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**Estimations of US debt dynamics: Growth cum debt and the savings glut
in Kouri's model**

Thomas Ziesemer

Estimations of US debt dynamics: Growth cum debt and the savings glut in Kouri's model

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Abstract:

We derive the central differential equation of the neoclassical growth model for the case of a CES (constant elasticity of substitution) production function with perfect capital movement in terms of the debt/GDP ratio and estimate it in several ways for the United States and in a later step the whole model. Debt data are derived from the accumulation of differences between investment and savings. The result is that at least since 1960 the US debt/GDP ratio follows the pattern of a stable differential equation, which will lead to a long-run debtor position. The debt/GDP ratio will approach a value between 50% and 60% (depending on the specification used) unless a structural break increases the world interest rates or, similarly, US spreads reduce the US demand for foreign debt. A value of 50% will be achieved around 2040. We also find short-run deviations from this long-run path, which are characterized by non-sustainable explosive debt growth. These phases are characterized by high interest rates and followed by devaluations of the dollar. Our simple method allows detecting such phases early on by way of testing a stability condition rather than working with arbitrary threshold levels. The estimation of the whole model yields an elasticity of substitution for capital and labour of .155 with autocorrelation correction (and 1/3 without), a growth rate of labour-augmenting technical change of 1.65% (1.5%) and a corresponding initial level of labour productivity as of 1959 of about 350 (320). As a complement to the growth model we estimate the Kouri model using time-varying coefficients obtaining a forecast of the debt/GDP ratio of about 57% for 2050 as for the growth model. As the rest of the world is catching up with the USA in terms of wealth this is a long-run version of Bernanke's savings glut idea.

UNU-MERIT Working Papers intend to disseminate preliminary results of the research carried out at the institute to stimulate discussion on the issues raised.

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1. Introduction

Since some years the business press reports about more and more people worrying about the increasing foreign debt of the USA. In this paper we look at the dynamics of debt per unit of GDP for the United States from a theoretical and an empirical perspective, starting with some descriptive work and then estimating the debt dynamics as derived from the neoclassical growth model with perfect capital mobility and Kouri's asset market model. In Section 2 we look at some basic data in order to show that the debt problem is not yet serious in terms of current levels. In Section 3 we derive the dynamic equation for debt per unit of GDP from the neoclassical growth model using a CES function. In section 4 we estimate that central differential equation in three different ways in order to see whether or not the current trend is sustainable and which short-term deviations from the trend caused short-term problems in the past. In section 5 we estimate the whole model using a CES production function. In section 6 we show that a similar prediction for the debt/GDP ratio can be obtained by way of estimating an equation derived from Kouri's model.

2. Some characteristics of current US debt

In Table 1 the third column (the difference of columns one and two) indicates net assets of the USA since 1980. In 1989 the U.S.A. became a net debtor. However, columns three and four indicate that the net income from this debtor position has been positive up until now. Columns five and six calculate the interest rates paid and received. The difference is between 0.6 and 1.8 percentage points, which may reflect differences in risk perceptions or valuation effects in their denominator. After 1997 net investment income has become negative according to the IMF IFSY (2001), but the data from IMF IFSY (2004) show still a positive net investment income. Revaluations (claims of the USA are often denominated in EUROs) clearly play an important role here (see Tille 2003; Cavallo 2004; Lane and Milesi-Ferretti 2001). As the net international

investment position yields small differences in interest income paid and received, the debt position of the United States is far from being dangerously large.

TABLE 1 ABOUT HERE

As financial data suffer from (re-)valuation problems we prefer to construct the data from the difference between investment and savings, which is the driving force behind the debt increase and less bothered by valuation problems. This difference in nominal terms is presented in the first column of Appendix B Data. Since 1976 this value has always been positive. In our view this is the essence of debt accumulation, because any more detailed explanation of debt must run via that of savings and investment – or the current account deficit respectively - wherever the ultimate reasons originate. Since 1960 we see an increase in the debt/GDP ratio.[2] Running a regression of the natural log of investment minus savings, (I-S), on the time variable (see Table 2) reveals that the difference between nominal investment and savings has an average growth rate of almost 14%, which is slightly decreasing though. On the other hand the nominal GDP has a growth rate of only 7.6% (see second regression in Table 2). These simple results from regressions with nominal flow variables might suggest that the U.S.A. is on a non-sustainable debt path. In order to analyze this more rigorously we derive the debt dynamics of debt per unit of GDP from the neoclassical growth model with perfect capital movements and estimate it. As this model does not explain why savings are low, we resort to Bernanke's (2005) savings glut idea, and turn it into a long-run approach. Based on the fact that the catching up of many countries in terms of GDP per capita must also lead to catching up in terms of wealth, much of which may go to the leading country USA, leads us to putting the growth of wealth into Kouri's model, estimate it and provide a forecast for the debt.

TABLE 2 OVER HERE

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[2] Engel and Rogers present evidence in their Figure 3 that the debt/GDP ratio increases since 1930.

3. The Model

We return to an older literature that focuses on simple debt dynamics. The model by Amano (1965) is the simplest variant of a model type, which generates conditions under which the traditional idea of a debt cycle can be shown to hold or not to hold respectively. We will use the model to show analytically that a dynamic debt process can be derived graphically with several possible outcomes summarized in Figure 1.

These outcomes can be dichotomized as follows. First, a debtor country can be on a stable path towards a steady state. In the final steady state, the country will remain a debtor if its saving ratio is low. However, it can also become a creditor in the steady state if its saving ratio is high enough. Second, the country can be on an unstable path without ending up in a steady state. In that case, the country can become a permanent creditor – again if it's saving ratio is high enough or the initial value of the debt/GDP ratio is low enough. Amano (1965) emphasizes the latter case, in which the country's GNP is dominated by foreign income and grows at a rate higher than the natural rate. Alternatively, the country can be situated on the other arm of the unstable path and experience an exploding debt, inevitably leading to a crisis. We derive each of these cases and the underlying assumptions below from a linear differential equation in debt per unit of GDP. Subsequently, we estimate these equations for the United States for the whole period 1960-2002, for shorter periods, and for time-varying coefficients. One of our purposes is to show that a country's debt dynamics can switch empirically between the stable and unstable paths over time in response to unexpected shocks although the long-run trend may be sustainable.

We start from a simple neoclassical growth model with a fixed rate of interest, r. Due to the assumption of perfect international capital mobility the domestic interest rate is exogenous and, in the first instance, fixed at the level of the world interest rate. In addition, we assume that output (Y) is produced by the production factors capital (K) and labor (L) with labor-augmenting technology (A). The production function is linearly homogeneous with positive first and negative second partial derivatives of an argument (or CES with $1/1-\rho$ as the elasticity of substitution):

$$Y = F(K, AL), \qquad Y = [\beta K^\rho + (1-\beta)(AL)^\rho]^{1/\rho} \qquad (1)$$

The budget of the firm in terms of goods then is $Y - rK - wL - \delta K$, which will be zero under perfect competition in all markets. Profit maximization of the representative firm yields the marginal productivity condition

$$r + \delta = f'(k) = \beta K^{\rho-1} Y^{(1-\rho)} = \beta(Y/K)^{1-\rho} \text{ with } k \equiv K/AL \text{ and } \hat{K} = \hat{A} + \hat{L} \equiv g \qquad (2)$$

Since interest and depreciation, δ, are given, the marginal productivity condition determines the capital/efficient-labor ratio and the capital-output ratio. The capital stock K consists of domestic capital W and foreign capital or debt D. Conditional on the level of the domestic stock of capital W, the country can choose its desired level of foreign indebtedness D to make equation (2) hold at any moment. The labor force L grows at an exogenous rate n. In the steady state the capital stock K and output Y grow at the same constant rate g, keeping k constant. However, if interest and depreciation vary, K/Y and k could also vary.[3]

Households get net national income, which can be used for consumption C or savings S. This yields the budget $Y - rD - \delta K - C - S = 0$. Goods produced at home can be used either for consumption net of imported consumption, for investment net of imported investment or for exports: $Y = C - C_{imported} + I - I_{imported} + Ex$. Inserting this into the household budget yields $I + EX - C_{imported} - I_{imported} - \delta K - rD - S = 0$ or $I - \delta K - rD - S = Im - Ex$. This is the standard result that (gross) investment minus (gross) savings equals the current account deficit. We assume that this goods market equilibrium condition holds.

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[3] Irreversibility of capital is only a problem if the existing capital stock exceeds the optimal one by more than the rate of depreciation.

The transactions with other countries are goods trade, *Ex-Im*, paying interest on net foreign debt, *rD*, and incurring of new debt, *dD*. The balance of payments then is *dD = Im –Ex + rD*. Using the goods market equilibrium result we get *dD= I - S - δK*. In an open economy with capital mobility, the country then can finance domestic investment in excess of savings through the international capital market, which yields an increase in the country's international indebtedness, as reflected in equation (3):

$$\dot{D} = I - S - \delta K = \dot{K} - S, \tag{3}$$

Dots indicate the change in a variable per unit of time. The first formulation of (3) has gross investment minus gross savings, and the second has net investment minus net savings. Domestic savings is assumed to depend on the country's income net of foreign interest payments, with marginal propensity to save out of net national income equal to *s*:

$$S=s(Y-\delta K-rD), \tag{4}$$

Combining equations (2)-(4) yields a differential equation for the change in the country's external debt per period. For constant interest and depreciation rats this has a special form:

$$\dot{D} = gK(0)e^{gt} - s[Y(0)e^{gt} - \delta K(0)e^{gt} - rD] \tag{3'}$$

For a formal treatment of this differential equation we refer to appendix A. In the next section we provide a more intuitive explanation. Note that these debt dynamics are derived from the goods market equilibrium condition and therefore reflect only the 'fundamentals': investment and savings. But it has been derived using all budgets the

sum of which is the balance of payments. Effects coming from exchange rate or terms of trade (changes) and from portfolio decisions are suppressed by simplifying assumptions of a constant savings ratio, certainty and absence of home asset preferences. The effects of earlier and recent undervalued exchange rates of Asian countries are not emphasized by the model although they may have been important at times but probably not for the whole period since 1960 (let alone since 1930 (see Engel and Rogers (2006)). Mostly during this period developing countries where accused of having overvalued exchange rates, which then should have had the opposite effect. What remains than as an explanatory factor, are low savings rates, high productivity growth (although other areas have been catching up) and a favorable risk perception of foreigners who think that their money is safer in the USA than elsewhere. Except for the risk differences the model does take the arguments into account and the risk issue might be reflected in the interest rate of the USA used below. However, production risks are normally an argument in favor of balanced two-way capital flows and sovereign risks are an argument in favor of loans rather than direct investment and for home bias. Other arguments, as those in our model, determine net flows and stocks (see Kraay et al. 2005). Risk therefore is not a plausible argument to explain high US net foreign debt and the remaining factors are well included in our model.[4] Note that this model shares the dominance of capital movements over the impact of the trade balance with those of the Keynes-Mundell-Fleming model and its successors under high capital mobility, and the Yaari-Blanchard-Weil type, because we assume perfect capital movements and an exogenous rate of interest.[5] Moreover, in spite of the large debt/GDP ratio of the USA presented below, there are no indications that they have had

[4] See also Dollar and Kraay 2006, whose arguments that China has fairly little debt would also imply that the USA have too much debt; both are outliers in their relevant regressions.
[5] For poorer countries models with imperfect capital markets may be more appropriate. For such models with fixed debt/GDP ratios see Verdier (2005) and for a model with flexible debt/GDP ratios see Ziesemer (1998).

any impact on the world market interest rates (see also Lane and Milesi-Ferretti 2002, Fig.11). Otherwise our assumption of a given world market interest rate would not be justified. Below we provide some evidence.

Debt cycle versus permanent debtor position

First, we slightly rewrite equation (3'). From the marginal productivity condition (2), we see in case of a CES production function that $r + \delta = \beta(Y/K)^{(1-\rho)}$. Substitution for $Y(0)$ leads to:

$$\dot{D} = K(0)[g + \delta s - s\left((r+\delta)/\beta\right)^{\frac{1}{1-\rho}}]e^{gt} + srD \qquad (3'')$$

Subsequently we express D in terms of *GDP (Y)*. To this purpose, we first divide both sides of equation (3'') by D, subtract g – the steady-state growth rate of Y – and then multiply by D/Y.[6] The result is as follows:

$$\dot{d} = \underline{k}[g + \delta s - s\left((r+\delta)/\beta\right)^{\frac{1}{1-\rho}}] - [g - sr]d \qquad (5)$$

d denotes the ratio of debt over GDP (D/Y). In equation (5), $\underline{k}$ is the capital-output ratio corresponding to equation (2). The sign and size of the slope and the intercept depend on the (unobserved) parameters g, s, β, δ, and r and can be positive or negative. The three possible cases – corresponding to the different stable and unstable paths of the differential equations – are shown in figure 1. In this figure, d is on the horizontal axis, while $\dot{d}$ is on the vertical one. If

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[6] Alternatively, we may express D as a percentage of the stock of efficient labor (*AL*). Equation (5) is unaltered by this transformation, apart from the definition of d, which would be *D/AL*. We prefer to use GDP as the denominator as both GDP and D are generally available in US\$, whereas *AL* is only available in real terms. Hyperinflationary periods make conversion much more problematic when *AL* is used to normalize.

interest and depreciation are not assumed to be constant, the first g-term has to be replaced by the growth rate of K and the second by that of Y.

If $s((r+\delta)/\beta)^{1/(1-\rho)} > g+\delta s > g > sr$, assuming $(r+\delta)/\beta < 1$ and $\rho < 0$,[7] both the slope and the intercept are negative; the corresponding line in Figure 1 is indicated by SC. The stable stationary point (I) is at negative values of d, implying that the country becomes a creditor in the long run. In point I, the ratio of foreign wealth (D) to GDP is constant if there are no more shocks. Both D and Y grow at the same rate g. Since D is negative, it implies the country holds positive net foreign assets in the steady state. Suppose the country starts out as a debtor at positive values of d. Subsequently, d will decrease to converge to the steady state along line SC. It can be easily shown that initially the level of external debt D will still grow – though at a slower pace than GDP – corresponding to a current account deficit. Over time, the current account deficit is reduced and turns into a surplus with reduction in D. This case is the traditional view on the debt cycle in which every country eventually becomes a creditor (non-debtor).

<div align="center">FIGURE 1 ABOUT HERE</div>

If $g+\delta s > s((r+\delta)/\beta)^{1/(1-\rho)}$, and $g > sr$, the slope remains negative, but the intercept becomes positive. This case is captured by line SD in figure 1. Now, the country converges to a steady state (II) where d is positive. The economy remains a debtor because of its low savings ratio and/or the low world interest rate. The low savings rate forces the country to externally finance its domestic investment, while the low world interest rate helps to reduce the cost of doing so. In the steady state, D again grows at the rate g, which implies that the country remains a capital importer. The current account is not balanced in the long run unless the horizontal intercept of the differential equation is at the origin.[8]

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[7] In case of a Cobb-Douglas function we have $\rho=0$ and $s(r+\delta)/\beta > sr$, with beta as the elasticity of production of capital. For the empirically relevant negative CES parameters with an elasticity of substitution smaller than unity, $-\infty < \rho < 0$ and assuming $(r+\delta)/\beta < 1$, which seems empirically reasonable, we get $s(r+\delta/\beta)^{1/1-\rho} > sr$.

[8] In the literature it is often assumed that the current account must be balanced in the long run, based on the assumption that creditors will try to increase their welfare by not allowing other countries to borrow permanently (see Cohen

If *g-sr < 0*, the slope is positive and the vertical intercept is negative for ρ, $\delta=0$ and values in their neighbourhood. This is the unstable case, labeled *U* in figure 1. By implication, *g–sr >0* is a stability condition, but obviously, there is no reason to a priori impose a stability assumption. The stationary but unstable point (III) is at a positive value of *d*. If *d(0)* is less than in the stationary point, the economy starts moving to the region of negative and continuously declining *d*. In that case, the country will be a creditor in the long run, where GNP grows faster than GDP. This was the point Amano (1965) tried to make.

A starting point to the right of the stationary point implies that *D(0)* sufficiently exceeds *K(0)* if ρ, $\delta=0$, as shown in appendix A. The country then starts out with more debt than capital and will have negative current wealth for all periods. Clearly this is unsustainable in the longer run. A country on such explosive path will be forced to adjust, one way or the other.[9] As *D(0) > K(0)* seems to be an unrealistic case from an empirical point of view, we would interpret explosive debt dynamics as a case of non-sustainable economic policy when it appears in our estimates and as a temporary deviation from the assumptions of our model, which are those of a good debtor. *D(0) < K(0)* can be reconciled with instability only if the elasticity of substitution is larger than unity (see appendix). Our estimate below does not support the empirical relevance of this case.

In this type of model, a switch from one dynamic process – say the SD line – to another – say the SC line – can only arise due to changes in the underlying parameters of the system, *g, s, β,* and *r*.[10] As an example, we will discuss a non-anticipated and permanent upward jump in the world interest rate. If this happens, the marginal product of capital will increase and the optimal amount of capital *K* to be used will fall. For a given national wealth *W*, a fall in the desired level of capital *K* implies that less external debt *D* is required. As a result less debt is obtained from the

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1991). Whether this is the case, however, is an empirical as well as a theoretical question. In our model we do not explicitly impose the restriction of a balanced CA.

[9] Neher (1970) assumes that debt *d* grows at the same rate as the capital stock. As we have shown, this is only the case in the steady states of the stable cases. Amano (1965) has treated only the left arm of the unstable case. Our model integrates all these cases.

[10] Of course, there can also be exogenous shocks to *d* without changing the prevailing differential equation. This moves the economy along the relevant line in figure 1 rather than changing the position of the line itself.

international capital market. Alternatively stated, it implies that more domestic wealth is invested abroad at the higher world interest rate level. Such a decrease in capital inflows or increase in capital outflow – even capital flight – thus can be rooted in the economy's fundamentals and need not be a purely monetary phenomenon of irrational speculators. Domestically, the resulting lower marginal product of labor leads to a fall in real wages.

We can also describe these dynamics in terms of differential equation (5) and Figure 1. Starting from a situation where the country is a long-run debtor (the SD line) a jump in the interest rate leads to both a counter-clock wise rotation (flatter slope) and a downward shift of the SD line. The economy can move from the SD in figure 1 to a situation SC or even to a situation *U,* depending on the size of the interest rate shock.

A similar analysis can be given for other exogenous events changing the slope and intercept of the differential equation, like for instance a jump in the savings rate *s* or in the rate of population growth *n* or technical change which in turn changes *g.*

In the empirical part of our paper, we will present evidence on the extent to which the United States have switched between the different regimes of debt dynamics over the period 1960-2002. The empirical equivalent of equation (5) that we use in the first estimation is:

$$\dot{d} = \alpha_0 + \alpha_1 d(-1) \tag{6}$$

with $\alpha_0 = \underline{k}(g+\delta s - s((r+\delta)/\beta)^{1/1-\rho})$, $\alpha_1 = sr\text{-}g$. Clearly, the regression coefficients are complex functions of behavioral parameters like the savings rate and exogenous but time-varying variables like the world interest rate. Without additional information on these parameters and variables, we can just report the regression estimates and account for potential time-variation. When more information is made below under additional assumptions, an extended regression of the model is possible. Note also that the explanatory variable in equation (6) is lagged one year. This is due to

the switch from the continuous-time derivation of our theoretical model to the discrete-time formulation of the empirical model.

4. Data, Estimation and results

Cumulating the differences of gross investment and savings, starting from the net international investment position of 254.57 billion US dollars in 1980 according to IMF IFSY 2001, we get the values for cumulated debt D in Appendix B. Savings is gross savings (GDP minus consumption) according to the World Development Indicators plus net current transfers plus net income from abroad in order to have it as disposable income minus consumption. We could get almost identical results from the accumulation of current account deficits. However, these deficits can be financed at times by reductions of reserves. For the most recent years the cumulated debt calculated is identical but in other years there are deviations.

Dividing the cumulated debt by the value of the GDP we get the debt/GDP ratio. In Figure 2a we plot these data together with the Net international investment position (from Table 1) per unit of GDP. The values are identical in the beginning just by assumption. They are lower for later years but become equal again in 2001. With debt data from cumulated investment-savings differences, cumulated current account data and NIIP being very similar in the beginning and in the end of the period considered and differing in between because of effects from exchange rate valuation to be discussed next, the consequences of imperfections in the calculations of savings do not seem to be overly important here.

FIGURE 2 OVER HERE

The difference between our data and the NIIP can be explained by the official exchange rates of the UK, Germany and Japan (local currency units per US$ from World Development Indicators) indicating (at least partly) the valuation effects. The regression result with dependent variable (debt/GDP – NIIP/GDP) is ('tsq' is a squared time trend):

	Coefficients	Standard Error	t Stat	P-value
Intercept	0.099596	0.045414	2.193057	0.041681
Germany	-0.13543	0.061942	-2.18646	0.042232
United Kingdom	0.345361	0.094013	3.673552	0.001738
Japan	-0.00053	0.000276	-1.90951	0.072266
Tsq	-0.00016	4.03E-05	-4.03995	0.000768

Adjusted R Square   0.843989382

All of the variables used here have a unit root. These variables are not cointegrated. Therefore we re-estimate the above equation in first differences getting the following result:

Dependent Variable: D(DMINNIIP), Method: Least Squares
Sample (adjusted): 1981 2002, Included observations: 22 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.004729	0.004891	-0.966849	0.3464
D(EGER)	-0.140701	0.067808	-2.074976	0.0526
D(EJ)	-0.000540	0.000313	-1.726158	0.1014
D(EUK)	0.390266	0.139891	2.789789	0.0121
Adjusted R-squared	0.383099	Akaike info criterion		-4.711
Schwarz criterion	-4.513375	F-statistic		5.347035
Durbin-Watson stat	2.297910	Prob(F-statistic)		0.008236

Results for first differences are very similar to those for levels. As the NIIP values are fluctuating and so are the exchange rate values (see Figure 2b), whereas our debt/GDP ratio data are fairly smooth, our data can be viewed as a version of NIIP data cleaned from fluctuations in the valuations.[11] Note that only for the British pound do we get the expected sign of the valuation channel: an appreciation of the dollar (more pounds per dollar) should lead to a lower NIIP in dollars because US claims are to a large extent in other currencies (see Tille 2004) and a higher difference between our debt/GDP data and the NIIP. The negative sign for the YEN and the Euro is consistent with a dominance of the trade channel if an appreciation of the dollar worsens the current account and increases the difference between investment and savings and this has a

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[11] For Japan the regression uses data yen/$ whereas figure 2b uses 100yen/dollar.

stronger impact on our debt/GDP data than on the NIIP data. Our data capture what we expect to be continued steadily through the future because of the differences between savings and investments. It is much harder to have a clear cut expectation on the exchange rate valuations. A long-run forecast therefore can more easily be based on our data because they depend less on unpredictable valuations.[12] It should be clear though that a permanent devaluation of the dollar lead to values of NIIP that tare lower than our values of cumulated dollars, because the devaluations result in higher values of the US assets for example in Euros (Cavallo and Tille 2006).

Using the debt/GDP variable we can run a regression indicated by equation (6). We use the least squares approach with the Newey-West HAC[13].

Long-run dynamics

The result is summarized in Regression (1) of Table 3. According to this estimate the intercept is positive, the slope is negative and we are in regime 'stable debtor' (SD) of Figure 1. The country would approach a constant debt/GDP ratio in the long run. The equation suffers from autocorrelation; adding an ARMA(1,1) process the error probability of rejecting the hypothesis of no autocorrelation is between 28% for 5 lagged residuals and 71% for 3 lagged residuals. Therefore we do not reject the hypothesis. Standard unit root tests indicate that the slope may be zero. The error probability of rejecting this hypothesis in an ADF test with a time trend is about 30%. This would indicate that the differential equation is not even stabilized at high values. However, the time trend is highly insignificant. If we drop it, we can use the estimate to determine the coefficients of equation (6). The result is the third regression in Table 3. Although the lagged dependent variables are usually introduced in order to remove autocorrelation, the

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[12] Readers particularly interested in portfolio and valuation effects on and from exchange rate movements are referred to Tille (2004), Roubini and Setser (2004) and Blanchard et al. (2005). We'll have a minor digression to the trade balance and devaluations below.

[13] HAC means 'heteroscedasticity and autocorrelation correction'. In our case non-correction gives about the same significance.

Breusch-Godfrey test tells us that they do not do this. Trying to add a quadratic regressor does not yield a significant result. This leaves us with three very similar results some of which have autocorrelation.

TABLE 3 OVER HERE

For the first, the second and the third regression the stationary value of the debt/GDP ratio will be 58.6%, 55.5% and 50% respectively.[14] This implies that it might become about twice its 2002 value of .27 according to the estimated model. Of course, this will only happen if the world market interest rate is not driven up and there are no substantial spreads above LIBOR/EURIBOR coming up for the United States. Both of these effects might dampen the increase of the debt/GDP ratio. Nothing like that has been observed so far. Therefore they are not included in the model. If this does not happen the question is: How long does it take until the 50% level is reached?

FIGURE 3 OVER HERE

Making a dynamic forecast of all equations from Table 1 we find that the first regression is the best according to most of the forecast criteria, in particular the high covariance proportion and the zero bias proportion (see Figure 3). From the forecast for the debt/GDP ratio one finds that the USA will reach the 50% mark of a debt /GDP ratio by 2040. The forecast in Figure 3 has a Theil index of .026, which is very close to its ideal value of zero, which would indicate a perfect fit. The bias proportion and the variance proportion of the mean squared error are almost zero and almost the whole error is in the unexplained part called covariance proportion. For the static one-period forecast the results are even better (not shown). In particular, the real development never is outside the margin of two standard deviations. The CUSUM and CUSUM of squares tests are passed at the 5% level. In the static forecast there are three observations indicating non-constancy of parameters and in the dynamic forecast only one. All of these forecast properties indicate a

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[14] Cline (2006) finds a debt/GDP ratio of about 50% for 2010 already and 135% for 2024 based on the worst case assumption of a continuing current account deficit of about 8% of GDP as of 2005.

quite good estimation result. The results for the change in the debt/GDP ratio in the lower part of figure 3 are slightly worse but the bias proportion is zero here.

FIGURE 4 OVER HERE

TABLE 4 OVER HERE

Short-run deviations

In order to detect short-run problems we run 7-years rolling windows regressions. The results are summarized in Table 4 and Figure 4. When the slope is positive in the period beginning in 1973 the USA is not even a debtor. The trade balance does not even dis-improve. But the interest rate increases in the period 1972-74 by 5.5% (see IMF, International Financial Statistics Yearbook, 2000). For the 7-years periods beginning in 1979-82 we also find positive slopes. Although the USA are still a net creditor, the combination of restrictive monetary and expansive fiscal policy drives up the interest rate and attracts much foreign debt leading to a net debtor position in 1985 according to our data.[15] Since 1983 it is broadly recognized that the dollar is overvalued and will have to depreciate. This happens in 1985. The stock market crashes in 1987. Our method signals increasing or positive slopes for the 7-years periods beginning 1976-1982 (ending 1984-88), that is warnings are available since 1982 or 1983 when the data become available. Our method would have warned against financial instability in the sense of an unstable differential equation of the debt/GDP ratio through devaluation of the dollar - and as a consequence a possible downturn of the stock exchange. Moreover, we find a positive slope for the periods beginning in 1988-91 (ending 1994-1997). In this period there is a recession in 1991 and an increase of the interest rate by 2.5 percentage points in the period 1993-1995. The positive slope with observations of the data corresponding to the upper right quadrant of Figure 1 suggests that the USA stop being a good debtor in this phase. After these periods the interest rates are decreasing again, production increases and the slope coefficients are going down again. Finally, for the last three intervals

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[15] Also according to the data of Sinn (1990) this happens in 1985; according to Lane and Milesi-Ferretti (2006) this happens in 1986 and according to NIIP data of the IMF in 1989.

beginning 1993-95 and ending 1999-2002 we get again a positive coefficient at an order of magnitude that is similar to those before 1987.

Does all this imply that there is no problem? Not completely so: The fact that the United States did become a net debtor around 1985 according to our data and in 1989 according to the financial data may have long-term consequences. The increase in the debt/GDP ratio cannot go on forever at a constant rate. Our model indicates the problem: According to the long-run trend there is a decrease in the growth rate of the debt/GDP ratio; but current or future trends or economic policy can follow a path with an increasing growth rate of the debt/GDP ratio as in the periods beginning 1979-1982 (ending 1986-1988)) and intervals beginning 1993-95 and ending 1999-2002 or even going beyond it.

Time-varying coefficients

What's the risk that the United States move into the direction of explosive debt? The slope coefficients in Figure 4 suggest that the slope may be a time-varying parameter. Therefore we apply the state-space technique in order to get time-dependent slope and intercept values (see Hamilton 1994, section 2.6).[16] The model is as follows:

$$d(debtgdp) = sv1 + sv2*debtgdp(-1)$$

$$sv1 = c(2)+c(8)*(sv1(-1)-c(2)) + c(9)*(sv2(-1)-c(3))$$

$$sv2 = c(3)+c(10)*(sv1(-1)-c(2))+c(11)*(sv2(-1)-c(3))$$

Here *sv1* is the intercept of the debt equation (6) and *sv2* is the slope. The first equation reproduces the debt equation. The second and third equations are a difference equation system in

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[16] We did not find any good result for Markov regime switching model in which a stable debtor and an unstable debtor would be the two regimes. This might change if there are more observations through the use of quarterly data.

the slope and intercepts coefficients in equation (6). $c(2)$ is the average or steady-state value of all intercept estimates and $c(3)$ is the average or steady-state value of all slope estimates. The values of the intercept and the slope depend on their previous values and are assumed to react to a deviation from their steady-state value in a way indicated by the coefficients $c(8)$-$c(11)$. All equations are assumed to have error terms with variance $exp(c(5))$, where $c(5)$ is a coefficient to be estimated. The coefficients $c(i)$ are estimated. We assume that all equations have an error term and that their variance is identical. We did try to estimate with different variances, and also without error terms in the second and third equation. Results were much worse than with identical variances.

<div align="center">TABLE 5 OVER HERE</div>

The estimation output and the details of its analysis can be found in Table 5 and Appendix C[17] (see working paper version). It is shown there that the system has a stable focus. The coefficients $c(9)$ and $c(11)$, both for the time-varying slope parameter, are highly insignificant. An F-test shows that they can be dropped. The second regression then has $c(8) = .52$ and $c(10) = 2.11$. As this means that $sv2$ is dropped from the right-hand side of the system it is obvious from looking at the system that the equation of $sv1$ is a stable one, which is not influenced by that for $sv2$, but rather is determining the change of $sv2$. It is a stable node. The implied absence of cyclical movements is unable to explain the ups and downs in the rolling windows regressions. Therefore it may be better not to drop $c(9)$. If only $c(11)$ is dropped from the first regression, in the third regression $c(9)$ has a marginal significance level of almost 19% and the result is a negative trace, a positive determinant and complex roots again. Again we get a stable focus and coefficients converging to their steady state in a spiraling path, which resembles those of the rolling windows regressions. Results for the one-period-ahead prediction of intercept, $sv1$, and the slope, $sv2$, are presented in Figure 5. The intercept varies around 2% and the slope around -4%. Interestingly,

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[17] We found only one good result. Results obtained depend strongly on the initial values. However, all bad results we looked at were qualitatively the same in the sense that never any positive prediction for the slope appeared.

positive slopes as they did appear in the rolling windows are never predicted. Probably, this is due to the fact that the estimates take on board the information of the good times of the sixties and seventies and therefore give relatively little weight to the periods where we found positive slopes for the rolling windows. The average values and the steady-state values for the intercept and slope are about as large as the estimates of the least squares equations. The system converges to its long-run steady-state value $a=sv1-c(2)=0$ and $b=sv2-c(3)=0$. The prediction for the long run then is dd= 0.020596-0.036434d, which yields a steady state value of $d^* = 0.565$, and similarly so for the other two regressions in Table 5. This is in the higher end of the range for the regressions of Table 3. But there is no instability problem.

<div align="center">FIGURE 5 OVER HERE</div>

Much recent literature is interested in the development of the trade balance. The balance-of-payments equation for this model can be written as stated before equation (3):

$$\dot{D} = TD + rD, \quad or, \dot{d} = TD/Y - (\hat{Y} - r)d$$

TD indicates the trade deficit. The second version has been obtained by dividing the first by D, subtracting the growth rate of Y and multiplying by $d=D/Y$. Equating it with (6) and solving for the trade deficit yields

$$TD/Y = (\hat{Y} - r)d + \alpha_0 + \alpha_1 d(-1)$$

Under the additional assumption of a growth rate equal to the interest rate, the trade deficit as a share of the GDP equals the change in the debt/GDP ratio as forecasted in the lower part of Figure 3. This predicts a soft landing for the trade deficit going to zero because of the absence of financial problems in the model. If the interest rate is larger than the growth rate of the GDP as is the assumption in much of economic theory, trade deficits have to be even lower or, alternatively, can be higher where the opposite case applies. If the whole adjustment of the trade balance for the benchmark case has to be achieved by a devaluation of the real exchange rate and if this requires a devaluation of 7-10% for each percent of TD/Y starting at 5% in 2004 and going to .005 in 2025

as predicted in figure 3, this requires a real devaluation of 30-45% during the next 20 years, which is roughly one or two percent a year.[18] If we do this consideration using the time-varying parameter system, parameter constancy is achieved in 2025 but the fluctuations before are very weak. If the long-run interest rate is at a low value of 2% as a possible consequence of the savings glut argument, which we treat as a long-run catching up phenomenon below, and the nominal GDP growth rate remains at 5%, and the long-run value of the debt/GDP ratio goes to .57, then the trade deficit will be about 1.7% in the long run. Starting from the 5% mentioned above it has to be reduced by 3.3%, which requires devaluations of 20-35% during the next 20 years, which is between one and 1.5% a year. With such a trend in the exchange rate of the dollar the net debt measured according to the NIIP will be lower than our measure of cumulating differences of investment and savings in current dollars (Cavallo and Tille 2006). The usual caveats for forecasts apply. In particular, if foreign financial investors expect this real devaluation to appear in the form of the nominal devaluation of the dollar they may want to reduce their financial position in the USA according the scenario by Roubini and Setser (2004) and the nominal devaluation may be quicker than the real one predicted above for the soft-landing scenario. However, as they know that they cause a lower valuation of their own assets they may want to do this only softly. And, of course, other adjustments rather than only one of the exchange rates are likely to come about.[19] In order to avoid a hard landing, Roubini (2006) has suggested that the USA increase their savings ratio through a reduction of their budget deficit; that Asian countries revalue their currencies and that Europe improves its growth rate through structural reforms. Obstfeld and Rogoff (2006) point out though that a productivity increase in Europe's tradable goods reinforces the adjustment problem of the USA.

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[18] Obstfeld and Rogoff (2006) also find a value of more than 30% for the closing up of the current account using a different model.
[19] For several interesting related scenarios see Corden 2006.

5. Extension of the basic regressions

Next, we will estimate the complete model for three reasons. First, when we did the previous regressions also with less data available, the predicted long-term debt/GDP ratio was a bit smaller. The recent years therefore seem to have some impact. This indicates that our procedure does not yield perfectly stable results, possibly because it does not take on board all relevant variables. Second, fluctuations in the slope and intercepts suggest that some of the variables contained in the intercept and slope have some variation. Third, some of the empirical work presented so far could also have been done from the perspective of an autoregressive process in the debt/GDP ratio without any theory. Therefore we should test in how far the model is not just a camouflage for an autoregressive process but rather yields reasonable estimates.

For purposes of regression analysis more information is obtained when leaving the marginal productivity condition as a separate equation. The model then consists of the equation for debt dynamics, the marginal productivity condition for capital (we do not use one for the labour market) and the production function. We take the last two in log form:

$$d(t) - d(t-1) + s(1 - rd(-1) - \delta K(-1)/Y) = c(1) + c(2)*K(-1)/Y - c(3)*d(-1)$$

$$log\ (K(-1)/Y) = (\ log(r+\delta) - log\beta)/(-1+\rho)$$

$$log\ Y = (\ 1/\rho\ )\ log[\beta K(-1)^\rho + (1-\beta)(A_0 e^{bt}L)^\rho]$$

The first equation is obtained by writing the version of equation (3) using net-expressions in discrete form, dividing by $D(t-1)$, subtracting the growth rate of GDP, multiplying by $d(-1)$, and taking some terms to the left-hand side. Then, $c(1)$ should be zero as it has been added only for econometric reasons. $c(2)$ should equal the growth rate of K, and $c(3)$ should equal $1- Y(t-1)/Y(t)$, which is approximately the growth rate of GDP. All terms on the left-hand side of the first

equation are taken in the form of data, which means in particular that we deviate from the model by assuming no longer that the rates of saving, depreciation and interest and the capital-output ratio are given constants.

FIGURE 6 OVER HERE

Additional Data

In order to get the output variable, we divide the GDP in current US dollars by the GDP deflator as obtained from the NIPA data of the BEA.[20] We deflate current gross fixed capital formation using the fixed investment deflator, cumulate it according to $K(t) = K(t-1) + I(t) - \delta(t)K(t-1)$. The rate of depreciation needed for this, is obtained through division of data for nominal depreciation by a nominal capital stock. The latter is obtained by cumulating nominal gross fixed capital formation minus the nominal depreciation data with a starting value of gross fixed capital formation divided by an assumed depreciation rate of 5% of capital and an initial growth rate of .048.[21] Employment is taken in thousands of workers. Interest rates are the average of the last two columns of Table 1 minus the rate of GDP inflation. The savings rate is calculated as Gross national savings minus depreciation both in current dollars divided by (nominal gross national income minus depreciation). Some of these data are plotted in Figure 6. A remarkable trend is the fall in the savings ratio since the early 1980s, where also the much discussed wage inequality process started. For a trace to literature on the explanation of the falling savings ratio see Chanda (2005). Trying to endogenise the savings ratio using all the arguments in that literature would make the model intractably complex.[22] Therefore we treat the savings ratio as an exogenous variable here.

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[20] NIPA and BEA stand for 'National Income and Product Accounts' of the 'Bureau of Economic Analysis'. Data from this section are from this source unless stated otherwise.

[21] This value is obtained form a simple growth accounting exercise assuming a Cobb-Douglas function with a production elasticity of capital of 1/3 and labour 2/3, employment growth of .02, technical change of .017, nominal GDP growth of 5.5 percent, GDP inflation of .014 and fixed investment inflation of .0054.

[22] Engel and Rogers (2005) discuss problems of interpreting the evidence by use of an intertemporal optimization framework with perfect foresight and infinite horizons.

The interest rates are fairly low compared to the early eighties and fall during the late nineties when the debt/GDP ratio increases. In particular, we would have preferred to subtract investment income debit from investment income credit and divide it by the debt variable from Appendix A. However, when the USA goes from a creditor to a debtor position, interest rates calculated in this way move above 60% due to the effect of the denominator crossing zero. In other periods the difference often is negative and would have resulted in a negative nominal interest rate for all years in which the USA were a debtor already. This comes from the revaluation of the US claims, which are held in EURO (see Cavallo 2004). Again this does not provide any indication of interest rates driven up by net foreign US debt.

Econometric method and results

In related literature that is close to the model used, imperfect capital mobility is captured by the assumption that the interest rate depends positively on the debt incurred per unit of GDP (see Bardhan 1967). In order to test the perfect capital mobility assumption for our data, we find that a vector autoregressive (VAR) model for the interest rate and the debt/GDP ratio has one lag according to the Schwarz criterion and three lags according the Hannan-Quinn criterion. The Johansen cointegration test shows cointegration for the period from 1983 onwards for both cases. In both cases the estimate of a VECM shows the wrong sign. Therefore there is no evidence that the debt/GDP ratio has an impact on the real interest rate (see Appendix D of the working paper version for the last two regressions), which justifies our assumption of perfect capital movements. Of course, this can only be a very preliminary test but adding other countries' debt/GDP ratios or other variables is beyond the scope of this paper.

Unlike the simplified model in section 3 there is a negative trend in the interest rate since the 1980s and a positive trend in the capital-output ratio at a rate smaller than one percent. Real interest rates follow inflation and are negative at least before 1979. Therefore we start the regression only in 1983, when inflation has come down to a value below 4%. A second reason for

this starting period is the information that capital mobility was increasing strongly after 1980 (see Obstfeld and Taylor 2002).[23] The regression output is presented in Table 6 and Appendix E (see working paper version). The system is estimated using the general method of moments with heteroscedasticity and autocorrelation correction for the standard deviations t-value and p-values (GMM-HAC), because this takes on board dependence among the residuals of the equations and allows for the use of instruments in order to correct for endogeneity. Table 6 contains three versions of the regression.

TABLE 6 OVER HERE

The first regression does not use instruments in addition to the regressors and does not make any correction for the potentially existing autocorrelation bias. It is similar to a non-linear estimate using the SUR (seemingly unrelated regression) method with constraints relating the second and third equation. But under GMM residuals are not assumed to be distributed normally as they are under the assumptions of SUR estimation. The coefficients for the capital-output ratio and the debt/GDP ratio in the first equation have been restricted to equal each other as they would in the steady state of the model. $c(2)$ is about 5%, which is a bit higher than one would expect in a steady state. The result $\beta = .34$ is about what one would also expect in case of a Cobb-Douglas function for the production elasticity of capital. The CES parameter has a value of $\rho = -1.85$, which implies an elasticity of substitution of 1/2.85. The level of labour augmenting productivity as of 1959 *(t=0)* is $c(11) = 321$ per worker. The rate of labour-augmenting technical change is about *1.5% = c(12),* which is fairly close to the GDP per capita growth rate of the USA for the period 1960-1988 mentioned in Jones (1997). As the model has five restrictions on the coefficients the J-statistic goes away from the value of zero, which it would have when only regressors are used as instruments and there were no constraints (see Greene 2003, 548-9). The

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[23] The classical counter-argument comes from the Feldstein-Horioka cross-country regressions of investment and savings for data before 1975. For recent evidence of high capital mobility based on modern econometrics see Hoffmann (2004), Coakley et al. (2004) and Fattouh (2005). Note that this literature is dealing with responses to stochastic shocks whereas ours is dealing with long-term trends.

adjusted R-squared is remarkably low for the first equation and the Durbin-Watson indicates autocorrelation. It should be noted here that in the second equation the sum of interest and depreciation as well as the capital variable have unit roots according to a standard augmented Dickey-Fuller test. They are not cointegrated. Basic econometric theory would suggest estimating in first-differences. However, if we would do so, the intercept would drop out of the equation and this implies a loss of important information because the parameter appears also in the third equation and the corresponding constraint would therefore vanish as well. Sticking to the model then implies sticking to levels in the estimation. Similarly, in the third equation output and capital have unit roots, but employment does not. Therefore we estimate an unbalanced equation, which is controversial in itself (see Maddala and Kim 1998, 252-2). However, again sticking to the model is our preference.

In the second regression of Table 6 we have added lagged regressors as instruments in order to correct for endogeneity. Table 6 indicates that the number of instruments in addition to the regressors is six, zero and three respectively. In the first equation the capital/output and the debt/GDP ratio are endogenous. As these are cumulative variables, we would want to add many lagged instruments. We add three lags for each of the two right-hand side variables. The second equation has only the real interest and depreciation rate as a regressor, which we consider to be exogenous. Therefore no instruments are used here except for the regressors. The third equation has only the capital variable as endogenous regressor. We can add three lags as instruments. The major impact of the use of instruments is in the t-values. All other results change only marginally. The elasticity of substitution is now 1/3. The J-statistic increases only marginally and the product of the number of observations and the J-statistic is clearly below the chi-square value (not shown), which justifies our number of instruments.

In the third regression we have followed one of the standard advices in regard to autocorrelation that is to add lagged dependent variables (the other advice is "don't'' (see Greene 2003) and stay with the second regression). For the debt equation and the marginal productivity

equation we have employed lagged dependent variables as autocorrelation correction. In particular it was not possible to add lagged dependent variables to the third equation. As the lagged dependent variables are also endogenous regressors now, we have also added them to the instruments. The Durbin-Watson statistic for the production function and the marginal productivity condition still indicate high autocorrelation. Adjusted R-squares are high now and marginal significance levels are fairly low. The only exception is the intercept, which should be zero according to the theoretical model; it is indeed highly insignificant. The other major change coming from this autocorrelation correction is that the parameters of the CES function have become much smaller and therefore the elasticity of substitution is much lower than before, *1/6.45*. The rate of labor-augmenting growth goes to 1.65%, which is fairly close to the famous *1.7%* mentioned by Mankiw, Romer and Weil (1992). The level of labour-productivity is 350, which means that a worker is 350 times more productive than his counterpart in a hypothetical country that has never had any technical change. These numbers are compatible with the fact that people in developed countries are several hundred times richer than those in the poorest countries. Under the additional assumption that the rate of technical change has always been 1.65% (1.5%), technical progress must have started about 427 (398) years ago (from 2002), that is slightly after 1575 (1600).[24]

The autocorrelation correction reduces residuals (see plot in Appendix F in the working paper version) by 50% in the first two equations. It is obvious from the plot that it mirrors the business cycle deviations from the growth model. For example in the 1991 recession, the residuals are negative for the GDP, positive for the capital-output ratio and negative for the change of the debt/GDP ratio, the latter result reflecting that investment tends to go back during recessions and more so than GDP.

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[24] Kremer (1993) has a nice description of world wide technical change phenomena of earlier times with a rate of technical change below .3% though for the world as a whole before 1600. Of course, during history these rates will not have been constant but probably were higher during the industrial revolution and lower at other times.

It is hard to decide, whether or not the third regression is more convincing than the second. A decision will depend on the readers view concerning autocorrelation correction. The values of the estimates are quite reasonable values in both cases, with the expected signs and high significance. This may justify our feeling that the model is a reasonable choice for the USA in the period in question.

6. The savings glut in the long run: catching up in wealth and debt dynamics in the Kouri model

Bernanke (2005) has pointed out that the above growth model has no explanation for the low savings ratio. His argument is that the increase in savings in many emerging market economies during the last ten years induces low interest rates and therefore high investment and low savings in the USA.[25] We want to extend the argument to a longer period as we see the increase in the debt/GDP ratio already since 1960 in the data and Engel and Rogers do so since 1930. The growth literature has shown that many countries are catching up with the USA in terms of GDP per capita. When they get away from a subsistence minimum level, their savings ratios increase and indeed savings ratios have a positive time trend in current panel data. With increasing or constant savings rates and a stronger growth of GDP per capita they should also catch up in terms of wealth, and the wealth of the USA relative to the rest of the world should decrease. This can increase US debt irrespective of the investments taking place in these countries as one can see from the *Kouri model* as used by Blanchard et al. (2005; henceforth BGS[26]). A starting point of that model is that debt, *D*, equals assets, *X*, minus wealth, *W*:

$$D = X - W \qquad (7)$$

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[25] A related argument is that interest rates are asset prices and therefore a model with two-way capital movements should complement the growth model.
[26] BGS formulate their model in real terms. We use nominal ones.

The USA invests a share α of their wealth in their own assets and the rest of the world invests a share 1-α* of their wealth W* in US assets:

$$X = \alpha W + (1-\alpha^*)W^* \tag{8}$$

Insertion of (8) into (7) yields

$$D = (\alpha-1)W + (1-\alpha^*)W^* \tag{9}$$

The invested shares in the BGS model depend on the relative rates of interest. Assumptions like decreasing absolute risk aversion and increasing relative risk aversion make these coefficients dependent on *W* and *W**, and globalization can be interpreted as making the coefficients dependent on a time trend capturing the sequence of liberalizations in international capital markets (see Obstfeld and Taylor 2002). As relative interest rates, *W* and *W** vary over time and may have a trend, the investment shares are assumed to be time-varying coefficients. Therefore we will use the methods of Hamilton (1994) again. We assume that the capital market for the rest of the world is the UK. As we are dealing with a long-run issue and yearly data, expected relative profit rates will be constant at unity.[27] But there may be a positive standard deviation for relative real interest rates, *Rfse*, which can be estimated by a forecast. The model to be estimated then is[28]

$$D = (\alpha_t-1)W + (1-\alpha^*_t)W^*$$

$$\alpha_t = c(21)+c(22)*\log(W/W^*)+c(23)*(@trend)^{\wedge}(-2) + c(24)*Rfse$$

$$\alpha^*_t = c(31)+c(32)*\log(W/W^*)+c(33)*(@trend)^{\wedge}(-2) + c(34)*Rfse.$$

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[27] Trying to include them (or the relative nominal dollar rates) yields pertinent problems of unexpected signs. If we take the standard deviation of nominal relative dollar rates rather than that of real rates, the investment shares do not remain between zero and unity, and we would jump out of the model.

[28] We add a stochastic term with the same variance assumption to each of the equations.

One difficulty of our approach is to find values of the shares invested at home, which are always between zero and unity; a second is that the intercept, which is zero according to the theory in the first equation, may have any sign, but all other coefficients except for *c(32)* then should be positive if we expect the risk aversion properties mentioned above and a globalization trend, $t^{(-2)}$, that approaches zero. A higher risk variable, *Rfse*, should induce both countries to invest more wealth at home. The third difficulty is that the estimates depend on initial values, which can result in many different outcomes.

The data used for net debt are the same as before. In order to get wealth data, we take net savings, *S(t),* from the World Development Indicators and cumulate them according to $W(t) = W(t-1)+S(t)$.[29] We take the data for real interest rates also from the WDI. We consider the United Kingdom as the capital market of the rest of the world, use the real interest rate of the UK as that of the rest of the world and generate the standard deviation of the relative interest rate by running an OLS regression on its own past value and make a static forecast from it.[30] The standard deviation from this forecast is our variable *Rfse*. Results are summarized in Table 7.

The first two regressions do not take the risk variable on board. The first has been found starting from initial values of zero for all the coefficients in the Table. The second has been found randomly and has a slightly better likelihood result. We get the expected signs at high levels of significance. The next three regressions include the risk variable. The likelihood is only slightly better. The risk induces the USA and the ROW to keep more money invested at home. The three results differ in that a lower constant is associated with a higher coefficient of the risk variable.

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[29] This requires an initial value for W, which we get from $W(1) = W(0)+S(1)$ through subtraction of and division by W(0), implying $S(1)/W(0) = [W(1)-W(0)]/W(0) \equiv g$, we get W(0)=S(1)/g. Then, as getting g requires knowledge of the initial value itself, we approximate it by the growth rate of the GDP. Data for debt, wealth and GDP are taken in current \$US. These values differ from those of the FED, used by Blanchard et al., because the latter are higher through asset inflation (see Arthur 2005). With an asset inflation factor of three with base year 1985 as in Arthur (2005) our data for wealth in 2003 are about what Blanchard et al. take from the FED statistics.

[30] OLS is known to be biased in this case of a lagged dependent variable. But the forecast from a Two-stage least squares regression is much worse according to all standard criteria.

The lower part of Table 7 is constructed to obtain a forecast of the debt/GDP ratio. To that end we need a forecast for US wealth, W, world wealth W^*, and the GDP of the USA. We regress the log of W, W^* and GDP respectively on their own two lags (only one for the GDP) and the square of their first lag. These results are summarized in Table 8 together with those of the forecast properties. The mean squared error (RMSE is its root) of the forecast has a small bias and variance proportion and most of the error consists of the unexplained part. As a matter of catching up, relative US wealth (not shown) falls by an average rate of -1.4%, which is higher in the beginning and slowing down later as is typical of catching up results. Table 7 then shows that we arrive at debt/GDP ratios between 50 and 75% in 2050. The last regression with the highest likelihood generates a debt/GDP ratio of 57%, which is about the same as that of the previous sections of the paper. A further detailed look would show that only around 2100 would the coefficients leave the zero-one range that is required by Kouri's model. In particular, the US home bias is falling towards zero, whereas the home bias of ROW is increasing towards unity, as the USA is getting small relative to the ROW, which follows from the estimates of the dynamic coefficients in Table 7.[31]

What will the long-term real rate of interest be? In the last four lines of Table 7 we have calculated the value of the marginal product of capital from the growth model above, $\beta(Y/K)^{(1-\rho)}$, that would be obtained if we interpret the parts of the assets from (8) as capital, K. These assets contain also money, say M1, and those parts of government bonds that are not covered by a public capital stock. This latter part seems to be large for developing countries, because the unweighted panel average of the bonds[32]/GDP ratio is above five. As the capital output ratio is usually said to be in the order of magnitude of three to four, much of government debt seems to be not covered by public capital. As it is hard to find a percentage value though, we postpone taking government debt into account here. M1 for the USA is about 16% of the GDP and 17% for

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[31] These results can be read as answers to Cordon (2006, p.14).
[32] We consider 'Claims on governments and other public entities (IFS line 32an + 32b + 32bx + 32c)' from the WDI. Central government debt is available only for a much smaller sample and therefore cannot be used here.

our panel average of countries (not shown). Dividing the time-series of the M1/GDP ratio by the

wealth/GDP ratio of the USA, results in a M1/wealth ratio of about 16% for the recent years. As a

first rough approximation we therefore assume that 84% of US assets as calculated from the

estimated model are capital, K, thus ignoring the public debt part that is not covered by public

capital. Next, we assume that a similar number may be true in the long run, say, 2050, for the rest

of the world. For MPK1-3 in Table 7 we use the values for the parameters of the CES function as

estimated according to Table 6. The result is a preliminary marginal product of capital between

zero and two percent. This value is admittedly too low because the share of assets that is assumed

to consist of physical capital is too high when bonds uncovered by public debt as a share of assets

are ignored. Neither do we know how large that is currently nor do we have any indication how

large it will be in 2050 for which we have presented the calculation. Lowering the share of assets

that consists of physical capital would yields a higher marginal product of capital that can more

plausibly equal the sum of depreciation and rate. What should have become clear though is that

developing countries' catching up in terms of GDP and wealth, when bringing a share 1- α^*_t into

the USA, create a demand for US assets or a supply of capital that decreases real interest rates in

accordance with the idea of Bernanke. Foreign wealth invested in US assets is partly an

investment in physical capital. It is a savings glut if foreign wealth grows more quickly than its

investment share in the USA, falls, which is the case according to our estimates and calculation of

the Kouri model. When the non-money share of the predicted value of US assets is inserted into

the marginal product of capital, it determines the rate of interest that was assumed to be given in

the growth cum debt model of perfect capital movements. And it is fairly low for 2050 probably

even if corrected for the difference between public debt and public capital.[33] Independent of the

issue of the exact number of the marginal product of capital, the Kouri model yields the same

prediction of the debt/GDP ratio as the growth cum debt model of about 57% for 2050 and it can

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[33] MPK1 and MPK2 get to 5% if the .6 of all assets is considered to be physical capital. MPK3 requires .35 to get there.

serve conceptually as the interest determination part of the growth model with perfect capital movements.

7. Summary and Conclusion

In this paper we have analyzed the dynamics of US foreign debt from the perspective of the central differential equation in the debt/GDP ratio of the neoclassical growth model with perfect capital movements and a CES production function. Our preliminary test shows that US debt does not drive up the interest rate and therefore the assumption of perfect capital movements may seem to be justified.

The US debt/GDP ratio is increasing. Our concept reveals that it is of utmost importance whether this happens at an increasing or a decreasing growth rate, because this is the difference between the stable and the unstable cases of the model.

We construct debt data from the accumulated differences of investment and savings, the problem that is likely to continue, and show that the difference between our data and the NIIP data can be largely explained by exchange rate movements, which are hard to predict.

We find that the long-run debt dynamics can be characterized by a decrease in the growth rate of the debt/GDP ratio, which leads to a stable value of the latter in the long run in the range of 50% and 60%, depending on the details of the methods. Using the simple OLS regression for a forecast, the debt/GDP ratio will reach a value of 50% under the linear specification at about 2040 and then approach its steady state value of about 58%. A savings ratio, which is lower in the second half of the period under consideration, seems to be the major 'culprit'.

In the short run, however, there are phases where the long-run path is left and debt becomes explosive for short periods. The unstable phases happen to occur when economic policy drives up interest rates. These phases are likely to be followed by devaluations of the dollar. The United States will have to make sure that they have low interest rates and exchange rates and a high growth rate. Increasing government deficits are not a contribution to this aim.

Given the great variability of the slope coefficients for the differential equation in the debt/GDP ratio we have also estimated the differential equation using methods of time-varying coefficients. The law of motion for the time-varying parameters shows that there is no instability and the intercept and the slope converge to their long-run average values with a debt/GDP ratio of about 56%, which is very similar to the value obtained with constant coefficients.

The trade balance is predicted to go towards zero in the long run if the GDP growth rate approximately equals the interest rate. Going from 5% to 1.7 percent until 2025 requires real devaluations of 20-35%, which is between 1 and 1.5% per year.

The analysis with time-varying parameters would suggest that there is no problem in the debt dynamics. However, this may be due to the fact that the method used takes the past strongly into account. If there is really a structural break the warning from the rolling window method may be more informative because it drops all information that is backward more than, in our case, 7 years.

We have estimated the neoclassical growth model with perfect capital movements - a CES production function and a marginal productivity condition for capital jointly with the debt dynamics - and find the expected signs of the coefficients at least at the five percent significance level. For the USA in the time period in question the model seems to be reasonably good for the question treated. Results for the level and growth rate of labor-augmenting technical change are 350 and 1.65% respectively.

Finally, it might be useful to think of possible reasons for overthrowing our prediction. According to our model differences between investment and savings drive the debt process, and this in turn would determine the trade balance. If the debt accumulation though would generate expectations of devaluations, the expected valuation effects might lead to a withdrawal of foreign capital. This type of self-fulfilling prophecy is not taken into account in our model because in general it can go also the other way around. However, it may not last very long and the economy may return to the predicted path. A more critical question therefore might be what the future

values of savings ratios and interest rates will be. This seems to be most difficult to forecast and is the subject of several areas of research. The short-run analysis using rolling windows and the estimation with methods of time-varying parameters are made possible by the simplicity of the model. More complex models with more complex stability conditions would not work here because of the low number of observations. The simplicity comes at some costs. Our model keeps the savings ratio exogenous and does not take into account its determinants such as population age structure and budget deficits. Repercussions from the balance of payments on investments and savings such as changes coming via nominal exchange rate movements and terms of trade and are assumed to be absent. Forces explaining the investment-savings difference and the current account via the capital account such as asymmetries in asset preferences and several types of risk are suppressed. All of these effects need to have an impact on the investment-savings difference though. In short, we keep the model simple enough to avoid that it works as a straightjacket but rather allows us to do what we want to do. Therefore the simplifications are not meant to deny any of the above mentioned arguments. We recognize that the trend in the debt/GDP ratio is present since 1960 as our data in the appendix show or since 1930 according to Engel and Rogers (2006), with particularly strong growth in some short phases. Caballero et al. (2006) focus on the period since 1990. Obstfeld and Rogoff (2006) draw a similar future scenario for the debt/GDP ratio as we do in one of their figures but do not apply it to the years since 1960 or 1930, but rather it follows from an assumption of 6% growth in the GDP and a 6% current account deficit. Blanchard et al. (2005) focus on Kouri's portfolio theory of shifts in US preferences towards foreign assets and goods. An interesting twist to their theory is Bernanke's idea that there are more and more nations coming up relative to the USA, whose increasing wealth and number makes the USA more and more indebted.

We explore this idea by estimating one part of the Kouri model using a time-varying coefficient method. We show that the investment shares of the USA and the rest of the world depend on relative wealth, a non-linear negative time trend representing globalization and the standard

deviation of the relative interest rate of the USA. Together with a forecast for the GDP, and the wealth of the USA and ROW the debt/GDP ratio is again predicted to be about 57%. The implied prediction for US assets and US capital stocks show that real interest rates are likely to be very low in the future similar to what they were in the 1960s and 1970s, because of the catching up of many countries in the world in terms of wealth accumulation. This is a long-run version of Bernanke's savings glut idea.

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Appendix A: Solution of the differential equation

The differential equation has the solution

$$D(t) = D(0)e^{srt} + K(0)B(e^{gt} - e^{srt}) \tag{A1}$$

with $B \equiv (g + \delta s - s((r+\delta)/\beta)^{1/(1-\rho)})/(g-sr)$. In the long run the larger of the two growth rates will dominate the process:

$$\lim D(t)_{t\to\infty} =$$

$$e^{gt} K(0)B = K(t)B \quad if \quad g - sr > 0, or \quad e^{srt}[D(0) - K(0)B] \quad if \quad g - sr < 0 \tag{A2}$$

As $s[(r+\delta)/\beta]^{1/(1-\rho)} > sr$ for $\rho<0$ as assumed here, we can distinguish the four cases of Figure 1. An unstable case with exploding debt/GDP ratio would require $sr > g$ and $D(0)-K(0)B >0$. For δ and ρ both close enough or equal to zero we would have $B >1$ and therefore $D(0) >> K(0)$ would be required. This case is fairly unrealistic because it implies that the country has negative current wealth from the beginning and through eternal times while we see from the data that all countries have positive savings at almost all times with only very short periods of exceptions.

The more general case is more complicated. Combining g-sr<0 and B<1 in order to allow for $D(0)<K(0)$ requires $\rho>1$, which means that substitution goes the wrong way, or the elasticity of substitution is larger than unity. This can be shown as follows:

$B \equiv (g + \delta s - s((r+\delta)/\beta)^{1/(1-\rho)})/(g-sr)<1$, with g-sr <0 implies

$g + \delta s - s((r+\delta)/\beta)^{1/(1-\rho)}> g-sr$; cancelling and switching terms to the other side yields:

$(r+\delta)s > s((r+\delta)/\beta)^{1/(1-\rho)}$; canceling s and taking natural logs on both sides yields

ln $(r+\delta) > [1/(1-\rho)]$ ln$((r+\delta)/\beta)$

As the case 1-ρ<0 takes substitution the wrong way, we assume 1-ρ>0. Multiplication by (1-ρ) then yields

(1-ρ)ln $(r+\delta) > $ ln$(r+\delta)$-lnβ or -ρln $(r+\delta) > $ -lnβ $\qquad (A.3)$

Case 1: As the log expressions are negative, a negative ρ leads to a negative left-hand side and a positive right-hand side and therefore the larger sign cannot hold and we have a contradiction. Therefore a negative ρ has to be ruled out.
Case 2: Suppose $(r+\delta) < \beta <1$ and therefore ln$(r+\delta)<$ ln$\beta<0$ and -ln$(r+\delta) >$- ln$\beta > 0$, a value of ρ slightly below unity yields a result free of contradiction. But it implies an elasticity of substitution larger than unity.
*Case 3*: If $(r+\delta)> \beta > 0$, ln$(r+\delta) > $ lnβ, -ln$(r+\delta) < $ - lnβ.. The condition A.3 still requires a positive ρ larger than unity to turn the sign of the inequality around into that of A.3, and substitution goes again the wrong way.

Appendix B: Data

United States	dD=I-S	D	GDP	debt/gdp ratio
1960	509020000	-4.24E+11	5.2012E+11	-8.16E-01
1961	-369305000	-4.25E+11	5.3811E+11	-7.89E-01
1962	421020000	-4.24E+11	5.7887E+11	-7.33E-01
1963	-663410000	-4.25E+11	6.1068E+11	-6.96E-01
1964	-2883380000	-4.28E+11	6.5632E+11	-6.52E-01
1965	-1067600000	-4.29E+11	7.1243E+11	-6.02E-01
1966	1718050000	-4.27E+11	7.8114E+11	-5.47E-01
1967	2337571000	-4.25E+11	8.2546E+11	-5.15E-01
1968	4862434673	-4.20E+11	9.019E+11	-4.66E-01
1969	5161860000	-4.15E+11	9.7386E+11	-4.26E-01
1970	3300000000	-4.12E+11	1.0255E+12	-4.01E-01
1971	11000100000	-4.01E+11	1.114E+12	-3.60E-01
1972	12600000000	-3.88E+11	1.2247E+12	-3.17E-01
1973	-600000000	-3.89E+11	1.3693E+12	-2.84E-01
1974	3100000000	-3.85E+11	1.4844E+12	-2.60E-01
1975	-3600100000	-3.89E+11	1.6178E+12	-2.40E-01
1976	15900100000	-3.73E+11	1.8053E+12	-2.07E-01
1977	30799900000	-3.42E+11	2.0117E+12	-1.70E-01
1978	31100000000	-3.11E+11	2.2749E+12	-1.37E-01
1979	33900000000	-2.77E+11	2.5444E+12	-1.09E-01
1980	22799800000	-2.55E+11	2.7715E+12	-9.19E-02
1981	21200200000	-2.33E+11	3.1045E+12	-7.52E-02
1982	3200000000	-2.30E+11	3.2286E+12	-7.13E-02
1983	79100000000	-1.51E+11	3.502E+12	-4.31E-02
1984	1.056E+11	-4.55E+10	3.8966E+12	-1.17E-02
1985	1.228E+11	7.73E+10	4.1749E+12	1.85E-02
1986	1.847E+11	2.62E+11	4.4118E+12	5.94E-02
1987	1.554E+11	4.17E+11	4.6989E+12	8.88E-02
1988	70900000000	4.88E+11	5.0619E+12	9.65E-02
1989	1.03E+11	5.91E+11	5.4397E+12	1.09E-01
1990	99700200000	6.91E+11	5.7508E+12	1.20E-01
1991	4800300000	6.96E+11	5.9307E+12	1.17E-01
1992	82101000000	7.78E+11	6.2618E+12	1.24E-01
1993	1.368E+11	9.15E+11	6.5829E+12	1.39E-01
1994	1.666E+11	1.08E+12	6.9933E+12	1.55E-01
1995	1.245E+11	1.21E+12	7.3384E+12	1.64E-01
1996	1.438E+11	1.35E+12	7.5511E+12	1.74E-01
1997	1.528E+11	1.50E+12	8.2565E+12	1.82E-01
1998	1.689E+11	1.67E+12	8.7202E+12	1.92E-01
1999	2.373E+11	1.91E+12	9.2128E+12	2.07E-01
2000	2.671E+11	2.18E+12	9.7621E+12	2.23E-01
2001	2.595E+11	2.44E+12	1.002E+13	2.43E-01
2002	3.722E+11	2.81E+12	1.0383E+13	2.70E-01

Source: own calculations from data in IMF IFS Yearbook

Figure 1 **Three different regimes for debt dynamics**

$\dot{d}$

SD

SC

I

III

II

U

d
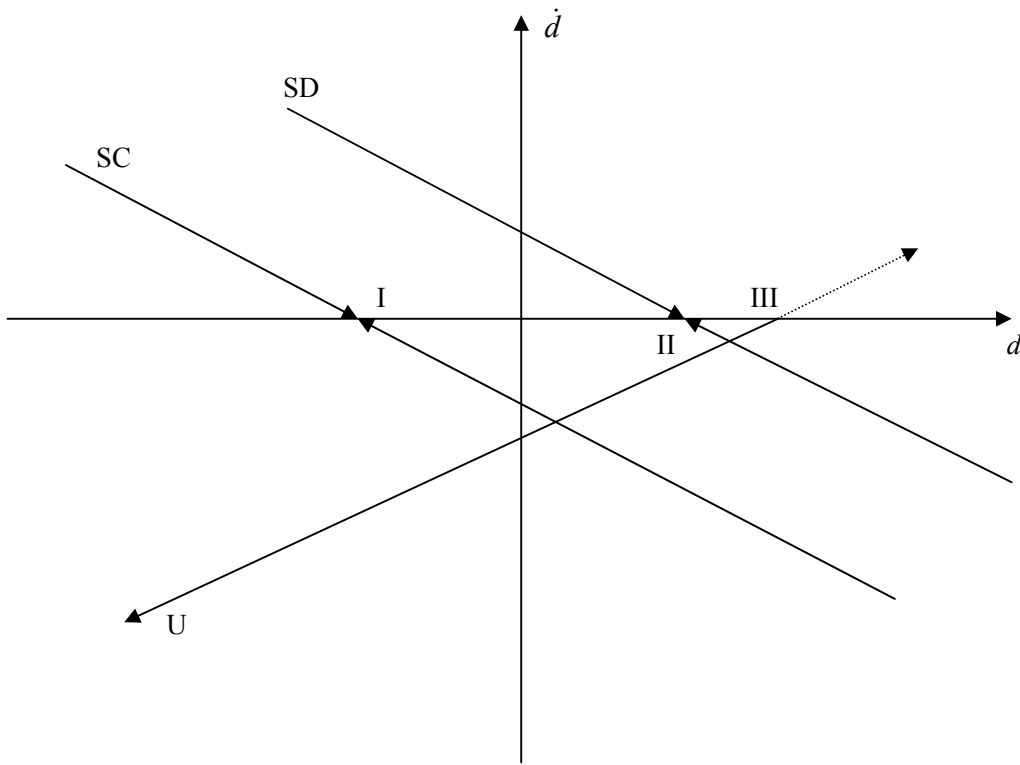
Figure 2
(a) Debt/GDP ratios
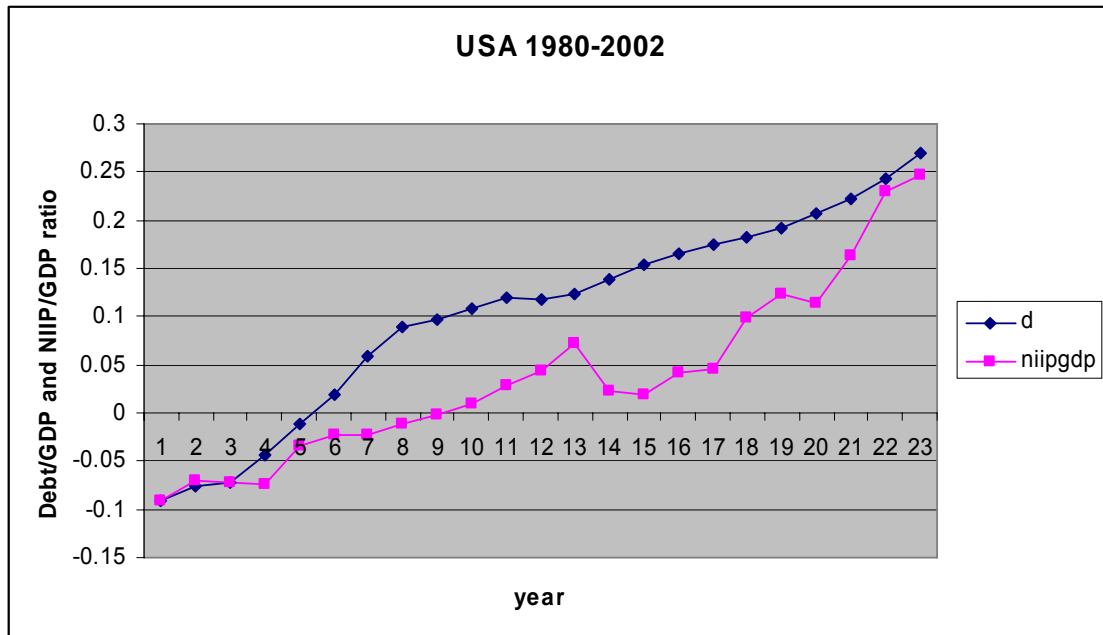
(b) Official exchange rates
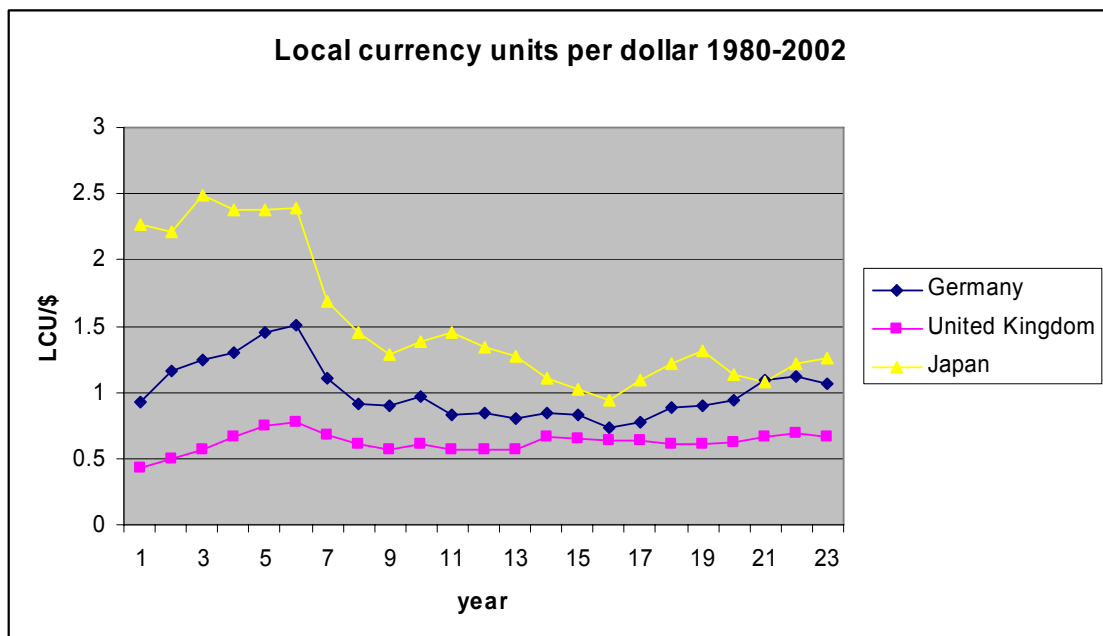
Figure 3
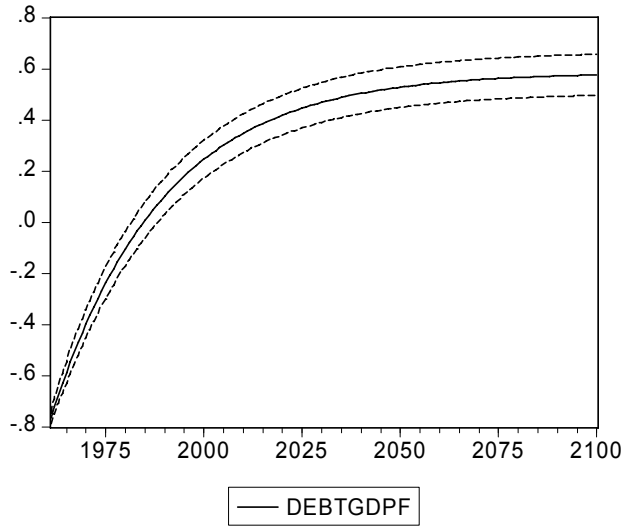
Forecast: DEBTGDPF	
Actual: DEBTGDP	
Forecast sample: 1960 2100	
Adjusted sample: 1961 2100	
Included observations: 42	
Root Mean Squared Error	0.017194
Mean Absolute Error	0.014215
Mean Abs. Percent Error	11.44113
Theil Inequality Coefficient	0.026062
Bias Proportion	0.001374
Variance Proportion	0.003496
Covariance Proportion	0.995130
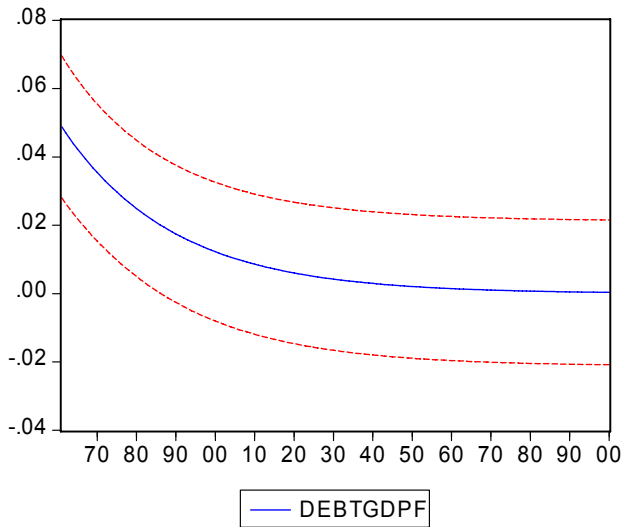
Forecast: DEBTGDPF	
Actual: D(DEBTGDP)	
Forecast sample: 1960 2100	
Adjusted sample: 1961 2100	
Included observations: 42	
Root Mean Squared Error	0.009693
Mean Absolute Error	0.008140
Mean Abs. Percent Error	61.59453
Theil Inequality Coefficient	0.168028
Bias Proportion	0.000006
Variance Proportion	0.138787
Covariance Proportion	0.861207

45

Figure 4
Intercept and slope of 7-years rolling windows of the basic regression

Figure 5

One-step-ahead SV1 State Prediction

One-step-ahead SV2 State Prediction
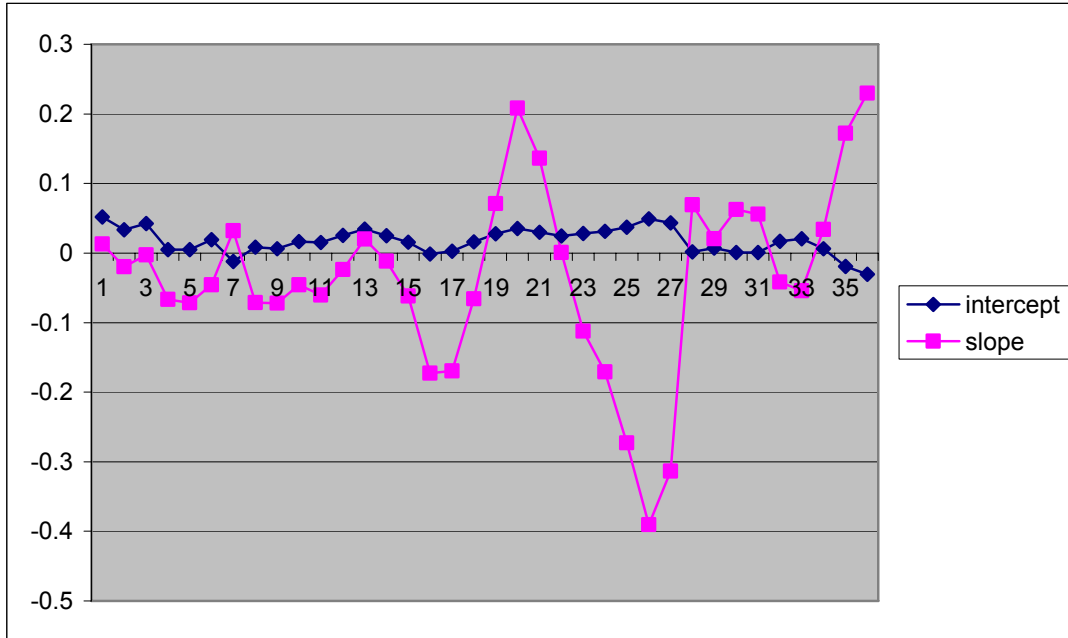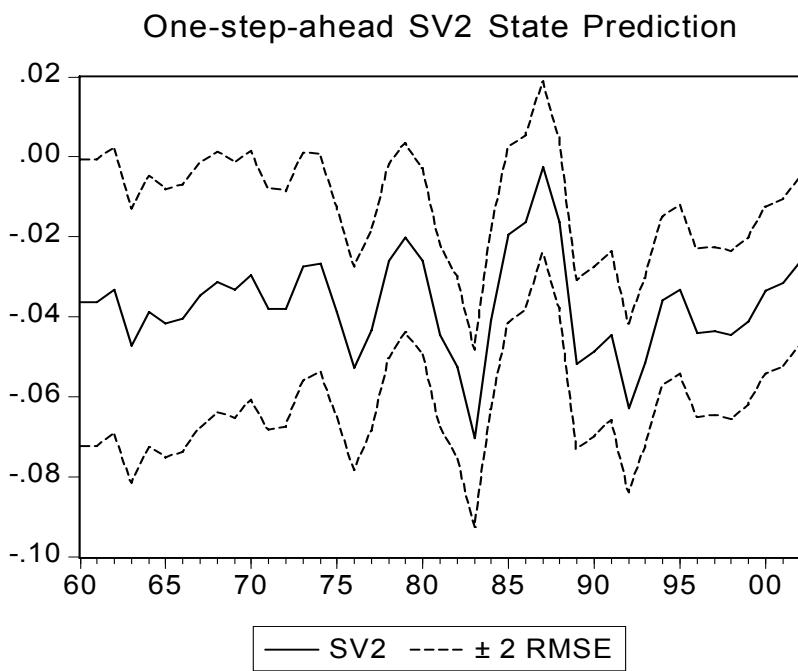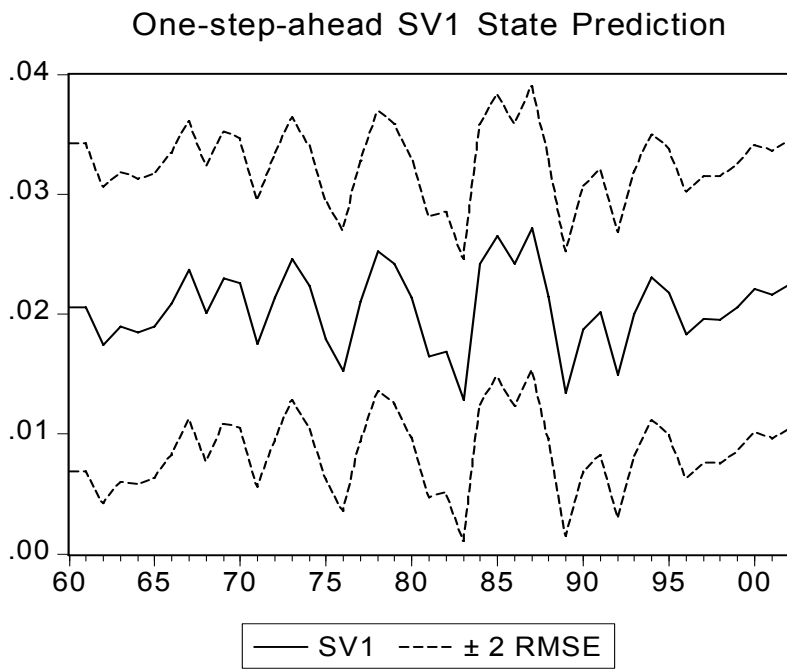
Figure 6: US Data (1960 onwards) for the growth model estimation
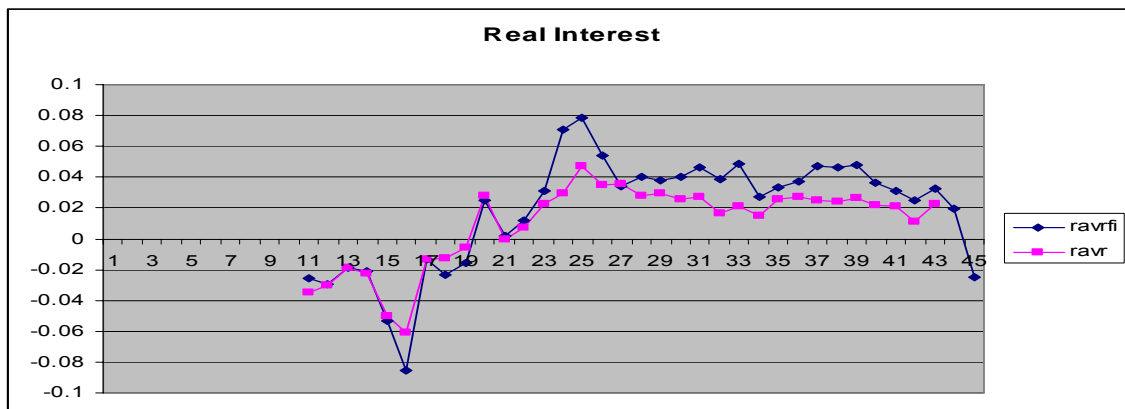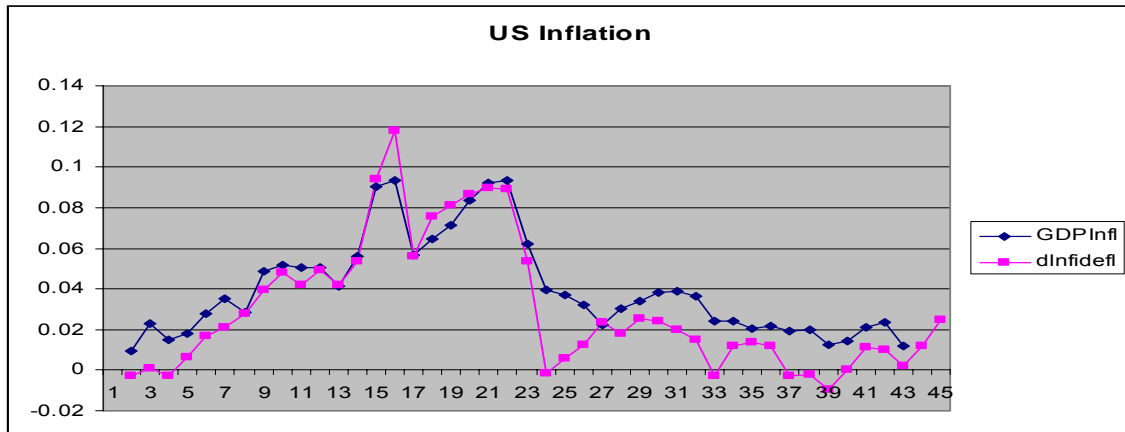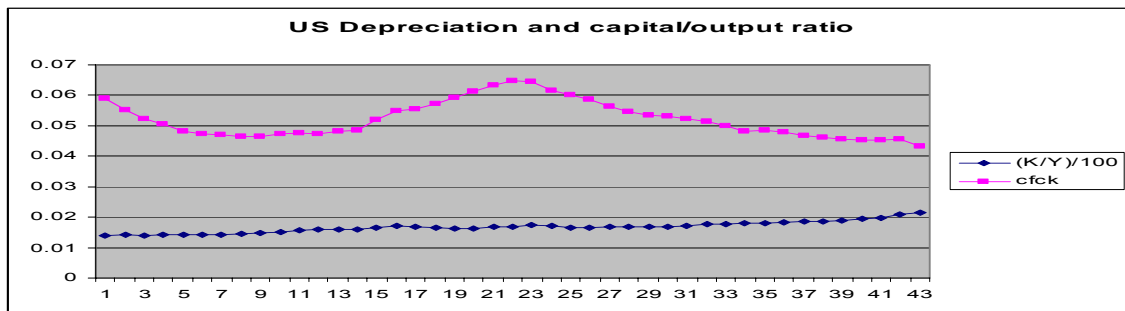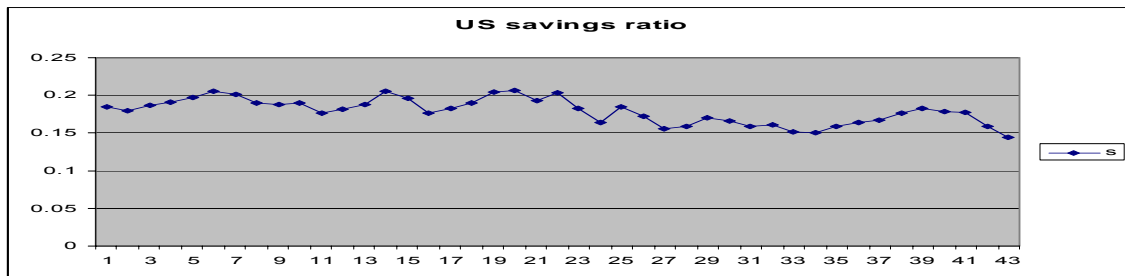
Abbreviations. *dlnfidefl*: growth rate of fixed investment deflator; *ravrfi*: interest rate deflated using fixed investment deflator. *ravr*: interest rate using GDP deflator.

Table 1

Data on Financial US Capital Stocks and Income (Billions of U.S. dollars)

year	Assets	Liabilities	NIIP[a]	Investment income Credit	Investment income Debit	Interest rate Credit[b]	Interest rate Debt[c]
1980	755.41	500.84	254.57	72.67	43.09	0.096	0.086
1981	802.09	583.37	218.72	86.65	54.26	0.108	0.093
1982	961.02	725.08	235.94	86.32	57.14	0.090	0.079
1983	1129.67	872.28	257.39	85.26	54.45	0.075	0.062
1984	1127.13	993.04	134.09	104.86	74.86	0.093	0.075
1985	1302.71	1205.82	96.89	93.73	73.94	0.072	0.061
1986	1594.65	1493.87	100.78	97.06	81.58	0.061	0.055
1987	1758.71	1708.18	50.53	108.18	93.89	0.062	0.055
1988	2008.37	1997.9	10.47	136.71	118.04	0.068	0.059
1989	2350.24	2397.23	-46.99	161.29	141.49	0.069	0.059
1990	2294.09	2458.58	-164.49	171.75	143.19	0.075	0.058
1991	2470.63	2731.45	-260.82	149.24	125.11	0.060	0.046
1992	2466.46	2918.79	-452.33	133.76	109.52	0.054	0.038
1993	3091.4	3235.71	-144.31	136.08	110.74	0.044	0.034
1994	3315.13	3450.39	-135.26	166.52	149.38	0.050	0.043
1995	3964.56	4270.39	-305.83	210.26	189.36	0.053	0.044
1996	4650.84	5010.86	-360.02	226.13	203.81	0.049	0.041
1997	5379.13	6201.86	-822.73	256.82	244.2	0.048	0.039
1998	6174.52	7249.9	-1075.4	261.32	257.55	0.042	0.036
1999	7390.46	8437.12	-1046.7	293.22	280.04	0.040	0.033
2000	7393.64	8982.2	-1588.6	350.45	329.86	0.047	0.037
2001	6898.71	9206.87	-2308.2	286.69	263.12	0.042	0.029
2002	6613.32	9166.73	-2553.4	266.8	259.63	0.040	0.028
2003	7863.97	10515	-2651	294.39	261.1	0.037	0.025

Source: IMF IFSY 2004 (years 2003-1992) and older.

a Difference of columns one and two.
b Column four, inv. Income credit, divided by column one, assets, end of one period earlier.
c Column five, inv. income debit, divided by column two, liabilities.
End of period data.

Table 2

Regression on the time index (t-values in parantheses)

Independent →

Dependent↓variable	intercept	t(ime)	time squared	Adjusted R-squared
log (I-S)	20.7418	0.14043	-	0.77729
	(61.64)	(11.25)	-	
log (I-S)	19.7596	0.25482	-0.0025	0.80342
	(37.99)	(5.15)	-(2.38)	
logGDP	26.9038	0.07671	-	0.98823
	(824.49)	(59.38)	-	
logGDP	26.7118	0.10231	-0.0006	0.9955
	(835.95)	(30.54)	-(7.88)	

Table 3

Regressions for the Debt/GDP ratio, 1961-2002
(dependent variable: d(debt/GDP), t-values in parantheses)

Regression	intercept	debt/ GDP(-1)	adj.R^2	BG prob(lags).	SIC
LS-NW-HAC	0.020 (9.32)	-0.035 -(6.31)	0.557	.12(1)	-6.29
LS-NW-HAC[a]	0.020 (8.98)	-0.037 -(6.84)	0.694	.33(1)	-6.51
unit root regression [b]	0.021 (3.74)	-0.041 -(3.33)	0.634	0.080386(1)	-6.29

a,b: Additional variables

Variable	Coefficient	Std. Error	t-Statistic	Prob.
a: AR(1)	-0.49	0.11	-4.44	0.00
a: MA(1)	0.94	0.04	22.59	0.00
b: D(DEBTGDP(-1))	0.41	0.17	2.43	0.02
b: D(DEBTGDP(-2))	-0.18	0.17	-1.01	0.32
b: D(DEBTGDP(-3))	0.05	0.17	0.32	0.75
b: D(DEBTGDP(-4))	-0.28	0.14	-1.93	0.06

LS: least squares; ; NW: Newey-West; HAC: heteroscedasticity and autocorrelation correction.

Table 4: Rolling windows for seven years

Years	intercept	slope	p-val. interc.	p-value slope	adj.R-squared
1961 1967	0.05	0.01	0.21	0.82	-0.19
1962 1968	0.03	-0.02	0.25	0.64	-0.14
1963 1969	0.04	0.00	0.12	0.95	-0.20
1964 1970	0.01	-0.07	0.83	0.14	0.25
1965 1971	0.00	-0.07	0.83	0.14	0.26
1966 1972	0.02	-0.05	0.45	0.39	-0.02
1967 1973	-0.01	0.03	0.15	0.79	-0.18
1968 1974	0.01	-0.07	0.63	0.14	0.26
1969 1975	0.01	-0.07	0.71	0.16	0.22
1970 1976	0.02	-0.05	0.38	0.41	-0.03
1971 1977	0.02	-0.06	0.35	0.27	0.09
1972 1978	0.03	-0.02	0.13	0.67	-0.15
1973 1979	0.03	0.02	0.02	0.65	-0.15
1974 1980	0.03	-0.01	0.06	0.83	-0.19
1975 1981	0.02	-0.06	0.15	0.28	0.07
1976 1982	0.00	-0.17	0.84	0.01	0.75
1977 1983	0.00	-0.17	0.78	0.05	0.48
1978 1984	0.02	-0.07	0.21	0.56	-0.11
1979 1985	0.03	0.07	0.03	0.52	-0.10
1980 1986	0.04	0.21	0.00	0.04	0.52
1981 1987	0.03	0.14	0.00	0.13	0.28
1982 1988	0.02	0.00	0.01	0.99	-0.20
1983 1989	0.03	-0.11	0.00	0.13	0.28
1984 1990	0.03	-0.17	0.00	0.04	0.54
1985 1991	0.04	-0.27	0.00	0.01	0.71
1986 1992	0.05	-0.39	0.00	0.00	0.89
1987 1993	0.04	-0.31	0.03	0.06	0.43
1988 1994	0.00	0.07	0.94	0.69	-0.16
1989 1995	0.01	0.02	0.72	0.90	-0.20
1990 1996	0.00	0.06	0.96	0.66	-0.15
1991 1997	0.00	0.06	0.96	0.66	-0.15
1992 1998	0.02	-0.04	0.12	0.52	-0.09
1993 1999	0.02	-0.05	0.08	0.39	-0.02
1994 2000	0.01	0.03	0.62	0.63	-0.14
1995 2001	-0.02	0.17	0.02	0.00	0.83
1996 2002	-0.03	0.23	0.00	0.00	0.92

Table 5   **Maximum likelihood estimation results**
              of the state-space model (p-values in parantheses)

	Regr.1 Coeff.(p-val.)	Regr.2 Coeff.(p-val.)	Regr.3 Coeff.(p-val.)
C(2)	0.021 (0.00)	0.021 (0.00)	0.021 (0.00)
C(3)	-0.036 (0.00)	-0.035 (0.00)	-0.036 (0.00)
C(6)	-10.64 (0.00)	-10.39 (0.00)	-10.60 (0.00)
C(8)	0.97 (0.08)	0.52 (0.01)	0.82 (0.00)
C(9)	-0.21 (0.37)	-	-0.16 (0.19)
C(10)	3.11 (0.10)	2.11 (0.10)	2.52 (0.08)
C(11)	-0.41 (0.59)	-	-
	Final State	Final Stat	Final State
SV1	0.021 (0.00)	0.025 (0.00)	0.024 (0.00)
SV2	-0.036 (0.05)	-0.020 (0.04)	-0.017 (0.10)
Log likelihood	141.0	139.7	140.939
Parameters	7	5	6
Diffuse priors	0	0	0
AIC	-6.38	-6.42	-6.42
SIC	-6.09	-6.21	-6.21
HQC	-6.28	-6.34	-6.34
Val.Obs.	42	42	42

AIC: Akaike info criterion; SIC: Schwarz criterion; HQC: Hannan-Quinn criter.

Table 6: GMM-HAC Regression results for the non-linear growth model

	Coeff. (t-val.)	Coeff. (t-val.)	Coeff. (t-val.)
C(1)	0.08	0.08	0.00
	(1.69)	(4.80)	-(0.09)
C(2)	0.05	0.06	0.05
	(1.78)	(5.01)	(5.39)
c(3)	=c(2)	=c(2)	0.10
			(7.10)
β	0.34	0.38	0.11
	(3.88)	(11.03)	(11.94)
ρ	-1.85	-2.06	-5.42
	-(3.82)	-(12.86)	-(19.76)
A-level	321311.70	318821.70	351159.80
	(29.71)	(60.43)	(153.26)
A growth rate	0.0152	0.0150	0.0165
	(37.92)	(83.50)	(73.31)
lag. Depend.	0/0/0	0/0/0	2/3/0
No.instr (a)	0/0/0	6/0/3	4/2/4
J-statistic	0.33	0.38	0.36
Adjusted R-squared	0.00	0.00	0.61
	0.70	0.71	0.96
	1.00	1.00	0.99
Durbin-Watson stat	0.59	0.60	1.84
	0.66	0.60	1.27
	0.59	0.58	0.41
Obs.	19/19/20	19/19/20	19/19/20

(a) Instruments in addition to the regressors: number of lagged endogenous variables.

Table 7 Time-varying coefficients for US debt

Regression	(1)	(2)	(3)	(4)	(5)
intercept	0.74	0.79	0.02	-1.42	-0.74
	(18.68)	(12.31)	(0.30)	-(20.00)	-(5.71)
log(w/w*)	0.24	0.25	0.25	0.26	0.28
	(3.49)	(2.64)	(2.44)	(3.87)	(2.81)
time^(-2)	94.84	96.04	96.08	97.85	97.58
	(10.69)	(12.12)	(11.88)	(9.50)	(13.49)
Rfse	0.00	0.00	35.02	94.61	64.76
			(17.49)	(26.36)	(7.81)
intercept	0.62	0.64	0.30	-0.39	-0.08
	(12.39)	(8.96)	(5.86)	-(7.31)	-(1.08)
log(w/w*)	-0.10	-0.09	-0.08	-0.10	-0.11
	-(5.07)	-(3.44)	-(4.37)	-(5.46)	-(7.88)
time^(-2)	94.44	97.18	98.25	99.03	98.73
	(13.78)	(13.09)	(12.39)	(17.56)	(13.70)
Rfse (a)	0.00	0.00	15.43	42.88	29.34
			(25.33)	(70.24)	(8.41)
log likel.	-923.23	-922.97	-918.51	-916.04	-915.84
log(w/w*) (2050)	-2.56	-2.56	-2.56	-2.56	-2.56
α-1	-0.86	-0.84	-0.80	-0.89	-0.94
1-α*	0.12	0.12	0.12	0.11	0.12
W(2050) in bill.$	52439.98	52439.98	52439.98	52439.98	52439.98
W*(2050)	675851.22	675851.22	675851.22	675851.22	675851.22
US GDP (2050)	55775.82	55775.82	55775.82	55775.82	55775.82
debt/GDP(2050)	0.63	0.71	0.73	0.52	0.57
MPK1	0.0206	0.0199	0.0208	0.0215	0.0189
MPK2	0.0187	0.0180	0.0190	0.0196	0.0171
MPK3	0.0002	0.0002	0.0002	0.0002	0.0002

z-statistics in paranthesis

Table 8 Auto-regressive estimates of Wealth and GDP

dep.: logW	coeff./t-val.	dep.: logW*	coeff./t-val.	dep.: logGDP	coeff./t-val.
C	-23.597	C	-9.838	C	-21.068
	-(4.82)		-(2.87)		-(5.93)
@TREND	0.012	@TREND	0.014	@TREND	0.009
	(6.37)		(3.40)		(3.85)
LOG(W(-1))	3.616	LOG(W*(-1))	2.548	LOG(GDP(-1))	2.595
	(10.74)		(8.18)		(9.66)
LOG(W(-1))^2	-0.036	LOG(W*(-1))^2	-0.016	LOG(GDP(-1))^2	-0.030
	-(5.62)		-(3.16)		-(5.89)
LOG(W(-2))	-0.781	LOG(W*(-2))	-0.753		
	-(4.90)		-(6.74)		
Adj.R-sq.	0.9996		0.9999		0.9997
DW stat	2.0588		1.6392		1.8264
Forecast					
RMSE	1.98E+11		7.85E+11		9.44E+10
Mabs.Err.	1.38E+11		5.29E+11		7.55E+10
MAbs.Perc.Err.	2.14		2.18		2.17
Theil Ineq.Coeff.	0.02		0.02		0.01
Bias Prop.	0.01		0.00		0.02
Var. Prop.	0.05		0.04		0.07
Covar.Prop.	0.93		0.96		0.92

t-values in paranthese
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

Appendix C (NOT FOR PUBLICATION)

Sspace: UNTITLED
Method: Maximum likelihood (Marquardt)
Date: 12/09/04   Time: 14:12
Included observations: 141
Valid observations: 42
Estimation settings: tol= 0.00010, derivs=accurate numeric
Initial Values: C(2)=0.02101, C(3)=-0.03671, C(6)=-9.80720,
     C(8)=0.60117, C(9)=-0.13181, C(10)=2.37148, C(11)=-0.26260
Convergence achieved after 31 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(2)	0.020772	0.002404	8.639336	0.0000
C(3)	-0.036316	0.005967	-6.086191	0.0000
C(6)	-10.64260	0.398508	-26.70615	0.0000
C(8)	0.971941	0.559465	1.737269	0.0823
C(9)	-0.206401	0.227941	-0.905503	0.3652
C(10)	3.111039	1.892069	1.644253	0.1001
C(11)	-0.407724	0.747180	-0.545685	0.5853
	Final State	Root MSE	z-Statistic	Prob.
SV1	0.020772	0.007103	2.924427	0.0035
SV2	-0.036316	0.018532	-1.959583	0.0500

Log likelihood	141.0412	Akaike info criterion	-6.382914
Parameters	7	Schwarz criterion	-6.093303
Diffuse priors	0	Hannan-Quinn criter.	-6.276760

Wald Test:
SSpace: SS01

Null Hypothesis:	C(9)=0		
	C(11)=0		
Chi-square	0.920880	Probability	0.631006

$c(6)$ indicates that the variance is $\exp[c(5)] = \exp\{-10.64260\}= -2.38769E{-}05$, which is very close to zero. $c(8) = .97$ and $c(9)= -.2$ means that a positive (negative) deviation of the intercept sv1 from its mean $c(2)$ in period t enhances or diminishes sv1 by .97 times that deviation and a positive (negative) deviation of the of the slope coefficient sv2 from its means reduces (enhances) the intercept by -.2 that deviation. A similar interpretation holds for $c(10)$ and $c(11)$ in regard to the equation for the slope coefficient. Note that here the coefficients are negative. The trace of the corresponding differential equation system is, $c(8){-}1{+}c(11){-}1 = .97{-}1{+}({-}0.41{-}1)={-}1.44$, and the determinant $= (.97{-}1)x({-}0.41{-}1){-}[3.11x{-}0.2]= .66$. With a negative trace and a positive determinant the system is stable. As four times the determinant is larger than the squared trace (see Chiang 1984, p.642/643) we have a stable focus. The stationary loci of the above system then are (writing *a* for sv1 and *b* for sv2, both diminished by there means $c(2)$, $c(3)$ respectively):

[c(8)-1]a +c(9)b =0, implies $a = bxc(9)/[1-c(8)] = b(-.2/.03)$

$c(10)\ a + [c(11){-}1]b{=}0$, implies $a = [1-c(11)]b/c(10) = b(-.59/3.11)$.

The eigenvalues of the differential equation system are -.72+.38197i,-.72-.38197i.

When $c(9)$ and $c(11)$ are dropped because of the F-test result given above we get:
Sspace: SS02
Method: Maximum likelihood (Marquardt)
Date: 12/14/04   Time: 16:25
Included observations: 43
Valid observas: 42
Estimation settings: tol= 0.00010, derivs=accurate numeric
Initial Values: C(2)=0.02077, C(3)=-0.03631, C(6)=-10.6426,

C(8)=0.97195, C(10)=3.11074
Convergence achiions

	Coefficient	Std. Error	z-Statistic	Prob.
C(2)	0.021054	0.002525	8.339595	0.0000
C(3)	-0.035412	0.006705	-5.281660	0.0000
C(6)	-10.38912	0.301912	-34.41110	0.0000
C(8)	0.520494	0.213758	2.434971	0.0149
C(10)	2.114453	1.299043	1.627700	0.1036
	Final State	Root MSE	z-Statistic	Prob.
SV1	0.024786	0.005921	4.186388	0.0000
SV2	-0.020252	0.010076	-2.009843	0.0444
Log likelihood	139.7476	Akaike info criterion		-6.416554
Parameters	5	Schwarz criterion		-6.209689
Diffuse priors	0	Hannan-Quinn criter.		-6.340730

The trace of the corresponding differential equation system is, c(8)-1+c(11)-1 = .52-1+(0-1)= -1.48, and the determinant = (.52-1)x(0-1)-[2.11x0]= .48 . With a negative trace and a positive determinant the system is stable. The first equation then is sv1=c(2)+.52(sv1(-1)-c(2)). This a stable difference equation. The second equation is sv2 = c(3) +2.11(sv1(-1)-c(2)). As sv1 approaches c(2), sv2 goes to c(3).
The stationary loci of the differential equation system then are (writing *a* for sv1 and *b* for sv2, both diminished by there means c(2), c(3) respectively):

da/dt = [c(8)-1]a +c(9)b =0, implies *a = bxc(9)/ [1-c(8)] = 0xb*

db/dt = c(10) a + [c(11)-1]b=0, implies *a = [1-c(11)]b/c(10) = 1b/2.11.*

If *a* is on vertical and *b* on the horizontal axis, then the first locus is just the horizontal axis. The second is positively sloped. The eigenvalues of the differential equation system are -1.0,-.47951. We have a stable node in a-b-space.

The implied absence of cyclical movements is unable to explain the ups and downs in the rolling windows regressions. Therefore it may be better not to drop c(9). If only c(11) is dropped from the first regression the result is as follows:

Sspace: SS02
Method: Maximum likelihood (Marquardt)
Date: 12/16/04   Time: 13:08
Included observations: 43
Valid observns: 42
Estimation settings: tol= 0.00010, derivs=accurate numeric
Initial Values: C(2)=0.02105, C(3)1, C(6)=-10.3891,
      C(8)=0.52050, C(9)=-0.20642, C(10)=2.11428
Convergence achieved after 18 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(2)	0.020596	0.002161	9.529071	0.0000
C(3)	-0.036434	0.005920	-6.154081	0.0000
C(6)	-10.59786	0.335409	-31.59680	0.0000
C(8)	0.824803	0.231715	3.559562	0.0004
C(9)	-0.162270	0.122581	-1.323779	0.1856
C(10)	2.517992	1.458854	1.726006	0.0843
	Final State	Root MSE	z-Statistic	Prob.
SV1	0.024121	0.006025	4.003712	0.0001
SV2	-0.017109	0.010496	-1.629948	0.1031
Log likelihood	140.9390	Akaike info criterion		-6.425667
Parameters	6	Schwarz criterion		-6.177429
Diffuse priors	0	Hannan-Quinn criter.		-6.334678

The variance is again very close to zero. The trace of the corresponding differential equation system is, c(8)-1+c(11)-1 = .825 -1+(0-1)=-1.175, and the determinant = (.825-1)x(0-1)-[2.52x-0.16]= .578 With a negative trace and a positive determinant the system is stable. As four times the determinant is

larger than the squared trace (see Chiang 1984, p.642/643) we have a stable focus. The stationary loci of the above system then are (writing *a* for sv1 and *b* for sv2, both diminished by there means c(2), c(3) respectively):

[c(8)-1]a +c(9)b =0, implies *a = bxc(9)/ [1-c(8)] = b(-.16/.175)*

c(10) a + [c(11)-1]b=0, implies *a = [1-c(11)]b/c(10) = b(1/2.52).*

The eigenvalues of the differential equation system are -.5876+.48838i, -.5876-. 48838i.

Appendix D (NOT FOR PUBLICATION)

Vector Error Correction Estimates
Date: 04/20/05   Time: 16:30
Sample (adjusted): 1983 2002
Included observations: 20 after adjustments
Standard eors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
RAVR(-1)	1.000000	
DEBTGDP(-1)	0.369567	
	(0.11008)	
	[3.35740]	
@TREND(60)	-0.004121	
	(0.00145)	
	[-2.84379]	
C	0.065933	

Error Correction:	D(RAVR)	D(DEBTGDP)
CointEq1	-0.484849	-0.596727
	(0.15885)	(0.19627)
	[-3.05227]	[-3.04038]
D(RAVR(-1))	-0.401761	-0.026816
	(0.16748)	(0.20694)
	[-2.39881]	[-0.12959]
D(RAVR(-2))	0.116134	0.083118
	(0.19836)	(0.24508)
	[0.58548]	[0.33914]
D(DEBTGDP(-1))	0.118037	0.669306
	(0.14545)	(0.17971)
	[0.81155]	[3.72440]

D(DEBTGDP(-2))	-0.085928	0.110832
	(0.15202)	(0.18783)
	[-0.56524]	[0.59006]
C	-0.000554	0.004586
	(0.00265)	(0.00327)
	[-0.20949]	[1.40274]

R-squared	0.620762	0.702316
Adj. R-squared	0.485320	0.596001
Sum sq. resids	0.000436	0.000666
S.E. equation	0.005581	0.006896
F-statistic	4.583235	6.605957
Log likelihood	78.95400	74.72348
Akaike AIC	-7.295400	-6.872348
Schwarz SC	-6.996680	-6.573629
Mean dependent	1.48E-05	0.017065
S.D. dependent	0.007780	0.010850

Determinant resid covariance (dof adj.)	1.26E-09
Determinant resid covariance	6.19E-10
Log likelihood	155.2753
Akaike information criterion	-14.02753
Schwarz criterion	-13.28073

Vector Error Correction Estimates
Date: 04/20/05   Time: 16:39
Sample (adjusted): 1983 2002
Included observations: 20 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
RAVR(-1)	1.000000
DEBTGDP(-1)	0.060032
	(0.05982)
	[1.00351]
@TREND(60)	-0.000105
	(0.00088)
	[-0.11882]
C	-0.029215

Error Correction:	D(RAVR)	D(DEBTGDP)
CointEq1	-0.874358	0.024760
	(0.19690)	(0.39745)
	[-4.44058]	[0.06230]
C	1.48E-05	0.017065
	(0.00123)	(0.00249)
	[0.01198]	[6.84706]
R-squared	0.522784	0.000216
Adj. R-squared	0.496272	-0.055328
Sum sq. resids	0.000549	0.002236
S.E. equation	0.005522	0.011146
F-statistic	19.71878	0.003881
Log likelihood	76.65593	62.60840
Akaike AIC	-7.465593	-6.060840
Schwarz SC	-7.366020	-5.961267
Mean dependent	1.48E-05	0.017065
S.D. dependent	0.007780	0.010850

Determinant resid covariance (dof adj.)	3.37E-09
Determinant resid covariance	2.73E-09
Log likelihood	140.4206
Akaike information criterion	-13.34206
Schwarz criterion	-12.99355

Dropping the time trend does not change the sign and order of magnitude of the debt term but makes it more significant.

Appendix E (NOT FOR PUBLICATION)

Abbreviations:
cfck: consumption of fixed capital as a share of the capital stock
debtgdp: debt/ GDP ratio
empl: Employment in thousand worker
GDPR: real GDP
KR2: capital stock
lnrplusd: natural log of the sum of real interest rate and depreciation rate
log: natural logarithm
RAVR: real interest rate
s: savings ratio

GMM-HAC Estimate of the model <u>without</u> instruments and autocorrelation correction
System: SYS13
Estimation Method: Generalized Method of Moments
Date: 04/13/05   Time: 17:35
Sample: 1983 2002

Included observations: 20

Total system (unbalanced) observations 58

Estimation settings: tol=0.00010, derivs=analytic

Initial Values: C(1)=0.04220, C(2)=0.07846, C(4)=0.42502, C(5)=
    -2.29362, C(11)=310439., C(12)=0.01525

Kernel: Bartlett, Bandwidth: Fixed (2), No prewhitening

Iterate coefficients after one-step weighting matrix

Convergence achieved after: 1 weight matrix, 8 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.082520	0.048851	1.689214	0.0972
C(2)	0.053447	0.030009	1.781051	0.0807
C(4)	0.343265	0.088386	3.883725	0.0003
C(5)	-1.853490	0.485363	-3.818773	0.0004
C(11)	321311.7	10815.83	29.70755	0.0000
C(12)	0.015154	0.000400	37.92057	0.0000

Determinant residual covariance	7.12E-12
J-statistic	0.333996

Equation: DDEBTGDP +S*(1 - RAVR*DEBTGDP(-1)-CFCK*KR2(-1)
    /GDPR) = C(1)+C(2)*KR2(-1)/GDPR - C(2)*DEBTGDP(-1)

Instruments: C KR2(-1)/GDPR DEBTGDP(-1)

Observations: 19

R-squared	0.057176	Mean dependent var	0.168757
Adjusted R-squared	0.001716	S.D. dependent var	0.014246
S.E. of regression	0.014234	Sum squared resid	0.003444
Durbin-Watson stat	0.592442		

Equation: LOG(KR2(-1)/GDPR(+0)) = (LNRPLUSD - LOG(C(4)))/(-1
    +C(5))

Instruments: C LNRPLUSD

Observations: 19

R-squared	0.720735	Mean dependent var	0.541292
Adjusted R-squared	0.704308	S.D. dependent var	0.057766
S.E. of regression	0.031412	Sum squared resid	0.016774
Durbin-Watson stat	0.663484		

Equation: LOG(GDPR) = (1/C(5))*LOG(C(4)*(KR2(-1)^C(5))+(1-C(4))
    (C(11)(EXP(C(12)*T))*EMPL)^C(5))

Instruments: C KR2(-1) T EMPL

Observations: 20

R-squared	0.996304	Mean dependent var	25.04202
Adjusted R-squared	0.995611	S.D. dependent var	0.187023
S.E. of regression	0.012391	Sum squared resid	0.002456
Durbin-Watson stat	0.592138		

GMM-HAC Estimate of the model with instruments but without autocorrelation correction

System: SYS13
Estimation Method: Generalized Method of Moments
Date: 04/14/05   Time: 12:26
Sample: 1983 2002
Included observations: 20
Total system (unbalanced) observations 58
Estimation settings: tol=0.00010, derivs=analytic
Initial Values: C(1)=0.07746, C(2)=0.05659, C(4)=0.38046, C(5)=
    -2.06240, C(11)=317057., C(12)=0.01516
Kernel: Bartlett, Bandwidth: Fixed (2), No prewhitening
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 3 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.079559	0.016565	4.802917	0.0000
C(2)	0.055240	0.011016	5.014384	0.0000
C(4)	0.379642	0.034433	11.02561	0.0000
C(5)	-2.055264	0.159857	-12.85692	0.0000
C(11)	318821.7	5275.564	60.43368	0.0000
C(12)	0.014975	0.000179	83.49802	0.0000

Determinant residual covariance	8.25E-12
J-statistic	0.384913

Equation: DDEBTGDP +S*(1 - RAVR*DEBTGDP(-1)-CFCK*KR2(-1)
    /GDPR) = C(1)+C(2)*KR2(-1)/GDPR - C(2)*DEBTGDP(-1)
Instruments: C KR2(-1)/GDPR DEBTGDP(-1) KR2(-2)/GDPR(-1)
    DEBTGDP(-2) KR2(-3)/GDPR(-2) DEBTGDP(-3) KR2(-4)/GDPR(
    -3) DEBTGDP(-4)
Observations: 19

R-squared	0.057294	Mean dependent var	0.168757
Adjusted R-squared	0.001840	S.D. dependent var	0.014246
S.E. of regression	0.014233	Sum squared resid	0.003444
Durbin-Watson stat	0.597822		

Equation: LOG(KR2(-1)/GDPR(+0)) = (LNRPLUSD - LOG(C(4)))/(-1
    +C(5))
Instruments: C LNRPLUSD
Observations: 19

R-squared	0.728871	Mean dependent var	0.541292
Adjusted R-squared	0.712923	S.D. dependent var	0.057766
S.E. of regression	0.030951	Sum squared resid	0.016285
Durbin-Watson stat	0.596007		

Equation: LOG(GDPR) = (1/C(5))*LOG(C(4)*(KR2(-1)^C(5))+(1-C(4))
    (C(11)(EXP(C(12)*T))*EMPL)^C(5))
Instruments: C KR2(-1) T EMPL KR2(-2) KR2(-3) KR2(-4)
Observations: 20

R-squared	0.996241	Mean dependent var	25.04202
Adjusted R-squared	0.995536	S.D. dependent var	0.187023
S.E. of regression	0.012496	Sum squared resid	0.002498
Durbin-Watson stat	0.581363		

GMM-HAC Estimate of the model with instruments and autocorrelation correction

System: SYS14
Estimation Method: Generalized Method of Moments
Date: 04/15/05   Time: 14:36
Sample: 1983 2002
Included obsvations: 20
Total system (unbalanced) observations 58
Estimation settings: tol.00010, derivs=analytic
Initial Values: C(1)=0.00281, C(2)=0.04483, C(3)=0.09800,
    C(6)=0.86852, C(7)=-0.27857, C(4)=0.10977, C(5)=-5.47842,
    C(8)=0.81616, C(9)=-0.30459, C(10)=0.38719,)=351595.,
    C(12)=0.01649
Kernel: Bartlett, Bandwidth: Fixed (2), No prewhitening
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 4 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.001514	0.017017	-0.088981	0.9295
C(2)	0.048399	0.008984	5.387269	0.0000
C(3)	0.100570	0.014171	7.097155	0.0000
C(6)	0.855877	0.083868	10.20509	0.0000
C(7)	-0.273396	0.071719	-3.812059	0.0004
C(4)	0.110437	0.009251	11.93839	0.0000
C(5)	-5.415859	0.274054	-19.76203	0.0000
C(8)	0.789403	0.036404	21.68425	0.0000
C(9)	-0.272284	0.054988	-4.951730	0.0000
C(10)	0.378495	0.026645	14.20487	0.0000
C(11)	351159.8	2291.267	153.2601	0.0000
C(12)	0.016529	0.000225	73.30994	0.0000

Determinant residual covariance	5.01E-13	
J-statistic	0.357456	

Equation: DDEBTGDP +S*(1 - RAVR*DEBTGDP(-1)-CFCK*KR2(-1)

/GDPR) = C(1)+C(2)*KR2(-1)/GDPR - C(3)*DEBTGDP(-1) + C(6)*(
    DDEBTGDP(-1) +S(-1)*(1 - RAVR(-1)*DEBTGDP(-2)-CFCK(-1)
    KR2(-2)/GDPR(-1))) + C(7)(DDEBTGDP(-2) +S(-2)*(1 - RAVR(
    -2)*DEBTGDP(-3)-CFCK(-2)*KR2(-3)/GDPR(-2)))

Instruments: C KR2(-1)/GDPR DEBTGDP(-1) (DDEBTGDP(-1) +S(-1)
    *(1 - RAVR(-1)*DEBTGDP(-2)-CFCK(-1)*KR2(-2)/GDPR(-1)))
    (DDEBTGDP(-2) +S(-2)*(1 - RAVR(-2)*DEBTGDP(-3)-CFCK(-2)
    *KR2(-3)/GDPR(-2))) KR2(-2)/GDPR(-1) DEBTGDP(-2)
    (DDEBTGDP(-3) +S(-3)*(1 - RAVR(-3)*DEBTGDP(-4)-CFCK(-3)
    KR2(-4)/GDPR(-3))) (DDEBTGDP(-4) +S(-4)(1 - RAVR(-4)
    *DEBTGDP(-5)-CFCK(-4)*KR2(-5)/GDPR(-4)))

Observations: 19

R-squared	0.697539	Mean dependent var	0.168757
Adjusted R-squared	0.611121	S.D. dependent var	0.014246
S.E. of regression	0.008884	Sum squared resid	0.001105
Durbin-Watson stat	1.837222		

Equation: LOG(KR2(-1)/GDPR(+0)) = (LNRPLUSD - LOG(C(4)))/(-1
    +C(5))+C(8)*LOG(KR2(-2)/GDPR(-1)) +C(9)*LOG(KR2(-3)/GDPR(
    -2)) +C(10)*LOG(KR2(-4)/GDPR(-3))

Instruments: C LNRPLUSD LOG(KR2(-2)/GDPR(-1)) LOG(KR2(-3)
    /GDPR(-2)) LOG(KR2(-4)/GDPR(-3)) LOG(KR2(-5)/GDPR(-4))
    LOG(KR2(-6)/GDPR(-5))

Observations: 19

R-squared	0.972500	Mean dependent var	0.541292
Adjusted R-squared	0.964643	S.D. dependent var	0.057766
S.E. of regression	0.010862	Sum squared resid	0.001652
Durbin-Watson stat	1.272446		

Equation: LOG(GDPR) = (1/C(5))*LOG(C(4)*(KR2(-1)^C(5))+(1-C(4))
   *(C(11)*(EXP(C(12)*T))*EMPL)^C(5))

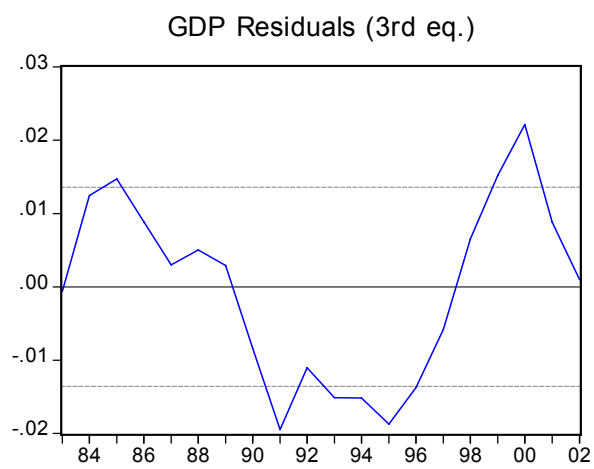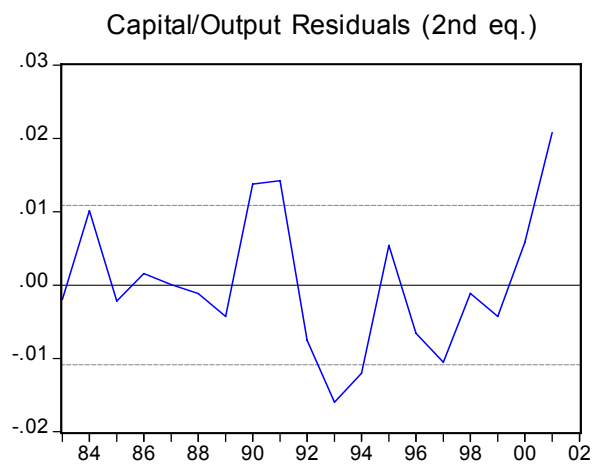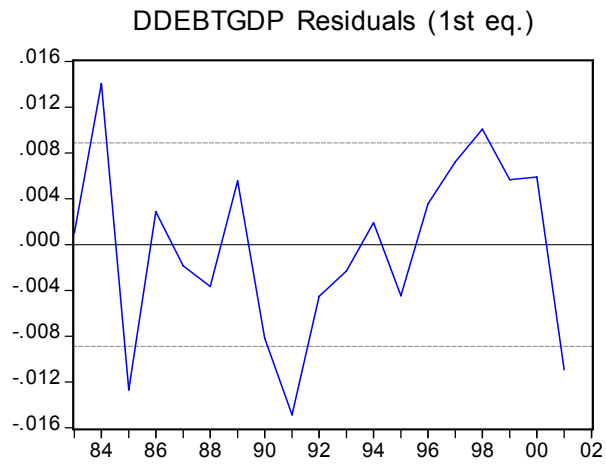Instruments: C KR2(-1) T EMPL KR2(-2) KR2(-3) KR2(-4) KR2(-5)

Observations: 20

R-squared	0.995577	Mean dependent var	25.04202
Adjusted R-squared	0.994748	S.D. dependent var	0.187023
S.E. of regression	0.013554	Sum squared resid	0.002939
Durbin-Watson stat	0.410307		

Appendix F (NOT FOR PUBLICATION):

Residuals of Regressions 2 in Table 6

DDEBTGDP Residuals (1st eq)

Capital/Output Residuals (2nd eq.)

GDP Residuals (3rd eq.)
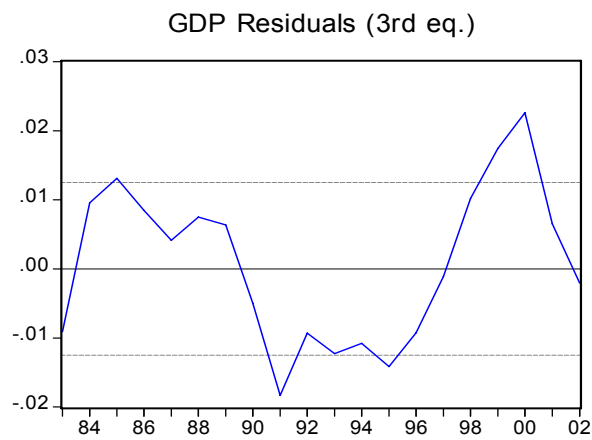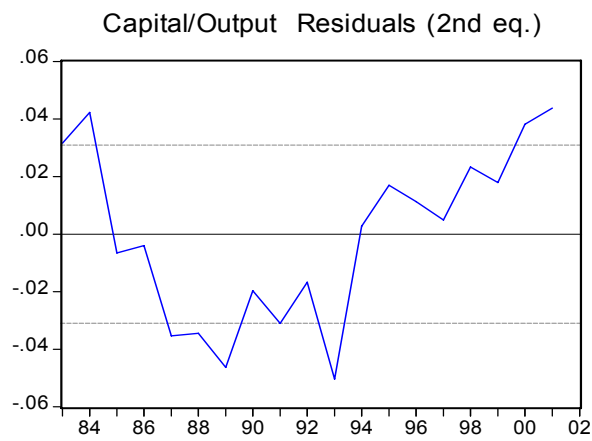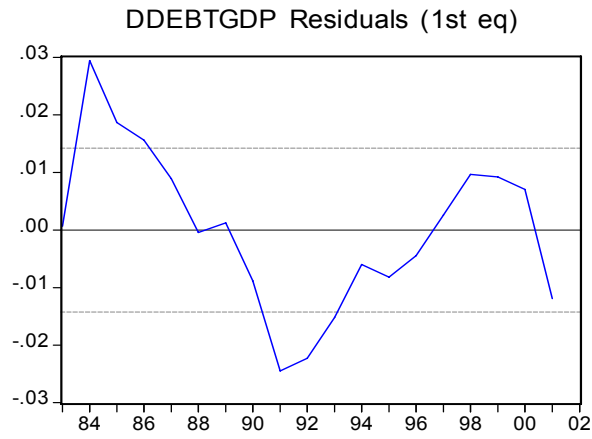
Residuals of Regressions 3 in Table 6

DDEBTGDP Residuals (1st eq.)

Capital/Output Residuals (2nd eq.)

GDP Residuals (3rd eq.)

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