation

$$\left|\frac{\partial p_n}{\partial v_n}\right| \gg \left|\frac{\partial p_n}{\partial v_j}\right|, \tag{2.10}$$

$\forall j \neq n$.

Consider the linearization of function $p_n(\mathbf{V}^s_t)$. If the requirement (2.10) is satisfied, as a first approximation, I can disregard the effects of lower orders of magnitude and write a linearized function as $p_n(\mathbf{V}^s_t) \approx \gamma v^s_{n;t}$. This permits us to write the expected change in $x^s_{n;t}$ as

$$\Delta x^s_n = \alpha v^s_n, \tag{2.11}$$

where α $(= \gamma\zeta)$ can be interpreted as the rate of habit formation.[4]

Substituting equation (2.11), allows us to write the system specified in section 2.3 as

$$\Delta v_n^s = \alpha v_n^s + \frac{\mu}{2H}\left[\sum_{h=1}^{H}(v_n^{s+h} + v_n^{s-h}) - 2Hv_n^s\right]. \tag{2.12}$$

From (2.12) it is clear that the law of motion of valuation for every good for any agent depends on the agent's own valuation of that good, and on the valuations of the agent's neighbours of that same good.

For the demonstration of the solution to the system, assume that each agent has exactly two neighbours ($H = 1$), and that there are only two goods available on the market ($N = 2$).[5] The model reduces to a system of S pairs of equations of the form

$$\Delta v_1^s = \alpha v_1^s + \frac{\mu}{2}\left(v_1^{s+1} + v_1^{s-1} - 2v_1^s\right) \tag{2.13}$$

$$\Delta v_2^s = \alpha v_2^s + \frac{\mu}{2}\left(v_2^{s+1} + v_2^{s-1} - 2v_2^s\right), \tag{2.14}$$

where $s = 1, 2, 3, \ldots, S$.

Continuous time and space. I seek to obtain the solution to the system given by (2.13) - (2.14). In the two-good system, what drives the dynamics at any point in time is the difference in the probabilities that each of the goods is chosen (by each consumer). We can thus re-write the system in terms of the difference in valuations of the two goods. Define the valuation difference $z^s = v_1^s - v_2^s$ and rewrite the system (2.13)-(2.14) as

$$\Delta z^s = \alpha z^s + \frac{\mu}{2}\left(z^{s+1} + z^{s-1} - 2z^s\right). \tag{2.15}$$

Next step is to approximate the discrete system (2.15) with its continuous counterpart. To do this I define a new variable δ which is the distance between two neighbouring consumers on the circle. Using δ we can rewrite equation (2.15) in continuous time and space

$$\frac{\partial z(s)}{\partial t} = \alpha z(s) + \frac{\mu}{2}\left(z(s+\delta) + z(s-\delta) - 2z(s)\right). \tag{2.16}$$

Then we can make a second order Taylor approximation in space around s for the terms $z(s+\delta)$ and $z(s-\delta)$. This will result in

$$z(s+\delta) \approx z(s) + \delta\frac{\partial z(s)}{\partial s} + \frac{\delta^2}{2}\frac{\partial^2 z(s)}{\partial s^2} \tag{2.17}$$

[4] In what follows I drop the expectation sign, although it should be remembered that all the discussion in this section is about the expected values of the variables.

[5] Both of these assumptions are relaxed at a later stage in propositions 2.5 and 2.6.

and

$$z(s-\delta) \approx z(s) - \delta\frac{\partial z(s)}{\partial s} + \frac{\delta^2}{2}\frac{\partial^2 z(s)}{\partial s^2}. \tag{2.18}$$

Substituting equations (2.17) and (2.18) into equation (2.16) collapses our system into one partial differential equation

$$\frac{\partial z}{\partial t} = \alpha z + \tilde{\mu}\frac{\partial^2 z}{\partial s^2}, \tag{2.19}$$

where $\tilde{\mu} = \mu\delta^2/2$.

In the following section I investigate the long run equilibrium behaviour of the system (2.19). Some insights to the behaviour of the model in the short-run will be provided in section 2.5.

2.4.1 Distribution of behaviour over space

It simplifies the analysis to separate the dynamics of $z(s;t)$ into the dynamics of the average over the population $\bar{z}(t)$; and the dynamics of the deviations from this average $\tilde{z}(s;t) = z(s;t) - \bar{z}(t)$.

Proposition 2.1. *The cross-agent average of valuation-differences ($\bar{z}_t$) evolves according to*

$$\bar{z}(t) = e^{\alpha t}\bar{z}(0).$$

Proof. In the continuous case the average over space can be defined as $\bar{z} = (1/S)\int_0^S z ds$. This implies that

$$\frac{\partial \bar{z}}{\partial t} = \frac{1}{S}\int_0^S \frac{\partial z}{\partial t} ds.$$

Then, using equation (2.19) we can write

$$\frac{\partial \bar{z}}{\partial t} = \alpha\frac{1}{S}\int_0^S z ds + \tilde{\mu}\frac{1}{S}\int_0^S \frac{\partial^2 z}{\partial s^2} ds. \tag{2.20}$$

As space in our system is a periodic lattice the second summand in equation (2.20) is zero.[6] Then, using the definition of average again we can write equation (2.20) as

[6] To see more easily why the second summand is zero, one can discuss the discrete case and thus use equation (2.15) instead of equation (2.19). In the discrete case the second summand is $\sum_s \left((z^{s+1} - z^s) - (z^s - z^{s-1})\right)$. As consumers are indexed by s around a circle, it is obvious that this sum is zero.

$$\frac{\partial \bar{z}}{\partial t} = \alpha \bar{z}. \tag{2.21}$$

This is an ordinary differential equation with the solution described in the proposition. □

Proposition 2.2. *With time, deviations of valuation-differences ($\tilde{z}_t^s$) in system (2.19), converge to*

$$\tilde{z}(s;t) = e^{\sigma t} \cos\left(k \frac{2\pi}{l} s\right) \tilde{z}(0;0),$$

where l is the length of the circle on which consumers are placed, while σ is the amplitude growth rate and $k (\in \mathbb{Z}_+)$ is the frequency of the sinusoid $\tilde{z}$.[7]

Proof. Omitted. □

The comprehensive proof of this proposition can be found in Turing (1952); here I give the basic intuition. The general solution to differential equations of this type can be represented as the (possibly infinite) sum of exponential functions of the form Ae^{bt}, where A and b are (possibly complex) coefficients. The real part of each summand in the solution can be represented as the dynamic sinusoid (in our case around the lattice on which consumers are located). The real part of each b will be the growth rate of the amplitude of the corresponding sinusoid. As a result, as $t \to \infty$ one summand will dominate all the others. This will be the term with the largest real part of b. Consequently the dynamics of the solution will converge to one sinusoid.

Proposition 2.3. *The amplitude growth rate of the dominant sinusoid of system (2.19) is*

$$\sigma = \alpha - \tilde{\mu} k^2 \left(\frac{2\pi}{l}\right)^2.$$

Proof. From proposition 2.1 and 2.2, I know that

$$z(s;t) = e^{\alpha t} \bar{z}(0) + e^{\sigma t} \cos\left(k \frac{2\pi}{l} s\right) \tilde{z}(0;0).$$

Substituting this into equation (2.19) and noticing that

$$\partial^2 \cos(\beta x)/\partial x^2 = -\beta^2 \cos(\beta x),$$

allows us to solve for σ. □

[7] Note that as consumers are located on a periodic lattice, the identity of agent zero is arbitrary, and thus can be placed anywhere on the circle. To write down proposition 2.2 I have set label 0 such that $s_0 = \underset{x \in [0, \frac{l}{k}]}{\arg\max} \cos\left(k \frac{2\pi}{l} x\right)$, which effectively means that I label agents such that the sinusoid identified in proposition 2.2 reaches its maximum at agent number zero.

Propositions 2.1 through 2.3 fully characterizes the solution to the system (2.19). Following subsections investigate the implications of the solution.

2.4.2 Temporal stability of clustering

In order describe the behaviour of the model in equilibrium we have to combine the results of propositions 2.1 and 2.2. For making interpretations of the results transparent, it is useful to go back to the discrete space and time. Thus, I move back to treat s as the serial number of an agent.[8] This makes $\tilde{\mu} = \mu/2$ and $l = S$. In this case we can write the complete solution to my system as

$$z_t^s = e^{\alpha t}\bar{z}_0 + e^{\sigma t}\cos\left(k\frac{2\pi}{S}s\right)\tilde{z}_0^0, \tag{2.22}$$

where

$$\sigma = \alpha - 2\mu\frac{\pi^2}{S^2}k^2. \tag{2.23}$$

Equation (2.22) determines the value of the difference in valuations (z) for every agent for every $t \gg 0$. The first summand in the equation describes the evolution of the average valuation difference, while the second summand describes the deviation from this average. As we can notice, the distribution of z along the circle has the form of a wave in space around the average, which points to the fact that in some neighbourhoods $\tilde{z}$ is positive, while in some other neighbourhoods it is negative (this is easiest so see if we assume that $\bar{z}_0 = 0$). This means that some neighbourhoods are more likely to buy one product, while some other neighbourhoods are more likely to buy the other with a gradual transition between them. Thus the general result is that the clustering in demand is an emergent property of my system.[9]

Our concern in this section is whether any observed clustering is persistent over time. Consider the case where $\bar{z}_t \neq 0,\ \exists t \geq 0$. That is, at some point in time one of the products is perceived as superior on average. In this case propositions 2.1 and 2.2 have an important corollary:

Corollary 2.1. *If $\exists t$ such that $\bar{z}_t \neq 0$, then as $t \to \infty$, $\exists i$ such that $v_i^s > v_j^s$ $\forall s$, where $i \neq j$ and thus in equilibrium there will be one cluster of size S in the economy.*

Proof. Consider the situation when $\bar{z}_t > 0$. Define $z^{min} \equiv \min_s (z^s)$ as the valuation difference of an agent with the lowest z.

[8] This effectively means that I fix $\delta = 1$. This move does not undermine the results of propositions 2.1 through 2.3. Moving back to consumer addresses is convenient for relating parameters in the solution to the parameters of the model.

[9] Our model can be applied to any type of economic behaviour that involves the choice among exclusive options at a constant cost. Thus clustering in this system will be a property of not only demand but of any similar economic activity.

Case 1: $z^{min} > 0$. This implies that $\forall s\ z^s > 0$, thus there is one cluster of size S. This is a stable pattern as both forces (interaction and habit formation) work to reinforce it.

Case 2: $z^{min} < 0$. In this case some of the consumers prefer the relatively "inferior" product.

Case 2a: $\sigma < 0$. Proposition 2.2 tells us that if $\sigma < 0$, with time, the amplitude of the wave goes to zero, which implies that $\forall s\ z^s = \bar{z}$. This, together with proposition 2.1, results in $z^s > 0\ \forall s$ as $t \to \infty$.

Case 2b: $\sigma > 0$. From proposition 2.2 we know that the amplitude of the wave around the average increases at rate σ. At the same time, proposition 2.1 suggests that the average over agents of the valuation-difference rises at the rate α. Therefore z^{min} is rising at the rate $\alpha - \sigma$. Equation (2.23) establishes that this rate is positive.[10] $\alpha - \sigma > 0$ ensures that as $t \to \infty$, $z^{min} > 0$. $z^{min} > 0$ implies that $\forall s$ $z^s > 0$. Thus case 2b with certainty collapses into case 1 at some point in time.

These intuitions hold for the situation when $\bar{z}_t < 0$.[11] □

In relation to market structure, we can have another corollary:

Corollary 2.2. *If* $\exists t$ *such that* $\bar{z}_t \neq 0$*, as* $t \to \infty$*,* $\exists i$ *such that* $v_i^s - v_j^s \to \infty\ \forall s$*, where* $i \neq j$ *and in equilibrium everybody will purchase only one of the products.*

Proof. Proof of corollary 2.1 directly implies not only that $v_i^s > v_j^s\ \forall s$ in equilibrium, but also that $v_i^s - v_j^s \to \infty$, which on its own implies that as long as the choice probability function is a positive monotonic mapping of valuations to choice probabilities, the probability of any agent purchasing product i converges to 1. □

Thus, $\bar{z}_t \neq 0$ is a relatively trivial case, and implies that ultimately only one product is consumed in the population, no matter the dynamics of the deviations from the average, and that clustering is a stable pattern.

Far more interesting is the case in which $\forall t\ \bar{z}_t = 0$, which permits both products to co-exist on the market indefinitely. Intuitively the stability of the cluster should depend on its size. For example, if one individual constitutes a cluster she is susceptible to influence from both her neighbours, both proponents of the choice contrary to hers. This cluster is less likely to be stable than a larger cluster where most of the members of the cluster (the ones away from its boundaries) receive information that reinforces their choices. Thus, there should be some minimum cluster size for which clustering will be persistent. When $\forall t\ \bar{z}_t = 0$ we know that behaviour of the system is governed by the pattern sine wave, which implies that all the clusters are of an equal size in the equilibrium.

[10] Unless $\mu = 0$, which is not a very interesting case as it implies no social influence. In this case the existing consumption pattern is reinforced indefinitely.

[11] This proof can be easily generalized to a multiproduct case.

Proposition 2.4. *In system (2.19), if* $\forall t\ \bar{z}_t = 0$, *clustering in demand is stable if and only if the pattern wave of the system results in the clusters of size* $c \geq \underline{c} = \frac{\pi}{\sqrt{2}}\sqrt{\frac{\mu}{\alpha}}$.

Proof. From equation (2.22) it can be readily seen that, when $\bar{z}_t = 0\ \forall t$, temporal stability of clustering depends on the sign of σ. If $\sigma < 0$, as $t \to \infty$, $z^s \to 0\ \forall s$, which implies that $v_1^s \to v_2^s\ \forall s$. This means that valuations of products converge, so in the case of probabilistic purchases every agent decides on her purchase by tossing a (fair) coin. This, clearly, will result in no clustering pattern.

However, if $\sigma > 0$ the amplitude of the pattern wave increases exponentially with time, thus clustering becomes more and more pronounced. If $\sigma = 0$, the amplitude of the wave does not change with time, and clustering is still stable.

Given the parameters of the model, the sign of σ depends on the frequency of the wave in the initial condition. We can pin down the critical frequency of the pattern wave (k), for which clustering will be stable, by simply solving $\alpha - \mu k^2 \frac{2\pi^2}{S^2} = 0$, for k. This results in $\bar{k} = \frac{S}{\pi}\sqrt{\frac{\alpha}{2\mu}}$. And $k \leq \bar{k}$ ensures that $\sigma \geq 0$. The inverse of the frequency is the wave length, and the size of the cluster is half of the wave length. Since the size of the economy is S, the size of the cluster(s) is $S/(2k)$. Thus, given $\bar{k}$, we can find the size of the smallest cluster that will persist over time: $\underline{c} = \frac{\pi}{\sqrt{2}}\sqrt{\frac{\mu}{\alpha}}$. Any pattern wave exhibiting clusters larger than $\underline{c}$, would ensure $\sigma \geq 0$, and thus will result in stable clustering. □

The important property of the minimum stable cluster size is that it does not depend on the size of the economy. However, as σ depends on S, a larger economy (*ceteris paribus*) increases the likelihood that the pattern wave of the system will support clusters of any given size c, thus it also increases the likelihood of clustering. I also point out that the minimum stable cluster size depends on the ratio of two parameters, habit formation and information transmission: μ/α.

The analysis so far has assumed that there were two goods ($N = 2$) and each agent has 2 neighbours ($H = 1$ on either side). It is also interesting whether these two variables have any influence on minimum stable cluster size.

Proposition 2.5. *In the case of arbitrary neighbourhood size* $2H$ *minimum sustainable cluster size is*

$$\underline{c}_H = \frac{\pi}{2\sqrt{3}}\sqrt{2H^2 + 3H + 1}\sqrt{\frac{\mu}{\alpha}}.$$

Proof. See appendix. □

Proposition 2.6. *In the case of a multi-product environment,* N *being the number of products, minimum sustainable cluster size is* $\underline{c}_N = \underline{c} = \frac{\pi}{\sqrt{2}}\sqrt{\frac{\mu}{\alpha}}$.

Proof. See appendix. □

From proposition 2.6, it is obvious that an increase in the number of products does not affect the stability properties of the system. However, proposition 2.5 implies that as neighbourhoods grow in size so does the minimum sustainable cluster. The intuition is that a larger neighbourhood facilitates the information diffusion process: each agent receives information from relatively distant agents. This works to homogenize the information structure across the population, and so works against small clusters.

We can analyze how minimum sustainable cluster size changes with enlargement of the neighbourhood. It is obvious from proposition 2.5 that $\underline{c}_{H+1} - \underline{c}_H$ is increasing with H. Moreover, it turns out that

$$\lim_{H \to \infty} \left(\underline{c}_{H+1} - \underline{c}_H \right) = \frac{\pi}{\sqrt{6}} \sqrt{\frac{\mu}{\alpha}}. \tag{2.24}$$

Equation 2.24 implies that for any value of μ/α, minimum sustainable cluster size increases linearly with the size of the neighbourhood, as long as H is sufficiently high.

2.5 Short-run analysis

Analysis of the model in section 2.4 characterizes its long-run, equilibrium dynamics. However, as those are asymptotic results, which might take a long time to emerge, short run behaviour of the system is also worth investigating. In particular how do clusters emerge and develop? What is the relation between the average cluster size and the parameters of the model? In this section I address these questions numerically.

Recall that the discrete nature of habit formation posed a problem for the mathematical analysis of the model. To address issues of tractability I assumed probabilistic purchases, and linearized the probability. With numerical simulations I am not so constrained and I can directly analyze the original model. However in order to ensure that the simulation and analytic results are in general agreement, initially I present the results for the linearized model as analyzed in sections 2.4.1 and 2.4.2.

2.5.1 Linearized model

I set the number of goods to $N = 10$;[12] and the population size to $S = 100$. The population is located on a one-dimensional periodic lattice, so the neighbours of agent 1 are agents 2 and 100. The specific parameters for habit formation, α and interaction μ are $\alpha = 0.0005$ and $\mu = 0.01$. Finally, each agent has one neighbour on either side, $H = 1$.[13] To read the figures below, agents are arrayed along the

[12] We expand the number of goods for reasons of generality. According to proposition 2.6, this does not affect the stability of the system for given neighbourhood size.

[13] Note that for this constellation of the parameters $\bar{k} \approx 5.04$ and $\underline{c} \approx 9.93$, as derived above.

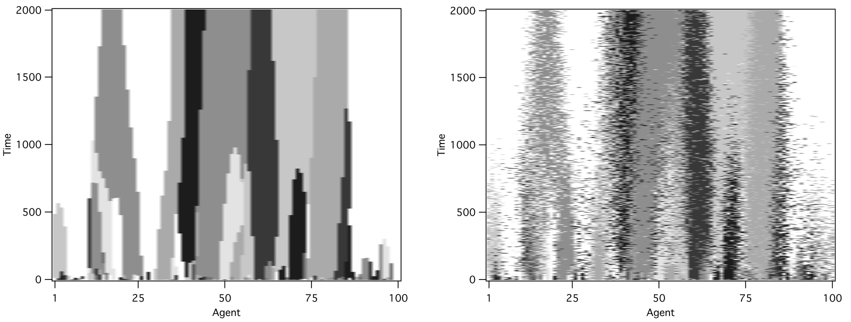

Figure 2.1: Most preferred products (left), and actual purchases (right) in the linearized model.

abscissa, remembering that the axis is a circle, so the right-most and left-most agent are neighbours. Time is read on the ordinate, from the initial period, $t = 0$ to the final period, $t = 2000$. Each good is assigned a different shade of gray. The ordering of the goods, and therefore the shades of gray, is arbitrary. At each point in time the choice (or the good with the highest valuation) for each agent is shown by the colour corresponding to that good.

For completeness, I show not only probabilistic purchases (driven by valuations), but also actual purchaces. Thus we have to specify the function mapping valuation to the probability of choice. Here I simply adopt the multinomial logit, from discrete choice theory:

$$p_n(V_t^s) = \frac{e^{v_{n;t}^s}}{\sum\limits_{i \in \mathbf{N}} e^{v_{i;t}^s}}, \tag{2.25}$$

where $\mathbf{N}$ is the set of available products. Note that $\partial p_n/\partial v_n = p_n(1-p_n)$ and that $\partial p_n/\partial v_j = -p_n p_j$, $\forall j \neq n$. As in the multi-product case $|p_n(1-p_n)| \gg |-p_n p_j|$ ($\Rightarrow 1 - p_n \gg p_j$) is true, probability function (2.25) satisfies the requirement (2.10).

Figure 2.1 shows the dynamics of the most preferred products and actual purchases in a representative run of the linearized model with random initial conditions: for each agent-product pair a $v_{n;0}^s$ is drawn from the uniform distribution over the interval $[0, 20]$.[14] As one can see the clustering pattern in "most preferred goods" is clearly identifiable after just a few periods. The same pattern is replicated (although with some noise) by the actual purchase. Actual choices differ from the preferred good only due to the probabilistic choice function (equation (2.25)). This difference is especially marked near the borders of a region, since here agents receive contradictory information about products, which tends to re-

[14]Changing the uniform distribution to other standard symmetric distributions does not change the numerical results.

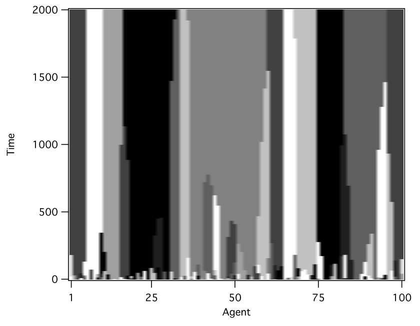

Figure 2.2: Most preferred products and actual purchases in the original model.

duce the difference between their valuations of the most preferred good and other goods. This makes the probability choice function relatively flat for agents near the borders of clusters, and choices less correlated with those of their neighbours.

I must point out that clustering patterns identified in figure 2.1 are only metastable. The reason is that stability of the multiple clusters requires $\bar{z}_t = 0 \ \forall t$ (corollary 2.1), the multi-product equivalent of which is $\bar{v}_{i;t} = \bar{v}_{j;t} \ \forall i, j, t$. Although this requirement can be imposed on the system while simulating the linearized model, it can not be guaranteed for the original model. (In fact there will always be some finite time at which mean valuations of two goods will differ: $\exists t < \infty$ such that $\bar{v}_{i;t} \neq \bar{v}_{j;t}$.) To make the examples comparable I do not impose the $\bar{v}_{i;t} = \bar{v}_{j;t} \ \forall i, j, t$ constraint on simulation of the linearized model either. Thus we know that the equilibrium of all my runs is the state which results in only one cluster (corollary 2.1). However, my experiment shows that in the short-run multiple clusters emerge and persist for relatively long periods.

2.5.2 Original model

Recall that there are two major differences between the linearized and the original model. One is that in the original model there are no probabilistic purchases, thus utility maximization implies that each consumer purchases her most preferred product in each period. Another is that in this case we have a habit formation *step* ζ instead of a habit formation *rate* α. We know that $\zeta = \alpha/\gamma$ where γ is the constant coming from the linearization of the choice probabilities. Unfortunately there is no way to pin down the value of γ. For this reason we cannot make a judgment about the relative magnitudes of α and ζ (apart from the fact that they are proportional), and thus the choice of the value of ζ is somehow arbitrary. I choose $\zeta = 0.01$ and use the same values for all other parameters as in the previous run. The result of the representative run of the original model is presented in figure 2.2. As one can see the clustering in purchases is clearly visible and relatively stable.

These numerical exercises also permit us to make a comment about what re-

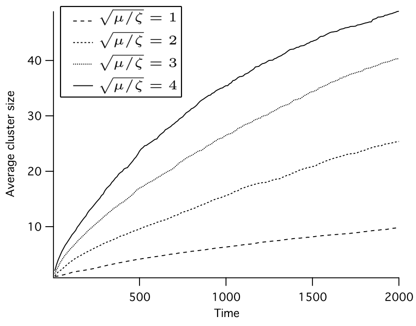

Figure 2.3: Dynamics of average cluster size for different parameter constellations in the original model.

vealed preferences cannot reveal. Revealed preferences give us information only about the most preferred product, namely which it is, and completely neglect the story that is going on in the background. By this I refer to the fact that agents do have preferences over, and information about the goods they do not in fact consume. Without acknowledging the importance of those "unexpressed" preferences it is difficult to understand a sudden change in consumption which is not simply imitating neighbours. This is something that is possible in my approach, and in fact is observed in figure 2.2 as well as in figure 2.1.[15] We can observe several cases of an agent adopting a new good which neither she nor her neighbours have consumed in the past. The explanation lies in the fact that an agent close to the border of a cluster can receive contradictory signals. Consider the following simple example. Agent $s-1$ ranks good 1 first and good 3 last; agent $s+1$ ranks good 3 first and good 1 last. Both agents, though, rank good 2 second. It is clear that agent s, based on her external information, could easily rank good 2 before either 1 or 3. If the high rankings of good 2 by $s-1$ and $s+1$ have emerged (due to information received by their neighbours) at roughly the same time, agent s can then switch to good 2, regardless of what he was doing in the past. This explains the emergence and growth of such neighbourhoods in my framework. Thus, my model is consistent not only with shrinking and disapearance of smaller clusters, but also with the emergence and growth of new ones.

As clustering in these simulations is only meta-stable, average cluster size should be steadily increasing over time until it reaches the equilibrium size $\bar{c} = S$. It is interesting to see how the rate of increase depends on the parameters of the model. As the amplitude growth rate, σ, of the dominant wave controls the speed of convergence to the equilibrium, intuitively it should also control the growth rate of the average cluster. Besides it's partial dependency on initial conditions

[15] In figure 2.2 the best example of this sort is agent 62 at period 70, who switches to consuming a product never consumed in her neighbourhood before. In the left panel of figure 2.1 a similar pronounced example is agent 39 at period 50, who is the pioneer of a new product consumption in her neighbourhood. In both cases products introduced survive and spread.

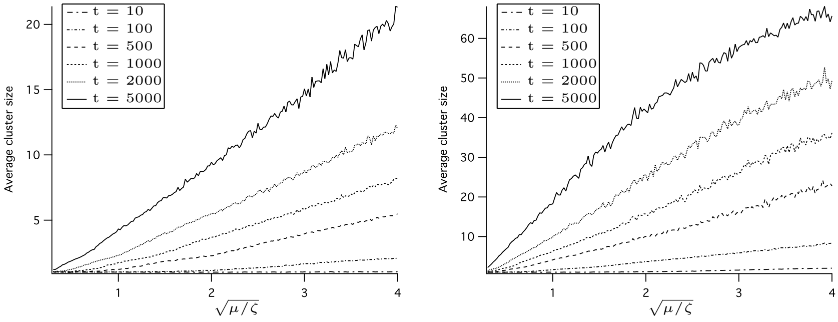

Figure 2.4: Average cluster size in the linearized model (left) and in the original model (right).

(due to k, which is the frequency of the dominant wave as determined by initial conditions), σ also depends on habit formation, α, interaction intensity, μ and population size, S. Or in the case of the original model on ζ, μ and S. Figure 2.3 shows the dynamics of the average cluster size under different parameter constellations. As the effects of these parameters are similar and it is only their joint effect which is important, I omit the size of the economy from the analysis and report the results for the different values of $\sqrt{\mu/\zeta}$ (for the sake of compatibility with the later results presented in figure 2.4). Here I present the average cluster size further averaged over 500 simulations. As expected[16] higher $\sqrt{\mu/\zeta}$ implies a higher rate of increase of average cluster size.

But equilibrium analysis can also be exploited to predict the behaviour of the system in the short-run. First I examine average cluster size in the equilibrium of linearized model. In equilibrium we have either one cluster of size S (corollary 2.1), or we have many clusters of the same size, with some minimum possible cluster size $\underline{c}$ (proposition 2.4). Thus the size of the representative cluster is bounded by $\underline{c}$ and S. The realized cluster size depends of course on the initial conditions, so I discuss expected cluster size given some distribution of initial conditions.

How expected cluster size scales with $\underline{c}$ (assuming a fixed population size S), depends on how initial conditions map to equilibrium outcomes. Proposition 2.4 suggests that in the linear model minimum sustainable cluster size will be $\underline{c} = \frac{\pi}{\sqrt{2}}\sqrt{\frac{\mu}{\alpha}}$. Without a formal proof, the law of large numbers suggests that expected cluster size should be roughly the average of the minimum and maximum cluster sizes. Thus mean cluster size should scale linearly with $\underline{c} = \frac{\pi}{\sqrt{2}}\sqrt{\frac{\mu}{\alpha}}$. Similarly, in the original, non-linearlized, model, since $\zeta \propto \alpha$, it would follow that the mean cluster size is proportional to $\sqrt{\mu/\zeta}$.

To examine whether this intuition carries over to describe clustering behaviour in the short run, I make 500 runs of both the linearized and the original model for 200 equally spread values of $\sqrt{\mu/\alpha}$ and $\sqrt{\mu/\zeta}$ in interval $(0, 4]$ and show the

[16] Recall from equation 2.23 that $\sigma = \alpha - 2\mu\frac{\pi^2}{S^2}k^2$.

results in figure 2.4. Here I present the average cluster size at different points in time averaged over the 500 simulations. These are essentially the same plots as in figure 2.3 but with a different abscissa. A sense of time in these plots comes from the differences between curves along the ordinate for each point on abscissa.

The results indicate that average cluster size increases with time (which we have already seen in figure 2.3). They also indicate that the linear relationship predicted above for equilibrium state is also present in the transition to equilibrium, at least in the linearized model. However, in the original model, we observe a linear relationship during the early periods, but this disappears as the system gets close to the equilibrium in which average (over agents) valuations of the goods differ. We can conclude from this that the linearized version of the model is a very good approximation of the original model except for the near-equilibrium dynamics.

The reason for this discrepancy between the original and linearized models close to equilibrium is that the linear model exaggerates the effect of habit formation when valuations are sufficiently high. To see this recall that in the original model habit formation parameter ζ is additive to the valuation and is constant (equation 2.7). Consequently, as valuations increase the relative habit formation effect will decrease. However, in the linear model, the contribution of habit formation depends linearly on the *level* of valuation (equation 2.11). Thus the change in valuations does not change the relative size of the habit formation effect. As valuations are monotonically increasing in time, this is seen in the difference between the two panels of figure 2.4 at high values of t.

We have two effects in this model: habit formation, which drives the evolution of the average valuations in the economy, and information exchange, which controls the idiosyncratic deviations from this average. These effects are completely separable in the solution (2.22), and each dominates the dynamics under different conditions. When the average difference between the valuations of the two goods becomes large (infinity in the limit) the information exchange effect becomes negligible (proposition 2.1), and habit formation dominates. However, if this difference is small (zero in the limit), habit formation is weak, and the dynamics are driven by the information exchange effect (proposition 2.2). Because I start from random initial conditions, ensuring that $\bar{v}_{i;0} \approx \bar{v}_{j;0}\ \forall i, j$, the effect of habit formation will initially be small, and the dynamics will be driven by information exchange. In this case, the linear model provides a good approximation. However, after a sufficiently long time, the dynamics come to be dominated by habit formation, and as the linearized model exaggerates its effect, we get distortion in the picture: with the linearized model predicting a linear relationship (left panel), while the original model shows a sub-linear relationship (right panel). This distortion persists until the system reaches the equilibrium implied by the corollary 2.1.

2.6 An extension to the model

The model discussed in the previous section assumes a very specific structure for habit formation, which governs the movement of the average of the valuation differences ($\bar{z}$). This system implies that consumers' valuations increase without bound with consumers' experience. Thus the average of the deviations either stays at zero forever or increases without bound (proposition 2.1). Although mathematically convenient, this assumption is not very realistic. It is more plausible that habits can be formed to a certain point, but no further. In this case the average of valuation differences ($\bar{z}_t$) will be bounded. For understanding the implications of this extension I return to the two good case of the linearized model, but the results generalize straight-forwardly.

Because the solution to my model is separable into the average and deviations from the average, it is possible to incorporate a finite bound on the average valuation difference. Unfortunately in this case it is not feasible to pin down the exact relation between σ and parameters of the model. However, we can characterize the set of possible equilibrium states.

When we impose a bound on average valuations, the discrete version of the solution[17] to the system (2.19) becomes

$$z_t^s = \bar{z} + e^{\sigma t} \cos\left(k \frac{2\pi}{S} s\right) \tilde{z}_0^0. \tag{2.26}$$

where $\bar{z}$ ($\neq 0$)[18] is the equilibrium level of the average valuation difference. The solution here differs from the unbounded case only in the first term: in the unbounded case, the first term can grow without bound eventually dominating the solution, whereas when the valuations are bounded, this term converges to a constant.

In the model with bounds on valuations, there are three regimes, characterised by the sign of σ, the growth rate of the dominant sinusoid. The three regimes exhibit qualitatively different behaviour with respect to clustering.

$\sigma > 0$ **:** In case when amplitude growth rate of the dominant sinusoid is positive ($\sigma > 0$),[19] qualitative results with respect to clustering do not differ from the baseline model (proposition 2.4): distinct clusters emerge and are stable, with the share of consumers preferring a certain product being equal across products.

For the other two cases adding bounds to average valuations changes the qualitative results of the baseline model discussed in section 2.4.

[17] To recall, the discrete version of the solution to the unbounded model is $z_t^s = e^{\alpha t}\bar{z}_0 + e^{\sigma t}\cos\left(k\frac{2\pi}{S}s\right)\tilde{z}_0^0$.

[18] The case $\bar{z} = 0$ is equivalent to the case $\bar{z}_t = 0$ $\forall t$, implying that the bounds make no difference to the solution. This case was discussed above in section 2.4.2.

[19] Recall that with time the constant part of the solution (2.26), $\bar{z}$, is dominated by one wave, as its amplitude goes to infinity. The effect is that z_t^s converges to $e^{\sigma t}\cos\left(k\frac{2\pi}{S}s\right)\tilde{z}_0^0$.

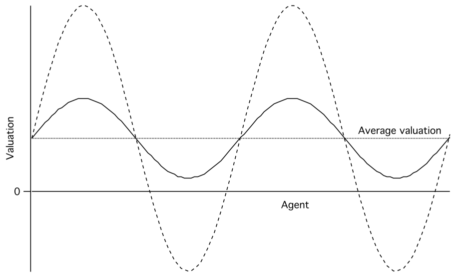

Figure 2.5: Difference between the cases with one stable cluster (solid line) and multiple different sized stable clusters (dashed line) when amplitude of the dominant sinusoid does not change.

$\sigma < 0$: Where the amplitude growth rate of the dominant sinusoid is negative, we know that even the dominant sinusoid vanishes in equilibrium. This implies that, in the limit, the second term of (2.26) vanishes, and for each agent, valuation difference collapses to the average: $z_t^s \to \bar{z}$. In the baseline model this would imply either no clustering (if $\bar{z}_0 = 0$), or one cluster with everybody purchasing the same product in equilibrium (if $\bar{z}_0 \neq 0$).

However, in extended case no clustering is not an option (as $\bar{z} \neq 0$). In this case every consumer's valuation difference converges to $\bar{z}$, thus there will be one big cluster. But, again unlike the baseline model, none of the products will attain 100 percent of the market in equilibrium, as choice probability of the dominant product will be bounded by some value below 1.

$\sigma = 0$: When the growth rate of the amplitude of the dominant sinusoid is zero, clustering is stable, as it is in the baseline model, but here the bounds imply a richer set of possible outcomes. In general there are two types of possibilities, illustrated in figure 2.5. If the amplitude of the dominant sinusoid is small relative to the average difference in valuations ($|\bar{z}|$), then all agents prefer one product over the other, though the strength of preference varies (solid line). A single cluster emerges in preferences, but again, similar to the case when $\sigma < 0$, no product attains 100 percent of the market. If, however, the the amplitude is relatively large, we have stable clustering with multiple clusters with different sizes (dashed line in figure 2.5). The relationship between the value of $\bar{z}/\tilde{z}_0^0$, and the frequency of the wave, determines the size of the clusters and share of the individuals preferring one product over another. More precisely, I can state

Proposition 2.7. *If $\bar{z} \neq 0$, $\sigma = 0$ and $\tilde{z}_0^0 < |\bar{z}|$, the share of consumers preferring one of the products will be $\frac{s_1 - s_2}{S}k$, where s_1 and s_2 are the solutions to*

$$s = \frac{S}{2\pi k} \arccos\left(-\frac{\bar{z}}{\tilde{z}_0^0}\right),$$

such that $s_1 \geq s_2$ *and* $s_1, s_2 \in (0; S/k]$.

Proof. $\tilde{z}_0^0 < |\bar{z}|$ guarantees that the equilibrium wave crosses the abscissa, thus for some consumers $z^s > 0$, while for others $z^s < 0$. We have to find the share of one of these groups of consumers. For this we have to solve the equation

$$\bar{z} + \cos\left(k\frac{2\pi}{S}s\right)\tilde{z}_0^0 = 0 \tag{2.27}$$

for s. This results in

$$s = \frac{S}{2\pi k}\arccos\left(-\frac{\bar{z}}{\tilde{z}_0^0}\right).$$

Denote the two solutions on one cycle of the wave with s_1 and s_2 ($s_1, s_2 \in (0; S/k]$) and order them such that $s_1 \geq s_2$. This implies that within each cycle, $s_1 - s_2$ agents have a valuation difference of one sign (and comprise one cluster), $S/k - (s_1 - s_2)$ the other. Thus $\frac{s_1 - s_2}{S}k$ will be the share of one kind of agents in the whole population. □

This extension of the model results in richer equilibrium patterns which allow for stable clustering patterns with clusters of different sizes co-existing in the equilibrium, whereas in "unbounded" version of the model, all clusters were of the same size in equilibrium. This extension could explain the existence of temporally stable geographical or social neighbourhoods of different sizes engaging in similar activities (e.g. voting for the same party).

2.7 Conclusion

In this essay I have argued that interaction with peers over social networks can have important effects on the social distribution of demand. This external force, together with internal forces such as inertia, generates rich demand dynamics for markets containing goods that are close substitutes. Information diffusion through fixed social networks naturally generates clustering in demand: some neighbourhoods collectively prefer one good over another, while other neighbourhoods do the reverse. Depending on the characteristics of the society, this pattern can be either fragile or stable. In essence, several parallel informational cascades can result in persistent spacial distributions, where clearly identified neighbourhoods have higher concentrations of one particular type of information, or to put it differently, where the peaks of different positive informational cascades (Hirshleifer, 1993) are located in different places in social space.

It is worth noting that stable clustering phenomena can also be obtained with simpler models. For example one can model consumers as cellular automata, who base their decisions purely on neighbours' current states (for example Greenberg and Hastings, 1978; or for an economics application, Cowan and Miller, 1998). The present model model differs from these specifications in two ways: firstly, I can discuss the importance of communication intensity, which is impossible in cellular automata and secondly, in my model consumers exchange information about the merits of (all) the products with their friends instead of just observing their actual consumption baskets.

This second difference leads to an important observation of the kind of dynamics my model can produce. Here, because the information transmission is rich, we can observe endogenous, apparently spontaneous emergence of behaviour. That is, behaviour that is in principle possible (part of the choice set) but which has not yet been observed, can suddenly emerge. This arises because agents transmit information beyond simply their revealed preferences. When information flow is restricted to the observation of behaviour (and thus to revealed preferences), "new" behaviour can emerge, but only if agents' experience with their actual behaviour is negative, and they downgrade their valuations of it. It cannot emerge due to information transmission from neighbours, because no information is passed about goods that are not being consumed. But agents typically do have preferences over goods they do not consume or actions they do not pursue, and there is no particular reason to believe they do not share this information, at least to some extent. When this is the case, even in the absence of bad experiences, agents can switch to new goods, even if those goods are not being consumed by their neighbours.

In the present essay I model the dynamics of valuations through local interactions. There has been an interest in the literature about the difference between local and global information flows, or interaction more generally (Ellison, 1993; Brock and Durlauf, 2000; Glaeser and Scheinkman, 2000). This issue can be addressed in my model by looking at its behaviour as neighbourhoods become very large ($H \to S/2$). Increasing the neighbourhood size (H) puts an upward pressure on minimum stable cluster size $\underline{c}$ (see proposition 2.5), and for larger region of parameter space pushes it above the threshold ($\underline{c} > S/2$) beyond which clustering is unstable in the long run (in case when the differences between average valuations are zero).[20] Thus, in line with Glaeser and Scheinkman (2000), my model demonstrates that local interactions result in richer and more complex dynamics than do global interactions.

The model presented in this essay can be applied not only to choices between substitute products, but also to any mutually exclusive decision. For example the valuations in this essay can be easily interpreted as the level of satisfaction one gets from voting for a certain party. Similarly, the difference between valuations (in case of the two product model) can be interpreted as the satisfaction from various economic and social behaviour (e.g. bribery or other forms of criminal activity). In this respect the present model which can not only explain the emergence of the (geographical) clusters in which similar behaviours prevail but also provide the conditions under which this phenomenon will be temporally stable.

[20] For example, in the small economy that I have simulated ($S = 100$), $H = 49$ implies that the speed of habituation, α, must be roughly 80 times higher than the influence of neighbours, μ, in order the system to be stable for the largest possible cluster ($\underline{c} = S/2$)

Essay 3

Returns to Product Promotion when Consumers Are Learning How to Consume*

Abstract. This essay presents a computational model of consumer behaviour. I consider two sources of product-specific consumer skill acquisition: learning by consuming and consumer socialization. Consumers utilize these two sources in order to derive higher valuations for products they consume. In this framework I discuss the behaviour of returns to advertising relative to changes in product characteristics, such as quality and user-friendliness. The main finding is that in case of duopoly dependence of returns to advertising on product quality is not monotonic as had been suggested by earlier studies. Rather, returns have an inverted U shape, given the quality of the competing product.

3.1 Introduction

It is the nature of the capitalistic free market that the results of producer actions are ultimately anchored to consumer behaviour. It is consumers who decide when to buy and what to buy, how to respond to price or quality changes in products supplied to them. Thus, I believe that the analysis of any economic phenomenon should start with the analysis of consumer decisions.

The process of consumer decision-making is complex. It is affected by numerous

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forces, some of them more important than others. Our everyday decisions about which products to purchase are largely influenced by our own consumption history, by the information coming from our social network, as well as by the information received from media.

At the same time, in today's world of advanced technologies, many products need specific consumption skills in order to be utilized to their maximum capacity. I argue that the level of these skills plays an important role in consumer decision about the composition of a significant share of their consumption baskets. Thus, effects of producer decisions are heavily influenced by the process of skill accumulation by consumers.

This essay looks at the influence of the skill acquisition process on consumers' decisions. I consider the problem of choosing a product among multiple alternatives. I discuss two sources of consumer skill acquisition: learning by consuming and consumer socialization process. Learning by consuming means that consumers acquire skills along the consumption process, while consumer socialization implies that consumers obtain skill spillovers from their social network.

In this framework, as a new product enters the market it is met with some initial skill distribution over the population of consumers. Consumers who purchase new products will acquire more skills through the consumption process, and these skills will be further diffused through a socialization process. Thus, consumer purchasing decisions have temporal effects on average skill levels over the population. High rates of initial market penetration will ensure fast acquisition and diffusion of skills for new products. This implies that a new product will be able to grab higher market shares during the transition to equilibrium. This framework is suited for analyzing the effects of producer policies that can influence consumer skill levels. I discuss the effects of such policies using the example of product promotion (advertising).

Advertising is recognized as one of the essential activities of a modern firm. Economists, as well as business and marketing academicians, have shown interest in its effects. The main concern has been whether advertising can be used to create barriers to entry and thus generate a long-run comparative advantage. Views on this issue are not unified: some researchers find that advertising can be effectively used for creating barriers to entry (e.g. Comanor and Wilson, 1967; 1979), while others find the opposite (e.g. Erickson and Jacobson, 1992). In this paper I tackle a similar problem. I use an innovative framework where effects of producer actions are rooted in the micro behaviour of consumers. I investigate how returns to advertising (measured as the gain in market share) depend on quality and user-friendliness of the product when consumers are making their product purchases taking into account their product-specific skills.

Products that I have in mind are those that are relatively durable. They are repeatedly purchased by households and are technologically sophisticated to some extent. This description fits the group of products called "consumer electronics" well. This class of products has one more characteristic that makes it particularly interesting for investigation: the products in this category are widely advertised. According to AdvertisingAge, in year the 2007 producers of consumer electronics in the USA spent 50% of their profits on advertising.[1] This indicates that producers of consumer electronics rely heavily on advertising as a way of promoting their products.

The essay is organized as follows. The first section briefly reviews the existing related literature about learning and socialization among heterogeneous consumers. Section two formulates the model of consumer behaviour, while section three presents the analysis and results of the model. The last section summarizes the essay and provides some concluding remarks.

3.2 Individual learning and socialization by consumers

An important characteristic of the model analyzed in this essay is that consumer behaviour evolves through individual learning. Consumers learn individually through experience as well as from social interactions. Individual learning implies heterogeneity among agents, as skill levels might differ across consumers. Models with heterogenous agents are quickly gaining recognition within the discipline. They present an alternative to models characterized by a representative agent. In the late 1980s criticism of a representative agent emerged concerning its ability to correctly describe the behaviour of an economy populated by heterogeneous agents (Kirman, 1992). The model presented in this paper does not have a representative agent. In fact, in the first essay of the present thesis I have shown that the representative agent cannot be constructed for an arbitrary economy described here.

Similar to agent heterogeneity, individual learning is not new to economics. It has been extensively discussed in the literature. Learning takes many different forms. Detailed discussion of them is outside the scope of the current essay; a comprehensive survey can be found in Brenner (1999; 2006). One of the most widespread forms of individual learning is learning-by-doing. It has been widely used in economics to explain the effects of innovation and technical change (e.g. Arrow, 1962b). The idea is that one becomes better at doing something by simply doing it. I have a similar concept in my essay applied to consumer learning: consumers are becoming better at utilizing products that they are frequently using. The path of skill level of an individual is called a learning curve (Ebbinghaus, 1885/1964). Learning curves have mostly been used in economics to study the rate of producer cost reduction along with the increasing experience (Spence, 1981; Cabral and Riordan, 1997). In this essay I use the learning curve idea to describe consumer learning.

[1] Source: 2007 advertising to sales ratios for 200 largest advertising spending industries, www.adage.com.

Apart from learning-by-consuming, agents can acquire consumption skills in other ways. For example, they can help each other out and share the skills that they have accumulated. This calls for (non-market) interactions among agents. Modeling non-market interactions among economic agents has a long tradition, but it has become increasingly important in the last two decades. There are various models analyzing interactions among consumers (e.g. Eshel et al., 1996; Cowan et al., 1997). In general, interactions generate feedback loops that affect the decisions of the economic agents. As noted by Glaeser and Scheinkman (2000) the structure of these interactions does matter for the outcome obtained at the end. In particular, they show that in case of local interactions systems generate more interesting dynamics, having multiple equilibria and possibility of moving from one equilibrium to another. More contextualized works show that interaction can explain certain interesting phenomena in economics and other social sciences, like standardization processes (e.g. Arthur 1989; Cowan 1991; Eshel et al. 1998), waves in consumption across the population classes (Cowan et al. 2004) or contagious justice (Alexander and Skyrms, 1999).

Non-market interaction among consumers is usually modeled as a socialization process. Consumer socialization has been identified as being important for various social processes first in sociology (e.g. Roszak, 1969) and then in the business literature (e.g. Moschis and Churchill, 1978). Sociologists have been concerned with consumer skill acquisition by adolescents through interaction with peers as well as parents, but aspects of life-time-long learning have been also discussed (Ward, 1974). Marketing academicians have also studied consumption skill acquisition of young people, as the learning process is more pronounced in this age-group (Moschis and Churchill, 1978). Although some aspects of consumer socialization processes have been discussed in economics, to the best of my knowledge, consumer skill sharing through social processes has not been studied. This essay contributes to filling this gap.

In this essay I combine learning by consuming with learning through socialization and discuss the consumer skill upgrading along the learning curve. The idea of skill acquisition through consumption has been introduced to economics by Witt (2001) under the notion of "learning to consume." The author makes a distinction between the two aspects of learning through consumption: cognitive and non-cognitive. Witt (2001) discusses the subject through the lens of changing preferences and argues that both types of learning (cognitive, as well as non-cognitive) change the consumer preferences and, as a result, the future pattern of consumption of an individual. In this essay I formalize a part of Witt's learning to consume ideas. In line with Witt (2001), I claim that consuming certain products gives incentives to consume this product again. The mathematical modeling of learning forces in this essay takes quite a general form which can accommodate cognitive (purposeful) learning, as well as non-cognitive learning, which, in my context, might be an accidental discovery of new (unknown to a consumer) features of a product in the process of consumption. The latter part is profoundly different from the definition of non-cognitive learning by Witt (2001), who looks at the matter from the angle of associative learning. There are additional distinctions between forces modeled in this essay and forces considered by Witt (2001). Here I discuss learning in a single product context, rather than learning to consume in general, which is equivalent to forming a habit of consuming. I concentrate on consumers acquiring skills to better utilize separate products. To emphasize these similarities and distinctions, the learning process discussed in this essay is named "learning how to consume."

In the next section I present a model that uses heterogeneous agents (Kirman, 1992), that interact locally outside the market (Cowan et al., 1997). Consumer skills that are acquired through consumption (Witt, 2001) are diffused among agents through social interactions (Ward, 1974). In this environment results of any action by producers are anchored to consumer behaviour through their effects on individual skill levels of separate consumers. There are temporal feedbacks present in the scheme which determine the size of any effect. I use this model to study the returns on advertising (Mariel and Oribe, 2005) and its impact on the market shares of advertisers.

3.3 The model of consumer behaviour

Consider an economy with many heterogeneous agents, who have to choose one product every period from an available product set. Each consumer (s) has an idiosyncratic valuation (v) for every product (n) at every time period (t). Valuation of a product for a consumer is the maximum price this consumer is willing to pay for it.

On the supply side, assume there are many (substitute) products with different qualities (λ) offered on the market. I assume that λ can be measured in monetary units. I abstract from the differences in prices as well as from the possibility of their temporal change and fix the prices of all the products to be equal to a constant over time.

Consumers are myopic: they make decisions by maximizing one-shot utility. Although I am aware of the shortcomings of the concept of utility maximization, I still use it in this work due to its advantages for the tractability of the formal model. I follow the standard discrete choice literature and model consumer choices probabilistically (Anderson et al., 1992). The probability that consumer s will choose product n at time t is a function of the vector of valuations ($\mathbf{V}_t^s$) that a given consumer holds for a given time period.

Assume that the valuation is multiplicative in two parts: one is the quality of the product (λ), the other is the consumer skill level ($k \in [0, 1]$), which I assume to be product-specific. If the level of consumer skill is 1, she can utilize the given product to its maximum capacity, thus her valuation of the product will be equal to the product quality.

Skill levels change over time: consumers learn through consumption and socialization. In modeling individual learning I follow the literature about learning curves pioneered by Ebbinghaus (1964). I assume that learning by consuming occurs at a decreasing rate and specify the learning function as:

$$k_m^s = 1 - (1 - k_0^s)e^{-\delta m}, \tag{3.1}$$

where δ is the speed of learning, k_0^s is the initial skill level of agent s for the product under discussion and m is the number of times a product has been consumed prior to (and including) the current one.

From equation (3.1) we can derive the change in skill levels between two subsequent consumptions of the same product

$$k_{m+1}^s - k_m^s = \gamma(1 - k_m^s), \tag{3.2}$$

where $\gamma = 1 - e^{-\delta}$.

Using equation (3.2) one can write the law of motion for the valuations of a product while abstracting from the consumer skill sharing process. Recall that $v_t^s = k_t^s \lambda$, thus multiplying both sides of the equation (3.2) by λ will yield:

$$v_{m+1}^s - v_m^s = \gamma(\lambda - v_m^s). \tag{3.3}$$

Every time period t agent s chooses product n for purchase with the probability $p_{n;t}^s(V_t^s)$. This implies that

$$E_t(v_{n;t+1}^s) = v_{n;t}^s + \gamma_n(\lambda_n - v_{n;t}^s)p_{n;t}^s, \tag{3.4}$$

where $E_t(x)$ denotes the expectation about the value of variable x at time t.

An important point to note in equation (3.4) is that besides the product-specific quality level, expected dynamics of valuations also depend on the product specific speed of learning (γ_n). This parameter can be interpreted as the user-friendliness of the product. If γ_n is high, the skill acquisition for the product is fast, while in case of a low γ_n it takes a lot of time before the skill level of a consumer converges to its maximum.

Regarding the socialization process, consider each consumer interacting with a small and constant group of other consumers. I assume that through this interaction some of them can acquire product specific consumer skills. For the sake of tractability, assume that consumers are aligned on a unidimensional lattice (circle) and that each of them interacts with only two neighbours (one on each side). The consumer can learn about (upgrade her skills for) a product through socialization if, and only if, there is at least one consumer in her neighbourhood with higher consumer skills for this particular product than that of her own. I assume that there is a constant rate of learning (μ) through socialization. I restrict consumers to be able to learn about any product from only one neighbour in any single time period and assume that they are choosing the most skillful consumers in their social networks as their mentors. The rationale for this is that there is a cost of communication and the rate of learning is constant. Thus, maximization of utility implies that one would choose the most skillful neighbour to learn from. Ignoring learning through consuming for a moment, the effect of consumer socialization on valuation can be written as

$$k^s_{n;t+1} = k^s_{n;t} + \mu \left(\max(k^{s-1}_{n;t}; k^s_{n;t}; k^{s+1}_{n;t}) - k^s_{n;t} \right). \tag{3.5}$$

It is important to note that μ is neither product nor consumer specific. In principle μ could well be consumer specific, which would reflect the differences in the absorptive capacity of consumers. However, that would further increase the already large parameter space of the model. Instead one can think of μ as the interaction intensity, which can be thought of as a characteristic of the society.

To combine two forces of consumer learning, I assume that inspite of the product choices, socialization affects the valuations of all the products in every time period. This means that consumers acquire some skills for every product at every time period (given that they have not reached the highest skill level and they are not the most highly skilled consumers in their neighbourhood). Multiplying both sides of equation (3.5) by λ_n and combining it with the equation (3.4) gives the full specification of the model

$$E_t(v^s_{n;t+1}) = v^s_{n;t} + \gamma_n(\lambda_n - v^s_{n;t})p^s_{n;t} + \mu \left(\max(v^{s-1}_{n;t}; v^s_{n;t}; v^{s+1}_{n;t}) - v^s_{n;t} \right) \tag{3.6}$$

It is important to note that the expected law of motion of product valuations is product-specific, as well as consumer-specific. Thus, I have $N \times S$ of these equations (where N is the number of products, and S is the number of consumers in the economy). It is impossible to obtain an analytic solution for this system for any reasonable shape of the probability function. Due to this complication I use numerical simulations to address the research questions.

3.4 Returns to product promotion

The model specified in section 3.3 describes the dynamics of purchasing probabilities of every product for every consumer in the economy. This property seems to be particularly appealing for studying market share dynamics of products on markets with fixed sizes. An important aspect of competing for market shares is product promotion. Producers can promote their product and affect the purchase probabilities of consumers. One widespread tool for product promotion is advertising.

The ground for theoretical work on advertising was laid by Nelson (1974; 1975). He considered advertising as a signal of product quality and speculated about the effects of advertising and its differences across types of products. He split the product space in two: "experience goods," whose characteristics can only be learned through experience, and "search goods," whose characteristics are observable prior to purchase. Nelson claimed that advertising would have a higher impact on "experience goods," thus expected experience goods to be advertised more. Some time later, Nelson's speculative ideas were formalized by Milgrom and Roberts (1986). They discussed only experience goods and concentrated on the impact of advertising across product quality. Milgrom and Roberts found that high quality brands would have higher incentive to advertise than low quality brands. Empirical support for this finding was provided by Nichols (1998) for the automobile industry. It is evident that academicians have mostly regarded advertising as a tool for signaling the product's high quality, thus the intuitive finding that products with higher quality should have higher incentive to advertise. The following theoretical work (e.g. Landes and Rosenfield, 1994) has been built on this intuition.

Unlike product quality, virtually no work has been done to analyze the effects of variance in product user-friendliness on returns to product promotion. However, one can hypothesize that products that are less user-friendly should benefit more from advertising because they require more extensive use by consumers to acquire an adequate share of their consumption baskets.

In this section I analyze the effects of parameter changes (product quality and user-friendliness) on returns to product promotion implied by the model of consumer behaviour presented in section 3.3.[2] I test the two intuitions specified above (about the effects of product quality and user-friendliness) in environments where consumers are learning how to consume.

In order to discuss the returns to product promotion I have to introduce a couple of notions and specify the ways in which I measure important variables. I do this in the following section.

3.4.1 Measurement

Measuring a market share. As I am studying markets with constant sizes, I can, without a loss of generality, normalize their size to unity. Then, the market share of a product will simply be the cross-average of its purchase probabilities:

$$h_{n;t} = \frac{1}{S} \sum_{s=1}^{S} p_{n;t}^{s}. \tag{3.7}$$

Following the discrete choice literature (Anderson et al., 1992), I assume that the probability of product n to be chosen by agent s at time t is described by the multinomial logit function

[2] Although in some places throughout the essay I refer to this phenomena as advertising, the modeling takes a general form so the intuitions can be applied to any other type of product promotion.

$$p^s_{n;t} = \frac{e^{v^s_{n;t}}}{\sum_{i=1}^N e^{v^s_{i;t}}}. \quad (3.8)$$

Thus, ultimately market share dynamics depend on the dynamics of valuations. It is easy to verify that as $k^s_n \to 1$, $\forall s, n$, market share distribution becomes time-invariant:

$$\bar{h}_n = \frac{e^{\lambda_n}}{\sum_{i=1}^N e^{\lambda_i}}. \quad (3.9)$$

Equation (3.9) implies that $\bar{h}_n \propto e^{\lambda_n}$, which effectively means that products with higher quality are guaranteed higher equilibrium market shares.[3] Equilibrium market share distribution does not depend on any other parameter of the model. The rest of the parameters influence only the transition path to the time invariant distribution.

Measuring returns to product promotion. As argued in section 3.2, in this model the effects of product promotion are anchored to skill acquisition. If the average consumer skill level in the population has not reached its maximum for product n, advertising will influence not only the probability of its purchase at the time period when it is advertised, but also during subsequent periods. Higher purchase probability today ensures higher rate of skill acquisition, which in turn influences the purchase probability for the next period. Thus, as long as advertising is undertaken before the average skill level reaches unity, it has a long-lasting effect (Landes and Rosenfield, 1994) and influences transitional dynamics to the time invariant market share distribution. On the other hand, if advertising takes place after everybody has learned how to utilize the product to its maximum capacity, it will not have any effect on purchasing probability in subsequent periods.

At this point we have to recognize that there are many ways to advertise a product, by which I mean that there are many strategies for spending the budget allocated to product promotion. Recent literature puts the emphasis on the search for the optimal temporal advertising policies. It has been found that pulsation advertising policies[4] are more efficient than uniform advertising policies[5] on a wide range of markets (vande Kamp and Kaiser, 2000). Mesak and Zhang (2001) provide the theoretical support for this finding on monopolistic markets. In general, however, the search for the optimal temporal policy has not yet yielded any clear recommendation for businesses.

Due to the lack of theoretical work on the subject I am confronted with the choice of advertising strategy in order to undertake the investigation into returns

[3] A model with similar outcome has been analyzed by Kihlstrom and Riordan (1984), while Schmalensee (1978) has presented a model where products with lower qualities can have higher equilibrium market shares.

[4] Spending large chunks of money at discrete periods of time.

[5] Distributing the advertising budget uniformly over extended period.

to product promotion. Thus, I consider the choice to be whether to advertise or not, rather than how much to advertise, or how to advertise. I assume that for some fixed cost, which is constant across producers, the producer of product n can influence every consumer's purchase probability in the following manner: if without advertising consumer s would buy the product n with the probability given by equation (3.8), with advertising the probability would be

$$\hat{p}^s_{n;0} = \frac{A + e^{v^s_{n;0}}}{A + \sum_{i=1}^{N} e^{v^s_{i;0}}}, \tag{3.10}$$

where A is the effect of advertising, which is constant.

It takes simple algebra to notice that $\hat{p}^s_{n;0} > p^s_{n;0}$ as long as $A > 0$, which I assume is the case. To see the further effects of this mechanism notice that the probability of purchase positively depends on the valuation of the product (equation (3.10)), which is proportional to the skill for the product (as $v^s_t = k^s_t \lambda$). The upgrade of those skills positively depends on the purchase probability (equation (3.4)). Thus, an exogenous increase in one of these variables creates a feedback loop which increases all the other variables (and ultimately increases itself) in subsequent periods. This is the mechanism by which consumers respond to actions by producers in my framework.

From $\hat{p}^s_{n;0} > p^s_{n;0}$ we can deduce that $\hat{h}_{n;0} > h_{n;0}$. Following the temporal effect argument earlier, I can argue that as long as $(1/S)\sum_{s=1}^{S} k^s_{n;0} < 1$, $\hat{h}_{n;1} > h_{n;1}$, and that, in general, $(1/S)\sum_{s=1}^{S} k^s_{n;t-1} < 1 \Rightarrow \hat{h}_{n;t} > h_{n;t}$. So, advertising results in market share gain over an extended period of time if the producer advertises during the first period when her product was put on the market.[6] Then we can measure the return to advertising as

$$r_n = \sum_{t=0}^{\infty} \left(\hat{h}_{n;t} - h_{n;t} \right), \tag{3.11}$$

where r_n is the return on advertising for product n.

3.4.2 Analysis

By looking at the structure of my model we can derive certain expectations about the effects of model parameters on returns to product promotion in environments where consumers are learning how to consume. The effect of advertising on market share distribution depends on parameter γ - the user-friendliness of a product. If γ is high, the probability gain of a certain size will result in higher average skill level and thus in (on average) higher valuation of the product during the next period compared to when γ is low. But at the same time a higher γ would also directly imply higher skills as well as valuation of the product in the next period

[6] The only case when this statement is not true is when $\forall s \; v^s_{n;0} = \lambda_n$, which we can rule out as it does not involve any learning, thus is not interesting.

compared to a lower γ. Thus, the size of market share gain due to advertising in case of a lower or higher γ is not clear right away.

Returns to advertising also depend on product quality (λ). The higher the quality, the more time it takes to reach the equilibrium market share for a given initial valuation and γ. Therefore, there is potential for higher return on advertising. But at the same time consider the situation when only two products are competing on the market. Assume that their qualities and user-friendliness levels are the same. Then without advertising both products will have half of the market share (if I also assume they start from equal average initial valuations). In this case advertising of one of the products will result in market share gain, which is the measure of return to advertising. Now assume one of the products is of a much higher quality, *ceteris paribus*. Consider the case when the difference between λs is so high that the equilibrium share of the better product is 95%. Will this better product have a higher return to advertising? It is not clear as the product would quickly acquire its market share even without advertising, so returns to advertising for this scenario would be marginal.

As a result I can hypothesize that neither product quality nor the level of user-friendliness has a monotonic effect on returns to advertising in my model. I use the r measure to analyze the effects of changes in values of these parameters on returns to product promotion.

There are few possible competition environments in which one can analyze these factors. A market where there is only one active product (i.e. a product that can increase its market share by advertising for an extended period) is the simplest.[7] With one active product on the market returns to advertising completely depend on the product's own characteristics and the model yields somewhat trivial results: lower product quality and higher user-friendliness result in higher returns to product promotion.

Notice that the effect of the product quality is at odds with intuitions in the earlier literature. The reason is that in this case competition is not present: although there are incumbent products on the market they are inactive. To understand why this environment should result in higher returns for lower quality products consider two products: one of a high quality, the other of a low quality. When they are put on the same market advertising in both cases will result in an equal size jump in their market shares. After this jump, market share dynamics start converging to the original (without advertising) transition path towards the equilibrium. Due to the model specification the convergence speed will be higher for the high quality product. This means that the lower quality product will stay off the original transitional path for longer, which in principle is the source of returns to advertising.

[7] Recall that if the product is on the market long enough for the average consumer skill to be sufficiently close to 1, it has no incentive to do costly advertising and it becomes inactive. Thus, this case simply means that there is only one new product on the market. This is equivalent to monopolistic competition, which I can not discuss in its classical form as returns to advertising are measured as the gain in market share.

The intuition behind the result with user-friendliness is simpler: the higher the user-friendliness of a product, the easier the skills are to acquire. Therefore the equilibrium market share will be reached faster. Because returns to product promotion are measured as the difference between the two scenarios (with advertising and without advertising), reaching equilibrium faster guarantees higher returns.

Including more new products brings new insights as returns now depend not only on a product's own characteristics, but also on competitors' characteristics. At the same time they increase the burden of managing the model as the introduction of every additional competitor increases the number of parameters. As a result, rigorous analysis of markets with many competitors becomes impossible in this framework. Thus I choose to analyze in greater detail the market with only two new products in order to detect intimate links between the parameter changes and returns to product promotion.

To demonstrate the results, it is convenient to assume that the number of incumbents on the market is zero, meaning that there are no other rivals to the two new entrants that are engaged in competition for market share. As a consequence, I discuss the case of duopoly.

Consider a new industry arising, having two firms that enter simultaneously. The two firms produce substitute products but their characteristics (λ and γ) might differ. One of the problems in discussing the effects of advertising in this setup is that both of the firms can advertise simultaneously. Therefore, if I choose one of the firms and discuss returns to its advertising I will have virtually two regimes to analyze: one when the competitor does not advertise and the other when the competitor does advertise. These two regimes might produce not only quantitatively but also qualitatively different responses to advertising. Yet, numerous simulations show that this is not the case in the model: although these two regimes produce quantitatively different results, the qualitative behaviour of returns is similar across those regimes. Meaning that, if one plots the size of returns against model parameters, the profile has the same shape no matter whether the competitor advertises or not. Their profiles might differ quantitatively, but these differences are extremely small, often negligible.

The reason for this is the following. Consider these two products having the same characteristics (λ and γ). In this case if none of the producers advertise both of the products will have half of the market share from start to end.[8] If one of the producers advertises, she takes additional market share from the competitor in that period. As a result, skills for that product are accumulating faster and this will give temporal advantage to the advertiser's product. This will last for some period until the average skill levels converge to 1 and market shares of both competitors converge to 50%.

Now consider what happens if the other producer also advertises. Of course both of them stay with 50% market share, thus I get the same dynamics of market shares as when neither of the producers was advertising. In this situation, gains

[8] Here I am assuming that the averages of initial skill levels for both products are equal.

from advertising when the competitor advertises and when the competitor does not advertise (in terms of market share gain due to advertising) are exactly equal. The situation becomes asymmetric when characteristics of the products start to differ from each other. However, as long as the differences between the product characteristics are not extremely large, returns to product promotion when the competitor advertises and when she does not are sufficiently close to each other. Thus, in the rest of this section the discussion of one of the scenarios will suffice. Due to simulation simplicity I choose the scenario when the competitor does not advertise.[9]

If we have two active products on the market we have pairs of λs and γs to work with. But as these products are competing only with each other, intuitively important parameters would be the ratio of λs and the ratio of γs rather than the values of single parameters themselves. Thus, I work with these ratios. This complicates the reporting of results. To solve this problem I work with peculiar scales for the presentation of simulation outcomes. The axes for the parameter ratios are constructed in such a way that they reach 1 in the center, which means that the two parameters under discussion have equal values. This splits the axes in two. The right half is a linear scale and reaches some maximum value (e.g. 5, which would mean that the value of the parameter is 5 times that of the value of the corresponding parameter of the competitor's product), while the left half symmetrically follows the right half and takes values of 1 over the corresponding value from the right half (thus in this case the left half of the axes would go to 1/5, which would mean that the value of the parameter is 5 times lower compared to the corresponding parameter of the competitor). To eliminate the differences in results due to the differences in absolute values of the parameter, parameter ratios are created by holding the average of the parameter values constant across the axes. This means that if the ratio of xs being equal to one is created by $x_1 = 2$ and $x_2 = 2$, then the ratio of 3 is created by $x_1 = 3$ and $x_2 = 1$ and the ratio of 1/3 is created by $x_1 = 1$ and $x_2 = 3$.

The left panel of figure 3.1 reports returns to advertising for the different values of the ratios of product user-friendliness (γ) and quality (λ). In these simulations I fix the number of consumers to be 100 and the intensity of communication to be $\mu = 0.3$. The range of axes are chosen so that the picture presented displays the relevant portion of the profile. Moving closer towards zero or infinity results in an absolutely flat profile.[10] These are the averages of 40 runs, standard deviations are very small. Every run covers the whole spectrum of γ and λ ratios. In the beginning of every run I generate consumers and the initial skill distribution for each of the products, averages of which are equal to each other. After that I run the economy as long as it takes advertising returns to become negligible for

[9] Despite the slight difference in values, the scenario when the competitor advertises maintains the general shape presented hereafter.

[10] What I mean by the flat profile is that returns quickly go to zero as values of parameters increase or decrease further outside the range depicted on the figure 3.1. Outside portions of the parameter space would have been represented on the same figure with black shading.

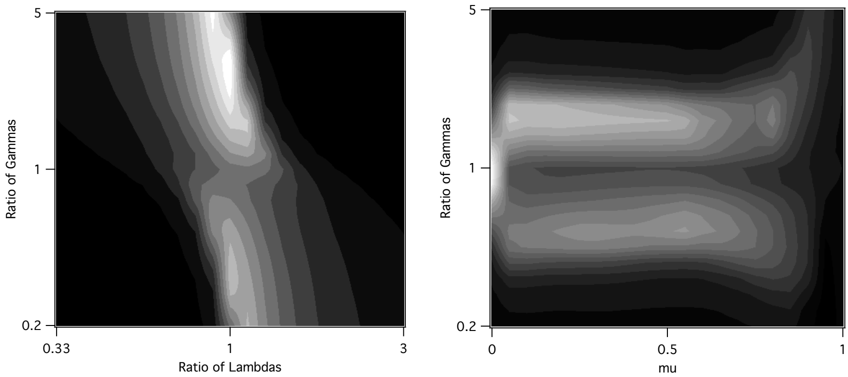

Figure 3.1: The case of duopoly: dependence of returns to advertising on quality and user-friendliness (left) and on user-friendliness and communication intensity (right). Lighter shades of gray indicate higher values.

each γ ratio - λ ratio pair. The next run starts by generating the new skill level distributions (of course, the averages of every run's skill distributions are equal).

There are a couple of observations one can make about the left panel of figure 3.1. The first is that, no matter what the user-friendliness of the product is, if the quality of the product is sufficiently higher or lower than the quality of the competitor, then returns to advertising are low as compared to the situation when qualities are equal.[11] The reason for this is that in both cases advertising can not affect the skill level development in the economy: if the product is doomed for 0.5% of the market share, there is little product promotion can do to affect the transitional dynamics to the equilibrium. The same reasoning applies to the symmetric situation: if the equilibrium market share of a product is 99.5%, product cannot gain much more by advertising during the transition.

The second observation is that if product qualities are sufficiently close to each other, the dependence of returns to product promotion on user-friendliness of the product has a double-humped shape: starting from the relatively non-user-friendly product, as user-friendliness increases, initially so do returns to advertising, but they fall after some time, reaching local minimum when user-friendliness parameters of the two products are equal, then they rise and fall again.

The explanation for this phenomenon can be found in the dual nature of the advertising in my model: besides the fact that advertising ensures more early consumers for the product, it also ensures fewer consumers, and slower skill accumulation, for the competitor. Thus, given the equal product qualities, if the product is more user-friendly (relative to the competitor) the major contribution to the returns comes from more consumers consuming this product, while the contribution due to less consumers consuming competitors product is minor. On

[11]Although here I report results for one particular μ, the shape of the profile is virtually the same for other values of communication intensity.

the other hand, if the product is (relatively) less user-friendly, the contribution from deterring some consumers from consuming competitor's user-friendly product becomes much larger, while the contribution from increased consumption of the own product is minor. The dependence of both of these effects on the level of user-friendliness is non-linear. And it seems that their joint effect is smaller when the levels of user-friendliness are equal to each other than when they are (not too) different. This explains the double peaked nature of the returns/gamma profile.

Recall that in the simulations reported in the left panel of figure 3.1 I fixed the communication intensity, μ. It is important to perform a robustness check to see whether the double-humped shape is due to some peculiar value of communication intensity or whether it persists for the different values of this parameter. In the right panel of figure 3.1 I present the results from similar simulations. In this case I fix product qualities and vary communication intensity instead. As we saw from the left panel, interesting dynamics are in place when competitors' products have quality levels which are sufficiently close to each other. So, in this simulation I fix the ratio of qualities to be 1, I vary γs in the same way as in previous simulations and explore the whole space of the values of parameter μ. Everything else stays the same as in the simulations reported in the left panel of figure 3.1. The right panel of figure 3.1 shows that a double-hump shape of returns-user-friendliness profile is present as long as communication levels are away from extremes. This suggests that the qualitative behaviour (the shape) of $r - \gamma$ profile is fairly robust to changes in consumer communication intensity.

3.4.3 Discussion

In the previous section I found that the dependence of returns to advertising on product quality and the level of user-friendliness is not monotonic. This contradicts previous theoretical (e.g. Milgrom and Roberts 1986), as well as empirical (e.g. Nichols 1998) contributions to the analysis of the effects of advertising, which claim that a higher quality would result in higher returns. Does this mean that the current model contradicts the empirical findings? This is an important concern which would imply that my base assumptions about the behaviour of consumers are not correct.

To test this we have to go deeper into the differences between markets used for empirical studies and the markets I analyzed. The monotonic relationship between quality and returns has been empirically found on markets with many diverse producers (e.g. automobiles in the case of Nichols, 1998), while I have discussed the case of duopoly. So, if we want to produce comparable results, we have to assume that there are many active products on the market and that all of them have different characteristics. As I argued before, there is no comprehensive way of thoroughly studying the multi-product cases using the current model. If one allows more products on the market each of them brings two additional parameters. This increases the burden of the model management. In addition, every extra product

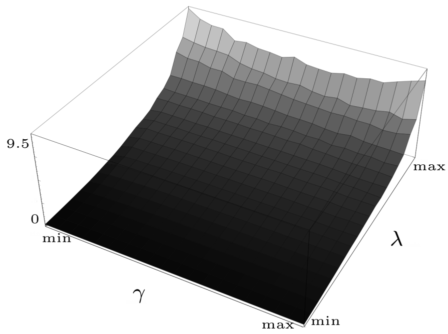

Figure 3.2: The case of competitive market: dependence of returns to advertising on quality and user-friendliness when the competitors do not advertise (left) and when they do advertise (right).

increases the discrepancy between the results when competitors do not advertise and when they do advertise. If one wants to allow for some of the competitors to advertise while others not, it would be better to study a model where products have three characteristics instead of two. Introducing a new characteristic for products in this framework will result in a complete reformulation of the model specified in section 3.3. A new model will be substantially less parsimonious than the model I am considering in the current essay. Therefore, I choose not to explore this alternative formulation here and keep to the baseline model in order to derive comparable results to Nichols (1998).

To derive the relevant results I perform the following exercise. Consider the case when there are 20 firms on the market. Assume that their qualities and user-friendliness levels are distributed normally around some means. First I generate these distributions along with the distribution of the skill levels for each product. Then the algorithm picks every combination of λ and γ to be the product under consideration. Each constellation of γ-λ is a different scenario. For each of the scenarios the remaining product qualities are coupled randomly and they comprise the competitor products. As I have 20 λs and 20 γs, I have $20 \times 20 = 400$ scenarios. All these 400 scenarios are run for the fixed distribution of λs and γs as well as for the fixed distribution of the initial skilled levels. Then I draw another 20 γs and 20 λs from the same normal distribution and another 20 skill distributions over consumers and run all 400 scenarios again. I repeat this exercise 40 times and report averages of returns to advertising.

When we have many firms, the quantitative difference between the two scenarios - one when competitors advertise and the other when they do not - becomes more pronounced. Thus, for this (multi-product) case I report both situations. The left panel of figure 3.2 reports returns to advertising when the remaining 19 producers do not advertise, while the right one reports the results for the case when all the remaining 19 producers do advertise. The axes on which I measure product characteristics, represent mere ordering in the relevant distribution, from the minimum to the maximum value of a given characteristic.

Figure 3.2 shows that on markets with multiple active products higher quality results in higher returns to advertising, no matter the level of user-friendliness. Thus, my model does not contradict the empirical findings. Rather, it highlights the importance of the market structure and warns that the relation between product quality and returns to advertising might be more complex than what has been believed before.

3.5 Conclusion

In this essay I have discussed two sources of consumer skill acquisition: learning by consuming and consumer socialization. I have analyzed a population of myopic consumers who socialize locally and utilize the above mentioned two forces to learn how to consume different products.

In my model each product on the market has two characteristics: quality and user-friendliness. Quality is the highest valuation consumers can extract from a given good; although it can vary over the product space, it does not vary from consumer to consumer. The long-term market share distribution depends solely on this characteristic. User-friendliness controls the speed of consumer skill acquisition through learning by consuming. This too is a product specific characteristic and does not vary from consumer to consumer for a given product. It does not affect the equilibrium market share distribution but it does affect the speed of transitional dynamics towards it.

The society is characterized by one parameter - communication intensity - which controls the speed of consumer skill diffusion through the socialization process. Another characteristic of the society is the size of the population, which has not been analyzed in this essay due to its straightforward effects. Larger population size creates greater challenge for the consumer skill diffusion as the socialization process is local.

In this framework I have discussed the dependence of returns to product promotion on product characteristics. I have analyzed two prior believes. One that products with higher quality have higher returns to advertising. The other, that products that are less user-friendly will also have higher returns to product promotion. The major conclusion is that irrespective of the level of product's user-friendliness, returns to advertising are higher when a product competes with another product of a similar quality. If the competitor is of a considerably higher or lower quality, returns to advertising fall. This contradicts earlier works (e.g. Milgrom and Roberts, 1986; Nichols, 1998) which claim that the relation between quality and returns is monotonic. I have shown that this model results in a similar pattern when there are many active products on the market. Therefore, I can conclude that earlier empirical findings (Nichols, 1998) do not contradict the results of the model presented in this essay. The fact that the current model results in different $r - \lambda$ profiles for different number of active products on the market suggests that dependence of returns to advertising on product quality is influenced by the market structure, a variable that has been omitted from previous analysis. This is in line with Becker and Murphy (1993), who warn that certain effects of product promotion might depend on the market composition.

Another finding of this essay is that the dependence of returns to advertising on the level of user-friendliness has a double-peaked shape when products have similar qualities. This is due to the dual effect of advertising, which means that advertising benefits producers not only by increasing the number of consumers using their products but also because it reduces the number of consumers using competitors' products. The sizes of these two effects change at different rates across the change of the levels of user-friendliness. In the case of a product being more user-friendly than competitor's the first effect dominates, while in the opposite case the second effect is the dominant one. What is important here is that the sum of these two effects is higher in each of the cases (when the product is more or less user-friendly compared to the competitor) than when the competitor's product is just as user-friendly.

Part II

Contributions to Modeling of Firm Behaviour

Essay 4

R&D Behaviour and the Emergence of Fat Tailed Firm Size Distributions*

Abstract. This essay presents a simple boundedly rational model of a firm and consumer behaviour. I formulate an entry game, where every firm decides on investing in R&D for inventing a new product that will appeal to certain group of consumers. The success depends on the amount of funds available for the project as well as firm's familiarity with the relevant proportion of taste space. I identify the section of parameter space where firms have an incentive to diversify. For these parameter constellations the model results in rich industrial dynamics. Equilibrium firm size distributions are heavy tailed and skewed to the right. The heaviness of the tail depends on one industry-level parameter.

4.1 Introduction

Research and development is an essential activity of a modern successful firm. Planning of this process is a complex problem as firms have to take into account many aspects, including dynamic external factors, like competition. R&D behaviour of firms has been an important topic in economics for the last three decades and there are important theoretical (Harris and Vickers, 1985), as well as empirical (Cohen and Klepper, 1992) contributions to its understanding. Successful R&D requires possession of special skills and knowledge by firms. It has also been suggested that there are certain increasing returns to doing R&D (Nelson, 1982).

*I am grateful to Robin Cowan, Steven Klepper, Ronald Peeters, Stefan Straetmans and Bart Verspagen for their insightful comments. Remarks from participants of various meetings in Barcelona, Jena, Maastricht, Milan and Sophia Antipolis, especially from Alex Coad, André Lorentz, Frank Neffke, Bulat Sanditov, Paolo Saviotti and Marco Valente are also appreciated.

Research and development is the main contributor to firm growth. Therefore, we should look at R&D behaviour when we try to explain the shape of firm size and growth rate distributions. However, to the best of my knowledge, this has not been done in the literature. Early models of firm growth, like Gibrat (1931), Kalecki (1945) and Simon (1955), were concerned with the firm size distributions but ignored deliberate efforts in R&D activities (de Wit, 2005). More recent models, like Nelson (1982), Cohen and Klepper (1992) and Klette and Kortum (2004), that explicitly model firm R&D decisions, do not analyze resulting firm size distributions.

This essay presents a model of firm R&D behaviour and studies firms' R&D incentives and resulting firm size distributions. The central notion in my model is that of a taste space. I assume that each product can be located at a point in a taste space. Its location in this space fully characterizes a product. Products that are located closer in taste space are better substitutes to each other, thus I can assume they are traded on a common market, I call this portion of the market a submarket. The definition of a submarket is, in principle, the outcome of the taste space discretization: each discrete unit is a separate submarket. Consequently, all the products traded on the same submarket are located at the same point in taste space.

In this environment I formulate an entry game where a constant number of firms decide on target submarkets for their potential products in order to grab the higher share of constant stream of aggregate profits. In order to come up with the suitable product, a firm has to invest in product development. R&D is a stochastic process and its success rate depends on the amount of money invested as well as the knowledge of the target submarket by the firm. Innovating on new markets results in acquisition of new knowledge by the firm, which increases productivity of its R&D. However, while deciding on the target market, firms fall short of perfect rationality: they do not take into account the effects of prospective new knowledge on their future R&D performance. This assumption is in line with empirical findings that suggest that expertise and knowledge that firms posses is often acquired in unplanned ways (Andrews, 1971; Porter, 1980; Nelson and Winter, 1982), in my case by innovating on new (sub)markets.

Proximity in taste space has two implications. One is that the shorter the distance, the better substitutes a pair of products are to each other. The other is that closely located submarkets will be somewhat similar, thus I can safely assume that knowledge of one of the submarkets can be used (with somewhat lower productivity) to innovate on nearby submarkets. These intuitions describe two important parameters that I use for the analysis: the submarket specificity of producer knowledge and the submarket specificity of consumer tastes.[1]

Starting from a homogenous cohort of entrants firm size heterogeneity arrises naturally in my model. It turns out that the equilibrium firm size distribution

[1]There are the discrete counterparts of location specificity of knowledge and preferences in taste space.

is fat tailed, which is consistent with empirical findings (Cabral and Mata, 2003; Coad, 2009). I further analyze the dependance of the tail index of this distribution on the parameters of the model. I show that for the relevant fraction of parameter space, the tail index depends only on the submarket specificity of firm knowledge.

The remainder of the essay is organized as follows. Section 4.2 presents a model of firm R&D behaviour and growth. Section 4.3 presents the results concerning R&D incentives and equilibrium firm size distributions. The last section concludes.

4.2 The Model

The model has a distinctive structure from most R&D models: the economy does not consist of intermediate and final good sectors. There is only one sector, thus one global market, on which substitute goods are traded. Each product on the market has only one property: the location in the taste space. I assume that although all the products existing in the economy are substitutes, degree of substitutability between any pair of products varies. More precisely, I assume that that the taste space is a uni-dimensional periodic lattice (a circle). Then higher (circular) distance between any pair of locations will be reflected in lower degree of substitutability between products located at those locations.

Each product is produced by one and only one producer. This is due to the existence of property rights. However, each firm can hold many patents and produce many goods. I assume that there is no market for intellectual property. I have two reasons for assuming that firms are not willing to sell their intellectual property. Firstly, imperfections on information market drives the offer price downwards (Arrow, 1962b). Secondly, because innovations are more valuable to the innovator than to other firms (Cohen and Levinthal, 1989), which makes the offer price to seem even lower.

As there is no transfer of property rights, the only way to grow is through R&D process. To model research and development, it is important to have an idea about the distribution of R&D intensities in a typical industry. This topic has attracted interest in economic literature (Cohen et al., 1987). Empirical work on various industries suggests that the cross-sectional correlation coefficient between firm's R&D expenditures and sales is nearly 0.8 (Cohen and Klepper, 1992). It also has been found that firm size has virtually no effect on R&D intensity (Cohen et al., 1987). This points to the fact that R&D intensity is relatively constant across the firms of different sizes. It has been reported that R&D intensity is not the result of maximizing behaviour of firms, but is rather based on rules of thumb and that its adjustment is quite sluggish (Silverberg and Verspagen, 1994). This suggests that R&D intensity is relatively constant across time too.

Therefore, I assume that R&D intensity is constant. Thus, I do not allow producers to choose the amount of money they want to spend on research. Rather, I restrict their choice to the decision of whether they want to do R&D or not. If they decide to do so, they allocate a constant share of their current profits to this activity. They can also decide on the target location for their prospective product. More precisely, they first decide on where in taste space they want to place the product and then perform the R&D aimed at coming up with the product that would be suitable to be placed at the target location.

Productivity of an R&D project depends on the amount of money invested in it (thus, on the current profits of the firm), as well as on the market knowledge (of the target location) of the producer. Recall that the lower distance between a pair of locations in taste space implies high substitutability between products placed at those locations. Then it seems intuitive to assume that familiarity with a certain location in a taste space might help to innovate in nearby locations. I assume that producers have good market knowledge of locations where they currently have products. I also assume that the productivity of their knowledge decreases with the shortest distance between the location on which a given producer operates and the target location.

Demand combines two well-known frameworks. The first is the well known ideal variety model by Lancaster (1979). In this framework a consumer has a preferred variety of a product on the market (her ideal variety). It also has preferences for other varieties. Then, taking into account prices and strength of preferences for each of the products consumer decides which product to buy. The other framework is constant elasticity of substitution utility function, which is a workhorse of modern neoclassical economics. In this framework elasticity of substitution between any pair of products is constant. To combine these two frameworks I make share coefficients of CES utility function proportional to the strength of preference of each consumer for the given location in taste space.

Consider consumers having heterogeneous tastes. Following Lancaster (1979), I assume that for each consumer there is a unique location in taste space which corresponds to her ideal variety. Her preferences decay with the distance from this location, reaching the lowest value on the opposite side of the circle representing taste space. Then, utility maximization implies that the demand of a consumer for a product depends on her preference for the location of this product in taste space, as well as on available alternatives and prices.

We analyze the industry of a fixed size. The consequence is that every time a new product enters the market it has a business stealing effect on incumbents, as now total demand has to be redistributed on more products. This is Schumpeterian creative destruction effect. However, this effect is not homogenous across products in my model. As the new product is a better substitute for products located in close proximity in taste space, it steals more business from them, relative to those at greater distances.

Given the description of the model we can analyze R&D incentives of firms. Due to the positive contribution of market knowledge to R&D productivity, firms have an incentive to innovate in close proximity to their products, as they have good knowledge of that portion of taste space. However, due to asymmetric creative destruction this might not be optimal as investing close to your old products might result in duplicating efforts in attracting consumers. This indirectly increases the cost of innovation. Therefore, at any point in time, given the market situation there will be an optimal point in taste space where a firm would want to innovate (provided that it wants to innovate).

4.2.1 General setup

So far we have thought about taste space to be continuous, however it is in many ways convenient to discretize it. In this case location of every product will be not a point, but rather one dimensional unit on a taste circle (a portion of a circle circumference with a positive length). Then we can construct a new unit of analysis.

Definition 4.1. *A submarket is as the collection of products that are located on the same discrete unit in taste space.*

Consider the finite number of submarkets located on a unidimensional periodic lattice. Locations on this lattice will be referred to as $i \in \{1, 2, \ldots, I\}$. The finite number of consumers are indexed by $s \in \{1, 2, \ldots, S\}$. There are also a finite number of firms indexed by $n \in \{1, 2, \ldots, N\}$ producing finite number of products. These products are indexed by $m \in \{1, 2, \ldots, M_t\}$ at any time. Each firm has to produce at least one product, therefore $M_t \geq N\ \forall t$.

Each firm produces as many products as many patents is holds. All the technologies used in the economy are neutral to scale. Profits of firm n at time t are given by

$$\pi_t^n = \sum_{j \in \mathscr{P}_t^n} (p_{j,t} - w_{j,t}) D_{j,t}, \tag{4.1}$$

where $p_{j;t}$, $w_{j;t}$ and $D_{j;t}$ are price, unit cost and total demand of product j at period t respectively and $\mathscr{P}_t^n$ is the set of products firm n produces.

I assume that producers are price takers and that prices are set somehow outside the market. I consider the case when all the products in the economy have the same price and it is constant over time $p_{i,t} = p_{j,t} = p_{i,\tau} = p,\ \forall i, j, t, \tau$. Moreover, I assume that production costs are homogenous across products and time (no process innovation). As a result, firms' profits are a constant share of their total demand $p_{j,t} - w_{j,t} = \omega,\ \forall n, j, t$. This reduces producer's profit function to $\pi_t^n = \omega \sum_{j \in \mathscr{P}_t^n} D_{j,t}$ and collapses producer's problem into the maximization of its total demand.

In this environment we can define an entry game. Producers engage in product innovation. If firm n decides to do R&D at time period t, it can choose an appropriate submarket and invest a constant share of its last period's profits into this activity $R_t^n = \beta \pi_{t-1}^n$.[2] Firms decide on the location of the prospective product

[2]Although empirical findings point to the fact that firms spend constant share of their rev-

in a taste space. Or, equivalently, on the target submarket. Given the amount of money allocated to R&D (R) and market knowledge of the producer for target submarket i, innovation success probability is given by $r_{i,R} \in [0, 1]$.

Recall that the success rate of R&D process depends on the knowledge base that a firm has. I have assumed that firms have good knowledge of the taste space close to the locations of their current products. Then successful innovation outside submarkets where firms currently operate results in new knowledge that can be used by firms in order to increase their R&D productivity on a certain portion of the taste space. Thus, firms can behave strategically in entering new markets for acquisition of knowledge. However, according to the discussion in the introduction to the essay we know that knowledge base of firms is acquired in unplanned ways. Therefore, I exclude these incentives for strategic behaviour from the analysis. I analyze producers that are not fully rational. Rather, they are myopic in this respect. In this case the game boils down to a constant number of producers stochastically placing new products on chosen submarkets in order to maximize their contemporaneous profits.

In order to analyze producer incentives recall that there is creative destruction in the model. Once a new product is placed on the market the incumbents' shares decrease. Therefore, while planning their actions, perfectly rational (and able) firms should take into account the effect of creative destruction that might come from other firms innovating. However, I assume that global information structure is such that producers do not observe each others incentives.[3] Hence, producers can not take into account this effect in a rigorous way. What they can do instead is to "anticipate" (estimate) the size of this effect on their markets and set their market attitudes accordingly. In this case we will have pessimistic firms, who anticipate destruction of large share of their markets (due to other producers innovating) and optimistic firms who anticipate the size of this effect to be small. Formation and updating of market attitudes are out of scope of this paper. Instead, I model optimistic firms who estimate the destruction of their market share due to innovation of other firms to be negligible.[4]

Let $\psi \in \{0, 1\}$ describe the decision of a producer to conduct R&D at a given time period or not. Then the problem of myopic, optimistic, risk-neutral producers can be written as

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enues/sales on R&D, my assumption of scale neutrality implies that constant share of revenues is constant share of profits.

[3]This might be, for example, due to the fact on a global anonymous market it is impossible to identify who produces which product.

[4]Alternatively I could make firms homogenous in this respect and introduce a parameter that would control their beliefs. For example, they would always anticipate that $\delta\%$ of their market share will get stolen. This enters the profit maximization problem (later equation (4.2)) in a trivial way and does not modify the incentive structure, but rather only the size of expected profits. my approach of modeling optimistic firms is equivalent to setting $\delta = 0$.

$$\max_{\psi,i} \gamma \left(\psi r_{i,R} \left(D_\iota^i + \sum_{j \in \mathscr{P}_{t-1}^n} D_j^i \right) + (1 - \psi r_{i,R}) \sum_{j \in \mathscr{P}_{t-1}^n} D_j \right) - \psi R, \tag{4.2}$$

where D_j describes the total demand for product j at current situation (if the producer does not innovate) and D_j^i describes the demand if the producer places a new product on the market i. ι is the index of a new product.

Taking into account the definition of R, we can see that value of γ does not affect the optimization problem (as long as $\gamma \neq 0$) and thus I can rewrite utility maximization as

$$\max_{\psi,i} \psi r_{i,R} \left(D_\iota^i + \sum_{j \in \mathscr{P}_{t-1}^n} D_j^i \right) + (1 - \psi(r_{i,R} + \beta)) \sum_{j \in \mathscr{P}_{t-1}^n} D_j. \tag{4.3}$$

Solution to this problem gives the answer to two questions, to engage in R&D or not, and if yes on which submarket. This fully characterizes the incentives of players of the game. Then, we can define the equilibrium of the game.

Definition 4.2. *Equilibrium of the game is reached when* $\psi_t^{n*} = 0$, $\forall n$.

This definition corresponds to the Nash equilibrium of the game in a sense that no player has any further incentives to unilaterally deviate from the current situation (invest in R&D). After this point, industry becomes stagnant and all the variables become time-invariant.

4.2.2 Functional forms: exponential decay in preferences and knowledge

Demand. Consider consumers having CES utility function of a form

$$U_t^s = \sum_{m=1}^{M_t} {k_m^s}^{\frac{1}{\gamma}} {C_{m,t}^s}^{\frac{\gamma-1}{\gamma}}, \tag{4.4}$$

where $C_{m,t}^s$ is the amount of product m consumed by the consumer s at time t, k_m^s is the parameter that reflects the strength of the preference of consumer s towards the product m.

Maximization of the consumer utility under equal prices implies that

$$\frac{k_m^s}{C_{m,t}^s} = \frac{k_j^s}{C_{j,t}^s}, \tag{4.5}$$

$\forall m, j \in \{1, 2, \dots, M_t\}$, which means that consumption of products by each consumer will be proportional to their consumer-specific preference coefficients.

Products traded on the same submarket are placed at the same location in taste space. Therefore, $\forall s$ $k_m^s = k_j^s$, if m and j are traded on the same submarket.

Then, optimality condition (4.5) implies that the total demand on the submarket is equally split by all the products on the submarket. This might seem to suggest that producers that are monopolists on their submarkets will never have incentives to invest in placing a new product on the submarket. However, this is not true, as coming up with the new variety of the product will attract consumers from neighbouring submarkets and increase the size of the submarket.

In order to clearly see this implication of the CES utility function consider the following example.

An example: 2×2×2 economy. There are only two producers and only two consumes in the economy. Each firm produces one product. Firm 1 produces product 1, firm 2 produces product 2. Consumer utility maximization implies condition: $k_1^1/C_1^1 = k_2^1/C_2^1$ for consumers 1 and $k_1^2/C_1^2 = k_2^2/C_2^2$ for consumer 2. Then, the consumption of product 1 by the consumer 1 is $C_1^1 = \frac{k_1^1}{k_1^1+k_2^1}\frac{Y^1}{p}$, where Y^1 is the income of consumer 1. Demand of each product by each consumer can be calculated similarly. Profits of two firms in this case will be $\pi_1 = \gamma\left(C_1^1 + C_1^2\right) = \frac{\gamma}{p}\left(\frac{k_1^1}{k_1^1+k_2^1}Y^1 + \frac{k_1^2}{k_1^2+k_2^2}Y^2\right)$ and $\pi_2 = \gamma\left(C_2^1 + C_2^2\right) = \frac{\gamma}{p}\left(\frac{k_2^1}{k_1^1+k_2^1}Y^1 + \frac{k_2^2}{k_1^2+k_2^2}Y^2\right)$.

Consider the situation when firm 1 places a new product (3) on the same submarket where its old product is traded. Then I have $k_3^1 = k_1^1$ and $k_3^2 = k_1^2$. In this case the profits of two firms are $\pi_1' = \gamma\left(C_1^1 + C_1^2 + C_3^1 + C_3^2\right) = \frac{\gamma}{p}\left(\frac{2k_1^1}{2k_1^1+k_2^1}Y^1 + \frac{2k_1^2}{2k_1^2+k_2^2}Y^2\right)$ and $\pi_2' = \gamma\left(C_2^1 + C_2^2\right) = \frac{\gamma}{p}\left(\frac{k_2^1}{2k_1^1+k_2^1}Y^1 + \frac{k_2^2}{2k_1^2+k_2^2}Y^2\right)$. It can be easily verified that $\pi_1' > \pi_1$. Therefore monopolists might have incentives to invest in R&D on own submarket. This, in principle, can be viewed as investment in expanding a submarket.

To understand the logic and economic relevance of the process that I am modeling consider the industry being defined quite broadly (e.g. defined at the 1 or 2 digit SIC level), so that even sub-markets are broad (e.g. at the 3 or 4 digit level). Intuitively, from the standard budget constraint argument, every product even in such a broad industry is a substitute to every other to a certain extent. According to my assumptions submarkets/subindustries with similar characteristics will contain products that are better substitutes to each other. Arrangement of these sub-industries along the circle offers a convenient way to organize and model elasticity's of substitution between the submarkets using the distance. Within the submarket, of course the products are better substitutes to each-other. However, I do not model at such a fine scale and simply assume that the demand attracted by the submarket is equally split by all the products contained by the submarket.

The consequence of using utility function (4.4) in this arrangement leads to two seemingly counter-intuitive results. One is the fact that increasing the number of products on the submarket makes this particular submarket more appealing to consumers. The other, which is the consequence of the first one, is that producers have incentives to innovate on the submarket where they are already active.

Although these results might seem at odds with common practices in economic modeling, both of them describe empirically identified properties. The first fact replicates the empirical regularity that larger shelf space leads to higher sales, which has been known to marketing academicians (Lee, 1959; Cox, 1964) and economists (Brown and Tucker, 1961) since 1960s. We can consider the constant shelf space being equally divided among all the products. Then a new product yields larger shelf space to the submarket and ensures higher popularity of that submarket. As of the second fact, once we define the submarket at the 4 digit SIC industry, it is not hard to think of an example of an actual company that has multiple products on the same submarket.

In order to incorporate the intuition that consumers' preferences decay with the distance from her ideal variety, I make share coefficient k dependent on distance in taste space. In particular, I assume that this dependence is exponential:

$$k_i^s = \frac{1}{(d_i^s + 1)^c}, \tag{4.6}$$

where d_i^s is the distance of submarket i from consumer s's favorite submarket and c is the parameter that controls the strength of submarket specificity of consumer tastes. It governs the rate of decay of preferences with distance: the higher c, the more submarket specific consumers' preferences are.

To see that these functional forms (equations (4.4) and (4.6)) imply that products located closer in taste space are better substitutes, we can look at the marginal rate of substitution

$$MRS_{jm} = \left(\frac{k_m^s}{k_j^s}\right)^{\frac{1}{\gamma}} \left(\frac{C_m^s}{C_j^s}\right)^{-\frac{1}{\gamma}}. \tag{4.7}$$

Assume product m is located at consumer s's preferred location in taste space (m is s's ideal variety). Then by substituting equation (4.6) into equation (4.7) it can be readily seen that $\partial MRS_{jm}/\partial d_j > 0$, which implies that with increasing distance from her ideal variety consumer needs bigger and bigger amounts of product j as a compensation for a unit of product m in order to stay at the same utility level. This means that with increasing distance product j becomes worse substitute to product m.

Equations (4.5) and (4.6) imply that product purchases of a single consumer are proportional to $(d_i^s + 1)^{-c}$. In this discussion, as well as in rest of the paper, I assume that the taste structure is the same for every consumer. In other words, that parameter c is constant across population, it is the characteristics of the society, rather than of an individual.

Now we can calculate the demand for product m (that is traded on submarket i) coming from agent s:

$$D^s_{m,i} = \frac{(d^s_i + 1)^{-c}}{\sum\limits_{j=1}^{M} (d^s_j + 1)^{-c}} \frac{Y^s}{p}. \tag{4.8}$$

Then, the total demand for the product m at time t would be

$$D_{m,i,t} = \sum_{s=1}^{S} \frac{(d^s_i + 1)^{-c}}{\sum\limits_{j=1}^{M} (d^s_j + 1)^{-c}} \frac{Y^s}{p}. \tag{4.9}$$

Supply. Producers can engage in R&D activity. For successful R&D they need two types of input: money and knowledge. I assume that the knowledge of the firm about specificities of the submarket declines with the distance from the submarket on which the firm currently operates. Formally:

$$\kappa^n_i = \frac{1}{(d^n_i + 1)^a}, \tag{4.10}$$

where d^n_i is the *shortest* distance from the submarket where the firm n currently operates to the submarket i. a controls the level of submarket specificity of producer knowledge. I assume a is a characteristics of a population of firms, thus it is constant across them.

The process described above implies that with successful innovation on new (to the firm) markets the firm gains additional knowledge. This is due to the fact that entering new markets guarantees better positioning of the firm for certain portions of the taste space, thus familiarity to new submarkets increases R&D productivity on part of the new submarkets where a firm can potentially diversify in later time periods.

I assume, that the probability that a firm will come up with the suitable product to be placed on at the submarket i after investing R into the R&D project during one period, is given by

$$r^n_i = \frac{\kappa^n_i}{1 + \exp(q_1 - q_2 R^n)}, \tag{4.11}$$

where R^n is the sum of money that firm n invested in R&D, q_1 (> 0) and q_2 (> 0) are the parameters controlling the productivity of investment in R&D, which satisfy the constraint $(1+\exp(q_1))^{-1} \approx 0$.[5] Equation (4.11) implies that $\partial r/\partial R > 0$, $\partial^2 r/\partial R^2 < 0$, $\partial r/\partial d < 0$, $\partial^2 r/\partial d^2 > 0$ and $r_{i,0} \approx 0\ \forall i$.

We restrict firms from taking more than one R&D project at a time and assume that if R&D is not successful the invested resources are lost without return and do not contribute to further knowledge accumulation.

[5] This is required in order to ensure that the probability of innovating is zero unless there is some money spent on R&D.

4.3 Analysis

The dynamic game presented in section 4.2 describes a complex, path-dependent process. Functional forms and the interdependence structure necessary to capture important intuitions make it analytically not tractable. Thus, I employ tools of numerical analysis in order to demonstrate the implications of the model. By definition, any complex system is sensitive to initial conditions. I analyze only a subset of initial conditions which is relevant for the research question. As I conduct numerical analysis with large number of parameters I am confronted with the problem of presenting the results. There are two major parameters: c and a. I present the results in the space of these parameters and examine the effects of changes in values of other parameters on these results.

4.3.1 Settings

Initial conditions. The main aim of the essay is to study the effect of the knowledge which is unpurposefully acquired through R&D process. The model endogenously generates the heterogeneity of firms with respect to their knowledge base, R&D incentives and size. As I want to understand only the contribution of the non-intended knowledge acquisition towards the shaping of firm size distribution I need to analyze industries where there is no initial heterogeneity in size and prospects of growth across firms. This requires several things. Firstly, that initial demand and amount of knowledge is the same for every producer. Secondly, that existing products at the onset of industry are uniformly distributed over submarkets. Thirdly, that the aggregate demand is uniformly distributed over submarkets. This limits the set of interesting initial conditions.

There is a simple setup that satisfies all of the requirements listed above: each producer initially has one and only one product, there is one and only one product traded on each submarket, and there is exactly one consumer per each submarket whose preferred variety is traded on this submarket. I use this simple constellation in my numerical exercise. There are several other variants of initial conditions satisfying requirements.[6] All of them produce similar results to the ones presented here.

Parameter values. As mentioned earlier, there are two important parameters in analysis of R&D incentives and firm size distributions. One is c - submarket specificity of consumer tastes, a demand-side parameter, which controls for the level of substitutability between products traded on different submarkets. Higher c implies that consumers' tastes are more submarket specific, which means that products traded on distant submarkets are poorer substitutes to each other. The other parameter is a - submarket specificity of producers' knowledge, a supply side

[6] For example when each firm produces multiple products but equal number of products. When these products are spread on the submarkets similarly for every firm. When there are twice more submarkets then are firms and every other submarket is initially empty.

parameter	description	value
γ	Profit margin	1
p	Price	1
Y	Per capita income per time period	10
β	R&D intensity	0.15
q_1	R&D productivity parameter (1)	6
q_2	R&D productivity parameter (2)	4
I	Number of submarkets	200
S	Number of consumer	200
N	Number of firms	200
M_0	Initial number of products	200

Table 4.1: Parameter values for the numerical analysis.

parameter, which controls for how useful the knowledge of a certain submarket is for the R&D process on other submarkets. Higher a implies that knowledge is more submarket-specific.

Contribution of other parameters to a qualitative analysis of R&D incentives and firm size distribution is of second order of importance. Thus, for presenting the results I keep the rest of the parameters constant. The parameter values used in the numerical analysis in following sections are given in table 4.1. The last four entries reflect the initial conditions of the industry. The selection of parameter values (except for q_1) is more or less arbitrary. However, the effects of their changes are examined. Regarding q_1, it was selected to be high enough in order for the innovation probability in case of no investment in R&D to be sufficiently close to zero even on markets where producers already operate.

4.3.2 Results

R&D incentives at the onset of homogenous industry

As discussed in section 4.3.1, initial conditions to the model have been selected to ensure the homogeneity of firms (except the location of their initial product). Because of this, initially every firm has the same problem to solve. In this section I analyze the effect of supply and demand parameters (a and c) on incentives of firms whether to do R&D and if yes, where (on which submarket) to do it.

To answer this question I rewrite producers problem taking into account equations (4.9) and (4.11) that specify the demand and distribution of firm's R&D productivity over submarkets. It is easy to verify that in case of parameter values given in table 4.1 initial demand for a product will be equal to $D = Y$, and that $R = \beta Y$. This, together with equations (4.9) and (4.11), allows us to represent R&D productivity (r) and demand for a product after innovation at a certain market (D^i) as the functions of only one variable - a distance from producer's initial submarket (d). Hence, we can rewrite D^i as D^d and consequently the equation

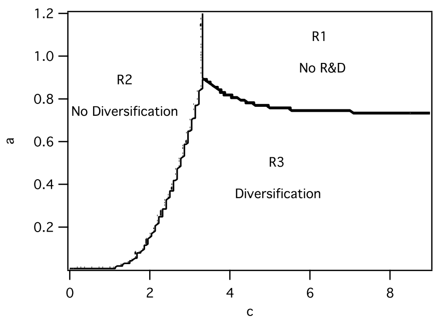

Figure 4.1: Borders of the three regimes.

(4.3) as

$$\max_{\psi\in\{0,1\}} \begin{cases} \max\limits_{d\in[0;S/2]} r^d(D^d + D^0) + (1 - r^d - \beta)Y & \text{if } \psi = 1 \\ Y & \text{if } \psi = 0. \end{cases} \tag{4.12}$$

For the sake of the analysis of behaviour implied by equation (4.12) we can differentiate between three regimes: (R1) $\psi^* = 0 \Rightarrow$ no research and development; (R2) $\psi^* = 1$ and $d^* = 0 \Rightarrow$ R&D on submarket where firm already operates (no diversification); and (R3) $\psi^* = 1$ and $d^* > 0 \Rightarrow$ R&D on unknown submarkets (diversification). These three regimes result in radically different market dynamics. R1 describes stagnant markets, where there are no new products. In this case the game starts off at equilibrium, thus there is no market share redistribution. In case of R2 everybody has an incentive to do R&D on the markets where they are currently operating. This means that every producer stays as a monopolist on its initial market.[7]

The most interesting case is R3. In this case every producer has an incentive to explore new markets. This results in diversified firms. From the market knowledge point of view, this is the only interesting regime, because in this case new market knowledge is acquired by innovating firms. New knowledge facilitates their R&D process, thus only this regime is characterized by increasing returns to innovation.

Simultaneously modeling heterogenous preferences and product location in taste space makes function D^d complicated even in this simple setup. Due to this it is impossible to solve the producers' profit maximization problem (4.12) analytically. Therefore I solve it numerically. Figure 4.1 presents the solution. It plots the borders of three regimes in c-a parameter space, while other parameters are set to the values specified in table 4.1. As one can see, higher values of both parameters simultaneously result in the least interesting regime R1. Combination of lower values of c with higher values of a results in R2, while higher values of c

[7]There are some exception to this, which will be discussed later in section 4.3.2.

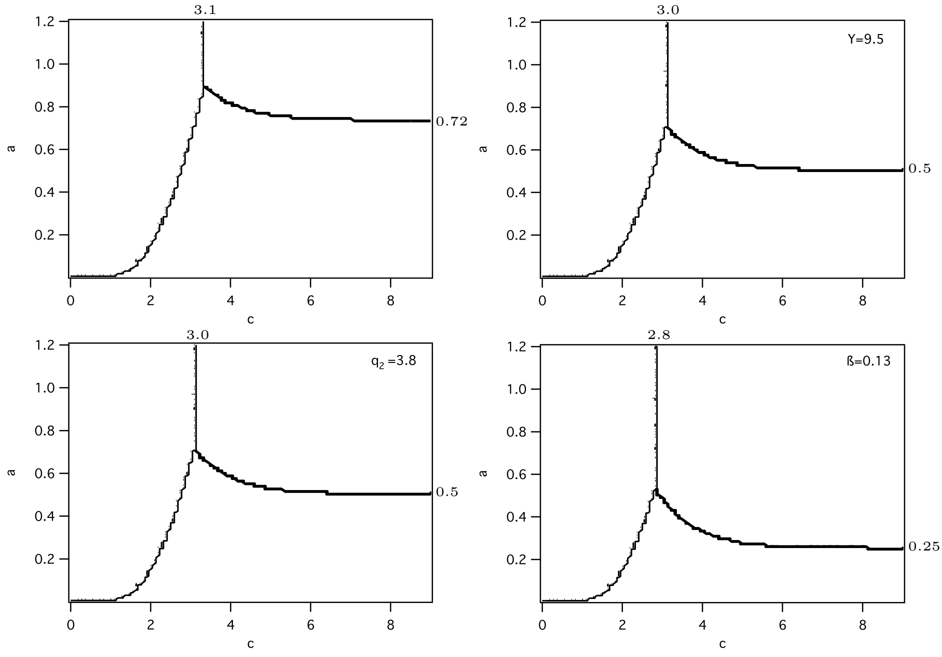

Figure 4.2: The effects of changes in Y, q_2 and β.

with lower values of a results in R3.

Intuition for this result is quite simple. High values of a imply that market knowledge is highly submarket specific. This, in turn, implies that R&D productivity on unknown submarkets is very low. As a result, the only option for producers is to innovate on their own markets (or not to innovate at all). On the contrary, low values of a imply productivity on remote submarkets that is only slightly lower compared to the productivity on markets where producers already operate. Thus, investing in remote submarkets becomes an available option.

On the demand side high values of c imply that consumers have very submarket specific tastes. In this situation, new product put on producer's own market will capture a big share of demand from producer's old product (as the new product is a good substitute to the old product of the same firm, but not a good substitute to products traded on other submarkets). This indirectly increases the cost of innovation on own market. Therefore, the only available option is to innovate on remote submarkets (or not to innovate at all). However, when c is low consumers' baskets are more evenly balanced and innovating on own market affects the demand on the producer's old product only marginally. In this case innovating on own submarket becomes an available alternative. The outcome is that interaction of these two forces (from supply and demand side) results in a pattern depicted in figure 4.1.

Changes of parameters besides c and a move borders between the regimes. The last four parameters in table 4.1 describe initial conditions, therefore I cannot freely experiment with their values as changing them modifies the essence of the problem. The same goes for q_1, the value of which ensures that producers do not innovate without spending money. Out of the remaining five parameters, γ does not have any impact on the producer's problem as demonstrated earlier (equation (4.3)). As price and income only affect the producer's problem through demand, looking at equation (4.8) suggests that increasing price will be equivalent to decreasing income. Hence, I discuss only the effect of one of them.

Figure 4.2 demonstrates the effect of changes in the three parameters to which the producer's problem is sensitive: Y, q_2 and β. This is a robustness check for the results presented in figure 4.1. In figure 4.2, upper left panel is the same as figure 4.1. The upper right panel demonstrates the effect of decrease of consumer's income, Y (from 10 to 9.5). Lower left panel demonstrates the effect of the decrease of R&D productivity parameter q_2 (from 4 to 3.8). Lower right panel demonstrates the effect of decrease of share of profits invested in R&D, β (from 0.15 to 0.13). As one can clearly see all these changes affect the results only quantitatively (moving borders). Thus I can conclude that the division of a-c parameter space in three fractions as presented in figure 4.1 is a robust finding.

Equilibrium firm size distributions

As I argued before, the initial R&D incentives and sizes are equal across firms. But although every firm has an incentive to innovate (if we are not in R1), the success rate of this process is not 100%. It can be expected that only a certain (say α) fraction of all firms will come up with the new product. This introduces the heterogeneity among the firms. Firstly, firms are becoming heterogeneous in size: α fraction of firms now has two products on the market (each), while $1-\alpha$ fraction is still producing only one product. Even more, as after innovation products will not be uniformly distributed over submarkets, the size of demand for one product will be potentially different from another. This implies that there will be heterogeneity even inside each of the groups (initial innovators and initial non-innovators). Secondly, there will be a heterogeneity in market knowledge (only in case of R3), those firms who innovated will acquire better knowledge about remote submarkets. Thirdly, there will now be a heterogeneity in further R&D incentives. This is due to heterogeneity in profits (size), in knowledge and also in non-uniform product distribution over submarkets. It is expected that incentives of some firms decrease (for instance of firms that did not innovate and whose nearby markets got populated with new products that led to the decrease of profits), while of others increase (for example of firms that innovated on remote markets and acquired new knowledge that increased their R&D productivity).

The structure of the present model is such that R&D incentives depend on the combination of total per period income and number of products available on the relevant submarkets. Thus, as my consumers' income is constant and while product space grows it is guaranteed that the additional demand for a firm, which comes with the new product on the market, decreases with time. As this demand is the only reward for costly R&D activity it is guaranteed that at some point it will become low enough for every firm to loose the incentive to invest in R&D. This, in turn, guarantees that the game reaches equilibrium in finite time. In this section I study these computed stochastic equilibria.[8] More precisely, I analyze the resulting firm size distributions.

Here I am not interested in parameter constellations that result in R1, as firms size distribution will not change with time. R2 is also relatively not interesting. In this case everybody has the incentive to innovate on their own markets, which does not result in any additional knowledge. If this incentive does not change I depart from the initial distribution only temporarily: ultimately every firm places an exactly equal number of products on their submarkets. This means that firms will again be homogenous in size. But if the parameter values are close enough to the border with R3 increase in profits might result in change of incentives for the innovator firm: now it might be optimal to invest in remote markets. Thus, some portion of R2 (close to the border with R3) might, at some later stage, become part of R3, and at the end result in interesting time invariant firm size heterogeneity. The most interesting regime from the perspective of equilibrium firm size distributions is R3, where profits as well as knowledge base can be changed due to the innovation process. In this section I concentrate my attention only on parameter constellations that fall under R3.

I reformulate the game as an agent-based model, where every producer is solving the profit maximization problem (of course these problems coincide during $t = 0$). I set the parameter values as listed in table 4.1, choose the constellation of a and c that falls in R3 (as given in figure 4.1) and run the model for consecutive time periods until it reaches equilibrium. Then I examine the shape of the firm size distribution in computed time invariant state.

The general result is that in R3 the entry game produces equilibrium firm size distributions that are heavy-tailed and skewed to the right. Normality test introduced by Anderson and Darling (1952) is rejected in all the runs I performed[9] with a very high confidence level. Quantile-quantile plots of the generated data suggest that the right tail of the resulting firm size distribution is Pareto-type, which is consistent to numerous empirical findings for developed countries (Coad, 2009). Next I examine the relation of the shape of equilibrium firm size distribution and the values of a and c parameters. More precisely I try to understand the contribution of variations in these parameters towards the fatness of the tail.

There are many ways to measure the fatness of the tail of a Pareto-type dis-

[8] By stochastic I refer to the fact that industrial dynamics depends on exact initial conditions that are random. Therefore, there is no way of predicting the equilibrium sizes of particular firms. If we run the model two times one firm might end up on opposite parts of the size distributions. However, we are not interested with single firms, but rather with the organization of the market.

[9] Which is over a million runs for various parameter constellations.

tribution. The methodology most widespread in the extreme value statistics literature is the one proposed by Hill (1975). Estimation of Hill index requires the explicit specification of tail size or more precisely, where I consider that tail of distribution starts. The procedure was designed for Pareto distributions, in case of which the methodology produces unbiased estimate of the tail index no mater where the tail starts. However, if the distribution is not exactly Pareto, Hill index is biased and the result that we get is sensitive to tail size selection. This problem is especially acute in small samples. This points to the importance of a careful tail size selection in case of analysis of empirical or experimental data.

There are several procedures for selecting the tail size. In this essay I use one of the most efficient procedures of adaptive tail selection proposed by Beirlant et al. (1999). More precisely, the simplification of the procedure proposed by Matthuys and Beirlant (2000). It selects the tail size by minimizing the asymptotic mean square error (AMSE) of the estimate. AMSE is additive in two parts: the variance of the estimate and the squared bias. It has been demonstrated that in small samples this procedure outperforms the alternatives for a large set of Pareto-type distributions (Matthuys and Beirlant, 2000).

There are alternative approaches to calculation of tail index. For example fitting a line to the Pareto quantile plots (Beirlant et al., 1996; Luttmer, 2007), moment estimators (Dekkers et al., 1989) or peak over threshold methods (Balkema and de Haan, 1974).[10] Like the usage of Hill estimator, each of these approaches has its disadvantages. These approaches are computationally more demanding and more suitable for testing for goodness of fit. As I are am trying to find the best fit to the data and in principle I am not even interested in the exact value of the tail index, but rather its dependence on model parameters, I opt for estimating the adaptive threshold and, consequently, Hill index.

The methodology is as folows. For each constellation of $c - a$ parameters in R3 I run the game until it reaches the equilibrium. To the data generated I apply the Matthuys and Beirlant (2000) procedure for tail selection and calculate Hill index. I repeat this 50 times and average Hill index over those 50 runs to obtain an estimate of tail index for a certain constellation.

The results for the relevant portion of R3 are presented in figure 4.3. Darker shades of grey represent fatter tails. In parts closer to the borders with other regimes results are noisy reflecting the regime change. However, in the interior of R3 Hill index profile is stable and it has a shape presented in figure 4.3. What I try to show on this figure is the qualitative relation between distribution tail and model parameters. I am not trying to draw the parallel between the magnitudes of tail indices observed in reality and in my numerical exercise. The average Hill index in my experiment is close to three, while real-life firm size data has been found to be fatter-tailed (index being slightly lower than two) (Coad, 2009).[11]

[10] For a good summary of these approaches see Beirlant et al. (1999).

[11] However, changing the initial conditions change average values of Hill index (without changing the qualitative picture), thus the model can be calibrated if necessary.

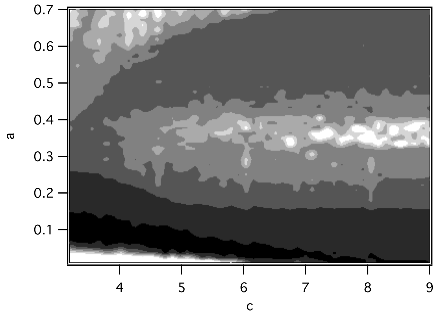

Figure 4.3: Hill indexes for the different parameter constellations in R3.

Rather I am trying to understand the contribution of the model parameters to the tail fatness of the actual firm size distributions.

The major finding in that respect is that in the interior of R3 thickness of the tail does not depend on demand parameter c: variation in tail index is explained by the variation in submarket specificity of producer knowledge (a).[12] I find that the dependance of the tail index on the relative knowledge productivity has the inverted U shape.

In order to understand the intuition note, that fatter tail means there are more firms of considerably larger size than the mean of the distribution. Due to the structure of the model, these are the firms who have more products on the market in equilibrium. As I do not have a market for the intellectual property, bigger firms are also the most successful innovators. These are usually firms who were lucky to have successful R&D projects at the onset of the run. Consider the effects a successful innovation has on incentives to innovate for the innovator and for the competitors who were not successful at this period. For the innovating firm this has two positive effects on R&D productivity: its profits have increased (i) and its knowledge base has expanded (ii). There is also the third effect, that affects both innovators and non-innovators. This is creative destruction (iii). Recall that creative destruction is asymmetric in my model, hence depending on new product placement it will affect firms asymmetrically.

Consider how the size of the effect (i) varies with a. The left panel of figure 4.4 presents a statistical picture of what happens to the size of innovators and non-innovators after the first period. These are averages of 200 runs and respective standard deviation bands. I set the value of c equal to $c = 9$, however, as I argued before, the results are not sensitive to the value of this parameter. Figure 4.4 shows the percentage growth of innovators and percentage decline of non-innovators. It says that when $a = 0.1$ a typical innovator grows by 25% during the first time period, while a typical non-innovator shrinks by 22%. One can easily see that these effects are linear in a, thus the difference between them (which is the measure of

[12]The similar behaviour as reported on figure 4.3 continues further for higher values of c.

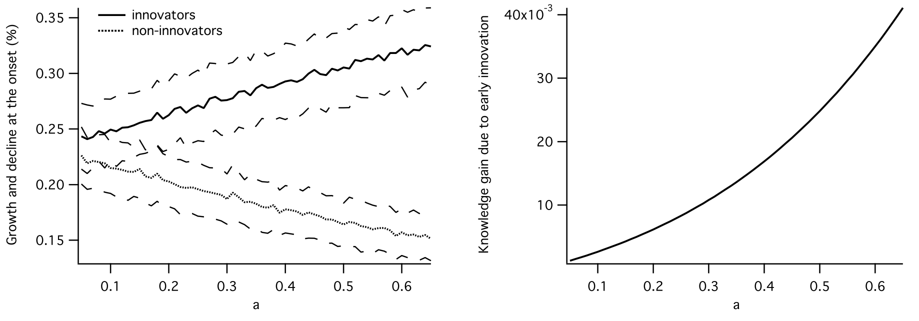

Figure 4.4: Changes in innovation incentives.

the difference in incentives to innovate between two groups) is also linear. The difference between these two values is slightly growing in a.

Now consider the size of the effect (ii). To measure the new knowledge acquired by innovative firms we can calculate the integral of their knowledge over the submarkets at period $t = 0$ and at period $t = 1$ and take the difference. Non-innovators do not gain any new knowledge. Note that as firms are homogenous at $t = 0$ I do not require statistical approach, we can calculate the value of the effect (ii). The right panel of figure 4.4 plots this value. One can see that it is increasing at higher rates with increasing a. Thus, this measure (as measure (i) above) suggests the explanation for the fatter tails when the value of a is high: innovation during the first period gives higher knowledge gain and higher incentives for further innovation. However, it does not offer the explanation for the fatter tails when the value of a is in lower range.

The reason for fatter tails in lower ranges of a can be found in the dependence of value of the effect (iii) on a. But there is no comprehensive way of measuring this effect. This effect is too sensitive to location of new products and attempts to measure it produce very noisy results. However, we can employ the statistical approach again and estimate the total difference in innovation incentives between early innovators and non-innovators. We can simply track the two groups until the equilibrium and record all the subsequent innovations. Then we can take the number of later innovations per early innovator and number of later innovations per early non-innovator and study the dependence of their difference on a. This measure simply looks at the realized innovation, thus takes all three effects into account.

Figure 4.5 plots the innovation frequency ratio between early innovators and non-innovators (as average of 200 runs) and standard deviation band. This value is calculated as the average number of new products introduced by a typical early innovator during the run (excluding the first time period) over the average number of new products introduced by early non-innovators. As one can see even though for lower values of a knowledge gain is small and that the size of the effect (i)

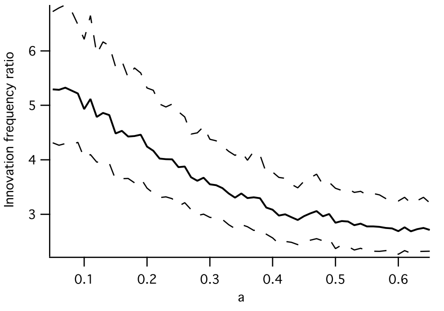

Figure 4.5: Changes in innovation freequency.

is relatively low, early innovators obtain higher invectives to continue innovation than for higher values of a. This should be the effect of the creative destruction (effect (iii)) that I cannot measure. The value plotted in figure 4.5 goes down steadily with increasing a, thus the difference in innovation frequency between a typical innovator and non-innovator is lower for higher values of a. However, innovations by innovators are better positioned in taste space as they possess good knowledge.

Therefore, I can conclude, that fatter tails for low values of a are due to higher innovation frequency by early innovators, however fatter tails for higher values of a are due to the considerable knowledge gain by early innovators. From figures 4.4 and 4.5 one can see that the size effect on innovation incentives is linear in a, the frequency of innovation in falling at somewhat decreasing rate and, most importantly, knowledge is growing at an increasing rate. Therefore, interaction of these three effects creates relatively thin tails for intermediate values of a.

Notice that parameter a is an industry level parameter. Thus, the shape of Hill index profile is an empirically testable result: one can look at the different industries, measure submarket specificity of their knowledge and tail indices of firm size distributions and see if the relationship has and inverted U shape. However, there are several issues with this procedure: it requires high quality data, it also calls for the control of other differences across industries. Most importantly, it is not very clear how one can measure submarket specificity of producer market knowledge in various industries on a comparable scale.

4.4 Conclusion

In this essay I have analyzed a simple model of boundedly rational producer behaviour in which heterogeneity of knowledge, R&D behaviour and firm size is generated endogenously. I have considered two key parameters of the model: submarket specificity of consumer tastes and submarket specificity of producer knowledge. I have studied the effects of these parameters on the R&D incentives at the onset of the industry populated by homogenous firms. Depending on parameter constellations I have identified three regimes in which optimal behaviour of a typical firm is qualitatively distinct: (i) no R&D, (ii) R&D only on familiar markets (no diversification) and (iii) R&D on unknown markets (diversification).

I have also analyzed the equilibrium firm size distributions which are the result of industry evolution in the regime where there are incentives to diversify. It turns out that the resulting firm size distributions are fat tailed and positively skewed, which is in accordance with what has been found empirically. Results indicate that for the interesting segment of parameter values the thickness of the tail of firm size distribution depends only on producer knowledge specificity parameter of the industry.

The conclusion is that successful innovation has two positive effects on innovators. Firstly, their profits increase. Secondly, their knowledge base increases. Both of these contribute to higher productivity of future R&D. On competitive constant size markets product innovation by a firm also has a negative effect. Due to Schumpeterian creative destruction new product reduces shares of incumbents. This negative effect influences innovators as well as non-innovators. Interplay of all these forces contributes to the shaping of industry's firm size distribution as a fat tailed one. Therefore, looking at actual R&D behaviour and at its effects on firms in certain industries can shed light to the process of emergence of actual fat tailed distributions.

Essay 5

Consumer Interaction and Innovation Incentives*

Abstract. In this essay I extend the model presented in essay 4 by adding global interaction to consumer behaviour. This interaction is modeled as network effects on submarket level: consumers adapt their tastes in order to take into account the popularity of the submarket the product is traded on. Consumer interaction is shown to result in higher innovation incentives during the transitional dynamics for two types of industries. However, the firms benefiting from the additional incentives are different. In one case larger firms are the ones gaining from consumer interaction, in the other case smaller firms are the ones collecting gains.

5.1 Introduction

Consumer interaction is slowly occupying a deserved important place in economic literature. Various current models are designed and explored in order to understand the effects of the consumer interaction for different socio-economic outcomes. The first three essays in this thesis can be regarded as the contribution to this literature. While the first essay is only concerned with the methodological issues, the following two present different applications for analyzing effects of the consumer interactions: one is concerned with the lateral distribution of economic behaviour, while the other by the advertising policies.

Many more examples where the dynamics of demand due to consumer interaction is analyzed from different angles can be found in literature. Few of these example are works by Murdie (1965) analysing travel decisions, Durlauf (1993) analyzing economic growth, Kirman (1993) analyzing opinion formation, An and Kiefer (1995) analyzing technology choice and McManus (2001) analyzing pricing policies. However, one thing that stands out, is that despite its importance for

*I am grateful to Robin Cowan and Ronald Peeters for their comments on this essay.

modern economics firms' innovation policies and resulting outcomes can hardly be included in the list above (exceptions to this rule are discussed in section 5.2).

The discipline has converged on the proposition that innovation is the driver of the economic growth. Growth is believed to be the major determinant of economic well-being. In its turn, well-being is based on the choices that individual consumers are making. Consumer choices directly correspond to demand, which is the primary consideration for firms that are thinking of innovating. Therefore, as consumer interaction influences consumer choices, it is also bound to have an effect on firm innovation policies, and ultimately on economic growth.

Although this essay does not make up the missing link between consumer interaction and economic growth, it contributes to the filling the gap by analyzing the effects of the consumer interaction on firms' innovation policies. Major concern of the work presented here is to understand how innovation incentives of different firms are modified due to the consumer interaction.

Interaction analyzed in this essay takes a different form from the ones discussed in essays 2 and 3. Interaction in previous essays was local: each consumer was sharing the information or skills with the small (constant) set of other consumers. In current essay, however, interaction is global. Every consumer's consumption decisions are affected by the same information. In this particular case it is the information about the sales of each of the submarkets. A substantial body of research has analyzed the differences in systems with local and global interactions (Ellison, 1993; Glaeser and Scheinkman, 2000). The general conclusion is that global interactions produce more ordered systems (Gonzalez-Avella et al., 2006) and that analysis of the equilibrium of these systems is more straightforward (Brock and Durlauf, 2000).

The remaining of the essay is organized as follows. Section 5.2 shortly reviews the relevant literature, section 5.3 presents the model, section 5.4 presents the results and the last section concludes.

5.2 Consumer interaction and innovation

Perhaps the most well-known group of global interaction models are the models of information cascades (e.g. Bikhchandani et al., 1992; Banerjee, 1992). These are models of sequential decision making where each decision-maker receives information about all previous decisions (aggregated in some way). Although there is a certain heterogeneity on information received due to the differences in decision timing, the idea that all the accumulated information is shared with the present decision-maker is at place. Thus, interaction in information cascades models is global (Bikhchandani et al., 1998).

Another type of well-known global interaction models are the ones that are characterized by some sorts of network effects. In these models a number of consumers (or share of spending) affects consumer decisions (Mayer and Sinai, 2003). Network effects can be positive (e.g. Rysman, 2004), as well as negative (e.g.

Mechoulan, 2007). In the former case consumer preferences are shifted towards popular products, while in the latter case preferences are shifted away from popular products. Under this class of models also fall models of "fad" behaviour (e.g. Young, 1993 and Bernheim, 1994).

Models with network effects can be divided in two sets: one where network effects are taken into account by the firms supplying the products, the other where firms neglect these effects and they remain as externalities (Liebowitz and Margolis, 1994). The model analyzed in this essay falls somewhere in between these two sets. In my model network effects have a time dimension: their effect is stretched over infinite time periods. Firms in my model maximize their profits over a finite horizon. Thus, although they try to take into account these effects they manage to do so only partially.

As global interaction models require either a high level of coordination among agents or sharing of a large amount of information, the legitimate question arises: are these kinds of processes applicable to consumer behaviour. An important thing to note in this respect is that global interactions do not necessarily imply the communication of all the individual decisions, but rather of some sort of aggregate signal. This aggregate signal might actually be easy to transfer. In fact, in today's age of information technologies and mass media communication of information becomes an easy task. Any information broadcasted using any of the publicly accessible utilities implies global interaction.

However, aggregation of information by agents themselves requires immense levels of coordination. Models of global interaction usually impose some sort of structure on the economy that guarantees the aggregation of information. For example in case of cascade models this structure is the sequentiality of choices, which offers a neat mechanism of aggregation and communication of the signal. Network models resolve the signal aggregation issue differently. In these models information aggregation is delegated to an aggregate institution (i.e. market) rather than to agents themselves. In most of the cases market collects the information on sales and makes it publicly available. Hence, there is no need of agent coordination in this respect.

Although the interaction literature is rich in economics, to the best of my knowledge, research on standardization is the only strand of economic work that has tried to analyze the effects of consumer interaction on innovation incentives (e.g. Farrell and Saloner, 1985). This stream of research is concerned by the innovation incentives remaining in the economy once the standard has been reached. Part of researchers believes that standardization will inhibit innovation (Farrell and Saloner, 1986). The argument goes as follows. The already accepted standard will have a well established consumer base. Therefore, it will be very hard for new entrants to compete with the firm controlling the standard. Thus, innovation incentives from the side of new entrants will be lower. As there will be no (or very low) competition in standardized industries, firms that are controlling the standard will feel less threatened by the entry possibilities and thus will not bother to further innovate and perfect the standard. According to these researchers, network effects result in lock-in of the industry into the standard, which might not be optimal route of the development (Farrell and Shapiro, 1989). However, there is an opposing view (e.g. Katz and Shapiro, 1994). These authors show that due to the phenomenon of "stranding" network industries may exhibit "insufficient friction" instead of "excess inertia" (Katz and Shapiro, 1986). Therefore, innovation incentives need not decrease with standardization.

The works referred to in the previous paragraph have one feature in common: they start the argument from an equilibrium situation, i.e. when a certain standard is in place. However, this is only part of the story. It is also important to analyze what happens to innovation incentives at the early stages of the industry development, when the standard is not yet at place. Current essay looks at this problem. I analyze not only the aggregate innovation incentives, but also the distribution of these incentives across heterogenous firms.

5.3 The Model

The model is a modification of the one presented in essay 4. Firm behaviour is the same: producers are myopic, optimistic and risk-neutral. They maximize the same profit function (4.3).

Consumers however behave in a slightly different way. In contrast to essay 4, here they take into account the popularity of the submarket when judging about their preferences. The mechanics is as follows. Consumer preferences consist of two parts: inherent, which are time invariant and are distributed over the submarkets exactly the same way as in essay 4 (equation (4.6)), and social, which are varying with time.

Just like inherent preferences, social preferences are defined at the submarket level. They are proportional to the popularity of the submarket, which is measured by the share of total spending on products on that submarket during the previous period:

$$\bar{k}_{i,t} = \frac{\bar{Y}_{i,t-1}}{SY}, \tag{5.1}$$

where $\bar{Y}_{i,t-1}$ is the total spending on submarket i at period $t-1$, S is the number of agents in the economy and Y is the income per capita.

From equation (5.1) we can infer that submarkets that are popular in one period will attract even more consumers during the next period. This is the network effect: the more people consume products on certain submarket, the more each consumer wants to consume. Social preferences are not consumer specific, and each consumer takes them into account the same way. Thus, interaction among consumers is global in this model.

I define the total preferences as the weighted average of inherent and social preferences:

$$\tilde{k}_{i,t}^{s} = (1-\bar{\mu})k_i^s + \bar{\mu}\bar{k}_{i,t}, \tag{5.2}$$

where k_i^s is the inherent preference of consumer s towards products on submarket i and $\bar{\mu}$ is the weight the consumer puts on social preferences. Notice that $\bar{\mu}$ is not consumer specific, it is the characteristic of a sociaty. In order to derive the values of $\tilde{k}_i^s$, variables are rescaled to satisfy $\sum\limits_i \bar{k}_{i,t} = \sum\limits_i k_i^s$, $\forall s, t$.

The rest of the model is identical to the one discussed in essay 4. Thus, in principle the only difference between these two models is that here I replace k_m^s by $\tilde{k}_{m,t}^s$ in equation (4.4). This modification gives us the opportunity to study the effects of communication on innovation incentives.

Adding network effects to the model alters its dynamics substantially. There are two central effects, sizes of which are changed by this modification: the prospect of market size gain due to innovation and the own cannibalization effect. The first effect is fairly straightforward to understand. Innovating on a submarket contributes to the popularity of that submarket, thus guarantees higher sales compared to the situation without network effects. Therefore, network externalities create additional incentives to innovate.

The second effect is more subtle. Innovating away from the own submarket results in a drop of popularity of the old sub-market (given that no other firm has innovated on the submarket where old product is traded.) Although this innovation results in an increase of the popularity of the new submarket, the net effect of innovation on the size of the innovator might be negative, as popularity of the new market should be shared with other players. Furthermore, given no further innovation, this effect will be reinforced with time as network effects are self-reinforcing. This intuition is better visible on the following example.

An example: $\mathbf{2 \times 2 \times 2}$ economy. Similar to an example in essay 4 consider the economy populated by two consumers and two firms. Each of the firms produces one product. These products are placed on different submarkets. For the purpose of demonstration assume consumers are not different from each other and that they do not have any preference for any of the submarkets. This effectively means that $k_1^1 = k_1^2 = k_2^1 = k_2^2 = k$. For the same purpose assume incomes of two consumers are equal ($Y^1 = Y^2 = Y$) and that prices of all products in the economy are also equal ($p_i = p_j = p$).

Consider the situation when firm 1 innovates on the submarket where firm 2 sells its product. In this case, in absence of network externalities, we can calculate the profits of the two firms. To do this first let's look at consumed quantities: $C_1^1 = C_1^2 = C_1 = \frac{1}{3}\frac{Y}{p}$ and $C_2^1 = C_3^1 = C_2^2 = C_3^2 = C_2 = C_3 = \frac{1}{3}\frac{Y}{p}$. Then we can calculate the profits of each of the firms: $\pi_1' = \frac{4}{3}\gamma\frac{Y}{p}$ and $\pi_2' = \frac{2}{3}\gamma\frac{Y}{p}$. If there is no more innovation, due to the fact that there is no consumption dynamics without innovation, π_1' and π_2' describes the equilibrium firm size distribution.

Now consider what happens if there are network effects on submarket level.

During the first time period after innovation by firm 1 social preferences are determined by previous time periods submarket sales. As prior to innovation market shares were split equally between firms nothing changes in firms' profits at the first period. However, due to dynamic network effects the picture changes in consequent period. As consumption on submarkets is now asymmetric so are social preferences. Consumption on submarket 1 is $\bar{C}_1 = 2C_1$ and consumption on submarket 2 is $\bar{C}_2 = 4C_2$. From this information we can infer social preferences for this period: $\bar{k}_1 = \frac{\bar{C}_1}{\bar{C}_1+2\bar{C}_2} = \frac{1}{5}$ and $\bar{k}_2 = \bar{k}_3 = \frac{\bar{C}_2}{\bar{C}_1+2\bar{C}_2} = \frac{2}{5}$.[1] Then we can calculate consumption of my consumers that take into account social preferences: $\tilde{C}_1 = \left((1-\bar{\mu})\frac{1}{3} + \bar{\mu}\frac{1}{5}\right)\frac{Y}{p}$ and $\tilde{C}_2 = \tilde{C}_3 = \left((1-\bar{\mu})\frac{1}{3} + \bar{\mu}\frac{2}{5}\right)\frac{Y}{p}$.

Given $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{C}_3$, I can also calculate new profits for both firms: $\tilde{\pi}'_1 = \left((1-\bar{\mu})\frac{4}{3} + \bar{\mu}\frac{6}{5}\right)\gamma\frac{Y}{p}$ and $\tilde{\pi}'_2 = \left((1-\bar{\mu})\frac{2}{3} + \bar{\mu}\frac{4}{5}\right)\gamma\frac{Y}{p}$. It is easy to verify that $\tilde{\pi}'_1 \leq \pi'_1$ and that $\tilde{\pi}'_2 \geq \pi'_2$. Thus, I can conclude that in presence of network effects the size gap between the innovator and non-innovator is smaller.

Further, I can take values of $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{C}_3$, calculate the social preferences for the next period and based on those calculate consumption baskets of consumers. If no further innovation takes place, this exercise will demonstrate that $\tilde{\pi}'_1$ is decreasing with time, while $\tilde{\pi}'_2$ is increasing. It is easy to verify that in equilibrium (as $t \to \infty$) $\tilde{C}_1 \to 0$ and $\tilde{\pi}'_1 \to \tilde{\pi}'_2$. From here I can conclude that the size advantage that the innovator gains (relative to the non-innovator operating on the target submarket) is only temporary when network effects are present. Innovators use this opportunity window and innovate persistently in order to maintain the advantage.

In the following section I discuss the changes in equilibrium and transitional dynamics that are brought about by the addition of network externalities to the model.

5.4 Results

5.4.1 Equilibrium Properties

The model presented in section 5.3 describes a complex system. By definition it is fragile to small perturbations and as R&D is a stochastic process, which introduces discontinuities in the behaviour of consumers, it is impossible to predict the exact evolution of the system. However, it is possible to characterize the equilibrium that the system will converge to.

If I define the equilibrium in the same way as in essay 4, its existence is straightforward to prove. However, again due to discontinuities, it is impossible to prove that the equilibrium state that I will discuss in this section is unique. But it is

[1] As prices are equal across products, consumption and income can be used interchangeably for this purpose.

possible to demonstrate that this is the only stable one. The properties of this equilibrium can be anticipated without mathematical analysis.

We know that due to the existence of network effects popularity of the submarket results in even higher popularity, *ceteris paribus.* Therefore, if there is a submarket emerging which is more popular than any other submarket, demand starts to switch towards the products traded on the popular submarket. This force will induce the desertion of unpopular markets one by one. Until the consumption basked of every consumer consists of products traded only on this popular submarket.

The only force that can prevent this from happening is innovation. Innovation on the less popular markets can induce people to spend more on those submarkets which will contribute to the growth of popularity of the competitor compared to the leader. It might well happen that the runner-up becomes the leader in this case. This kind of competition between submarkets will continue for some time. With time the pool of the popular submarkets will get smaller. Unfortunate submarkets, which did not attract enough innovation, will be permanently falling behind as the stop of innovation activities on those submarkets is guaranteed. Firms and consumers will follow each-other in putting their resources in more and more distilled set of popular submarkets.

If innovation introduces the discontinuity, jump of popularity of the submarket, one can imagine that this kind of behaviour will continue infinitely. However, the size of the discontinuities is constantly decreasing as market gets saturated. Thus, at some point innovation on the runner-up submarkets will become insufficient to keep up with the popularity of the leader. The leader submarket at that point will become the ultimate winner.

In principle it is possible for several submarket to have exactly the same level of popularity at that final stage. Then all the demand will start flowing to those submarkets at the same rate. However, this scenario is unlikely to persist for long. If there is an innovation, it will affect the submarkets asymmetrically (except exceptional cases). This will disadvantage some of the submarkets and narrow the pool of leaders. In general, if there is any type of perturbation in the system that will affect leaders asymmetrically it will induce the gap in popularity between submarkets, which, due to network effects, is growing until the sales on the laggard submarket reach zero. Therefore, as number of innovations during the transitional dynamics increases the chance of multiple submarkets being active in equilibrium goes to zero.

Thus, I formulate the following proposition.

Proposition 5.1. *If an industry starts offsuffi ciently far from the time-invariant state, every consumer will be consuming products traded on one and only one submarket in equilibrium.*

Proof. Omitted. □

This proposition has an important corollary.

Corollary 5.1. *As $t \to \infty$, the size of every firm that does not have a product on the unique popular submarket goes to zero.*

This corollary translates the submarket level equilibrium properties of the model to the firm-level, that is my main concern. Contrasting these properties to the ones analysed in essay 4 demonstrates the effect of the network externalities.

Corollary 5.1 implies that the market size of some of the firms goes to zero in the setup with network externalities, which was not the case without network externalities. If we are in the scenario with high submarket specificity of producer knowledge, shrinking firms will be the ones positioned at the opposite side of growing firms on the submarket space. The disadvantaged firms are simply unable, due to their knowledge profile, to jump on the running train and enjoy the popularity. In general, higher a would imply that in equilibrium there will be less firms surviving. If a is sufficiently low, it is possible that all the firms survive, as there might be enough time for each of them to innovate on the popular market as their knowledge profile permits them to do so.[2]

One more thing that can be said right away is that it will take longer to reach the equilibrium in case of network externalities, in contrast to the case without externalities. Economy also creates more innovation incentives in consumer interaction case. In the case of no network externalities preferences are fixed. Thus consumers contribute to the innovation at certain fixed locations. In the case of network externalities the picture is different. The same consumer can contribute to innovation incentives in different places at different times. This is due to the fact that tastes are changing. In order to see this consider the case where there are two submarkets competing for popularity and that consumers do not have inherent preferences for any of them. Then they split their budgets between those two submarkets creating innovation incentives on both of them. However, once one of the submarkets starts dominating the other, consumers put more and more of their money on the leader submarket. Thus, the stream of money that was creating innovation incentives on one submarket now goes to another submarket contributing innovation incentives there. This was not the case in the model without network effects. Also in general the competition between submarkets in the model in essay 4 was not that fierce as they could not steal each-others market to such large extent. In this model competition is fierce as it is only one submarket that has to survive. More competition drives the innovation incentives up.

This dynamics described here is an example of the known process of standardization (Katz and Shapiro, 1994). Although my producers do not look that far ahead to be concerned with their product to be the standard, they do look far enough to see that there are certain advantages of having the product on the popular submarket. Emergence of the standard happens as a result of competition for

[2] Time is crucial here. With time, and innovation, size of the firms that do not have products on popular markets decreases. Together with size, decrease the funds allocated to R&D.

the small portions of the market, rather than for the ultimate purpose of having a dominant design.

5.4.2 Out-of-Equilibrium Dynamics

As I argued before, it takes much longer for the economy to reach the equilibrium compared to the setup without network effects. Therefore, analysis of the off-equilibrium dynamics becomes more important in order to predict the equilibrium state. For example to understand which of the firms are more likely to be at the right tail of the distribution, which of them are more likely not to exist in the equilibrium. Although transition to equilibrium takes time, equilibrium outcomes are determined in early parts of transition. Once the economy is sufficiently close to the equilibrium firm size distributions change only marginally. The reason for this is that each new innovation will have a small effect on firm size redistribution and that consumer tastes will be gradually reinforcing the present popularity profile.

Due to the mechanics of network effects, based on corollary 5.1, we can also anticipate that as dynamics approaches the equilibrium the firm size distribution will depart from Pareto-like and become bimodal. This is due to the fact that the firms that are producing products traded on unpopular markets and do not have enough resources (either money or knowledge) to switch to popular markets start declining irreversibly. On the other hand, firms that are active on popular market will grow persistently. As claimed earlier, network effects will reinforce the distinction between two groups and push them further apart producing a bimodal distribution. The stream of firms that are steadily falling behind from the group will also be persistent. This will continue until the right tail of the distribution consists only of firms that have products traded on the unique popular submarket.

As equilibrium and near-equilibrium dynamics of the model are fairly predictable and linear, in a sense that there are no big changes in firm size distributions anticipated, it is important to analyze the behaviour of the model during earlier stages of transitional dynamics. Similar to the essay 4, I start the economy when it is populated by firms with homogenous prospects. As we have seen the equilibrium will result in heterogeneity in firms: some firms will be very successful while others will be forced to exit. This segregation between two groups is solely dependent on their innovation performance.

We have seen in section 5.4.1 that there are more innovation incentives in the economy with network effects. However, the out-of-equilibrium dynamics is stretched over a longer period, thus the comparison of the overall number of new products is not entirely appropriate. It is more relevant to analyze the innovation frequency in a time period of the same length. Innovation incentives in this model are somewhat different compared to the incentives in the model in essay 4. On the one hand, here successful innovation yields a higher market share gain for the innovator due to the popularity increase. This adds to the firm size and to the R&D funds, which help further innovation. On the other hand, products left on the unpopular markets shrink in size faster which decreases firm size and funds available for R&D. Thus the total effect is not clear right away.

Besides the innovation frequency, there is a more important question we can analyze. And it is how are these new innovation incentives distributed over the firms of different sizes. More precisely, does the firm size distribution becomes more or less fat-tailed in the setup with network effects compared to the setup in essay 4? If the distribution is thicker-tailed, we know that network effects are benefiting large firms. However, if the firm size distribution is thinner-tailed we know that new innovation incentives benefit smaller firms. Thus, by analysing distribution of innovation incentives at the onset of the industry we can conclude whether early innovators which belong to the right tail of firm size distribution during early dynamics are more or less likely to be the successful ones in the equilibrium.

To examine these questions I use the following methodology. I start the economy with exactly the same initial conditions as numerical examples in essay 4. As I have one more variable, $\bar{\mu}$, I report its value on each graph that I analyze. First I run the economy with the value of $\bar{\mu} = 0$ (which is equivalent to the economy in essay 4) for different constellations of a and c until it reaches the equilibrium. I record the number of time steps it took the economy to reach the equilibrium for each $a - c$ pair. I also record the number of new products in the economy and Hill index of the firm size distribution.[3] I record these latter values at two time points: at the equilibrium and when the economy is on half way to equilibrium. I run this scenario 30 times, so I end up with 30 observations for innovation frequency and Hill index.

I run the economy for different values of $\bar{\mu}$ for exactly the same number of time periods as it took my economy to converge to the equilibrium in the case $\bar{\mu} = 0$. I, again, take note of the number of new products and the value of Hill index at the end of the run and half way. I run these also 30 times. Then I contrast the two samples for each a-c constellation.

In order to understand whether values in presence of network effects are different from the ones in absence of network effects ($\bar{\mu} = 0$) I test for statistical significance of the difference between pairs of samples. I use two-sample Kolmogorov-Smirnov test proposed by Smirnov (1939). This is a very powerful non-parametric statistical test that, in presence of proper critical values, is superior to alternatives for relatively small samples. For evaluating the significance of the difference between two empirical distributions I use critical values of Kolmogorov distribution presented by Miller (1956).

For analysis it is useful to divide the parameter space in four regions. Consider that each of the two parameters (a and c) have two groups of values: high and low. Let H denote high values, and L denote low values. Then in $a - c$ space I will have four regions: HH, HL, LH and LL. In each of the pairs the first letter refers to the value of a.

[3] Hill index is calculated using the methodology described in essay 4.

Results for moderate magnitudes of network effects ($\bar{\mu} \leq 0.2$) are presented in figures 5.1 and 5.2. As one can see with even small doses of network effects innovation incentives start increasing for two areas on the graph. Innovation incentives rise for the economies with high a and low c (i.e. HL) and with low a and high c (i.e. LH).

In order to understand why those two locations benefit more from consumer interaction note how two effects of consumer interaction are dependent on the two parameters of the model. First consider the increasing innovation incentive that is due to the popularity of the new location (i). This popularity is transformed into size and further innovation incentives. The additional incentives could be used for innovating on further submarkets and obtaining more knowledge than during the scenario without consumer interaction. In fact this is a likely scenario for industries where a is low.[4] When a is low producers have enough knowledge to supplement abundance of R&D funds and innovate on remote submarkets. If producer knowledge is too specific to the submarkets (a is high) more money or higher demand prospects might not be enough to push producers on new submarkets to gain more knowledge. However, as network effect becomes stronger, industries with higher values of a will start gaining more knowledge. One more thing to notice here is that if a is sufficiently low ($a \approx 0$), network effects will not result in higher knowledge, as producers are already familiar to all the submarkets well.

Thus, we can anticipate that as long as a is not too small, industries with the lower values of the parameter will benefit from the additional knowledge due to network effects. This hypothesis is confirmed by the results presented in figure 5.3, which reports on the amount of accumulated knowledge in two scenarios: with and without network effects at the end of the run. These are values for $c = 5$, but graphs for different values of c are indistinguishable. Standard deviations are extremely small, usually not even visible if I plot them on the same graphs. As one can clearly see knowledge benefit becomes pronounced for higher values of a as $\bar{\mu}$ increases. This confirms my conjecture about the effects of increase in expected demand on new products.

The magnitude of the second effect, which is the decrease of popularity of the old submarket (ii), depends on the other parameter of the model - c.[5] To understand why, note that lower c means that consumers inherent preferences are better balanced. If there is no consumer interaction, in case of smaller values of c consumption baskets will contain similar quantities of all the products available on the market. On the other hand, if the value of c is high consumption baskets will contain large amounts of few products and small amounts of the rest. Different compositions of consumption baskets will be affected to a different extent by the network effect of the same magnitude. As inherent preferences are more balanced, I can anticipate that a network effect of the same magnitude will be more pro-

[4] Recall, a indicates the level of submarket specificity of the producers' market knowledge.
[5] Recall, c indicates the level of submarket specificity of the consumers tastes.

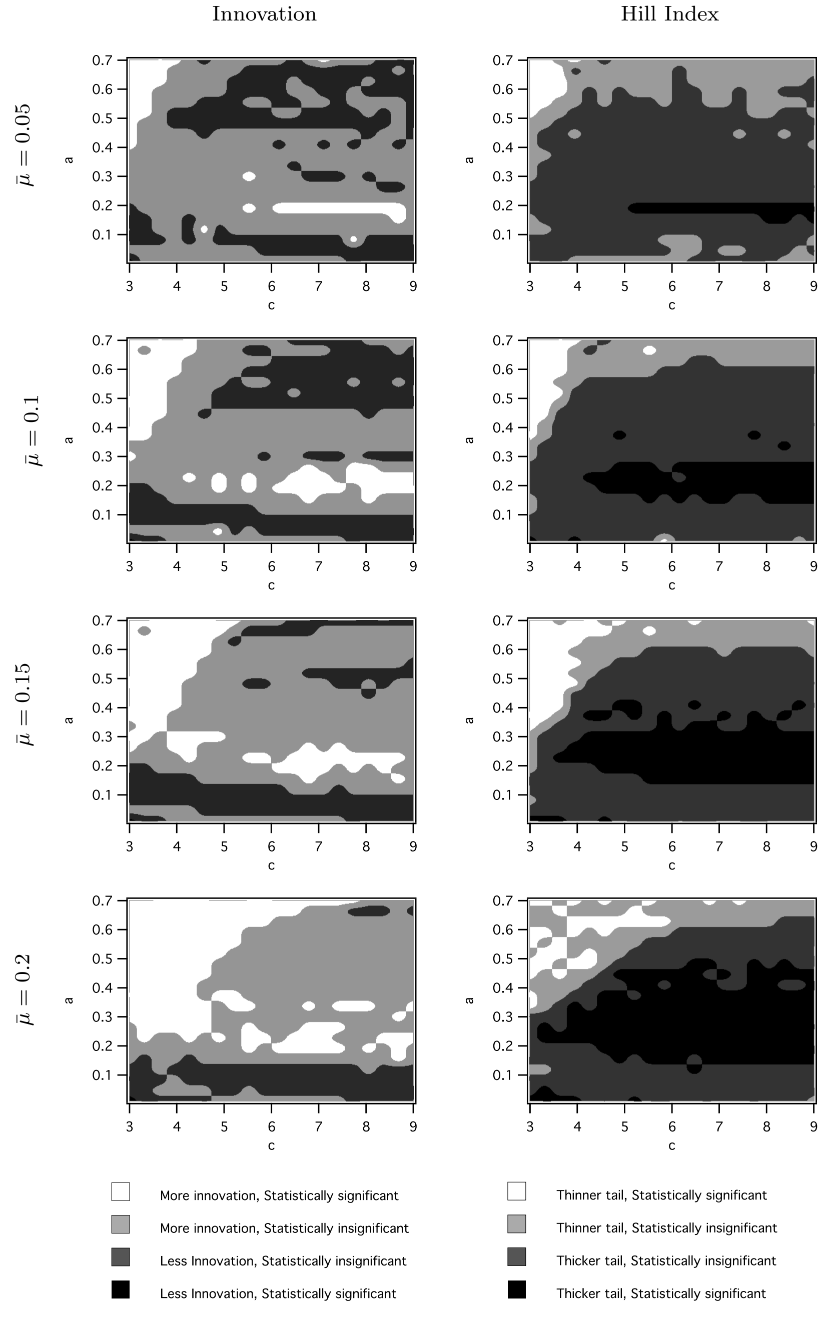

Figure 5.1: Comparison of industry dynamics in between scenarios with and without network effects half way to the equilibrium.

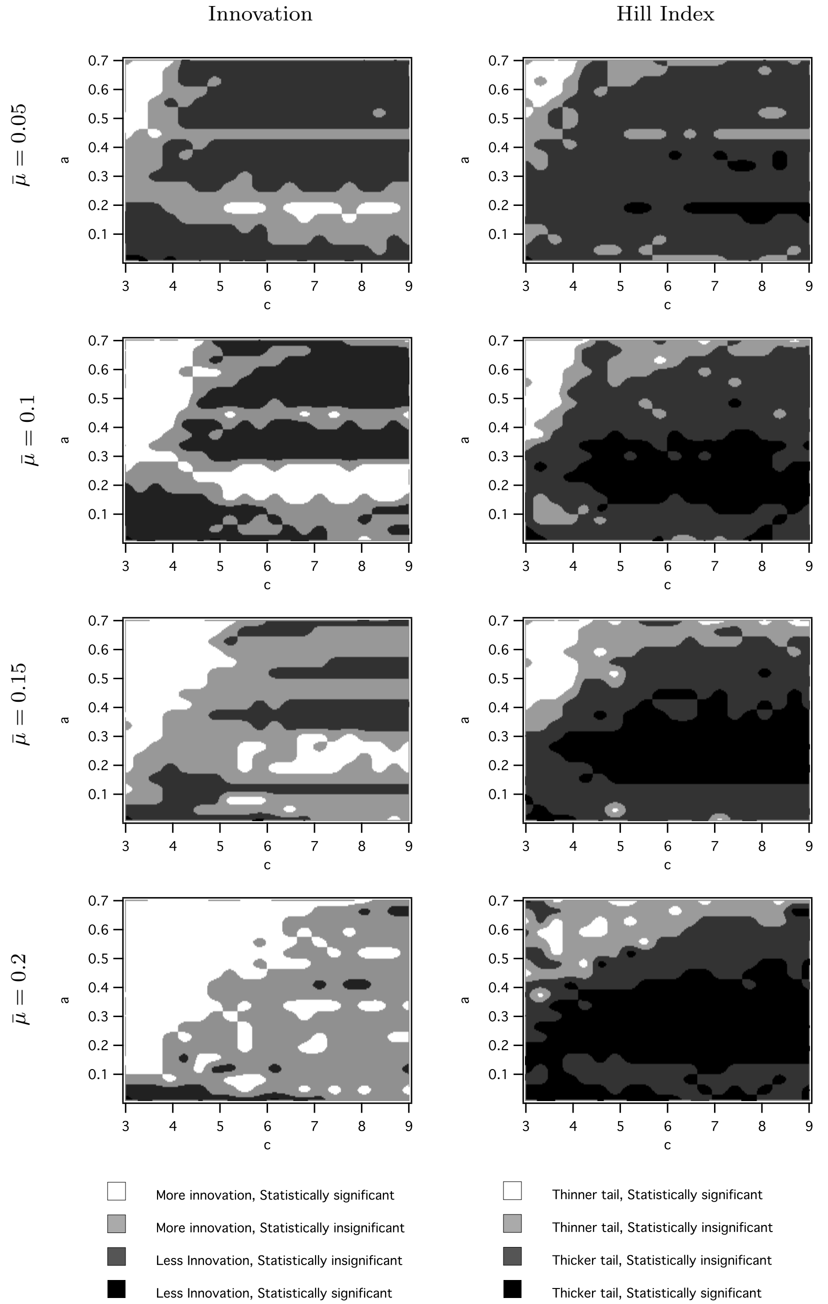

Figure 5.2: Comparison of industry dynamics in between scenarios with and without network effects in the equilibrium.

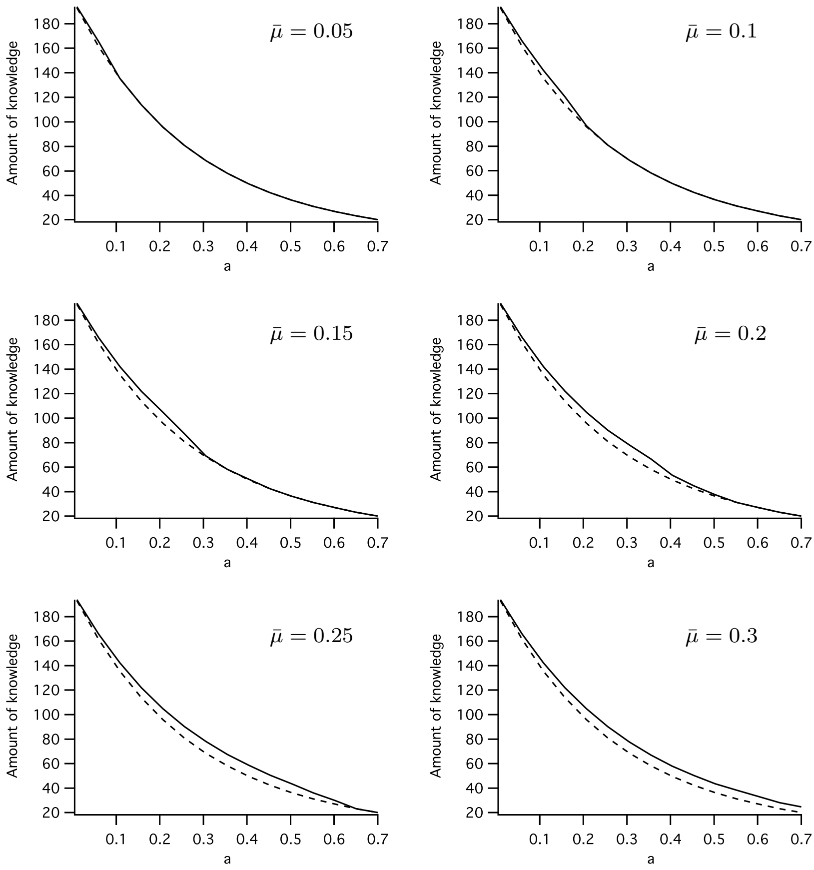

Figure 5.3: Amounts of knowledge accumulated by the typical firm in the industry with network effects (solid line) and without network effects (dashed line).

nounced for lower values of c, as in this case the variance in social preferences will be easily visible.

These two effects affect differently the firms of the different sizes (or early innovators and non-innovators). For demonstration purposes let's discuss these effects on the example of large firms. Lower values of a clearly give advantage to larger firms as they are better equipped to utilize additional incentives. However, lower values of c affect adversely larger firms as in this environment their products left on non-popular markets, consequently firms themselves shrink in size. But the same effect works positively for certain smaller firms as due to the increase in popularity their products shrink less compared to the scenario without network effects. Thus, the discrepancy in sizes between innovators and (at least portion of) non-innovators decreases, that increases the competition and drives the number of new products up.

Therefore, using the parameter space partition in four regions, I can interpret the results from figures 5.1 and 5.2 as follows. In the LL region large firms are disadvantaged by the effect (ii) and advantaged by the effect (i). Apparently these two effects balance each other as there is no statistically significant change in innovation intensity (left panel) and in firm size distributions (right panel) for lower values of $\bar{\mu}$. In the HH region none of the effects are at work: $\bar{\mu}$ is not high enough to push large firms to overcome their knowledge specificity, and it is not high enough for social tastes to affect the consumer decisions as they have very pronounced preferences towards certain submarkets. Again, there is no statistically significant change in the thickness of the tail of the firm size distribution.

However, when c is high enough for large firms not to get disadvantaged by the effect (ii) and a is low enough for them to be able to exploit higher expected demand for new products (LH), total innovation goes up and this is due to the large firms, as firm size distribution becomes significantly fatter-tailed. On the other hand when the producer knowledge is too submarket specific for firms not to be able to take advantage of the effect (i) while consumers' inherent tastes are balanced enough in order for social preferences to be more pronounced (HL) innovation goes up due to smaller firms, and the tail of the firm size distribution becomes thinner.

As the size of the network effects increases all the processes described above takes less time. And the two effects are at work for higher and higher values of a and c. As a result if the $\bar{\mu}$ is sufficiently high all the pictures depicting the frequency of innovation become monotonicly white indicating statistically significant increase of innovation frequency for any parameter constellation (as it was anticipated from the discussion of equilibrium). Due to the fast progression towards the equilibrium firm size distributions also depart faster from the power law distribution and become bimodal. Due to the bi-modality of the distribution my technique of calculating the Hill index fails and gives incorrect results. Thus it is not further possible to examine the evolution of the firm size distributions. However, I know that the industry is close to equilibrium and that the dynamics follows the scenario described earlier.

5.5 Conclusion

In this essay I have analyzed the effects of the consumer interaction on innovation incentives of producers. Consumer interaction is global and modeled as network effects. Except this feature the model of this essay is a replica of the one used in essay 4. Therefore, any difference between results of this model and the one analyzed in essay 4 can be safely attributed to consumer interactions.

When interaction intensity is sufficiently high innovation incentives increase for all industries. However, in case of moderate interaction intensity, innovation incentives increase only for certain types of industries. These are industries characterized simultaneously by the (i) high level of consumer taste specificity and the low level of producer knowledge specificity or by the (ii) low level of consumer taste specificity and the high level of producer knowledge specificity. In the case of the former type industries innovation incentives are increased for larger firms, while in case of the latter type industries innovation incentives of smaller firms are receiving a boost due to consumer interaction. As a result network effects increase the firm size distribution tail thickness of the first type of industries and decreases it for the second type of industries.

Another result is that due to consumer interactions the system spends more time out-of-equilibrium: starting from the same initial conditions as in essay 4, it takes considerably longer to converge to the equilibrium. The convergence time is increasing with decreasing intensity of interaction. As the model in essay 4 is only a special case of the one analyzed in this essay (when $\bar{\mu} = 0$), I can conclude that the convergence time is discontinuous at $\bar{\mu} = 0$: convergence time is constantly increasing with falling $\bar{\mu}$ until it reaches zero, where convergence time drops sharply.

Our result from section 5.4.1 that network effects increase aggregate innovation should not be viewed as the contradiction to the general believe that network effects might harm innovation (Farrell and Saloner, 1986). What I describe here is the innovation during the process of emergence of the standard. While the concern in the literature is about the innovation incentives once the standard is in place. From that prospective, also in my model, once the standard (dominant submarket) is identified innovation incentives, at least outside the submarket, decrease with time as demand gradually shifts away to the dominant submarket.

Concluding Remarks

In this thesis I have studied several applications of heterogenous interacting consumer models to various problems in industrial organization. In the first essay I propose a simple setup for the study of consumer behaviour. Consumer choices depend on consumption skill levels with respect to individual products. If the agent has good consumer skills for a product she can utilize it better, thus can derive higher level of utility from consuming it. Consumer skills are dynamic. They increase through experience as well as through interaction with other, better skilled consumers. I have examined whether the representative agent could describe the evolution of the average skill level in this simple economy. I have conducted two exercises that examine two forces of skill augmentation (experience and interaction) separately. It turned out, that the representative agent fails in both cases. As consumer behaviour in this essay is similar to the ones presented in later essays, failure of the representative agent has motivated the use of heterogenous agent models in the rest of the thesis.

In the second essay I have presented the discrete choice model of consumption. In this case consumers are making decisions based on the information they have about each of the alternatives. Similar to the first essay, there are two sources of information: consumption and communication. Communication takes the form of local interaction: agents talk to their friends and in the process convey their general impressions about available products. The network describing social linkages is not disconnected. I have analyzed whether this kind of decentralized consumer choice and communication could result in a stable distribution of behaviour over social networks. The result is that for large number of initial conditions clustering in economic behaviour emerges as an equilibrium outcome. I have also examined the out-of-equilibrium behaviour of the model which turned out to be accurately predictable given the equilibrium results.

The third essay modeled choices depending on skills rather than information, and maintained the local nature of consumer interaction. In this case consumers shared consumption skills. Besides interaction, accumulation of skills through experience was in place. Products in this setup had two characteristics: quality and the level of user-firendliness. I considered producers that are able to influence consumption decisions at the onset of the industry through their efforts in advertising. I have analyzed the producer incentives to advertise their products on a duopolistic market. I found that relation between the quality of the product and returns to advertising is not monotonic as suggested by earlier studies. Rather, returns have and inverted U shape, given the characteristics of the competing product.

In the fourth essay I have temporarily abandoned the interaction among consumers. I have described the consumer side relatively schematically. Consumer behaviour was modeled through well-known toolbox of utility maximization. Of crucial importance in this setup was the consumer taste heterogeneity. The main concern turned on the producer innovation incentives. Given the behaviour of consumers, firms engage in an uncertain process of research and development with the aim to create a new product that would appeal to a certain group of consumers. The industry is organized through sub-markets of products over which consumer tastes are defined. Starting from firms with equal prospects, I have analyzed the development of different industries. I found that fat-tailed firm size distributions emerged as an equilibrium outcomes for a large variety of industries. The thickness of the tail depended on only one industry-level parameter of the model.

In the fifth essay I have extended the model from the fourth to include consumer interaction. In this case interaction was modeled as a network effect and took a global form. Consumers adapt their tastes in order to take into account the popularity of the submarket on which the product are traded. The aim was to understand how innovation incentives change due to consumer interaction. I showed that the economies with the same potential support higher levels of innovation in case of consumer interaction. This is true not only in equilibrium but also during out-of-equilibrium dynamics, although in this case innovation frequency is increased only for certain types of industries. I have also analyzed the distribution of these additional innovation incentives across different firms. It turned out that depending on the characteristics of industry, innovation incentives might increase either for larger or for smaller firms.

In light of research presented in this thesis, I think it is safe to state that consumer interaction is important for understanding various important features of consumer behaviour on disaggregate and on aggregate levels. I have shown that change in the level of peer interaction can change aggregate behaviour (essays 1 and 2). Therefore, basing policy recommendations on models ignoring, or in the best case not accurately modeling, consumer interaction might lead to non-desirable outcomes.

On more disaggregate level, I have shown that diffusion of information through social interaction might generate empirically observed consumption patterns which would not be possible to generate with other types of models. Here I have in mind the emergence of behavioural practices in remote areas identified in essay 2. If interaction is not social and is rather based only on payoff realizations, this kind of patterns cannot emerge. These non-social, revealed preference communication models are characterized by the feature of must-see-to-adopt, which implies that for the information about the product to reach the consumer one of her neighbours has to try the product. Notice, that this kind of behaviour might not involve communication as such. In early models of this kind farmers were simply able to observe farming practices and consequent harvest of their neighbours. However, in case of social interaction information can be acquired and further passed on without validation. In this case uninterrupted chain of adopters is not necessary for the diffusion of the practice. This is crucial for modeling innovation as it guarantees that information will reach potential consumers no matter the adoption behaviour of people that are tied to them.

Furthermore, taking demand side seriously might help to explain certain regularities in industrial organization. Two such examples have been presented in essays 3 and 4 in relation to returns to advertising and firm size distributions respectively. Looking at the effects of consumer interaction might also help in designing industrial policies for achieving certain goals. For example, certain ways of facilitating consumer socialization might prove to be cheap ways to stimulate innovative activities of certain types of firms as shown in essay 5.

All in all, consumer interaction has large and yet unexplored potential to advance current state of the art in economics. Methodological difficulties with analysis of these types of systems are fast becoming reduced with the advancement of (social) complexity theory, agent-based modeling and computational power. Economics should take advantage of these current developments and try to create models of consumer behaviour that are richer and more reliable at the same time.

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Appendix

Proof of proposition 2.5.

Proof. Consider the case of arbitrary neighbourhood size of $2H$. In this case after assuming that the distance between two neighbouring consumers is δ and considering the two-good case, continuous version of equation (2.12) can be rewritten as

$$\frac{\partial z(s)}{\partial t} = \alpha z(s) + \frac{\mu}{2H}\left[\int_{-H}^{H} z(s+\delta h)dh - 2Hz(s)\right]. \tag{3}$$

Using second order Taylor approximation I can rewrite the part of (3) under the integral as

$$\int_{-H}^{H} z(s)dh + \int_{-H}^{H} \delta h \frac{\partial z(s)}{\partial s} dh + \int_{-H}^{H} \frac{\delta^2 h^2}{2}\frac{\partial^2 z(s)}{\partial s^2} dh.$$

Which, after integration of first two summands, is equal to

$$2Hz(s) + 0 + \frac{\delta^2}{2}\frac{\partial^2 z(s)}{\partial s^2}\int_{-H}^{H} h^2 dh.$$

To obtain more accurate values for smaller neighbourhood size, I go back to discrete space and replace the integral in expression above with the sum of squares of integer values.

Substituting this result back to (3) yields

$$\frac{\partial z(s)}{\partial t} = \alpha z(s) + \frac{\mu\delta^2}{4H}\sum_{h=-H}^{H} h^2 \frac{\partial^2 z(s)}{\partial s^2}.$$

Thus, it follows that the only modification that this generalization brings to the system can be captured by the definition of $\tilde{\mu}$ in the text being changed to

$$\tilde{\mu} = \frac{\mu\delta^2}{4H} \sum_{h=-H}^{H} h^2. \tag{4}$$

Going back to consumer addresses ($\delta = 1$), using new definition of $\tilde{\mu}$, and the identity $\sum_{n=1}^{x} n^2 = \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{6}$ I can rewrite equation (2.23) as

$$\sigma_H = \alpha - 2\mu \left(k\frac{\pi}{l}\right)^2 \left(\frac{H^2}{3} + \frac{H}{2} + \frac{1}{6}\right), \tag{5}$$

which results in

$$\bar{k}_H = \frac{S}{\pi}\sqrt{\alpha/\left(2\mu\left(\frac{H^2}{3} + \frac{H}{2} + \frac{1}{6}\right)\right)}, \tag{6}$$

and further in

$$\underline{c}_H = \frac{\pi}{2\sqrt{3}}\sqrt{2H^2 + 3H + 1}\sqrt{\frac{\mu}{\alpha}}. \tag{7}$$

□

Proof of proposition 2.6.

Proof. Consider the case of the arbitrary number of products (N) being available on the market, but each consumer still communicating with only immediate neighbours ($H = 1$). Continuous counterpart of equation (2.12) after applying a Taylor approximation procedure looks as follows

$$\frac{\partial v_n(s;t)}{\partial t} = \alpha v_n(s;t) + \tilde{\mu}\frac{\partial^2 v_n(s;t)}{\partial s^2}. \tag{8}$$

Define two $N \times N$ dimensional diagonal matrices: one $\mathbf{A}$ with only α's on the diagonal, the other $\tilde{\mathbf{M}}$ with $\tilde{\mu}$'s on the diagonal, and three vectors, $\mathbf{V}$ which is the vector of v_ns, $\partial\mathbf{V}/\partial t$ and $\partial^2\mathbf{V}/\partial s^2$ which contain first derivatives with time and second derivatives with space, the system defined in (8) can be written in a matrix form

$$\frac{\partial \mathbf{V}}{\partial t} = \mathbf{AV} + \tilde{\mathbf{M}}\frac{\partial^2 \mathbf{V}}{\partial s^2}. \tag{9}$$

The pattern wave solution to (9) is

$$\mathbf{V} = e^{\alpha t}\bar{\mathbf{V}}_0 + e^{\sigma t + ik\frac{2\pi}{l}s}\tilde{\mathbf{V}}_0^0, \tag{10}$$

where $\bar{\mathbf{V}}_0$ and $\tilde{\mathbf{V}}_0^0$ are vectors of initial values and just like in the paper agents are reindexed in a way that the wave reaches maximum at agent zero. The real part of (10) can be written as

$$\mathbf{V} = e^{\alpha t}\bar{\mathbf{V}}_0 + e^{\sigma t}\cos\left(k\frac{2\pi}{l}s\right)\tilde{\mathbf{V}}_0^0,$$

which is the same as the combination of propositions 2.1 and 2.2.

For the analysis of the stability of the system I again need to determine σ. Doing the same trick as in the text (taking the first derivative with time and the second derivative with space and plugging back to the original equation), I get the following expression

$$(\mathbf{A} - \mathbf{B})\,\mathbf{V}_0^0 = 0, \tag{11}$$

where $\mathbf{A}$ is the same matrix of coefficients, while $\mathbf{B}$ is a new diagonal matrix, which has $\tilde{\mu}w^2 + \sigma$ terms everywhere on the main diagonal. So I get a new $N \times N$ dimensional diagonal matrix of a form

$$\begin{pmatrix} \alpha - \tilde{\mu}w^2 - \sigma & 0 & \cdots & 0 \\ 0 & \alpha - \tilde{\mu}w^2 - \sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha - \tilde{\mu}w^2 - \sigma \end{pmatrix}, \tag{12}$$

determinant of which has to vanish for the nontrivial solution of the system. The determinant of the matrix above is easy to calculate: the determinant of a diagonal matrix is the product of its diagonal entries, so

$$Det = (\alpha - \tilde{\mu}w^2 - \sigma)^N. \tag{13}$$

Equating the determinant to zero and plugging the definition of w gives the opportunity to solve for σ

$$\sigma = \alpha - \tilde{\mu}k^2\left(\frac{2\pi}{l}\right)^2, \tag{14}$$

which is the same as the solution obtained for the $N = 2$ case. Thus, this system, of course, has N solutions but all of them are given by (14). As a result $\bar{k}_N = \bar{k}$ and $\underline{c}_N = \underline{c}$. □

Samenvatting*

In dit proefschrift, dat bestaat uit vijf gerelateerde essays, presenteer en analyseer ik verschillende toepassingen van zogenaamde *heterogeneous interacting consumer models*. Deze toepassingen zijn gefocust op een selectie van probleemstellingen die vallen binnen het domein van industriële organisatie. In het eerste essay stel ik een eenvoudige configuratie voor om het consumentengedrag te bestuderen. Keuzes van consumenten zijn afhankelijk van het consumptievaardigheidsniveau tot het betreffende product. Dit vaardigheidsniveau neemt toe met consumptie-ervaring en interactie met beter getrainde consumenten. Ik onderzoek of een representatief persoon, een persoon met het gemiddelde vaardigheidsniveau, de evolutie van het gemiddelde vaardigheidsniveau in deze eenvoudige economie kan beschrijven. Ik voer twee oefeningen uit die afzonderlijk de twee krachten van vaardigheidstoename (beleving en interactie) onderzoeken. De representatieve persoon blijkt in beide gevallen te falen.

In het tweede essay presenteer ik een *discrete choice model of consumption*. Consumenten nemen beslissingen op basis van de informatie die zij beschikken over elk van de alternatieven. Er zijn twee bronnen van informatie: de consumptie en communicatie met andere consumenten. Communicatie neemt in dit geval de vorm aan van interactie op lokaal niveau: consumenten praten met hun vrienden en brengen zodoende hun algemene indruk over van de producten die voorhanden zijn. Ik analyseer of deze vorm van gedecentraliseerde consumentenkeuzes en communicatie kan resulteren in een stabiele verdeling van gedrag over sociale netwerken. Het resultaat is dat voor een groot aantal van de oorspronkelijke voorwaarden clustering in het economisch gedrag naar voren komt als economisch evenwicht.

Het derde essay modelleert dat keuzes afhankelijk zijn van vaardigheden in plaats van informatie maar behoudt het lokale karakter van de consumenteninteractie. In dit geval delen de consumenten hun consumptievaardigheden. Naast interactie modelleer ik de accumulatie van vaardigheden door ervaring. Producten hebben in deze configuratie twee kenmerken: (i) de kwaliteit en (ii) het niveau van de gebruikersvriendelijkheid. Ik ga ervan uit dat producenten in staat zijn keuzes van consumenten te beïnvloeden door middel van reclame. Ik analyseer de producentenprikkels om te adverteren op een duopoly markt. Ik ontdek dat de

*I am grateful to Bram Timmermans for translating the summary into Dutch.

relatie tussen de kwaliteit van het product en inkomsten van reclame niet, zoals voorgesteld wordt door eerdere studies, monotoon is. In plaats daarvan hebben inkomsten, gebaseerd op de kenmerken van het concurrerende product, een inverse U-vorm.

In het vierde essay neem ik tijdelijk afstand van de consumenteninteractie. Consumentengedrag is schematisch gemodelleerd door middel van nuts-maximalisatie. Van cruciaal belang in deze configuratie is de heterogene voorkeur van de consument en de meeste aandacht gaat uit naar de innovatieprikkels voor producenten. Bedrijven houden zich bezig met onderzoek en ontwikkeling, een hoogst onzeker proces, met het doel om een nieuw product te creëren dat aantrekkelijk is voor een bepaalde groep consumenten. De industrie is georganiseerd via deelmarkten. Ik analyseer de ontwikkeling van de verschillende industrieën en neem het uitgangspunt in bedrijven met gelijkwaardige vooruitzichten. De uitkomst van deze analyse laat een *fat-tailed* verdeling van bedrijfsgrootte zien als een economisch evenwicht voor een grote verscheidenheid aan industrieën. De dikte van de *tail* hangt af van één industrieniveau-parameter in het model.

Het vijfde essay neemt het uitgangspunt in het in essay vier gepresenteerde model maar herintroduceert de consumenteninteractie. In dit geval wordt deze interactie gemodelleerd als een netwerkeffect met een globaal karakter. Consumenten passen hun voorkeur aan om rekening te houden met de populariteit van de deelmarkt waarop de producten worden verhandeld. Het doel van dit essay is om inzicht te verkrijgen in hoe innovatie en innovatie prikkels veranderen als gevolg van consumenteninteractie. In dit essay illustreer ik dat in het geval van consumenteninteractie een hoger niveau van ondersteuning voor innovatie in de economie blijkt te zijn. Daarnaast analyseer ik ook de verdeling van deze extra prikkels voor innovatie in verschillende bedrijven. Of deze prikkels toenemen voor grote of kleine bedrijven is echter afhankelijk van de kenmerken van de industrie innovatie.

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